

**MONITORING THE COEFFICIENT OF
VARIATION THROUGH A
VARIABLE PARAMETERS CHART**

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**MONITORING THE COEFFICIENT OF VARIATION THROUGH A
VARIABLE PARAMETERS CHART**

By

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ABSTRACT

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Statistical Process Control (SPC) is widely used in a variety of fields, such as in manufacturing industries, healthcare, environmental monitoring and so on. Traditional control charts mostly monitor the deviation in the process mean and/or process variance. However, in some processes, the mean is not constant, even when the process is in-control. As a further complication, in many of these processes, the variance is a function of the mean. As a result, it is not appropriate to monitor the mean and standard deviation for such processes, as it will result in dubious results. If such processes have a standard deviation which is proportional to the mean, monitoring the coefficient of variation as a source of variability is a better measure. The coefficient of variation (CV) chart monitors the ratio of the standard deviation, σ to the mean, μ . In this project, we improve the detection ability of CV charts by proposing the variable parameters chart. Variable parameters chart is a chart that varies the sample size, sampling interval, control limits and warning limits according to the prior sample information to improve the performance of the chart. Formulae for various performance measures, for example the Average Time to Signal (ATS), the Average Sample Size (ASS) and Average Sampling Interval (ASI) are derived. Optimization algorithms to optimize the out-of-control ATS, subject to

constraints in the in-control ATS , in-control ASS and in-control ASI , are proposed. Next, the performance of the proposed chart is compared with the Shewhart CV chart, the variable sampling interval (VSI) CV chart, the variable sample size (VSS) CV chart, the variable sample size and sampling interval (VSSI) CV chart, the Synthetic CV control chart and the EWMA CV^2 chart. In addition, the proposed chart is implemented on an actual industrial data. In general, a comparison with the VSSI, VSI, VSS, Synthetic and Shewhart CV charts shows that the VP CV chart gives better performance than all of these charts for all shift sizes, including small shift sizes, based on the ATS_1 performance criterion, and the proposed chart also outperforms all of these charts for all ranges of shift sizes, based on the $EATS_1$ performance criterion. Compared to the EWMA CV^2 chart, the VP CV chart outperforms this chart for moderate and large shift sizes. However, for small shift sizes, the EWMA CV^2 chart outperforms the VP CV chart. Through the industrial example, the VP CV chart performs better compared to other competitor charts.

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I would also like to express my gratitude to Universiti Tunku Abdul Rahman as it had provided me the opportunity to conduct this research. Finally, I would also like to thank my family and friends for providing support throughout this project.

DECLARATION

I hereby declare that the project report is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTAR or other institutions.

(LIM CHUEN YANG)

APPROVAL SHEET

This report entitled “**MONITORING THE COEFFICIENT OF VARIATION THROUGH A VARIABLE PARAMETERS CHART**” was prepared by LIM CHUEN YANG and submitted as partial fulfilment of the requirements for the degree of Bachelor of Science (Hons) Statistical Computing and Operations Research at Universiti Tunku Abdul Rahman.

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PERMISSION SHEET

It is hereby certified that **LIM CHUEN YANG** (ID No: **13ADB03410**) has completed this final year project entitled “MONITORING THE COEFFICIENT OF VARIATION THROUGH A VARIABLE PARAMETERS CHART” under the supervision of MS. NG PEH SANG from the Department of Physical and Mathematical Science, Faculty of Science.

I hereby give permission to the University to upload the softcopy of my final year project in pdf format into the UTAR Institutional Repository, which may be made accessible to the UTAR community and public.

Yours truly,

(LIM CHUEN YANG)

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LIST OF ABBREVIATIONS

The following are the list of abbreviations, notations and the symbols of input parameters used in this project:

| | |
|---------------|---|
| $a/ b/ c$ | Parameters ($a/ b/ c$) to compute the T_i |
| a_L | Value of a for large sample size |
| a_S | Value of a for small sample size |
| α_0 | Probability of Type-I error |
| AL | Adjusted loss function |
| ARL | Average run length |
| ARL_0 | In-control average run length |
| ARL_1 | Out-of-control average run length |
| ASI_0 | In-control average sampling interval |
| ASS_0 | In-control average sample size |
| ATS | Average time to signal |
| ATS_0 | In-control average time to signal |
| ATS_1 | Out-of-control average time to signal |
| \mathbf{b}' | Vector of starting probability |
| b_L | Value of b for large sample size |
| b_S | Value of b for small sample size |
| \mathbf{B} | Diagonal element with the h_j element |
| c_L | Value of c for large sample size |
| c_S | Value of c for small sample size |
| CCC | Joint three one-sided CUSUM chart (I , D and V charts) |
| cdf | Cumulative distribution function |

| | |
|--------------------------|--|
| CL | Center limit |
| <i>CRL</i> | Conforming run length |
| CUSUM | Cumulative sum |
| CV / γ | Coefficient of variation |
| CV^2 | Coefficient of variation squared |
| CVSSIDS | Variable sample size, sampling interval and double sampling features |
| χ^2 | Chi-squared |
| DEWMA | Double exponentially weighted moving average |
| <i>EARL</i> | Expected average run length |
| <i>EATS</i> | Expected average time to signal |
| <i>EATS</i> ₁ | Out-of-control <i>EATS</i> |
| EWMA | Exponentially Weighted Moving Average |
| $f_{\tau}(\tau)$ | Probability density function of τ |
| $F_t(\cdot c, d)$ | cdf of a noncentral t distribution with c degrees of freedom and noncentrality parameter d |
| $F_N^{-1}(r)$ | Inverse distribution function of the normal distribution with parameter (0, 1) |
| $F_t^{-1}(\cdot c, d)$ | Inverse cdf of $\hat{\gamma}$ |
| $\hat{\gamma}$ | Sample CV |
| $\hat{\gamma}_i$ | i^{th} sample CV |
| γ_0 | In-control CV |
| γ_1 | Out-of-control CV |
| h | Sampling interval |

| | |
|--------------------------|--|
| h_0 | In-control sampling interval |
| h_L | Long sampling interval |
| h_S | Short sampling interval |
| FSI | Fixed sampling interval |
| FVP-WV | Fuzzy variable parameters \bar{X} control charts by weighted variance method |
| I | Identity matrix |
| K | Control limit coefficient |
| K_L | Large control limit coefficient |
| K_S | Small control limit coefficient |
| λ | Weighting factor for EWMA CV chart |
| L | Threshold |
| LCL | Lower control limit |
| LCL_{CV} | LCL for the Shewhart CV chart |
| LWL | Lower warning limit |
| μ | Mean |
| $\mu_0(\hat{\gamma})$ | Mean of the sample CV |
| σ | Standard deviation |
| $\sigma_0(\hat{\gamma})$ | Standard deviation of the sample CV |
| m | Number of Phase-I samples |
| n | Sample size |
| n_0 | In-control average sample size |
| np | Number of non-conforming units |
| n_L | Large sample size |
| n_S | Small sample size |

| | |
|---------------|---|
| P | Transition probability matrix |
| P_{ij} | Transition probability from the previous state i to the current state j |
| Q | 2x2 transition probability matrix for the transient states |
| r | Selected quantile of the CV distribution |
| R | Range |
| S | Sample standard deviation |
| S_i | i^{th} sample S |
| $ s $ | Generalized sample variance |
| S^2 | Sample variance |
| $SDRL$ | Standard deviation of the run length |
| $SDTS$ | Standard deviation of the time to signal |
| $SDTS_0$ | In-control $SDTS$ |
| $SDTS_1$ | Out-of-control $SDTS$ |
| $SVSSI$ | Special VSSI |
| $SSGR$ | Side sensitive group runs |
| $SSVSS$ | VSS scheme with the side-sensitive synthetic rule |
| SPC | Statistical process control |
| $SynCV$ | Synthetic chart |
| τ | Shift size |
| τ_{\min} | Smallest shift size |
| τ_{\max} | Largest shift size |
| T_i | i^{th} transformed CV |
| \hat{T}_i | i^{th} sample T_i |
| \mathbf{t}' | Vector of sampling intervals |

| | |
|-------------|--|
| TSVP | VP control chart with three stage process |
| UCL | Upper control limit |
| UCL_{CV} | UCL for the Shewhart CV chart |
| UWL | Upper warning limit |
| VSI | Variable sampling interval |
| VSIFT | VSI with fixed times |
| VSS | Variable sample size |
| VSSI | Variable sampling size and sampling interval |
| VSSIIFT | VSSI \bar{X} charts based on fixed sampling time |
| VSSIWL | Variable sample size, sampling interval and warning limits |
| VP | Variable parameters |
| \bar{X} | Sample mean |
| \bar{X}_i | i^{th} sample mean |
| W | Warning limit coefficient |
| W_L | Large warning limit coefficient |
| W_S | Small warning limit coefficient |
| WLC | Weighted-loss-function-based scheme |
| z_i | Statistic for EWMA CV chart |

CHAPTER 1

INTRODUCTION

1.1 Research Background

Nowadays, quality has been established as one of the key competitive factors in the business world. Statistical Process Control (SPC) is a collection of tools to measure and control quality during the production process. There are seven major SPC tools which include the Pareto Chart, the Cause and Effect Diagram, the Defect Concentration Diagram, the Histogram, the Scatter Diagram, the control chart and the Check Sheet.

Among all those SPC tools, control chart is one of the widely used tools to monitor and identify the changes of the process effectively. For instance, control chart is used in industries to detect whether the process is out-of-control. If the process is in an out-of-control state, corrective action is taken to ensure the process is in-control.

One of the principles of SPC is that a normally distributed process cannot be claimed to be in-control until it has a constant mean and variance. Traditional control charts monitoring the mean and/or variance may not work in some processes. This is because some processes do not have a constant mean and the standard deviation may be a function of the mean, i.e. the standard deviation changes when the mean changes. When the standard deviation (σ) is directly proportional to the mean (μ), the coefficient of variation, $\gamma = \frac{\sigma}{\mu}$ is a constant.

For such processes, monitoring the coefficient of variation as a measure of variability is a better approach. There are several significances of monitoring the coefficient of variation (CV). In finance, CV is used to measure the risk and uncertainty met by investors. In chemical and biological assay quality control, the CV is used to validate results. In physiological science, the CV can be applied to assess the homogeneity of bone sample. In clinical and diagnostic areas, the CV is often used as a measurement to determine the amount of certain chemicals in a patient's urine and blood to diagnose the presence of a disease.

A robust control chart is able to detect the changes of the process in the shortest time and it should not have a high number of false alarms. In order to study the control chart's performance, some performance measures are needed. For examples, the measurements are average run length (*ARL*), the average time to signal (*ATS*), the expected average run length (*EARL*) and the expected average time to signal (*EATS*). The *ARL* is the average number of samples taken until an out-of-control signal is produced, while the *ATS* is the average time until the process signals an out-of-control condition. The difference between the *ARL* (*ATS*) and the *EARL* (*EATS*) is in the computation of the *ARL* (*ATS*), the shift size needs to be specified as a specific value, while in computation of the *EARL* (*EATS*), the shift size only needs to be specified as a particular range of possible values. A good control chart will have a small number of false alarms, which is shown through a large *ARL* (*ATS*) when the process is in-control, and will be able to detect changes in the process quickly, as shown by a small *ARL* (*ATS*) when the process is out-of-control.

By comparing the adaptive charts and standard control charts, adaptive charts are more efficient because it uses historical information to determine the next chart parameter. Adaptive charts are charts where the sample size, the sampling interval, the control limits or the warning limits are made to vary. An adaptive chart that varies the sample size is called the variable sample size chart, while the chart that varies the sampling interval is called the variable sampling interval chart. Meanwhile, the chart that varies both the sample size and the sampling interval is called the variable sample size and sampling interval chart, while the variable parameter chart is a chart which varies all the control chart's parameters, i.e. the sample size, sampling interval, control limit and warning limit.

In this project, a Variable Parameters chart (VP CV chart) is proposed to improve the performance of the traditional CV chart. VP CV chart is a chart that varies the sample size, sampling interval, warning limits and control limits according to the prior sample information. It is expected that this chart will result in better performance compared to the variable sampling interval (VSI) CV chart, the variable sample size (VSS) CV chart and the variable sample size and sampling interval (VSSI) CV chart.

1.2 Problem Statement

In the existing literature, it is shown that the VP chart outperforms the charts with fixed chart parameter. However, monitoring the CV using VP technique does not exist in the literature. Thus, this project proposes a Variable Parameters (VP) chart to monitor the CV.

1.3 Objectives

The objectives of this study are as follows:

- (i) To propose a variable parameters chart to monitor the CV.
- (ii) To propose optimization algorithms to obtain the chart parameters which result in optimal performance of the proposed chart.
- (iii) To compare the performance of the proposed chart with other charts.

1.4 Significance of this Research

This research proposes a VP chart to monitor the CV, which is not available in the literature prior to this research. It is expected that the proposed chart will show a significant improvement in detecting changes in the CV, compared to existing charts such as the Shewhart CV chart, the variable sampling interval (VSI) CV chart, the variable sample size (VSS) CV chart, the variable sample size and sampling interval (VSSI) CV chart, the Synthetic CV control chart and EWMA CV² chart.

Through this research, practitioners will be able to implement the VP CV chart easily as the step-by-step procedure which include the derived formulae of the VP CV chart is proposed. Furthermore, the optimization algorithms proposed in this project will enable practitioners to implement the proposed chart at the optimal level. Finally, comparisons are made with other existing charts in the literature to show the improvement of the proposed chart with other existing charts.

1.5 Limitations of the Research

The limitation of the research is the VP CV chart may not be the best chart to monitor the CV with the shift size is small. Apart from that, the data must be normal distribution when implement the VP CV chart. Next, there are many methods to compute the statistic, the VP CV chart use the T_i statistic to compute the chart.

1.6 Organization of the Report

This report is organized into five chapters. The problem statement, the objectives and the significance of this project are stated in Chapter 1. Chapter 2 reviews the existing literatures that are related to CV control charts and adaptive charts. The design of the VP CV chart, formulae derived and the optimization algorithms are introduced and discussed in the following chapter. Subsequently, Chapter 4 interprets and discusses all of the numerical results on the proposed VP CV chart. The VP CV chart's performance is compared with the performance of the Shewhart CV chart, the variable sampling interval (VSI) CV chart, the variable sample size (VSS) CV chart, the variable sample size and sampling interval (VSSI) CV chart, the Exponentially Weighted Moving Average (EWMA) CV^2 chart and the synthetic CV control chart. This is followed by an illustrative example from an actual industrial scenario. The last chapter is the conclusion of this project.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In this chapter, a review of past studies related to adaptive type charts and CV charts is given.

2.2 Adaptive Type Charts

When the control chart has at least one parameter, for example the sample size, the sampling interval or the control limits that vary according to the prior sample information, it is considered as an adaptive control chart.

2.2.1 Variable Sampling Interval (VSI) Charts

VSI control charts are charts that vary the sampling interval h . From the diagram shown in Figure 2.1, the VSI chart is divided into three regions, which are the central region, the warning region and the out-of-control region. The line in between the central region and the warning region is called warning limits, whereas control limits are the line in between the warning region and the out-of-control region. When the sample falls in the warning region, there is high possibility for the process to be out-of-control, so the next sample should be taken after a short sampling interval (h_S) to tighten the control. In contrast, if the sample falls in the central region, the next sample should be taken after a long sampling interval (h_L) as it is more unlikely for the process to go out-of-control. The process is considered as out-of-control when the sample falls in the out-of-control region.

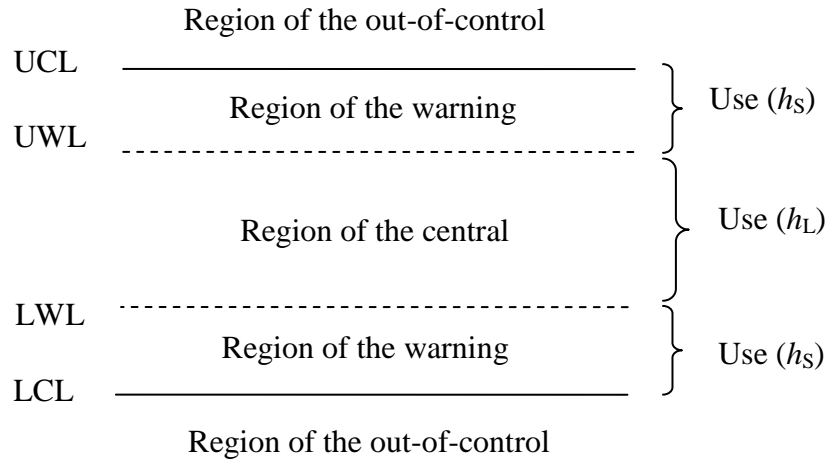


Figure 2.1: VSI chart's graphical view.

From the previous research about the VSI charts, Reynolds, et al. (1988) stated that the performance of the VSI \bar{X} chart is better than the standard Shewhart \bar{X} control chart. When the time between samples is varied, expressions for the optimal one-sided Shewhart control chart subject to some constraints are derived (Reynolds and Arnold, 1989). The one-sided control limit for a VSI Shewhart \bar{X} control chart was proposed and had shown improvement compared to the two-sided VSI Shewhart \bar{X} control chart (Runger and Pignonettiello, 1991).

Amin and Miller (1993) studied the VSI \bar{X} chart's performance when the data is not normally distributed. VSI for multi-parameter Shewhart chart showed that it performs better compared to the FSI control chart (Chengalur, Arnold and Reynolds, 1989). A VSI control chart with fixed times (VSIFT) was proposed and the performance is similar to the EWMA chart and the CUSUM chart. VSIFT charts showed better performance than FSI charts. However, when compared with the VSS charts, the VSIFT charts only showed better performance when detecting a large shift (Reynolds, 1996). Reynolds, Amin

and Arnold (1990) proposed the standard cumulative sum (CUSUM) control chart with VSI. In small and moderate shifts in terms of mean, it is more efficient than the VSI Shewhart \bar{X} control chart and the FSI CUSUM control chart. A double exponentially weighted moving average (EWMA) with VSI was proposed by Shamma, Amin and Shamma (1991). From the research, VSI-DEWMA chart is more efficient compared with FSI-DEWMA. The *ATS* properties of two-sided VSI EWMA charts were evaluated (Saccucci, Amin and Lucas, 1992). Castagliola, Celano and Fichera (2006) designed a Variable Sampling Interval (VSI) *R* Exponentially Weighted Moving Average (EWMA) control chart. VSI *R* EWMA monitors the range (*R*) from a normally distributed process and showed improvement in the performance compared with other charts. Next, the VSI S^2 EWMA chart was proposed to monitor the process in term of the variance (Castagliola, et al., 2007). Kazemzadeh, Karbasian and Babakhani (2013) proposed a variable sampling interval (VSI) exponentially weighted moving average (EWMA) *t* chart and found that it is more robust compared with the FSI EWMA \bar{X} chart. By comparing the FSI EWMA *t* and FSI EWMA \bar{X} charts, the VSI EWMA *t* chart performed well in terms of the *ATS* performance.

Lee and Khoo (2015) proposed a multivariate synthetic generalized sample variance $|s|$ chart with variable sampling interval (VSI). By detecting the shifts in the covariance matrix, the VSI multivariate synthetic control chart had outperformed other multivariate control charts. Next, a variable sampling interval (VSI) S^2 chart was proposed by Guo and Wang (2015). The VSI S^2

chart performed well compared with the FSI S^2 chart in terms of detecting the shifts in the process variance.

2.2.2 Variable Sample Size (VSS) Charts

VSS control charts vary the sample size n . When the sample falls in the warning region, it is suspected that the process had a higher chance to shift to the out-of-control region. Thus, a large sample size is taken to tighten control. In contrast, when the sample falls in the central region, a small sample size is taken as it is less likely for the process to go out-of-control. Figure 2.2 shows the VSS control chart, where n_S and n_L represent the small and large sample sizes, respectively.

Prabhu, Runger and Keats (1993) proposed a dual sample size scheme and the control limits and sampling interval are kept constant. Zimmer, Montgomery and Runger (1998) developed a three-state adaptive sample size. Based on the results from small shifts in the process mean, three-state adaptive control chart performs better compared to the two-state adaptive control. Annadi, et al. (1995) designed the VSS CUSUM charts. Comparisons are made between the average run length (ARL) for the adaptive sample scheme with the fixed sample scheme. In order to monitor the process mean, Aparisi (1996) developed the Hotelling's T^2 control chart with adaptive sample size. A new VSS EWMA control charts which determine the next sample size by the location of the prior EWMA statistic was proposed (Amiri, Nedaie and Alikhani, 2014). The new VSS EWMA chart showed better performance compared with the FSI EWMA chart and the traditional VSS EWMA chart.

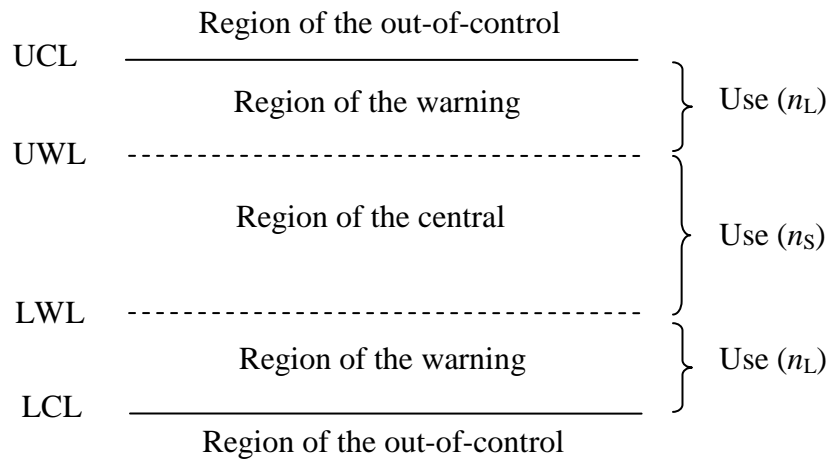


Figure 2.2: VSS chart's graphical view.

De Magalhães and Von Doellinger (2016) designed a variable sample size chi-squared control chart (VSS χ^2). The purpose of the VSS χ^2 control chart is to monitor the linear profiles. In general, the VSS χ^2 control chart has outperformed the fixed parameter chi-squared control chart in terms of *ARL*. Costa and Machado (2016) proposed the \bar{X} chart by combining the VSS scheme with the side-sensitive synthetic rule (SSVSS \bar{X} chart) and showed that it performed better compared with the VSS \bar{X} chart.

Aslam, Arif and Jun (2016) designed a variable sample size (VSS) chart with multiple dependent state sampling. This chart showed better performance than the existing VSS chart.

2.2.3 Variable Sample Size and Sampling Interval Charts (VSSI) Charts

The VSSI chart varies two parameters which are the sample size and the sampling interval. For the VSSI chart, when the sample falls in the warning region, the next sample to be taken should be a large sample (n_L) and it should

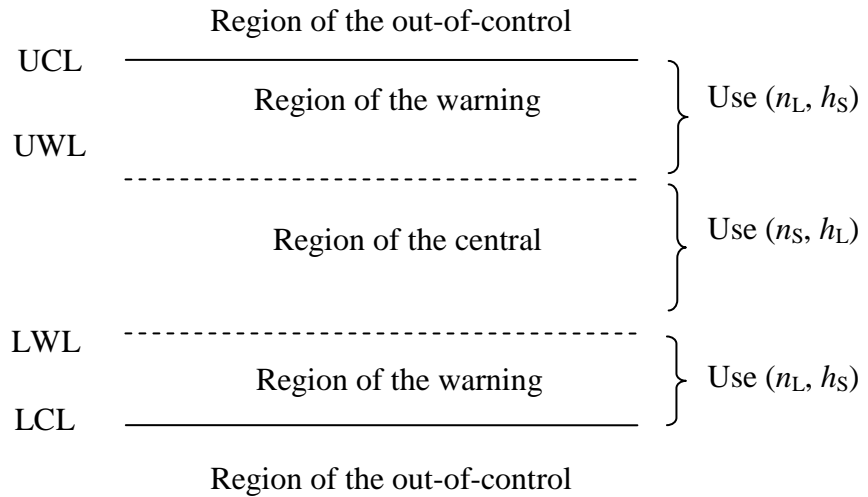


Figure 2.3: VSSI chart's graphical view.

be taken after a short sampling interval (h_S) due to a higher chance for the process to be out-of-control. However, when the sample falls in the central region, a small sample size (n_S) is taken after a long sampling interval (h_L) as it is less likely for the process to go out-of-control. Figure 2.3 shows the VSSI control chart.

There are several researchers that studied the VSSI charts. The VSSI \bar{X} chart was proposed by Prahbu, Montgomery and Runger (1994) and Costa (1997). The proposed chart by Prahbu, Montgomery and Runger (1994) is compared with the standard Shewhart \bar{X} chart, a VSS chart, and a VSI chart in terms of the *ATS*. The VSSI \bar{X} chart developed by Costa (1997) showed better performance than the VSI or VSS \bar{X} charts for moderate shifts in the process. VSSI \bar{X} charts based on fixed sampling time (VSSI_{FT}) were developed by Costa (1998a). VSSI \bar{X} chart with runs rules was proposed by Mahadik (2013a) to reduce the frequency of switches. Mahadik (2013b) developed the variable sample size, sampling interval and warning limits (VSSIWL) \bar{X} chart which

includes one more parameter of control chart (warning limits) to reduce the frequency of switches.

In order to have better performance in terms of *ATS*, Yang and Yang (2013) proposed the optimal VSSI mean squared error control chart. By modifying the VSSI mean squared error control chart, Cheng, Yang and Wu (2013) used the square root transformation for improvement. From the result shown, the optimal proposed control chart performed better than the other charts. The assumption for using control chart is the data is normally distributed, but the assumption may be violated in real life. From Lin and Chou (2005)'s paper, the VSSI \bar{X} chart under non-normality by using the Burr distribution were studied.

For the VSI, VSS and VSSI charts, there are two levels for each parameter. This leads Mahadik and Shirke (2009) to propose a special VSSI (SVSSI) \bar{X} chart which uses two sampling intervals and three sample sizes. In this paper, SVSSI \bar{X} chart had the same efficiency as the VSSI \bar{X} chart in small shifts and the same efficiency as the VSI \bar{X} chart in detecting large shifts.

A hybrid adaptive scheme used by Celano, Costa and Fichera (2006) considers both the VSI and VSS with runs rules. The combination of Costa's cost model with Yang and Hancock's correlation model was designed by Chen, Hsieh and Chang (2007) in the economic design of the VSSI chart to monitor data which are correlated. Moreover, Wu, Tian and Zhang (2005) proposed the Adjusted Loss Function (AL) with VSSI (VSSI AL chart).

Researchers applied the VSSI strategy into other charts since most of the studies were on \bar{X} charts from the past. Arnold and Reynolds (2001) designed the VSSI joint three one-sided CUSUM chart (CCC) scheme. From Wu, Zhang and Wang (2007)'s paper, the VSSI weighted-loss-function-based scheme (WLC) based CUSUM scheme was proposed and its operation is simpler than the VSSI CCC scheme. The VSSI CCC scheme consist of three individual CUSUM charts which are two for monitoring increasing and decreasing mean shifts and one for monitoring increasing variance shifts, whereas the VSSI WLC scheme comprises one two-sided chart to monitor the mean shift and another chart to monitor increasing shifts in the standard deviation.

Noorossana, Shekary and Deheshvar (2015) designed the \bar{X} chart by combining the variable sample size, sampling interval and double sampling features (CVSSIDS). The CVSSIDS used different levels for different parameter, for instance, three different levels for sample size, two different levels for the sampling interval, two control limits, and two warning limits. By comparison in terms of *ATS*, the CVSSIDS always showed smaller values compared with the other charts.

2.2.4 Variable Parameters (VP) Charts

VP chart varies four parameters which are the sample size, the sampling interval, the warning limits and the control limits. When the sample falls in the warning region, the chart parameters for the next sample to be taken is a large sample size (n_L), short sampling interval (h_S), small control limit coefficient (K_S) and small warning limit coefficient (W_S) to tighten the control. When the

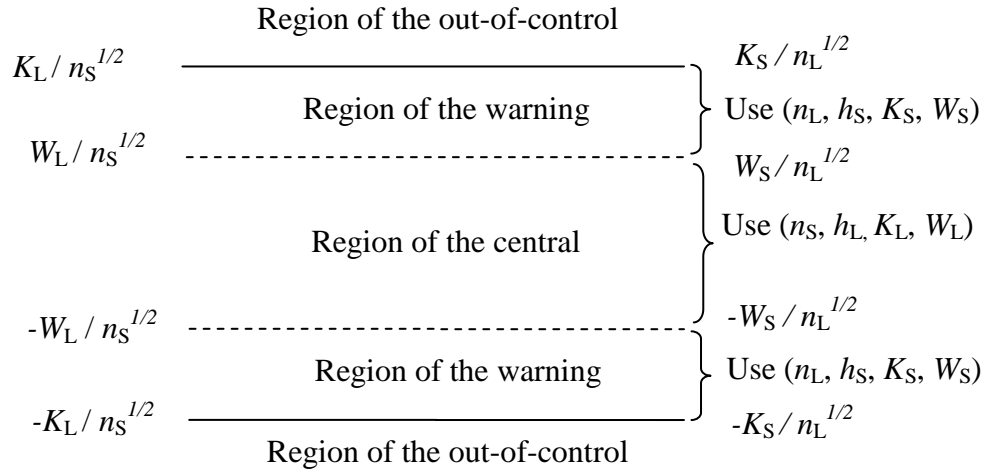


Figure 2.4: VP chart's graphical view.

sample falls in the central region, a small sample size (n_S), long sampling interval (h_L), large control limit coefficient (K_L) and large warning limit coefficient (W_L) are adopted. Figure 2.4 shows the VP control chart.

A joint \bar{X} and R charts with variable parameters was first proposed by Costa (1998b). The variable parameters chart improves the joint \bar{X} and R charts when detecting shifts in the process mean (Costa, 1998b). Apart from that, Costa (1999) proposed the \bar{X} charts with variable parameters. For detecting shifts in the process mean, the new VP \bar{X} chart is more powerful than the CUSUM scheme. Lin and Chou (2007) claimed the assumption of normally distributed quality variables may not work for some processes. This leads Lin and Chou (2007) to study the VP \bar{X} chart under non-normality. In the non-normal process of detecting small mean shifts, the VP \bar{X} chart performs better than other charts and reduce the false alarm rate. Chen and Chang (2008) studied the economic design of the VP \bar{X} control chart for fuzzy mean shifts. Lin (2009) studied the effect of autocorrelation on the features of the VP \bar{X} control chart. The VP \bar{X}

chart does not show a good performance when the data are strongly autocorrelated.

The Shewhart R control chart (R chart) performs poorly in detecting small process shifts in variability. Hence, Lee (2011) proposed the R chart with variable parameter and the chart shows significant improvement. A modified version of the VP control chart with three stage process (TSVP) was designed by Deheshvar, et al. (2013). However, a modified version of the VP control chart showed a larger complexity compared to the traditional VP control chart.

Hashemian, et al. (2016) proposed an adaptive np -VP control chart with estimated parameters and compared it with the np -VP control chart with known parameters in terms of ATS as the performance measure. The np -VP control chart with known parameters does not perform well when the parameters are estimated from the Phase I data.

Panthong and Pongpullponsak (2016) introduced fuzzy variable parameters \bar{X} control charts by weighted variance method (FVP-WV). Panthong and Pongpullponsak (2016) used triangular fuzzy numbers (a, b, c) to propose FVP-WV charts. From the results, FVP-WV chart is more efficient than the traditional VP \bar{X} control charts.

2.3 CV charts

Monitoring the CV was kick started by Kang, et al. (2007). Kang, et al. (2007) proposed a Shewhart CV control chart for monitoring the cyclosporine level in

organ-transplantation procedures using rational subgroups. Cyclosporine is an immunosuppressive drug that is used to prevent rejection of the implanted organ. Excessive use of cyclosporine will weaken the immune system and hence leads to dangerous infections, conversely insufficient amount of cyclosporine will cause organ rejection. The suitable amount of circulating drug varies from patient to patient. Therefore, frequent blood assays are required to monitor the drug level that best stabilizes every patient. The interest is on the CV as the means of these assays vary widely. As a result, this chart is only sensitive to large shifts but is less sensitive to small and moderate shifts. Because of this reason, several studies were carried out to improve this chart.

Hong, et al. (2008) proposed an Exponentially Weighted Moving Average (EWMA) CV control chart. From this paper, the comparisons of the EWMA CV chart with the Shewhart CV chart were performed. The EWMA CV chart proposed by Hong, et al. (2008) yields smaller out-of-control ARL (ARL_1) values than the control chart proposed by Kang, et al. (2007). However, the ARL is evaluated by using intensive simulations, and no theoretical method has been provided to compute the ARL .

This leads Castagliola, Celano and Psarakis (2011) to develop a new chart to monitor the CV (two one-sided EWMA charts). From this paper, the researchers used a Markov chain to compute the ARL of the EWMA CV² chart. Castagliola, Celano and Psarakis (2011) showed that using two one-sided charts, one to detect an increasing shift in the CV and another to detect a decreasing shift in the CV, results in an improved ability to detect shifts

compared to using a single two-sided chart to simultaneously monitor both an increasing and decreasing shift in the CV. By comparing the *ARL* values, the EWMA CV^2 chart shows smaller *ARL* values compared with the EWMA CV chart.

Next, Calzada and Scariano (2013) developed a synthetic chart (SynCV) to monitor the CV. The results showed that the ARL_{1s} obtained for the synthetic chart are clearly smaller than the ones for Kang, et al. (2007), but generally larger than the ones obtained by Castagliola, Celano and Psarakis (2011) for small shifts in the CV. However, for large shifts in the CV, the synthetic chart had shown good performance in terms of smaller *ARL* values compared with the EWMA CV^2 chart.

Subsequently, Castagliola, et al. (2013a) developed a Shewhart-type chart with supplementary run rules to monitor the CV. However, the runs rules charts for monitoring the CV does not outperform more advanced strategies like the chart proposed by Castagliola, Celano and Psarakis (2011) and Calzada and Scariano (2013). In general, Shewhart-type chart with supplementary run rules perform better in detecting small shift in the CV, but is not efficient in detecting large shift in the CV compared to the Shewhart CV chart proposed by Kang, et al. (2007).

Zhang, et al. (2014) proposed a modified EWMA CV chart which improves the performance of the EWMA CV chart proposed by Castagliola, Celano and

Psarakis (2011). The modified EWMA CV chart had outperformed the EWMA CV chart in terms of the *ARL* values from small to large shifts in the CV.

Next, Castagliola, et al. (2015b) proposed a one-sided Shewhart CV chart for short production runs. Subsequently, You, et al. (2015) proposed a side sensitive group runs (SSGR) chart to monitor the CV. Comparative studies show that the SSGR CV chart outperforms the Shewhart CV, runs rules CV, synthetic CV and EWMA CV charts.

Adaptive strategies, which vary the chart parameters according to the prior sample statistics, are proposed by Castagliola, et al. (2013b), Castagliola, et al. (2015b) and Yeong, et al. (2015). Castagliola, et al. (2013b) monitored the CV using a variable sampling interval (VSI) chart. From the results shown, the VSI CV chart does not perform well compared with the EWMA CV chart. After that, Castagliola, et al. (2015b) proposed a VSS CV chart (variable sample size control chart) by using the T_i statistic which approximates a normal distribution. The VSS CV chart is shown to outperform the VSI CV chart for shifts of less than 1.5 and outperform the Synthetic chart for shifts of less than 2 based on the out-of-control *ARL*. Yeong, et al. (2015) monitored the CV using the variable sample size (VSS) charts. It was found that varying the chart parameters results in an improved performance in detecting shifts in the CV. Yeong, et al. (2016) proposed a chart which monitors the CV for multivariate data. Finally, Khaw, et al. (2016) proposed VSSI chart to enhance the basic CV chart's performance.

CHAPTER 3
THE DESIGNS OF THE VARIABLE PARAMETER CONTROL
CHART

3.1 Basic properties of the sample CV

Let X be a random variable and let $\mu = E(X)$ and $\sigma = \sigma(X)$ be the mean and standard deviation of X respectively. The CV of X is defined as

$$\gamma = \frac{\sigma}{\mu}. \quad (3.1)$$

Assume that $\{X_1, X_2, \dots, X_n\}$ is a sample of size n from the normal distribution.

Let \bar{X} and S be the sample mean and sample standard deviation respectively, where

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i, \quad (3.2)$$

and

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\bar{X} - X_i)^2}. \quad (3.3)$$

The sample CV $\hat{\gamma}$ is defined as

$$\hat{\gamma} = \frac{S}{\bar{X}}. \quad (3.4)$$

According to Iglewicz, Myers and Howe (1968), $\frac{\sqrt{n}}{\gamma}$ follows a noncentral t distribution with $n-1$ degree of freedom and noncentrality parameter $\frac{\sqrt{n}}{\gamma}$ by concerning the probability distribution of sample CV.

Iglewicz, Myers and Howe (1968) noticed that the cumulative distribution function (cdf) of $\hat{\gamma}$ can be approximated as the following equation if $\gamma > 0$ is not too large ($\gamma \in (0, 0.5]$).

$$F_{\hat{\gamma}}(x | n, \gamma) \approx 1 - F_t\left(\frac{\sqrt{n}}{x} \mid n-1, \frac{\sqrt{n}}{\gamma}\right), \quad (3.5)$$

where $F_t(\cdot | c, d)$ refers to the cdf of a noncentral t distribution with c degrees of freedom and noncentrality parameter d . The inverse cdf of $\hat{\gamma}$, can be computed as

$$F_{\hat{\gamma}}^{-1}(\alpha | n, \gamma) \approx \frac{\sqrt{n}}{F_t^{-1}\left(1-\alpha \mid n-1, \frac{\sqrt{n}}{\gamma}\right)}, \quad (3.6)$$

where $F_t^{-1}(\cdot | c, d)$ refers to the inverse cdf of a noncentral t distribution with c degrees of freedom and noncentrality parameter d .

3.2 The VP CV chart

For the VP CV chart, the sample size, the sampling interval, the control limits and the warning limits vary between two levels. The reason the VP CV chart with dual sampling schemes is adopted is due to the administrative difficulty level if more than two levels of the parameter are used (Jensen, Bryce and

Reynolds, 2008). The sample size used by the VP CV chart is varied, which are the small sample size, n_S , and the large sample size, n_L . The interval of the sample size is $n_S < n_0 < n_L$, where n_0 is the in-control average sample size (ASS_0). The chart varies the sampling interval into the short sampling interval, h_S , and long sampling interval, h_L , where $h_S < h_0 < h_L$ and h_0 is the average sampling interval when in-control (ASI_0). Next, the control limits are varied into the small control limit coefficient, K_S , and the large control limit coefficient, K_L , whereas the warning limits are varied between the small warning limit coefficient, W_S , and the large warning limit coefficient, W_L .

The VP CV chart is the one-sided upward chart. The first reason is the CV follows a non-symmetric distribution. Thus, it does not make sense to assume that the control limits and the warning limits are symmetrical about different directions in the chart. The second reason for using one-sided upward chart instead of a two-sided chart is because it is usually of interest to detect an increase in the CV as it corresponds to increased variability in the process (Castagliola, Celano and Psarakis, 2011). Figure 3.1 shows the VP CV chart.

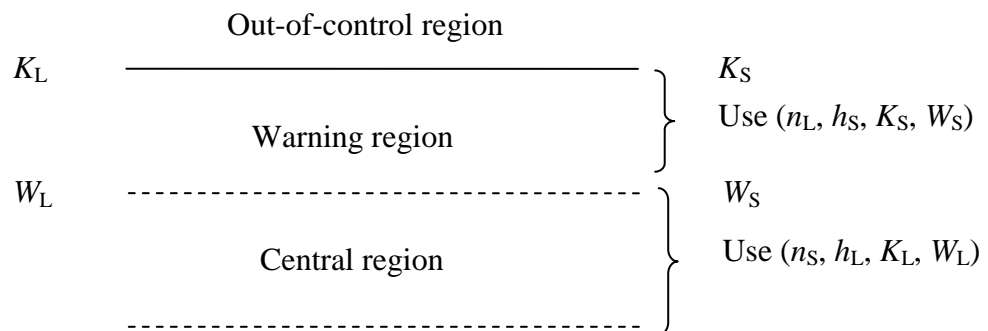


Figure 3.1: A graphical view of the one-sided upward VP CV chart.

Since the CV is a right skewed distribution, the VP CV chart used the T_i statistic instead of using CV directly (Castagliola, et al., 2015a). The purpose of using T_i statistic is the T_i statistic approximately follows a normal distribution with parameter (0, 1). From that, this indicates that the central line monitoring T_i is CL = 0. The T_i statistic is defined as

$$T_i = a + b \ln(\bar{y} - c), \quad (3.7)$$

where $a, b > 0$ and c are three parameters depending on $n(i)$ and γ_0 .

In order to obtain the parameters (a, b, c), the 3-parameter lognormal distribution are fitted from three selected quantiles of the CV distribution which are $r, 0.5$ and $1-r$ (Castagliola, et al., 2015a).

$$b = \frac{F_N^{-1}(r)}{\ln\left(\frac{x_{0.5} - x_r}{x_{1-r} - x_{0.5}}\right)}, \quad (3.8a)$$

$$a = -b \ln\left(\frac{x_{0.5} - x_r}{1 - \exp\left(\frac{F_N^{-1}(r)}{b}\right)}\right), \quad (3.8b)$$

and

$$c = x_{0.5} - e^{\frac{a}{b}}. \quad (3.8c)$$

where $F_N^{-1}(r)$ is the inverse distribution function of the normal distribution with parameter (0, 1).

Based on Castagliola, et al. (2015a), the suitable value of r is in between 0.01 and 0.1. The value of r could not be less than 0.01, to prevent too much contribution given from the tails of the CV distribution. However, the value of

r also could not be more than 0.1 to obtain enough information from the tails of the CV distribution. This can be proven by using normality test to compute the values of a , b , and c from different r values. Note that since the parameters (a , b , c) are the functions of the quantiles x_r , $x_{0.5}$ and x_{1-r} , and these parameters are depend on the values of n_i and γ_0 , and thus n_i and γ_0 will have an impact on the value of the transformed statistics T_i .

According to the Costa (1999), the operations of the VP CV chart strategy (Figure 3.1) are described as follows:

- The process is considered as an in-control process if the $(i-1)^{\text{th}}$ sample \hat{T}_i falls in the interval $CL \leq \hat{T}_i \leq UWL$, and n_S , h_L , K_L and W_L should be taken for calculating the current sample, \hat{T}_i .
- The process is considered as an in-control process if the $(i-1)^{\text{th}}$ sample \hat{T}_i falls in the interval $UWL \leq \hat{T}_i \leq UCL$, and n_L , h_S , K_S and W_S should be taken for calculating the current sample, \hat{T}_i .
- The process is considered as an out-of-control process if the $(i-1)^{\text{th}}$ sample \hat{T}_i falls out of the $CL \leq \hat{T}_i \leq UCL$ region, and immediate correction action need to be taken to determine the assignable cause of variation.

The measurement of the performance of the VP CV chart are derived to evaluate whether the control chart is performing well. In order to compute the *ATS*, the Markov chain approach is used. The Markov chain process is a process that transits from one state to another state. The Markov chain is

memoryless, where the probability of the next state occurring only depends on the previous state and do not depend on the sequence of the event which occurs prior to that (Costa, 1997). The Markov chain model for the VP CV chart with three states is defined as follows:

State 1: $\Pr (\text{CL} \leq \hat{T}_i \leq \text{UWL}),$

State 2: $\Pr (\text{UWL} \leq \hat{T}_i \leq \text{UCL}),$

State 3: $\Pr (\hat{T}_i \geq \text{UCL}).$

From the Markov chain theory, state 3 is known as absorbing state. State 3 is also known as the out-of-control state. The transition probability matrix shown below:

$$\mathbf{P} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}, \quad (3.9)$$

where P_{ij} is defined as the transition probability from the previous state i to the current state j . In order to obtain γ_1 , the shift size, τ is introduced and apply $\gamma_1 = \tau\gamma_0$. For example, if the τ is 1.1, it represents a 10% $[(1.1-1.0) / 1.0 \times 100\%]$ increase in the CV when the process is out-of-control. From the Castagliola, et al. (2015a), the formulas of the transition probabilities of the Markov chain are shown as follows:

$$P_{11} = P \left(\exp \left(\frac{-W_L - a}{b} \right) + c < \hat{\gamma}_i < \exp \left(\frac{W_L - a}{b} \right) + c \mid n_s, \gamma_1 \right), \quad (3.10a)$$

$$P_{12} = P\left(\exp\left(\frac{-K_L - a}{b}\right) + c \leq \hat{\gamma}_i < \exp\left(\frac{-W_L - a}{b}\right) + c \mid n_S, \gamma_1\right) + P\left(\exp\left(\frac{W_L - a}{b}\right) + c < \hat{\gamma}_i \leq \exp\left(\frac{K_L - a}{b}\right) + c \mid n_S, \gamma_1\right), \quad (3.10b)$$

$$P_{21} = P\left(\exp\left(\frac{-W_S - a}{b}\right) + c < \hat{\gamma}_i < \exp\left(\frac{W_S - a}{b}\right) + c \mid n_L, \gamma_1\right), \quad (3.10c)$$

and

$$P_{22} = P\left(\exp\left(\frac{-K_S - a}{b}\right) + c \leq \hat{\gamma}_i < \exp\left(\frac{-W_S - a}{b}\right) + c \mid n_L, \gamma_1\right) + P\left(\exp\left(\frac{W_S - a}{b}\right) + c < \hat{\gamma}_i \leq \exp\left(\frac{K_S - a}{b}\right) + c \mid n_L, \gamma_1\right). \quad (3.10d)$$

The average time to signal and the standard deviation of the time to signals proposed by Saccucci, Amin and Lucas (1992) computed as:

$$ATS = \mathbf{b}'(\mathbf{I}-\mathbf{Q})^{-1} \mathbf{t} - \mathbf{b}'\mathbf{t}, \quad (3.11)$$

and

$$SDTS = \sqrt{\mathbf{b}'\mathbf{Q}\mathbf{B}(2\mathbf{Q}-\mathbf{I})\mathbf{t} - (\mathbf{b}'\mathbf{Q}\mathbf{t})^2}, \quad (3.12)$$

where $\mathbf{b}' = (b_1, b_2)$ is the vector of starting probability and the summation of b_1 and b_2 is equals to 1. \mathbf{Q} is a 2X2 transition probability matrix for the transient states, \mathbf{B} is a diagonal element with the h_j element, \mathbf{I} is a 2X2 identity matrix with all values equal to 1 and $\mathbf{t}' = (h_L, h_S)$ is the vector of sampling intervals.

The formula to compute b_1 is shown below (Khaw, et al., 2016):

$$b_1 = \frac{F_{\hat{\gamma}}(UWL \mid n_S, \gamma_0) - F_{\hat{\gamma}}(LWL \mid n_S, \gamma_0)}{F_{\hat{\gamma}}(UCL \mid n_S, \gamma_0) - F_{\hat{\gamma}}(UCL \mid n_S, \gamma_0)}, \quad (3.13)$$

and

$$b_2 = 1 - b_1 . \quad (3.14)$$

The VP CV chart restricts the *ASS*, n_0 and *ASI*, h_0 below (Khaw, et al., 2016):

$$b_1 n_s + b_2 n_L = n_0 , \quad (3.15a)$$

and

$$b_1 h_L + b_2 h_s = h_0 . \quad (3.15b)$$

When compute the *ATS*, τ is needs to be specified. However, practitioners could not clearly define the shift size in real life. From the Castagliola, Celano and Psarakis (2011), *EATS* is introduced which can measure the performance of control chart without specifying the shift size.

$$EATS_1 = \int f_\tau(\tau) ATS_1(n_s, n_L, h_s, h_L, W_s, W_L, K_s, K_L, \gamma_0, \tau) d\tau , \quad (3.16)$$

where $f_\tau(\tau)$ is the probability density function of τ . As it is difficult to estimate the actual distribution for $f_\tau(\tau)$ is very hard to obtain the actual shape and distribution on it, one assumption which can be made is that τ follows a uniform distribution with the parameter $(\tau_{\min}, \tau_{\max})$, where the τ_{\min} is the smallest shift size and τ_{\max} is the largest shift size (Castagliola, Celano and Psarakis, 2011). Castagliola, Celano and Psarakis (2011) suggested using $\tau_{\min} = 1$ and $\tau_{\max} = 2$ for increasing shift in the CV. Due to the difficulty in evaluating the integral of *EATS* directly, the Gauss-Legendre quadrature is used to approximate it.

3.3 Optimization procedure

In this section, the algorithms to obtain the optimal VP CV chart's parameters $(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*)$ are explained. Two optimization criteria are considered. For the first criterion, we select the optimal chart's parameters which minimize ATS_1 , while for the second criterion, we select the optimal chart's parameters which minimizes $EATS_1$. The first criterion is used when τ is known, while the second criterion is used when τ is unknown. For both of these criteria, constraints in the in-control ATS (ATS_0), in-control sample size (n_0) and in-control average sampling interval (h_0) need to be satisfied.

For the first optimization criterion, in order to minimize the ATS_1 , $(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*)$ is obtained such that

$$(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*) = \underset{(n_S, n_L, h_S, h_L, W_S, W_L, K_S, K_L)}{\operatorname{argmin}} ATS_1(n_S, n_L, h_S, h_L, W_S, W_L, K_S, K_L, \gamma_0, \tau), \quad (3.17)$$

subject to the constraints

$$b_1 n_S + b_2 n_L = n_0, \quad (3.18)$$

$$b_1 h_L + b_2 h_S = h_0, \quad (3.19)$$

and

$$P[|T_i| > K_L] b_1 + P[|T_i| > K_S] b_2 = P[|T_i| > K_0], \quad (3.20)$$

where $P[|T_i| > K_0] = \frac{1}{ATS_0}$ of the standard Shewhart CV chart with fixed chart parameters. Constraints (3.18) to (3.20) ensures the n_0 , h_0 and ATS_0 constraints are satisfied and enables a fair comparison with other charts with fixed chart parameters (Costa, 1999).

The following steps are employed to minimize Equation (3.17) subject to constraints (3.18) to (3.20):

1. Set $\gamma_0, \tau, n_0, h_0, ATS_0$ and h_S . The value of h_S needs to be determined to avoid getting a value of h_S which is too small to be implemented in practical scenarios, as too frequent samplings are usually infeasible.
2. Set $n_S = 2$.
3. Set $n_L = n_0 + 1$.
4. From Equation (18) and since $b_1 + b_2 = 1$, $b_1 = \frac{n_0 - n_L}{n_S - n_L}$.
5. Substitute $b_1 = \frac{n_0 - n_L}{n_S - n_L}$ and $b_2 = 1 - b_1$ into Equation (3.19), then solve for h_L .
6. Substitute $b_1 = \frac{n_0 - n_L}{n_S - n_L}$ and $b_2 = 1 - b_1$ into Equation (3.20), then by using a non-linear equation solver, obtain the (K_S, K_L) combination which solves Equation (3.20) and choose the combination which results in minimum ATS_1 .
7. By using a non-linear equation solver, obtain the (W_S, W_L) combination which solves $b_1 = \frac{p_{11}^0}{p_{11}^0 + p_{12}^0} = \frac{n_0 - n_L}{n_S - n_L}$ and choose the combination which results in minimum ATS_1 .
8. Increase n_L by 1, and retain the same value for n_S .
9. Repeat Steps (4) to (8) until $n_L = 31$. Values of $n_L > 31$ are not considered as larger values of sample sizes are usually not preferred by practitioners.

10. Increase n_S by 1.
11. Repeat Steps (3) to (10) until $n_S = n_0 - 1$.

From the outputs computed from Steps 1 to 11, the combination of $(n_S, n_L, h_S, h_L, W_S, W_L, K_S, K_L)$ that provides the smallest value of ATS_1 is the optimal chart's parameters $(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*)$.

In this project, the maximum value of n_L is set as 31. In most applications, there is usually a maximum feasible value for n_L because of constraints on how many items can be conveniently handled in one sample (Reynolds and Arnold, 2001). The maximum number of items which can be handled usually depends on experience and on the particular process involved. In industrial context, it is a common practice to use moderately small sample sizes. Some of the recent papers which also adopted $n_L = 31$ as the maximum value for n_L are Castagliola, et al. (2015a), Yeong, et al. (2015) and Khaw, et al. (2016). However, setting $n_L = 31$ as the maximum value is only a guideline, and other maximum values of n_L can be adopted depending on the process. The same steps to obtain the optimal chart parameters can still be adopted, even though other maximum values of n_L are used.

For the second optimization criterion, in order to minimize the $EATS_1$, $(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*)$ is obtained such that

$$(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*) = \arg \min_{(n_S, n_L, h_S, h_L, W_S, W_L, K_S, K_L)} EATS_1(n_S, n_L, h_S, h_L, W_S, W_L, K_S, K_L, \gamma_0, \tau_{\min}, \tau_{\max}), \quad (3.21)$$

subject to the same constraints in Equations (18) to (20) and by adopting a similar procedure as in Steps (1) to (11). τ_{\min} and τ_{\max} are determined by the practitioner depending on the process. Note that $EATS_1$ is computed from Equation (16), and unlike ATS_1 , the exact value of τ does not need to be specified.

CHAPTER 4

NUMERICAL RESULTS AND DISCUSSIONS FOR THE VARIABLE PARAMETERS CONTROL CHART

4.1 Numerical result

By referring to the past reviews, the researchers normally use the *ARL* and the *SDRL* to measure the chart's performance when the sampling interval is fixed. However, the *ATS* and the *SDTS* are used to evaluate the performance of VP CV chart due to the sampling interval is varies. The in control average of the time to signals and the standard deviation of the time to signals are denoted as ATS_0 and $SDTS_0$ respectively. On the other hand, when the process is out-of-control the average of the time and the standard deviation of the time to signals are denoted as ATS_1 and $SDTS_1$ respectively.

Based on the VP CV chart, the sampling intervals, the sample sizes, the warning limits and the control limits are vary into two levels (i.e. the sampling intervals is vary into h_L and h_S) to minimize the ATS_1 . Firstly, the optimal chart's parameters $n_S, n_L, h_L, W_L, W_S, K_L, K_S$ are obtained to minimize the ATS_1 , subject to the constraints ATS_0, ASI_0 and ASS_0 with 370.4, 1 and n_0 . In order to detect the increasing CV shifts, $\tau \in \{1.1, 1.2, 1.5, 2.0\}$, $n_0 \in \{5, 7, 10, 15\}$ are considered for the Tables 4.1 to Table 4.4. The ATS_1 and $SDTS_1$ values are obtained for a specific shift size by using the Markov chain discussed in Chapter 3 after the charting parameters, $n_S, n_L, h_L, W_L, W_S, K_L, K_S$ are computed. Take note that an increasing value of τ such as $\tau \in \{1.1, 1.2, 1.5, 2.0\}$ in computing the ATS_1 will increase the value of γ_0 . For this project, we

Table 4.1: Values of $(a_s, b_s, c_s, a_L, b_L, c_L)$, optimal chart parameters $(n_s, n_L, h_L, W_L, W_s, K_L, K_s)$ and the corresponding out-of-control ATS_1 and $SDTS_1$ of the VP- γ chart when $\gamma_0 = 0.05$, $n_0 = \{5, 7, 10, 15\}$, $h_s \in \{0.01, 0.10\}$ and $\tau \in \{1.1, 1.2, 1.5, 2.0\}$.

| τ | $n_0 = 5$ | |
|--------|--|--|
| | $h_s = 0.01$ | $h_s = 0.10$ |
| 1.1 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.11, 1.63, 1.53, 2.86, 2.24 84.17, 85.48 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.10, 1.63, 1.53, 2.86, 2.24 84.40, 85.62 |
| 1.2 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.11, 1.63, 1.53, 2.93, 2.23 20.62, 21.82 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.10, 1.63, 1.53, 2.92, 2.23 20.77, 21.85 |
| 1.5 | 9.86, 4.14, -0.05, 30.31, 15.37, -0.09 3, 16, 1.18, 1.45, 1.39, 2.88, 2.45 2.96, 3.84 | 9.86, 4.14, -0.05, 31.34, 15.92, -0.09 3, 17, 1.15, 1.49, 1.42, 2.87, 2.44 3.05, 3.82 |
| 2.0 | 9.86, 4.14, -0.05, 16.91, 8.08, -0.08 3, 6, 2.98, 0.46, 0.44, 2.79, 2.77 0.57, 2.19 | 9.86, 4.14, -0.05, 18.69, 9.07, -0.08 3, 7, 1.90, 0.72, 0.68, 2.79, 2.75 0.65, 1.73 |
| τ | $n_0 = 7$ | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| 1.1 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.21, 1.44, 1.33, 2.97, 2.42 72.25, 73.68 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.21, 1.44, 1.33, 2.97, 2.42 72.25, 73.68 |
| 1.2 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.21, 1.44, 1.33, 2.99, 2.41 15.04, 16.35 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.21, 1.44, 1.33, 3.04, 2.41 15.04, 16.35 |
| 1.5 | 12.63, 5.69, -0.06, 31.34, 15.92, -0.09 4, 17, 1.30, 1.22, 1.18, 2.98, 2.58 1.72, 2.65 | 12.63, 5.69, -0.06, 32.34, 16.45, -0.09 4, 18, 1.25, 1.26, 1.22, 2.96, 2.56 1.82, 2.62 |
| 2.0 | 14.92, 6.98, -0.07, 20.31, 9.96, -0.08 5, 8, 2.98, 0.44, 0.44, 2.92, 2.83 0.24, 1.74 | 14.92, 6.98, -0.07, 20.31, 9.96, -0.08 5, 8, 2.80, 0.44, 0.44, 2.92, 2.83 0.30, 1.62 |
| τ | $n_0 = 10$ | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| 1.1 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.38, 1.20, 1.08, 3.08, 2.57 61.68, 63.23 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.34, 1.20, 1.08, 3.08, 2.57 62.15, 63.56 |
| 1.2 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.33, 1.19, 1.13, 3.39, 2.54 10.77, 12.11 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.30, 1.19, 1.13, 3.39, 2.54 11.02, 12.21 |
| 1.5 | 14.92, 6.98, -0.07, 31.34, 15.92, -0.09 5, 17, 1.71, 0.83, 0.81, 3.13, 2.73 0.92, 2.07 | 16.91, 8.08, -0.08, 33.30, 16.96, -0.09 6, 19, 1.40, 1.03, 1.01, 3.14, 2.65 1.02, 1.84 |
| 2.0 | 20.31, 9.96, -0.08, 24.55, 12.26, -0.09 8, 11, 2.98, 0.44, 0.43, 2.95, 2.89 0.08, 1.51 | 20.31, 9.96, -0.08, 24.55, 12.26, -0.09 8, 11, 2.80, 0.44, 0.43, 2.95, 2.89 0.11, 1.39 |
| τ | $n_0 = 15$ | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| 1.1 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.74, 0.84, 0.79, 3.43, 2.72 52.04, 53.76 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.68, 0.84, 0.79, 3.43, 2.72 52.74, 54.29 |
| 1.2 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.74, 0.84, 0.79, 3.57, 2.72 7.40, 8.99 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.68, 0.84, 0.79, 3.57, 2.72 7.74, 9.12 |
| 1.5 | 21.81, 10.78, -0.08, 33.30, 16.96, -0.09 9, 19, 2.49, 0.53, 0.52, 3.23, 2.84 0.38, 1.75 | 21.81, 10.78, -0.08, 35.15, 17.95, -0.09 9, 21, 1.90, 0.68, 0.67, 3.21, 2.80 0.48, 1.49 |
| 2.0 | 27.00, 13.59, -0.09, 30.31, 15.37, -0.09 13, 16, 2.98, 0.43, 0.43, 2.95, 2.94 0.02, 1.42 | 27.00, 13.59, -0.09, 30.31, 15.37, -0.09 13, 16, 2.80, 0.43, 0.43, 2.95, 2.94 0.03, 1.30 |

Table 4.2: Values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal chart parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ and the corresponding out-of-control ATS_1 and $SDTS_1$ of the VP- γ chart when $\gamma_0 = 0.10, n_0 = \{5, 7, 10, 15\}, h_S \in \{0.01, 0.10\}$ and $\tau \in \{1.1, 1.2, 1.5, 2.0\}$.

| τ | $n_0 = 5$ | |
|--------|--|--|
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.11, 1.63, 1.53, 2.88, 2.24 85.12, 86.44 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.10, 1.63, 1.53, 2.88, 2.24 85.36, 86.57 |
| 1.2 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.11, 1.63, 1.53, 2.95, 2.23 20.98, 22.17 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.10, 1.63, 1.53, 2.95, 2.23 21.13, 22.21 |
| 1.5 | 6.89, 4.04, -0.10, 19.30, 14.53, -0.17 3, 16, 1.18, 1.45, 1.39, 2.90, 2.45 3.02, 3.91 | 6.89, 4.04, -0.10, 19.94, 15.04, -0.17 3, 17, 1.15, 1.49, 1.42, 2.89, 2.44 3.11, 3.88 |
| 2.0 | 6.89, 4.04, -0.10, 11.11, 7.73, -0.14 3, 6, 2.98, 0.46, 0.44, 2.79, 2.78 0.59, 2.22 | 6.89, 4.04, -0.10, 12.18, 8.65, -0.15 3, 7, 1.90, 0.71, 0.68, 2.79, 2.76 0.67, 1.75 |
| τ | $n_0 = 7$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.21, 1.44, 1.33, 2.99, 2.42 73.23, 74.65 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.19, 1.44, 1.33, 2.99, 2.42 73.56, 74.87 |
| 1.2 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.21, 1.44, 1.33, 3.07, 2.41 15.36, 16.66 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.19, 1.44, 1.33, 3.07, 2.41 15.54, 16.72 |
| 1.5 | 8.54, 5.50, -0.12, 19.94, 15.04, -0.17 4, 17, 1.30, 1.22, 1.18, 3.00, 2.58 1.76, 2.70 | 8.54, 5.50, -0.12, 20.55, 15.54, -0.17 4, 18, 1.25, 1.26, 1.22, 2.98, 2.56 1.87, 2.66 |
| 2.0 | 9.91, 6.70, -0.14, 13.17, 9.48, -0.15 5, 8, 2.98, 0.44, 0.44, 2.94, 2.83 0.25, 1.75 | 9.91, 6.70, -0.14, 13.17, 9.48, -0.15 5, 8, 2.80, 0.44, 0.44, 2.94, 2.83 0.32, 1.64 |
| τ | $n_0 = 10$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.38, 1.20, 1.08, 3.11, 2.57 62.63, 64.18 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.34, 1.20, 1.08, 3.11, 2.57 63.10, 64.51 |
| 1.2 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.33, 1.19, 1.13, 3.42, 2.54 11.04, 12.38 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.30, 1.19, 1.13, 3.42, 2.54 11.30, 12.48 |
| 1.5 | 9.91, 6.70, -0.14, 20.55, 15.54, -0.17 5, 18, 1.62, 0.89, 0.87, 3.14, 2.71 0.95, 2.05 | 9.91, 6.70, -0.14, 21.15, 16.02, -0.17 5, 19, 1.50, 0.94, 0.92, 3.12, 2.69 1.05, 1.98 |
| 2.0 | 13.17, 9.48, -0.15, 15.75, 11.63, -0.16 8, 11, 2.98, 0.44, 0.43, 2.96, 2.90 0.08, 1.52 | 13.17, 9.48, -0.15, 15.75, 11.63, -0.16 8, 11, 2.80, 0.44, 0.43, 2.97, 2.89 0.12, 1.40 |
| τ | $n_0 = 15$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.74, 0.84, 0.79, 3.46, 2.72 52.94, 54.66 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.68, 0.84, 0.79, 3.46, 2.72 53.64, 55.19 |
| 1.2 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.74, 0.84, 0.79, 3.60, 2.72 7.62, 9.21 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.68, 0.84, 0.79, 3.60, 2.72 7.96, 9.34 |
| 1.5 | 14.08, 10.25, -0.16, 21.73, 16.48, -0.17 9, 20, 2.19, 0.61, 0.60, 3.24, 2.82 0.40, 1.65 | 14.08, 10.25, -0.16, 22.84, 17.38, -0.17 9, 22, 1.77, 0.74, 0.73, 3.21, 2.79 0.50, 1.45 |
| 2.0 | 17.26, 12.87, -0.16, 19.30, 14.53, -0.17 13, 16, 2.98, 0.43, 0.43, 2.95, 2.95 0.02, 1.42 | 17.26, 12.87, -0.16, 19.30, 14.53, -0.17 13, 16, 2.80, 0.43, 0.43, 2.95, 2.95 0.03, 1.30 |

Table 4.3: Values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal chart parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ and the corresponding out-of-control ATS_1 and $SDTS_1$ of the VP- γ chart when $\gamma_0 = 0.15, n_0 = \{5, 7, 10, 15\}, h_S \in \{0.01, 0.10\}$ and $\tau \in \{1.1, 1.2, 1.5, 2.0\}$.

| τ | $n_0 = 5$ | |
|--------|--|--|
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.11, 1.63, 1.53, 2.92, 2.25 86.71, 88.02 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.10, 1.63, 1.53, 2.92, 2.25 86.95, 88.16 |
| 1.2 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.11, 1.63, 1.53, 3.00, 2.24 21.58, 22.76 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.10, 1.63, 1.53, 3.00, 2.24 21.73, 22.82 |
| 1.5 | 5.16, 3.87, -0.14, 13.72, 13.79, -0.22 3, 17, 1.17, 1.48, 1.42, 2.93, 2.43 3.12, 3.99 | 5.16, 3.87, -0.14, 14.13, 14.23, -0.22 3, 18, 1.14, 1.52, 1.45, 2.91, 2.42 3.21, 3.97 |
| 2.0 | 5.16, 3.87, -0.14, 8.56, 8.04, -0.20 3, 7, 1.99, 0.71, 0.68, 2.81, 2.77 0.63, 1.85 | 5.16, 3.87, -0.14, 8.56, 8.04, -0.20 3, 7, 1.90, 0.71, 0.68, 2.81, 2.77 0.71, 1.78 |
| τ | $n_0 = 7$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.21, 1.44, 1.33, 3.04, 2.42 74.86, 76.28 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.19, 1.44, 1.33, 3.04, 2.42 75.19, 76.50 |
| 1.2 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.21, 1.44, 1.33, 3.13, 2.41 15.89, 17.20 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.19, 1.44, 1.33, 3.13, 2.41 16.08, 17.26 |
| 1.5 | 6.21, 5.20, -0.17, 14.13, 14.23, -0.22 4, 18, 1.27, 1.26, 1.22, 3.01, 2.56 1.84, 2.75 | 6.21, 5.20, -0.17, 14.53, 14.67, -0.22 4, 19, 1.23, 1.30, 1.26, 3.00, 2.55 1.94, 2.73 |
| 2.0 | 7.08, 6.29, -0.19, 9.21, 8.79, -0.21 5, 8, 2.98, 0.44, 0.43, 2.96, 2.84 0.27, 1.78 | 7.08, 6.29, -0.19, 9.21, 8.79, -0.21 5, 8, 2.80, 0.44, 0.43, 2.96, 2.84 0.34, 1.66 |
| τ | $n_0 = 10$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.38, 1.20, 1.08, 3.17, 2.58 64.22, 65.77 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.34, 1.20, 1.08, 3.17, 2.58 64.70, 66.11 |
| 1.2 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.33, 1.19, 1.13, 3.48, 2.54 11.50, 12.84 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.34, 1.20, 1.08, 3.27, 2.57 11.76, 13.04 |
| 1.5 | 7.08, 6.29, -0.19, 14.13, 14.23, -0.22 5, 18, 1.62, 0.89, 0.87, 3.17, 2.71 1.00, 2.10 | 7.08, 6.29, -0.19, 14.92, 15.09, -0.22 5, 20, 1.45, 0.98, 0.96, 3.14, 2.68 1.10, 2.01 |
| 2.0 | 9.21, 8.79, -0.21, 10.92, 10.72, -0.22 8, 11, 2.98, 0.44, 0.43, 2.98, 2.90 0.09, 1.53 | 9.21, 8.79, -0.21, 10.92, 10.72, -0.22 8, 11, 2.80, 0.44, 0.43, 2.99, 2.89 0.14, 1.41 |
| τ | $n_0 = 15$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.74, 0.83, 0.79, 3.51, 2.72 54.45, 56.17 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.68, 0.83, 0.79, 3.51, 2.72 55.16, 56.71 |
| 1.2 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.74, 0.83, 0.79, 3.67, 2.72 7.99, 9.58 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.68, 0.83, 0.79, 3.67, 2.72 8.34, 9.72 |
| 1.5 | 9.81, 9.48, -0.21, 14.92, 15.09, -0.22 9, 20, 2.19, 0.61, 0.60, 3.26, 2.82 0.43, 1.68 | 9.81, 9.48, -0.21, 15.68, 15.90, -0.23 9, 22, 1.77, 0.74, 0.73, 3.24, 2.79 0.53, 1.48 |
| 2.0 | 11.93, 11.83, -0.22, 13.30, 13.33, -0.22 13, 16, 2.98, 0.43, 0.43, 2.96, 2.95 0.02, 1.43 | 11.93, 11.83, -0.22, 13.30, 13.33, -0.22 13, 16, 2.80, 0.43, 0.43, 2.96, 2.95 0.04, 1.30 |

Table 4.4: Values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal chart parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ and the corresponding out-of-control ATS_1 and $SDTS_1$ of the VP- γ chart when $\gamma_0 = 0.20$, $n_0 = \{5, 7, 10, 15\}$, $h_S \in \{0.01, 0.10\}$ and $\tau \in \{1.1, 1.2, 1.5, 2.0\}$.

| τ | $n_0 = 5$ | |
|--------|---|---|
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.11, 1.63, 1.53, 2.99, 2.25 88.90, 90.21 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.10, 1.63, 1.53, 2.99, 2.25 89.14, 90.35 |
| 1.2 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.11, 1.63, 1.53, 3.09, 2.24 22.43, 23.62 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.10, 1.63, 1.53, 3.08, 2.24 22.58, 23.67 |
| 1.5 | 3.98, 3.65, -0.17, 10.19, 12.75, -0.25 3, 18, 1.15, 1.52, 1.45, 2.97, 2.42 3.26, 4.13 | 3.98, 3.65, -0.17, 10.19, 12.75, -0.25 3, 18, 1.14, 1.52, 1.45, 2.97, 2.42 3.36, 4.12 |
| 2.0 | 3.98, 3.65, -0.17, 6.26, 7.32, -0.24 3, 7, 1.99, 0.70, 0.68, 2.86, 2.77 0.68, 1.90 | 3.98, 3.65, -0.17, 6.26, 7.32, -0.24 3, 7, 1.90, 0.71, 0.68, 2.86, 2.77 0.76, 1.83 |
| τ | $n_0 = 7$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.21, 1.44, 1.33, 3.12, 2.42 77.12, 78.54 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.19, 1.44, 1.33, 3.12, 2.42 77.45, 78.76 |
| 1.2 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.21, 1.44, 1.33, 3.24, 2.42 16.65, 17.95 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.19, 1.44, 1.33, 3.23, 2.42 16.84, 18.02 |
| 1.5 | 3.98, 3.65, -0.17, 9.90, 12.36, -0.25 3, 17, 1.40, 1.11, 1.06, 3.10, 2.62 1.94, 3.04 | 4.66, 4.83, -0.20, 10.47, 13.13, -0.25 4, 19, 1.23, 1.30, 1.26, 3.05, 2.55 2.05, 2.84 |
| 2.0 | 5.25, 5.79, -0.22, 6.71, 7.98, -0.24 5, 8, 2.98, 0.44, 0.43, 2.99, 2.85 0.30, 1.81 | 5.25, 5.79, -0.22, 7.13, 8.58, -0.24 5, 9, 1.90, 0.69, 0.68, 2.98, 2.81 0.38, 1.35 |
| τ | $n_0 = 10$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.38, 1.20, 1.08, 3.27, 2.58 66.44, 67.98 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.34, 1.20, 1.08, 3.27, 2.58 66.92, 68.33 |
| 1.2 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.38, 1.20, 1.08, 3.40, 2.58 12.14, 13.58 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.34, 1.20, 1.08, 3.40, 2.58 12.41, 13.69 |
| 1.5 | 5.25, 5.79, -0.22, 10.47, 13.13, -0.25 5, 19, 1.55, 0.94, 0.92, 3.20, 2.70 1.07, 2.14 | 5.25, 5.79, -0.22, 10.75, 13.50, -0.25 5, 20, 1.45, 0.98, 0.96, 3.18, 2.66 1.17, 2.08 |
| 2.0 | 6.71, 7.98, -0.24, 7.91, 9.67, -0.25 8, 11, 2.98, 0.43, 0.43, 3.02, 2.90 0.11, 1.55 | 6.71, 7.98, -0.24, 7.91, 9.67, -0.25 8, 11, 2.80, 0.43, 0.43, 3.02, 2.90 0.15, 1.43 |
| τ | $n_0 = 15$ | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| 1.1 | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.74, 0.83, 0.79, 3.59, 2.72 56.57, 58.28 | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.68, 0.83, 0.79, 3.59, 2.72 57.28, 58.83 |
| 1.2 | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.74, 0.83, 0.79, 3.78, 2.72 8.52, 10.11 | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.68, 0.83, 0.79, 3.78, 2.72 8.88, 10.27 |
| 1.5 | 7.13, 8.58, -0.24, 11.02, 13.86, -0.25 9, 21, 1.99, 0.68, 0.67, 3.29, 2.80 0.47, 1.64 | 7.13, 8.58, -0.24, 11.55, 14.56, -0.26 9, 23, 1.68, 0.80, 0.79, 3.26, 2.77 0.57, 1.48 |
| 2.0 | 8.28, 10.17, -0.25, 9.59, 11.95, -0.25 12, 16, 3.97, 0.32, 0.32, 2.97, 2.96 0.02, 1.75 | 8.63, 10.64, -0.25, 9.59, 11.95, -0.25 13, 16, 2.80, 0.43, 0.43, 2.97, 2.96 0.05, 1.31 |

only concern the upward shifts. This is because an adaptive type chart is not a chart to monitor the downward shift in the CV (Castogliola, et al., 2013b). Tables 4.1 to Table 4.4 show the values of $(a_S, b_S, c_S, a_L, b_L, c_L)$ in the first row of each cell, followed by the optimal parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ in the second row of each cell while the corresponding $(ATS_1, SDTS_1)$ values are shown in the third row of each cell. Table 4.1 shows the $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the ATS_1 when $\gamma_0=0.05$ and $h_S \in \{0.01, 0.10\}$. For instances, the $(a_S, b_S, c_S, a_L, b_L, c_L) = (6.27, 2.21, -0.02, 43.22, 22.23, -0.09)$ and optimal parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S) = (2, 31, 1.11, 1.63, 1.53, 2.86, 2.24)$ are used to minimize the ATS_1 when the τ, n_0, h_S, γ_0 are 1.1, 5, 0.01 and 0.05 respectively. Table 4.2 shows the $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the ATS_1 when $\gamma_0=0.10$ and $h_S \in \{0.01, 0.10\}$. Table 4.3 shows the $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the ATS_1 when $\gamma_0=0.15$ and $h_S \in \{0.01, 0.10\}$. Table 4.4 shows the $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the ATS_1 when $\gamma_0=0.20$ and $h_S \in \{0.01, 0.10\}$. The optimal parameters obtained from Scicoslab optimization program are based on the steps mentioned in Chapter 3. Practitioners can refer to the tables of optimal charting parameters provided in this section to implement the VP CV chart, thereby facilitating a quick implementation of the proposed chart by practitioners. For ease of implementation, this project also provides the values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, which are the parameters required to obtain the

transformed statistics T_i and are defined by Equations (8a) to (8c). Note that (a_S, b_S, c_S) are adopted for n_S , while (a_L, b_L, c_L) are adopted for n_L .

From the Table 4.1 to Table 4.4, the value of ATS_1 and $SDTS_1$ increase when the τ approaches 1. As noticed that the $\tau = 1$ represents an in-control process. Thus, the VP CV chart will likely to take more time to detect the out-of-control sample when the shift size is smaller. Furthermore, a smaller value of $SDTS_1$ also shows that the variability in the time to signal decreases as τ becomes larger. Thus, the further τ is from 1, the larger the shift and the shift can be detected faster. Since larger values of τ are easier to be detected, the difference between n_S and n_L becomes smaller as τ increases. Apart from that, there is a slightly different in term of ATS_1 when the $h_S = 0.01$ and $h_S = 0.10$. From the table shown, $h_S = 0.01$ has smaller ATS_1 value compare to the $h_S = 0.10$ when the γ_0 remain constant. Different values of γ_0 have small impact on the ATS_1 and $SDTS_1$ values of the VP CV chart.

In overall, the larger the difference between h_S and h_L , the smaller the ATS_1 value. This is because the W_S and W_L decrease when the difference between h_S and h_L is increased. From that, the warning region tends to cover a large portion compare to the central region in the control interval. Therefore, this situation will improve the statistical properties of the VP CV chart due to the higher chance of tightened sampling. As expected, there is a decrease in the values of ATS_1 and $SDTS_1$ with an increase in the sample size (n_0). Therefore, it is recommended to use a larger sample size, since a larger sample size not only

results in less time to detect an out-of-control condition but also results in a smaller variability in the time to signal.

Apart from that, the VP CV chart is evaluated in term of $EATS_1$ value. Firstly, the optimal chart's parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ are obtained to minimize the $EATS_1$, subject to the constraints $ATS_0 = 370.4$, $ASI_0 = 1$ and the $ASS_0 = n_0$. Then, $EATS_1$ values are computed according their corresponding optimal parameters as shown in Table 4.5. From the table 4.5, the values of $(a_S, b_S, c_S, a_L, b_L, c_L)$ and the values of the VP CV chart's optimal parameter $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ in the first and second rows of each cell that minimize the $EATS_1$ are shown when $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$ and $h_S \in \{0.01, 0.10\}$ for $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$. For example, the optimal parameters are $(n_S, n_L, h_L, W_L, W_S, K_L, K_S) = (2, 31, 1.11, 1.63, 1.54, 2.79, 2.26)$ when the $\gamma_0 = 0.05$, $h_S = 0.01$, $n_0 = 5$, $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$ and $(a_S, b_S, c_S, a_L, b_L, c_L) = (6.27, 2.21, -0.02, 43.22, 22.23, -0.09)$ to minimize the $EATS_1$. The Scicoslab optimization program is used to calculate the $EATS_1$ value from optimal parameters combination. Table 4.6 shows the values of $(a_S, b_S, c_S, a_L, b_L, c_L)$ and the optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the $EATS_1$ when $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$, $h_S \in \{0.01, 0.10\}$ and $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$. Table 4.7 shows the values of $(a_S, b_S, c_S, a_L, b_L, c_L)$ and the optimal parameters of the VP CV chart $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ that minimize the $EATS_1$ when $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$, $h_S \in \{0.01, 0.10\}$ and $(\tau_{\min}, \tau_{\max}) = (1.50, 2.00)$.

Table 4.5: Values of $(a_s, b_s, c_s, a_L, b_L, c_L)$, optimal chart parameters $(n_s, n_L, h_L, W_L, W_s, K_L, K_s)$ and the corresponding $EATS_1$ for the VP CV chart when $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$, $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$ and $h_s \in \{0.01, 0.10\}$.

| $n_0 = 5$ | | |
|-------------------|---|---|
| | $h_s = 0.01$ | $h_s = 0.10$ |
| $\gamma_0 = 0.05$ | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.11, 1.63, 1.54, 2.79, 2.26 19.80 | 6.27, 2.21, -0.02, 43.22, 22.23, -0.09 2, 31, 1.10, 1.63, 1.54, 2.79, 2.26 19.88 |
| $\gamma_0 = 0.10$ | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.11, 1.63, 1.54, 2.81, 2.26 19.97 | 4.70, 2.18, -0.05, 27.30, 20.94, -0.17 2, 31, 1.10, 1.63, 1.54, 2.81, 2.26 20.06 |
| $\gamma_0 = 0.15$ | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.11, 1.63, 1.54, 2.85, 2.26 20.26 | 3.76, 2.13, -0.07, 18.69, 19.11, -0.23 2, 31, 1.10, 1.63, 1.54, 2.85, 2.26 20.34 |
| $\gamma_0 = 0.20$ | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.11, 1.63, 1.54, 2.91, 2.27 20.65 | 3.09, 2.07, -0.09, 13.43, 17.05, -0.26 2, 31, 1.10, 1.63, 1.54, 2.91, 2.27 20.74 |
| $n_0 = 7$ | | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| $\gamma_0 = 0.05$ | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.17, 1.49, 1.42, 3.01, 2.39 16.72 | 9.86, 4.14, -0.05, 43.22, 22.23, -0.09 3, 31, 1.15, 1.49, 1.42, 3.00, 2.39 16.83 |
| $\gamma_0 = 0.10$ | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.17, 1.49, 1.42, 3.02, 2.39 16.89 | 6.89, 4.04, -0.10, 27.30, 20.94, -0.17 3, 31, 1.15, 1.49, 1.42, 3.02, 2.39 17.00 |
| $\gamma_0 = 0.15$ | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.17, 1.49, 1.42, 3.05, 2.40 17.18 | 5.16, 3.87, -0.14, 18.69, 19.11, -0.23 3, 31, 1.15, 1.49, 1.42, 3.05, 2.40 17.29 |
| $\gamma_0 = 0.20$ | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.17, 1.48, 1.42, 3.10, 2.40 17.59 | 3.98, 3.65, -0.17, 13.43, 17.05, -0.26 3, 31, 1.15, 1.48, 1.42, 3.09, 2.40 17.70 |
| $n_0 = 10$ | | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| $\gamma_0 = 0.05$ | 12.63, 5.69, -0.06, 43.22, 22.23, -0.09 4, 31, 1.28, 1.24, 1.20, 3.10, 2.54 14.27 | 12.63, 5.69, -0.06, 43.22, 22.23, -0.09 4, 31, 1.26, 1.24, 1.20, 3.10, 2.54 14.40 |
| $\gamma_0 = 0.10$ | 8.54, 5.50, -0.12, 27.30, 20.94, -0.17 4, 31, 1.28, 1.24, 1.20, 3.11, 2.54 14.44 | 8.54, 5.50, -0.12, 27.30, 20.94, -0.17 4, 31, 1.26, 1.24, 1.20, 3.11, 2.54 14.57 |
| $\gamma_0 = 0.15$ | 6.21, 5.20, -0.17, 18.69, 19.11, -0.23 4, 31, 1.28, 1.24, 1.20, 3.14, 2.55 14.71 | 6.21, 5.20, -0.17, 18.69, 19.11, -0.23 4, 31, 1.26, 1.24, 1.20, 3.14, 2.55 14.85 |
| $\gamma_0 = 0.20$ | 4.66, 4.83, -0.20, 13.43, 17.05, -0.26 4, 31, 1.28, 1.24, 1.20, 3.17, 2.55 15.10 | 4.66, 4.83, -0.20, 13.43, 17.05, -0.26 4, 31, 1.26, 1.24, 1.20, 3.17, 2.55 15.24 |
| $n_0 = 15$ | | |
| | $h_s = 0.01$ | $h_s = 0.10$ |
| $\gamma_0 = 0.05$ | 14.92, 6.98, -0.07, 43.22, 22.23, -0.09 5, 31, 1.62, 0.89, 0.86, 3.26, 2.70 12.20 | 14.92, 6.98, -0.07, 43.22, 22.23, -0.09 5, 31, 1.56, 0.89, 0.86, 3.26, 2.70 12.37 |
| $\gamma_0 = 0.10$ | 9.91, 6.70, -0.14, 27.30, 20.94, -0.17 5, 31, 1.62, 0.89, 0.86, 3.27, 2.70 12.35 | 9.91, 6.70, -0.14, 27.30, 20.94, -0.17 5, 31, 1.56, 0.89, 0.86, 3.27, 2.70 12.52 |
| $\gamma_0 = 0.15$ | 7.08, 6.29, -0.19, 18.69, 19.11, -0.23 5, 31, 1.62, 0.89, 0.86, 3.29, 2.70 12.61 | 7.08, 6.29, -0.19, 18.69, 19.11, -0.23 5, 31, 1.56, 0.89, 0.86, 3.28, 2.71, 12.78 |
| $\gamma_0 = 0.20$ | 5.25, 5.79, -0.22, 13.43, 17.05, -0.26 5, 31, 1.62, 0.88, 0.86, 3.31, 2.71 12.97 | 5.25, 5.79, -0.22, 13.43, 17.05, -0.26 5, 31, 1.56, 0.88, 0.86, 3.31, 2.71 13.15 |

Table 4.6: Values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal chart parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ and the corresponding $EATS_1$ for the VP CV chart when $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$, $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$ and $h_S \in \{0.01, 0.10\}$.

| $n_0 = 5$ | | |
|-------------------|---|---|
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 9.86, 4.14, -0.05, 34.24, 17.46, -0.09 3, 20, 1.13, 1.57, 1.50, 2.96, 2.33 3.33 | 9.86, 4.14, -0.05, 35.15, 17.95, -0.09 3, 21, 1.11, 1.60, 1.52, 2.94, 2.32 3.41 |
| $\gamma_0 = 0.10$ | 6.89, 4.04, -0.10, 22.29, 16.94, -0.17 3, 21, 1.12, 1.60, 1.52, 2.97, 2.32 3.39 | 6.89, 4.04, -0.10, 22.29, 16.94, -0.17 3, 21, 1.11, 1.60, 1.52, 2.96, 2.32 3.47 |
| $\gamma_0 = 0.15$ | 5.16, 3.87, -0.14, 15.30, 15.50, -0.22 3, 21, 1.12, 1.60, 1.52, 3.01, 2.32 3.49 | 5.16, 3.87, -0.14, 15.68, 15.90, -0.23 3, 22, 1.11, 1.62, 1.54, 2.99, 2.30 3.56 |
| $\gamma_0 = 0.20$ | 3.98, 3.65, -0.17, 11.29, 14.21, -0.26 3, 22, 1.12, 1.62, 1.54, 3.05, 2.30 3.63 | 3.98, 3.65, -0.17, 11.55, 14.56, -0.26 3, 23, 1.10, 1.64, 1.56, 3.04, 2.29 3.70 |
| $n_0 = 7$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 12.63, 5.69, -0.06, 36.04, 18.42, -0.09 4, 22, 1.20, 1.40, 1.35, 3.08, 2.45 2.05 | 12.63, 5.69, -0.06, 36.90, 18.88, -0.09 4, 23, 1.17, 1.43, 1.37, 3.05, 2.44 2.13 |
| $\gamma_0 = 0.10$ | 8.54, 5.50, -0.12, 22.84, 17.38, -0.17 4, 22, 1.20, 1.40, 1.35, 3.10, 2.45 2.10 | 8.54, 5.50, -0.12, 23.38, 17.81, -0.17 4, 23, 1.17, 1.43, 1.37, 3.07, 2.44 2.18 |
| $\gamma_0 = 0.15$ | 6.21, 5.20, -0.17, 16.04, 16.28, -0.23 4, 23, 1.19, 1.43, 1.37, 3.12, 2.43 2.17 | 6.21, 5.20, -0.17, 16.39, 16.66, -0.23 4, 24, 1.16, 1.45, 1.40, 3.09, 2.42 2.25 |
| $\gamma_0 = 0.20$ | 4.66, 4.83, -0.20, 11.80, 14.89, -0.26 4, 24, 1.17, 1.45, 1.40, 3.16, 2.42 2.28 | 4.66, 4.83, -0.20, 12.04, 15.22, -0.26 4, 25, 1.15, 1.47, 1.42, 3.13, 2.41 2.36 |
| $n_0 = 10$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 16.91, 8.08, -0.08, 38.58, 19.77, -0.09 6, 25, 1.26, 1.26, 1.23, 3.27, 2.51 1.18 | 16.91, 8.08, -0.08, 39.39, 20.20, -0.09 6, 26, 1.23, 1.29, 1.26, 3.23, 2.51 1.26 |
| $\gamma_0 = 0.10$ | 11.11, 7.73, -0.14, 24.42, 18.64, -0.17 6, 25, 1.26, 1.26, 1.23, 3.28, 2.51 1.22 | 11.11, 7.73, -0.14, 24.92, 19.04, -0.17 6, 26, 1.23, 1.29, 1.26, 3.24, 2.51 1.29 |
| $\gamma_0 = 0.15$ | 7.86, 7.22, -0.20, 17.08, 17.40, -0.23 6, 26, 1.25, 1.29, 1.26, 3.30, 2.50 1.27 | 7.86, 7.22, -0.20, 17.41, 17.75, -0.23 6, 27, 1.21, 1.32, 1.28, 3.26, 2.49 1.35 |
| $\gamma_0 = 0.20$ | 5.25, 5.79, -0.22, 12.04, 15.22, -0.26 5, 25, 1.33, 1.16, 1.13, 3.31, 2.57 1.35 | 5.25, 5.79, -0.22, 12.52, 15.85, -0.26 5, 27, 1.26, 1.22, 1.19, 3.26, 2.55 1.43 |
| $n_0 = 15$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 21.81, 10.78, -0.08, 40.97, 21.03, -0.09 9, 28, 1.46, 1.01, 0.99, 3.41, 2.64 0.58 | 21.81, 10.78, -0.08, 42.48, 21.84, -0.09 9, 30, 1.36, 1.08, 1.06, 3.31, 2.63 0.65 |
| $\gamma_0 = 0.10$ | 14.08, 10.25, -0.16, 25.90, 19.83, -0.17 9, 28, 1.46, 1.01, 0.99, 3.42, 2.64 0.60 | 14.08, 10.25, -0.16, 26.84, 20.58, -0.17 9, 30, 1.36, 1.07, 1.06, 3.32, 2.63 0.67 |
| $\gamma_0 = 0.15$ | 9.81, 9.48, -0.21, 18.06, 18.44, -0.23 9, 29, 1.42, 1.04, 1.03, 3.43, 2.63 0.63 | 9.81, 9.48, -0.21, 18.69, 19.11, -0.23 9, 31, 1.34, 1.10, 1.08, 3.34, 2.62 0.70 |
| $\gamma_0 = 0.20$ | 7.13, 8.58, -0.24, 13.21, 16.76, -0.26 9, 30, 1.40, 1.07, 1.06, 3.46, 2.62 0.68 | 7.13, 8.58, -0.24, 13.43, 17.05, -0.26 9, 31, 1.34, 1.10, 1.08, 3.38, 2.61 0.75 |

Table 4.7: Values of $(a_S, b_S, c_S, a_L, b_L, c_L)$, optimal chart parameters $(n_S, n_L, h_L, W_L, W_S, K_L, K_S)$ and the corresponding $EATS_1$ for the VP CV chart when $(\tau_{\min}, \tau_{\max}) = (1.50, 2.00)$, $\gamma_0 \in \{0.05, 0.10, 0.15, 0.20\}$, $n_0 \in \{5, 7, 10, 15\}$ and $h_S \in \{0.01, 0.10\}$.

| $n_0 = 5$ | | |
|-------------------|--|--|
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 9.86, 4.14, -0.05, 23.22, 11.54, -0.09 3, 10, 1.40, 1.11, 1.06, 2.86, 2.63 1.40 | 9.86, 4.14, -0.05, 24.55, 12.26, -0.09 3, 11, 1.30, 1.19, 1.14, 2.85, 2.60 1.49 |
| $\gamma_0 = 0.10$ | 6.89, 4.04, -0.10, 14.94, 10.96, -0.16 3, 10, 1.40, 1.11, 1.06, 2.88, 2.63 1.44 | 6.89, 4.04, -0.10, 15.75, 11.63, -0.16 3, 11, 1.30, 1.19, 1.14, 2.86, 2.61 1.53 |
| $\gamma_0 = 0.15$ | 5.16, 3.87, -0.14, 10.92, 10.72, -0.22 3, 11, 1.33, 1.19, 1.14, 2.90, 2.60 1.50 | 5.16, 3.87, -0.14, 10.92, 10.72, -0.22 3, 11, 1.30, 1.19, 1.14, 2.90, 2.60 1.59 |
| $\gamma_0 = 0.20$ | 3.98, 3.65, -0.17, 7.91, 9.67, -0.25 3, 11, 1.33, 1.19, 1.14, 2.95, 2.61 1.58 | 3.98, 3.65, -0.17, 8.28, 10.17, -0.25 3, 12, 1.26, 1.25, 1.21, 2.93, 2.59 1.67 |
| $n_0 = 7$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 12.63, 5.69, -0.06, 24.55, 12.26, -0.09 4, 11, 1.74, 0.82, 0.79, 2.95, 2.74 0.75 | 12.63, 5.69, -0.06, 25.80, 12.94, -0.09 4, 12, 1.54, 0.91, 0.89, 2.93, 2.72 0.83 |
| $\gamma_0 = 0.10$ | 8.54, 5.50, -0.12, 15.75, 11.63, -0.16 4, 11, 1.74, 0.82, 0.79, 2.97, 2.74 0.77 | 8.54, 5.50, -0.12, 16.52, 12.27, -0.16 4, 12, 1.54, 0.91, 0.89, 2.95, 2.72 0.86 |
| $\gamma_0 = 0.15$ | 6.21, 5.20, -0.17, 10.92, 10.72, -0.22 4, 11, 1.74, 0.82, 0.79, 3.00, 2.74 0.81 | 6.21, 5.20, -0.17, 11.44, 11.29, -0.22 4, 12, 1.54, 0.91, 0.89, 2.98, 2.72 0.90 |
| $\gamma_0 = 0.20$ | 4.66, 4.83, -0.20, 8.28, 10.17, -0.25 4, 12, 1.59, 0.91, 0.88, 3.03, 2.72 0.87 | 4.66, 4.83, -0.20, 8.63, 10.64, -0.25 4, 13, 1.45, 0.99, 0.96, 3.00, 2.70 0.96 |
| $n_0 = 10$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 18.69, 9.07, -0.08, 25.80, 12.94, -0.09 7, 12, 2.49, 0.53, 0.53, 3.13, 2.82 0.34 | 18.69, 9.07, -0.08, 28.15, 14.21, -0.09 7, 14, 1.68, 0.80, 0.79, 3.09, 2.77 0.41 |
| $\gamma_0 = 0.10$ | 12.18, 8.65, -0.15, 16.52, 12.27, -0.16 7, 12, 2.49, 0.53, 0.53, 3.14, 2.83 0.35 | 12.18, 8.65, -0.15, 17.97, 13.45, -0.17 7, 14, 1.68, 0.80, 0.79, 3.10, 2.77 0.43 |
| $\gamma_0 = 0.15$ | 7.86, 7.22, -0.20, 11.93, 11.83, -0.22 6, 13, 2.32, 0.58, 0.57, 3.15, 2.82 0.38 | 8.56, 8.04, -0.20, 12.40, 12.35, -0.22 7, 14, 1.68, 0.80, 0.79, 3.12, 2.77 0.46 |
| $\gamma_0 = 0.20$ | 5.78, 6.60, -0.23, 8.63, 10.64, -0.25 6, 13, 2.32, 0.57, 0.57, 3.18, 2.83 0.42 | 5.78, 6.60, -0.23, 9.28, 11.53, -0.25 6, 15, 1.72, 0.77, 0.76, 3.14, 2.78 0.50 |
| $n_0 = 15$ | | |
| | $h_S = 0.01$ | $h_S = 0.10$ |
| $\gamma_0 = 0.05$ | 24.55, 12.26, -0.09, 30.31, 15.37, -0.09 11, 16, 4.96, 0.26, 0.25, 3.13, 2.92 0.10 | 25.80, 12.94, -0.09, 30.31, 15.37, -0.09 12, 16, 3.70, 0.32, 0.32, 3.10, 2.91 0.16 |
| $\gamma_0 = 0.10$ | 15.75, 11.63, -0.16, 19.30, 14.53, -0.17 11, 16, 4.96, 0.26, 0.25, 3.14, 2.92 0.11 | 15.75, 11.63, -0.16, 19.94, 15.04, -0.17 11, 17, 2.80, 0.43, 0.43, 3.10, 2.89 0.16 |
| $\gamma_0 = 0.15$ | 10.92, 10.72, -0.22, 13.30, 13.33, -0.22 11, 16, 4.96, 0.26, 0.25, 3.16, 2.92 0.12 | 10.92, 10.72, -0.22, 13.72, 13.79, -0.22 11, 17, 2.80, 0.43, 0.43, 3.12, 2.90 0.18 |
| $\gamma_0 = 0.20$ | 7.91, 9.67, -0.25, 9.59, 11.95, -0.25 11, 16, 4.96, 0.25, 0.25, 3.19, 2.92 0.14 | 7.91, 9.67, -0.25, 9.90, 12.36, -0.25 11, 17, 2.80, 0.43, 0.43, 3.15, 2.90 0.20 |

From the Table 4.5 to Table 4.7, the higher the gap between τ_{\min} and τ_{\max} , the larger the $EATS_1$ value. The higher the gap, the higher chance for the smaller τ value can be considered. From the three different ranges of the shifts, the τ_{\min} is varying such as 1.00, 1.25 and 1.50 where the τ_{\max} remain the same which is 2.0. When the τ_{\min} is small like 1.00, it will generate larger $EATS_1$ value compare to other. By taking the example with $n_0 = 5$, $h_S = 0.01$ and $\gamma_0 = 0.05$ from the Table 4.5 and Table 4.6, for the $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$, the $EATS_1$ value is 19.80, whereas the $EATS_1$ for $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$ is 3.33.

Besides that, when $(\tau_{\min}, \tau_{\max})$ is chosen as $(1.50, 2.00)$, the difference between n_S^* and n_L^* is the smallest, followed by $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$ and $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$. This shows that when $(\tau_{\min}, \tau_{\max})$ is more concentrated towards larger values of τ , the difference between n_S^* and n_L^* becomes smaller. Apart from that, there is a slightly different in term of $EATS_1$ when the $h_S = 0.01$ and $h_S = 0.10$. From the Table 4.5 to Table 4.7, $h_S = 0.01$ has smaller $EATS_1$ value compare to the $h_S = 0.10$ when the γ_0 remain constant. Different value of γ_0 have small impact on the $EATS_1$ values of the VP CV chart. Moreover, the bigger the n_0 value, the smaller the $EATS_1$ values.

4.2 Numerical comparisons

In this section, the VP CV chart is compared with the competitor charts, which are the Shewhart CV chart, VSS CV chart, VSI CV chart, VSSI CV chart, Synthetic CV control chart and EWMA CV² chart in term of performance measures. There are three criteria used to make a comparison. Firstly, we are

looking at the ATS_1 performance of the various charts. Secondly, the charts are compared based on the $SDTS_1$. Thirdly, the charts are compared based on the $EATS_1$. As mentioned before, the ATS_1 value is computed with the specify shift size, whereas, there is no need predefine specify shift size for computing the $EATS_1$ value. Both performance measures need to satisfy the constraints $ATS_0 = 370.4$, $ASI_0 = 1$ and $ASS_0 = n_0$. For the fair comparison, a control chart is reflected better than other competitor charts when it has smaller ATS_1 value or smaller $EATS_1$ value. Readers are suggested to refer to Khaw, et al. (2016), Castagliola, et al. (2013b), Castagliola, et al. (2015a), Castagliola, Celano, and Psarakis (2011), Calzada and Scariano (2013) and Kang, et al. (2007) for the designs of the VSSI CV, VSI CV, VSS CV, EWMA CV², Synthetic CV and Shewhart CV charts, respectively. The comparisons made in this section are important as it allows practitioners to identify the charts which took the least amount of time to detect a shift (through smaller values of ATS_1 and $EATS_1$) and to identify the charts which show least variability in the time to signal (through smaller values of $SDTS_1$).

4.2.1 ATS

Table 4.8 shows ATS_1 s of the VP CV chart, the Shewhart CV chart, VSS CV chart, VSI CV chart, VSSI CV chart, Synthetic CV control chart and EWMA CV² chart. The Table 4.8 presents the different charts ATS_1 value for the $\tau \in \{1.1, 1.2, 1.5, 2.0\}$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0=0.05$ and $h_s = 0.10$. Table 4.8 also shows the percentage of increase in the ATS_1 value for the six competitor charts compared with the ATS_1 value for the VP CV chart. For instance, the VSSI CV

Table 4.8: A comparison of the out-of-control ATS_1 values of the VP CV chart and competing charts when $\tau \in \{1.1, 1.2, 1.5, 2.0\}$, $\gamma_0 = 0.05$, $n_0 \in \{5, 7, 10, 15\}$ and $h_S = 0.10$.

| | | $n_0 = 5$ | | | | | | | | | | | | | |
|--------|-----|------------|---------|------------|---------|------------|---------|------------|---------|----------------------|---------|--------------|---------|-------------|--|
| | | VP CV | | VSSICV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | | ATS_1 | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | |
| τ | 1.1 | 84.40 | 96.58 | 14.43 | 103.10 | 22.16 | 98.80 | 17.06 | 51.03 | -39.54 | 115.42 | 36.75 | 159.86 | 89.41 | |
| | 1.2 | 20.77 | 25.16 | 21.14 | 38.90 | 87.29 | 27.20 | 30.96 | 20.38 | -1.88 | 37.67 | 81.37 | 64.69 | 211.46 | |
| | 1.5 | 3.05 | 4.07 | 33.44 | 6.10 | 100.00 | 5.20 | 70.49 | 5.76 | 88.85 | 5.76 | 88.85 | 10.57 | 246.56 | |
| | 2.0 | 0.65 | 1.65 | 153.85 | 1.50 | 130.77 | 2.30 | 253.85 | 2.36 | 263.08 | 1.97 | 203.08 | 2.34 | 260.00 | |
| | | $n_0 = 7$ | | | | | | | | | | | | | |
| | | VP CV | | VSSICV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | | ATS_1 | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | |
| τ | 1.1 | 72.25 | 82.89 | 14.73 | 87.60 | 21.25 | 84.50 | 16.96 | 39.32 | -45.58 | 97.69 | 35.21 | 141.22 | 95.46 | |
| | 1.2 | 15.04 | 18.62 | 23.80 | 29.60 | 96.81 | 20.70 | 37.63 | 15.44 | 2.66 | 27.65 | 83.84 | 50.26 | 234.18 | |
| | 1.5 | 1.82 | 2.89 | 58.79 | 4.00 | 119.78 | 4.00 | 119.78 | 4.26 | 134.07 | 3.95 | 117.03 | 7.21 | 296.15 | |
| | 2.0 | 0.30 | 1.30 | 333.33 | 1.00 | 233.33 | 1.80 | 500.00 | 1.80 | 500.00 | 1.51 | 403.33 | 2.05 | 583.33 | |
| | | $n_0 = 10$ | | | | | | | | | | | | | |
| | | VP CV | | VSSICV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | | ATS_1 | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | |
| τ | 1.1 | 62.15 | 71.19 | 14.55 | 73.50 | 18.26 | 72.40 | 16.49 | 30.09 | -51.58 | 78.87 | 26.90 | 120.27 | 93.52 | |
| | 1.2 | 11.02 | 13.72 | 24.50 | 21.70 | 96.91 | 16.20 | 47.01 | 11.60 | 5.26 | 19.24 | 74.59 | 37.09 | 236.57 | |
| | 1.5 | 1.02 | 2.02 | 98.04 | 2.40 | 135.29 | 3.10 | 203.92 | 3.16 | 209.80 | 2.71 | 165.69 | 4.78 | 368.63 | |
| | 2.0 | 0.11 | 1.11 | 909.09 | 0.60 | 445.45 | 1.50 | 1263.64 | 1.41 | 1181.82 | 1.22 | 1009.09 | 1.52 | 1281.82 | |
| | | $n_0 = 15$ | | | | | | | | | | | | | |
| | | VP CV | | VSSICV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | | ATS_1 | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | ATS_1 | % increase | |
| τ | 1.1 | 52.74 | 59.61 | 13.03 | 59.70 | 13.20 | 62.80 | 19.07 | 22.46 | -57.41 | 58.48 | 10.88 | 95.85 | 81.74 | |
| | 1.2 | 7.74 | 9.71 | 25.45 | 14.80 | 91.21 | 13.20 | 70.54 | 8.43 | 8.91 | 12.24 | 58.14 | 25.03 | 223.39 | |
| | 1.5 | 0.48 | 1.47 | 206.25 | 1.40 | 191.67 | 2.40 | 400.00 | 2.30 | 379.17 | 1.86 | 287.50 | 3.02 | 529.17 | |
| | 2.0 | 0.03 | 1.03 | 3333.33 | 0.40 | 1233.33 | 1.20 | 3900.00 | 1.15 | 3733.33 | 1.07 | 3466.67 | 1.19 | 3866.67 | |

chart's ATS_1 value shows an increase of 14.43% compared to the ATS_1 value of the VP CV chart for $n_0 = 5$ and $\tau = 1.1$. There are some competitor charts with constant sampling interval like the VSS CV chart, the Shewhart CV chart, the Synthetic CV control chart and the EWMA CV^2 chart. Those charts have fixed the sampling interval, $h = 1$ are assumed. From that, the ATS and ARL values are the same since $ATS = h \times ARL$.

From the Table 4.8, when the shift size, $\tau \in \{1.1, 1.2\}$ with $n_0 = 5$ and $\gamma_0 = 0.05$, it is clearly noticed that the best performance is the EWMA CV^2 chart. The second best performance chart is VP CV chart when the shift size, $\tau \in \{1.1, 1.2\}$ with $n_0 = 5$ and $\gamma_0 = 0.05$. However, when the shift size, $\tau \in \{1.5, 2.0\}$ with $n_0 = 5$, the best performance chart is the VP CV chart.

For the $n_0 \in \{7, 10, 15\}$ and $\gamma_0 = 0.05$, the EWMA CV^2 chart showed smallest ATS_1 value compared with other competing charts when $\tau = 1.1$. However, for the shift size, $\tau \in \{1.2, 1.5, 2.0\}$, the VP CV chart has showed best performance.

In general, except for the EWMA CV^2 chart, the VP CV chart shows lower ATS_1 values compared to the other existing CV charts when the smaller shift size is used. Next, the percentage of an increase for the six competing charts is considered high. Note that a positive percentage shows that the VP CV chart shows better performance than the competing chart, while a negative one shows that the competing chart shows better performance than the VP CV chart. The six competitor chart's percentage increase shows an increasing trend for the $\tau \in \{1.1, 1.2, 1.5, 2.0\}$. From that, a conclusion can be made which is the

VP CV chart performs well for the large shift size based on the percentage increase. For instance, for $\tau = 2.0$ and $n_0 = 15$, the percentage increase based on the ATS_1 of the VSSI CV, the VSI CV, the VSS CV, the EWMA CV², the Synthetic CV and the Shewhart CV charts are 3333.33%, 1233.33%, 3900.00%, 3733.33%, 3466.67% and 3866.67% respectively. Thus, the VP CV chart is compared to the other charts as there is a large improvement in term of ATS_1 .

4.2.2 SDTS

Next, the $SDTS_1$ of the VP CV chart is compared with the $SDTS_1$ of the VSSI CV, VSI CV, VSS CV, EWMA CV², Synthetic CV and Shewhart CV charts. Table 4.9 shows the $SDTS_1$ of these charts for $n_0 \in \{5, 7, 10, 15\}$, $\tau \in \{1.1, 1.2, 1.5, 2.0\}$ and $\gamma_0 = 0.05$. A smaller value of $SDTS_1$ indicates less variability in the time to signal. Table 4.9 also shows the percentage of an increase in the $SDTS_1$ value for the six competing charts compared with the $SDTS_1$ value for the VP CV chart. As in Table 4.9, a positive percentage shows that the VP CV chart shows a smaller $SDTS_1$ value, while a negative one shows that the VP CV chart shows a larger $SDTS_1$ value. From Table 4.9, the VP CV chart generally shows smaller values of $SDTS_1$ compared to the VSSI, VSI, VSS, Synthetic and Shewhart CV charts for small and moderate shifts, while the $SDTS_1$ value of the VP CV chart is larger for large shifts. The $SDTS_1$ value of the VP CV chart is generally larger than the $SDTS_1$ value of the EWMA CV² chart in most cases.

Table 4.9: $SDTS_1$ of the VP CV chart and the corresponding $SDTS_1$ of the VSSI CV, VSI CV, VSS CV, EWMA CV², Synthetic CV and Shewhart CV charts, for $n_0 \in \{5, 7, 10, 15\}$, $\tau \in \{1.1, 1.2, 1.5, 2.0\}$ and $\gamma_0 = 0.05$.

| $n_0 = 5$ | | | | | | | | | | | | | | |
|------------|----------|----------|------------|----------|------------|----------|------------|----------|----------------------|----------|--------------|----------|-------------|--|
| τ | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | $SDTS_1$ | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | |
| 1.1 | 85.62 | 96.55 | 12.77 | 102.60 | 19.83 | 97.70 | 14.11 | 41.33 | -51.73 | 151.46 | 76.90 | 159.36 | 86.12 | |
| 1.2 | 21.85 | 25.15 | 15.10 | 38.50 | 76.20 | 25.70 | 17.62 | 13.96 | -36.11 | 48.35 | 121.28 | 64.19 | 193.78 | |
| 1.5 | 3.82 | 3.57 | -6.54 | 5.90 | 54.45 | 3.90 | 2.09 | 3.68 | -3.66 | 6.29 | 64.66 | 10.06 | 163.35 | |
| 2.0 | 1.73 | 1.27 | -26.59 | 1.40 | -19.08 | 1.30 | -24.86 | 1.54 | -10.98 | 1.56 | -9.83 | 2.34 | 35.26 | |
| $n_0 = 7$ | | | | | | | | | | | | | | |
| τ | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | $SDTS_1$ | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | |
| 1.1 | 73.68 | 83.02 | 12.68 | 87.20 | 18.35 | 83.30 | 13.06 | 29.74 | -59.64 | 128.09 | 73.85 | 140.71 | 90.97 | |
| 1.2 | 16.35 | 18.74 | 14.62 | 29.20 | 78.59 | 19.2 | 17.43 | 10.23 | -37.43 | 35.13 | 114.86 | 49.76 | 204.34 | |
| 1.5 | 2.62 | 2.66 | 1.53 | 3.80 | 45.04 | 2.70 | 3.05 | 2.77 | 5.73 | 4.12 | 57.25 | 6.69 | 155.34 | |
| 2.0 | 1.62 | 1.62 | 0.00 | 0.90 | -44.44 | 0.90 | -44.44 | 1.07 | -33.95 | 0.95 | -41.36 | 1.48 | -8.64 | |
| $n_0 = 10$ | | | | | | | | | | | | | | |
| τ | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | $SDTS_1$ | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | |
| 1.1 | 63.56 | 71.49 | 12.48 | 73.10 | 15.01 | 71.00 | 11.71 | 20.93 | -67.07 | 103.22 | 62.40 | 119.77 | 88.44 | |
| 1.2 | 12.21 | 13.98 | 14.50 | 21.40 | 75.27 | 14.50 | 18.76 | 7.44 | -39.07 | 24.11 | 97.46 | 36.59 | 199.67 | |
| 1.5 | 1.84 | 1.67 | -9.24 | 2.30 | 25.00 | 1.80 | -2.17 | 1.98 | 7.61 | 2.54 | 38.04 | 4.25 | 130.98 | |
| 2.0 | 1.39 | 0.42 | -69.78 | 0.60 | -56.83 | 0.70 | -49.64 | 0.72 | -48.20 | 0.56 | -59.71 | 0.89 | -35.97 | |
| $n_0 = 15$ | | | | | | | | | | | | | | |
| τ | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
| | $SDTS_1$ | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | $SDTS_1$ | % increase | |
| 1.1 | 54.29 | 60.00 | 10.52 | 59.40 | 9.41 | 61.40 | 13.10 | 14.75 | -72.83 | 76.27 | 40.49 | 95.35 | 75.63 | |
| 1.2 | 9.12 | 10.06 | 10.31 | 14.50 | 58.99 | 11.60 | 27.19 | 5.20 | -42.98 | 14.95 | 63.93 | 24.53 | 168.97 | |
| 1.5 | 1.49 | 1.74 | 16.78 | 1.30 | -12.75 | 1.40 | -6.04 | 1.37 | -8.05 | 1.47 | -1.34 | 2.47 | 65.77 | |
| 2.0 | 1.30 | 0.20 | -84.62 | 0.50 | -61.54 | 0.40 | -69.23 | 0.42 | -67.69 | 0.29 | -77.69 | 0.48 | -63.08 | |

4.2.3 EATS

Table 4.10 to Table 4.12 show the $EATS_1$ s of the optimal VP CV, the Shewhart CV chart, VSS CV chart, VSI CV chart, VSSI CV chart, Synthetic CV control chart and EWMA CV^2 chart. Table 4.10 presents the different charts $EATS_1$ value for the $(\tau_{\min}, \tau_{\max}) = (1, 2)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$ and $h_S = 0.10$. Table 4.11 presents the different charts $EATS_1$ value for the $(\tau_{\min}, \tau_{\max}) = (1.25, 2)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$ and $h_S = 0.10$. Table 4.12 presents the different charts' $EATS_1$ values for the $(\tau_{\min}, \tau_{\max}) = (1.50, 2)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$ and $h_S = 0.10$. Table 4.10 to Table 4.12 also show the percentage of an increase in the $EATS_1$ value for the six competitor charts compared with the $EATS_1$ value for the VP CV chart. From Table 4.10, the $EATS_1$ value of the VSSI CV chart shows an increase of 12.58% compared to the $EATS_1$ value of the VP CV chart for $n_0 = 5$ and $\gamma_0 = 0.05$. The $EATS_0$ is fixed as 370.4 for all the charts under comparison. As in Table 4.10 to Table 4.12, a positive percentage shows that the VP CV chart shows better performance than the competing chart, while a negative one shows that the competing chart shows better performance than the VP CV chart.

The best performance chart is the EWMA CV^2 chart when the $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$, $h_S = 0.10$ and $(\tau_{\min}, \tau_{\max}) = (1, 2)$. The second best performance chart is the VP CV chart compared with other charts. From the Table 4.11 and Table 4.12, the VP CV chart basically superior over all the charts. From that, noticed that the smaller the gap between the τ_{\min} and τ_{\max} , the VP CV chart performs better than EWMA CV^2 chart. Apart from that, the percentage

Table 4.10: $EATS_1$ values of the VP CV chart and competing charts when $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$, and $h_S = 0.10$.

| n_0 | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
|-------|----------|----------|------------|----------|------------|----------|------------|----------|----------------------|----------|--------------|----------|-------------|--|
| | $EATS_1$ | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | |
| 5 | 19.88 | 22.38 | 12.58 | 24.75 | 24.50 | 23.25 | 16.95 | 15.72 | -20.93 | 27.18 | 36.72 | 38.06 | 91.45 | |
| 7 | 16.83 | 19.24 | 14.32 | 20.33 | 20.80 | 20.32 | 20.74 | 12.52 | -25.61 | 22.76 | 35.23 | 32.12 | 90.85 | |
| 10 | 14.40 | 16.56 | 15.00 | 16.67 | 15.76 | 17.86 | 24.03 | 9.89 | -31.32 | 18.70 | 29.86 | 26.66 | 85.14 | |
| 15 | 12.37 | 14.19 | 14.71 | 13.44 | 8.65 | 15.82 | 27.89 | 7.60 | -38.56 | 14.73 | 19.08 | 21.30 | 72.19 | |

Table 4.11: $EATS_1$ values of the VP CV chart and competing charts when $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$, and $h_S = 0.10$.

| n_0 | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
|-------|----------|----------|------------|----------|------------|----------|------------|----------|----------------------|----------|--------------|----------|-------------|--|
| | $EATS_1$ | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | |
| 5 | 3.41 | 4.63 | 35.78 | 6.28 | 84.16 | 5.28 | 54.84 | 5.53 | 62.17 | 6.16 | 80.65 | 10.82 | 217.30 | |
| 7 | 2.13 | 3.30 | 54.93 | 4.22 | 98.12 | 3.98 | 86.85 | 4.15 | 94.84 | 4.31 | 102.35 | 7.60 | 256.81 | |
| 10 | 1.26 | 2.37 | 88.10 | 2.73 | 116.67 | 3.09 | 145.24 | 3.12 | 147.62 | 2.99 | 137.30 | 5.19 | 311.90 | |
| 15 | 0.65 | 1.70 | 161.54 | 1.61 | 147.69 | 2.44 | 275.38 | 2.30 | 253.85 | 2.05 | 215.38 | 3.36 | 416.92 | |

Table 4.12: $EATS_1$ values of the VP CV chart and competing charts when $(\tau_{\min}, \tau_{\max}) = (1.50, 2.00)$, $n_0 \in \{5, 7, 10, 15\}$, $\gamma_0 = 0.05$, and $h_S = 0.10$.

| n_0 | VP CV | | VSSI CV | | VSI CV | | VSS CV | | EWMA CV ² | | Synthetic CV | | Shewhart CV | |
|-------|----------|----------|------------|----------|------------|----------|------------|----------|----------------------|----------|--------------|----------|-------------|--|
| | $EATS_1$ | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | $EATS_1$ | % increase | |
| 5 | 1.49 | 2.53 | 69.80 | 2.98 | 100.00 | 3.22 | 116.11 | 3.58 | 140.27 | 3.20 | 114.77 | 5.33 | 257.72 | |
| 7 | 0.83 | 1.85 | 122.89 | 1.89 | 127.71 | 2.49 | 200.00 | 2.68 | 222.89 | 2.28 | 174.70 | 3.66 | 340.96 | |
| 10 | 0.41 | 1.43 | 248.78 | 1.15 | 180.49 | 1.97 | 380.49 | 2.03 | 395.12 | 1.68 | 309.76 | 2.51 | 512.20 | |
| 15 | 0.16 | 1.16 | 625.00 | 0.66 | 312.50 | 1.54 | 862.50 | 1.53 | 856.25 | 1.30 | 712.50 | 1.72 | 975.00 | |

increase for six competing charts show an increasing trend except for the $(\tau_{\min}, \tau_{\max}) = (1, 2)$. For example, for the $(\tau_{\min}, \tau_{\max}) = (1.50, 2)$, $\gamma_0 = 0.05$, $n_0 = 15$, the percentage increase for the VSSI CV, the VSI CV, the VSS CV, the EWMA CV², the Synthetic CV and the Shewhart CV chart compared to the VP CV chart are 625.00%, 312.50%, 862.50%, 856.25%, 712.50% and 975.00% respectively. Practitioner is advised to use smaller gaps between the τ_{\min} and τ_{\max} as the VP CV chart shows better result based on the numeric comparison above.

4.3 Illustrative example

By using torque measurement in manufacturing car radios from Khaw, et al. (2016), the VP CV chart based on the case is established. The following steps shown are adopted to demonstrate the proposed chart.

- (1) There are $m = 20$ samples with $n = 5$ sizes in the Phase-I data.
- (2) For each of the samples, the summary statistic \bar{X}_i , S_i and $\hat{\gamma}_i$ are computed. The summary statistics of the Phase I data are listed in Table 4.13 (left side).

- (3) From the analysis of the Phase I data, $\hat{\gamma}_0 = 0.005645$ is obtained by

$$\text{using formula, } \hat{\gamma}_0 = \frac{\sum_{i=1}^{20} \gamma_i}{20} .$$

- (4) By using the Shewhart CV chart, the Phase I data is in-control from the Khaw, et al. (2016). By referring the Khaw, et al. (2016), all the Phase I samples fall between LCL_{CV} and UCL_{CV} . The LCL_{CV} and UCL_{CV} are obtained based on $\alpha_0 = 0.0027$.

- (5) For this industrial example, $n_0 = 5$ and $\hat{\gamma}_0 = 0.005645$ are assumed, the coefficients a , b and c needed to compute the transformed sample coefficient of variation, T_i by using the formula stated in Chapter 3. The coefficients a , b and c are 30.4614, 7.0694 and -0.0083 respectively. The T_i value is computed in Table 4.13 (left side).
- (6) After that, assume the n_0 , h_0 , ATS_0 and τ with 5, 1 hour, 370.5 and 1.1 respectively. The optimal parameter of the VP CV chart, n_s , n_L , h_s , h_L , W_L , W_S , K_L , K_S are 2, 31, 0.1 hour, 1.1038 hour, 1.6339, 1.5334, 2.8500 and 2.2440 respectively for detecting a shift from $\hat{\gamma}_0 = 0.005645$ to $\hat{\gamma}_1 = \hat{\gamma}_0 \times 1.1 = 0.0062095$ by using the Scicoslab optimization program.
- (7) The 20 new samples from the Phase II process are collected. The initial sample size, the initial sampling interval, the initial warning limit and the initial control limit are taken as n_L , h_S , W_S and K_S . The summary statistics \bar{X}_i , S_i and $\hat{\gamma}_i$ are computed. The summary statistics are shown in Table 4.13 (right side).
- (8) For the Phase-II data, the coefficients a , b , and c needs to be computed based on the small sample size and large sample size. For the small sample size is taken, $a_S = 11.1248$, $b_S = 2.2151$ and $c_S = -0.0028$ are used to calculate the T_i . Whereas $a_L = 93.1005$, $b_L = 22.6584$ and $c_L = -0.0108$ are used to compute the T_i for the large sample size is taken. For example, the first sample is large sample size, so $a_L = 93.1005$, $b_L = 22.6584$ and $c_L = -0.0108$ are used to compute the T_i .

In order to compute the T_i statistic, the formula is used as follow:

Table 4.13: Phase-I and Phase-II datasets.

| Phase-I | | | | | | Phase-II | | | | | | | |
|---------|-------------|-------|------------------|----------|----------|----------|-------------|-------|-------|--------|------------------|----------|----------------|
| i | \bar{X}_i | s_i | $\hat{\gamma}_i$ | Z_i^1 | τ_i | i | \bar{X}_i | s_i | n_i | h_i | $\hat{\gamma}_i$ | Z_i^1 | τ_i |
| 1 | 49.83 | 0.46 | 0.0093 | 0.005561 | 1.8732 | 1 | 50.86 | 0.21 | 31 | 0.1 | 0.0041 | 0.005231 | -2.0988 |
| 2 | 50.95 | 0.49 | 0.0096 | 0.005823 | 2.0192 | 2 | 50.18 | 0.148 | 31 | 0.1 | 0.0029 | 0.005083 | -3.9581 |
| 3 | 49.3 | 0.31 | 0.0063 | 0.005853 | 0.5640 | 3 | 50.05 | 0.191 | 31 | 0.1 | 0.0038 | 0.005002 | -2.5771 |
| 4 | 50.19 | 0.26 | 0.0052 | 0.005811 | 0.0153 | 4 | 50.06 | 0.101 | 31 | 0.1 | 0.0020 | 0.004809 | -5.5429 |
| 5 | 49.84 | 0.55 | 0.0110 | 0.006146 | 2.5437 | 5 | 50.21 | 0.321 | 31 | 0.1 | 0.0064 | 0.004911 | 1.0919 |
| 6 | 51.38 | 0.33 | 0.0065 | 0.006167 | 0.6571 | 6 | 50.22 | 0.302 | 2 | 1.1038 | 0.0060 | 0.004982 | 0.6396 |
| 7 | 50.07 | 0.26 | 0.0051 | 0.006098 | -0.0413 | 7 | 50.86 | 0.245 | 2 | 1.1038 | 0.0048 | 0.004972 | 0.3158 |
| 8 | 50.33 | 0.25 | 0.0050 | 0.006025 | -0.1080 | 8 | 50.64 | 0.829 | 2 | 1.1038 | 0.0164 | 0.005708 | 2.3634 |
| 9 | 49.32 | 0.42 | 0.0085 | 0.006186 | 1.5701 | 9 | 50.19 | 0.161 | 31 | 0.1 | 0.0032 | 0.005546 | -3.5375 |
| 10 | 50.03 | 0.28 | 0.0055 | 0.006143 | 0.1793 | 10 | 49.86 | 0.091 | 31 | 0.1 | 0.0018 | 0.005306 | -5.8845 |
| 11 | 50.64 | 0.42 | 0.0082 | 0.006278 | 1.4508 | 11 | 50.39 | 0.404 | 31 | 0.1 | 0.0080 | 0.005481 | 3.1324 |
| 12 | 50.74 | 0.19 | 0.0038 | 0.006115 | -0.7812 | 12 | 50.43 | 0.715 | 31 | 0.1 | 0.0142 | 0.006042 | 9.5365 |
| 13 | 50.02 | 0.15 | 0.0031 | 0.005919 | -1.1955 | 13 | 49.84 | 0.211 | 31 | 0.1 | 0.0042 | 0.005926 | -1.9411 |
| 14 | 50.03 | 0.16 | 0.0031 | 0.005739 | -1.1711 | 14 | 50.12 | 0.173 | 31 | 0.1 | 0.0035 | 0.005766 | -3.1476 |
| 15 | 50.4 | 0.18 | 0.0037 | 0.005604 | -0.8481 | 15 | 51.09 | 0.2 | 31 | 0.1 | 0.0039 | 0.005646 | -2.4255 |
| 16 | 50.51 | 0.16 | 0.0032 | 0.005452 | -1.0916 | 16 | 50.23 | 0.239 | 31 | 0.1 | 0.0048 | 0.005589 | -1.1662 |
| 17 | 50.03 | 0.17 | 0.0034 | 0.005320 | -0.9875 | 17 | 50.43 | 0.213 | 2 | 1.1038 | 0.0042 | 0.005501 | 0.1356 |
| 18 | 49.93 | 0.18 | 0.0035 | 0.005204 | -0.9231 | 18 | 50.34 | 0.513 | 2 | 1.1038 | 0.0102 | 0.005804 | 1.5004 |
| 19 | 50.23 | 0.21 | 0.0042 | 0.005140 | -0.5295 | 19 | 50.02 | 0.417 | 2 | 1.1038 | 0.0083 | 0.005967 | 1.1588 |
| 20 | 50.31 | 0.23 | 0.0047 | 0.005108 | -0.2788 | 20 | 50.43 | 0.362 | 2 | 1.1038 | 0.0072 | 0.006045 | 0.9151 |

Note: $^1 Z_i = (1-\lambda)Z_{i-1} + \lambda\hat{\gamma}_i$ is used for the EWMA CV chart.

$$T_i = a + b \ln(\hat{\gamma} - c)$$

For example, the first T_i statistic obtained by:

$$T_1 = 93.1005 + 22.6584 \ln\left(\frac{0.210}{50.86} - (-0.0108)\right)$$

For the sample 6, this sample is small sample size, so $a_S = 11.1248$, $b_S = 2.2151$ and $c_S = -0.0028$ are used to calculate the T_i .

- (9) As T_i is approximate normal distribution, so we define the fixed and symmetric warning limits

$$\text{UWL} = +W,$$

$$\text{LWL} = -W$$

and fixed and symmetric control limits.

$$\text{UCL} = +K,$$

$$\text{LCL} = -K$$

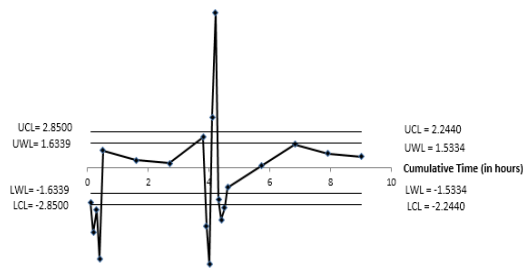
When the T_i falls in the interval $\text{LWL} < T_i < \text{UWL}$, the next small sample size is taken, the sampling interval, the warning limit and the control limit are 1.1038 hour, 1.6339 and 2.8500. When the T_i falls into the warning region, the next large sample size is taken, the sampling interval, the warning limit and the control limit are 0.1 hour, 1.5334 and 2.2440. For example, from the Table 4.13 (right side), the first sample is taken after 0.1 hour. As the sample 1 falls in the warning region of the VP CV chart, the sample 2 is large sample size after the short sampling interval is taken with using small control limit coefficient and small warning limit coefficient. Then as the sample 2 falls in the out-of-control region of the VP CV chart, the process is out-of control. The bold faced values in Table 4.13 are the out-of-control sample detected by the VP CV chart.

For comparison, the VSSI CV chart, the VSI CV chart, the VSS CV chart, the EWMA CV chart, the Synthetic CV chart and the Shewhart CV chart are plotted in Figure 4.1. The reason of the EMWA CV chart is used instead of the EWMA CV² chart for the comparison is because other competitor charts are two-sided control chart and choose EWMA CV chart is for fair comparison purpose.

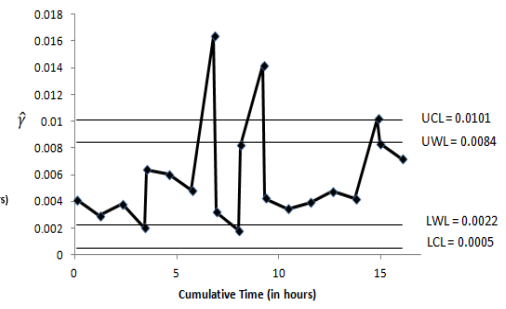
For the VP CV chart (Figure 4.1a), the left control limit coefficient and the warning limit coefficient are used when the next small sample size and the long sampling interval are considered. Whereas, the right control limit coefficient and the warning limit coefficient are used when the next large sample size and the short sampling interval are considered. The VP CV chart plotted detects several out-of control samples, which are at the 2nd, 3rd, 4th, 9th, 10th, 11th, 12th, 14th and 15th samples. The cumulative time for detecting first out-of-control sample is 0.2 hours (see Table 4.13). The process engineer should investigate possible cause(s). To ensure the process back to the in-control process after these out-of-control samples, process modifications are required.

For the VSSI CV chart, the optimal chart's parameters, n_S , n_L , h_S , h_L , W , K are 2, 31, 0.1 hour, 1.1038 hour, 1.6299 and 2.5160 from Scicoslab optimization program. The initial sample size and the initial sampling interval are taken as n_L and h_S . Based on Khaw *et al.* (2016), the $\mu_0(\hat{\gamma}) = 0.0053$ and the $\sigma_0(\hat{\gamma}) = 0.0019$ are substituted into LCL, LWL, UCL and UWL's formula as:

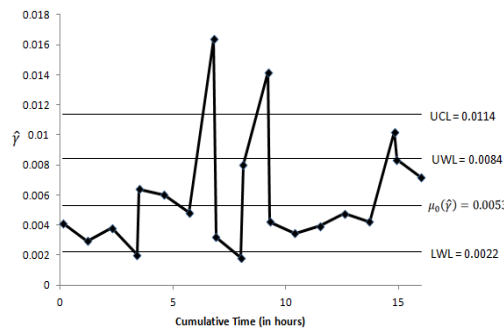
a) VP CV chart



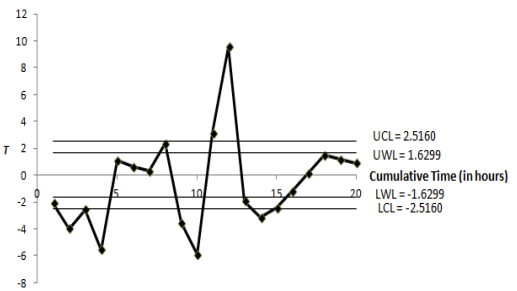
b) VSSI CV chart



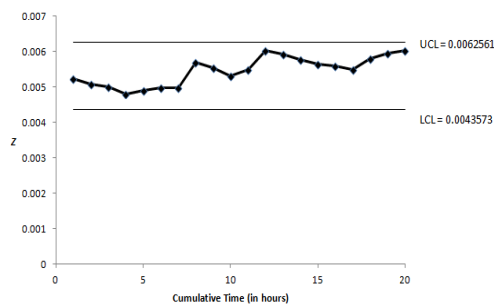
c) VSI CV chart



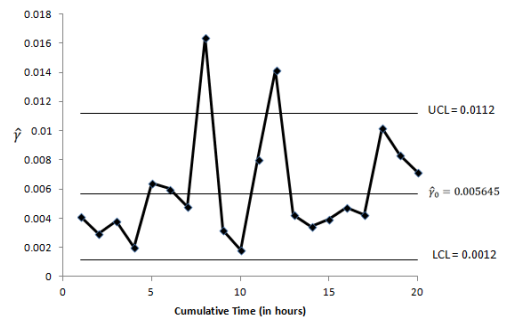
d) VSS CV chart



e) EWMA CV chart



f) Synthetic CV chart



g) Shewhart CV chart

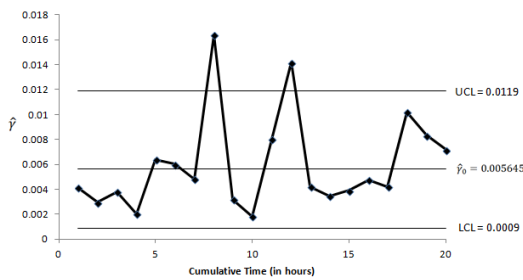


Figure 4.1: (a) VP CV, (b) VSSI CV, (c) VSI CV, (d) VSS CV, (e) EWMA CV, (f) Synthetic CV and (g) Shewhart CV charts are applied to the industrial example (Phase-II with the shift size, $\tau=1.1$).

$$\text{LCL} = 0.0053 - 2.5160(0.0019) = 0.0005$$

$$\text{UCL} = 0.0053 + 2.5160(0.0019) = 0.0101$$

$$\text{LWL} = 0.0053 - 1.6299(0.0019) = 0.0022$$

$$\text{UWL} = 0.0053 + 1.6299(0.0019) = 0.0084$$

From the chart plotted (Figure 4.1b), the VSSI CV chart detects the 8th, 12th and 18th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 6.8231 hours (see Appendix B (i)).

For the VSI CV chart, the short and the long sampling interval are fixed as $(h_s, h_L) = (0.1, 1.1)$. The optimal chart's parameters, W, K are 1.6097, 3.1527 respectively. The LCL, LWL, UCL and UWL are -0.0008, 0.0022, 0.0084 and 0.0114 respectively. The value of LCL will be rounded up to 0 as the CV is a positively defined statistic. From the Figure 4.1(c), the 8th and 12th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 6.8 hours (see Appendix B (ii)).

For the VSS CV chart, the optimal chart's parameters, n_s, n_L, W, K are 2, 31, 1.6299 and 2.5160 from Scicoslab optimization program. From the Figure 4.1(d), the VSS CV chart detects the 2nd, 3rd, 4th, 9th, 10th, 11th, 12th and 14th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 2 hours (see Appendix B (iii)).

For the EWMA CV chart, the optimal chart's parameters, λ and K are 0.0645559 and 2.6980157 respectively. Based on the equation to compute the

UCL and LCL from the Hong, et al. (2008), the UCL is 0.0062561 and the LCL is 0.0043573. From the Figure 4.1(e), the EWMA CV chart fails to detect any out-of-control condition (see Appendix B (iv)).

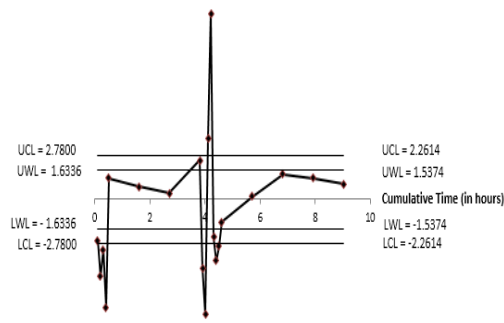
For the Synthetic CV chart, the optimal chart's parameters, L , LCL and UCL are 73, 0.0012 and 0.0112 respectively. From the Figure 4.1(f), the 8th and 12th samples are falling outside the control limit and only treated as non-conforming samples. When the number of samples between two successive non-conforming samples which include the ending non-conforming sample, this is known as the conforming run length, CRL . The Synthetic CV chart only signals an out-of-control when the conforming run length is less than or equal to a threshold L . In order to give the Synthetic CV chart a head start, the sample at time $t = 0$ known as a non-conforming sample. For this case, there are two conforming run length which are $CRL_1 = 8$ and $CRL_2 = 4$. As CRL_1 and CRL_2 are less than $L = 73$, the 8th and 12th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 8 hours (see Appendix B (v)).

For the Shewhart CV chart, the optimal chart's parameters, LCL_{CV} and UCL_{CV} are 0.0009 and 0.0119 respectively. From the Figure 4.1(g), the 8th and 12th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 8 hours (see Appendix B (v)).

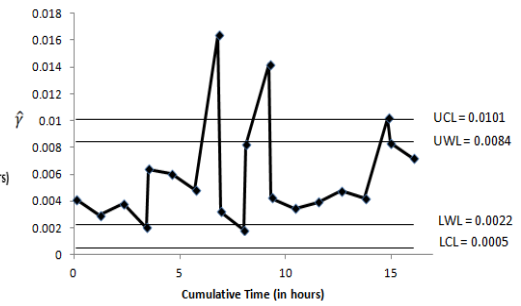
Among seven control charts, the VP CV chart performs better than the competitor charts based on this industrial example. The VP CV chart needs shortest time to detect the out-of-control sample compare to other charts.

From the Figure 4.1, the charts are plotted with the optimal parameters based on objective to minimize the ATS_1 . Anyhow, τ needs to be known in order to obtain these optimal chart's parameters. However, practitioners could not specify the shift size in reality. From that, the Phase II data also monitored based on the optimal parameters that to minimize the $EATS_1$, because the $EATS$ do not need to specify the shift size. The procedures to design the VP CV chart which minimize the $EATS_1$ are similar to the procedure to design the VP CV chart which minimize the ATS_1 . The only exception is in Step 6. The optimal parameters, $n_S, n_L, h_S, h_L, W_L, W_S, K_L, K_S$, to minimize the $EATS_1$ for the $(\tau_{\min}, \tau_{\max}) = (1, 2)$ is computed rather than the optimal parameters, $n_S, n_L, h_S, h_L, W_L, W_S, K_L, K_S$, to minimize the ATS_1 . Based on the formula stated in Chapter 3, the optimal parameters, $n_S, n_L, h_S, h_L, W_L, W_S, K_L, K_S$ of the VP CV chart when the $n_0 = 5$ are 2, 31, 0.1, 1.1038, 1.6336, 1.5374, 2.78 and 2.2614 respectively. For the VP CV chart (Figure 4.2a), the left control limit coefficient and the warning limit coefficient is used when the next small sample size and the long sampling interval is taken, whereas, the right control limit coefficient and the warning limit coefficient is used when the next short sampling interval and the large sample size is taken. The VP CV chart plotted detects several out-of control samples, which are at the 2nd, 3rd, 4th, 9th, 10th, 11th, 12th, 14th and 15th samples. The cumulative time for detecting first out-of-control sample is 0.2 hours (see Table 4.12).

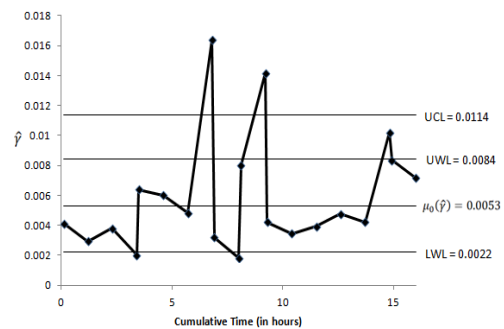
a) VP CV chart



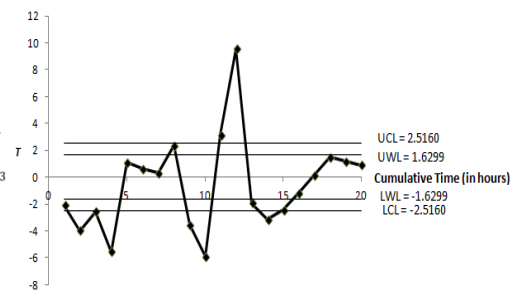
b) VSSI CV chart



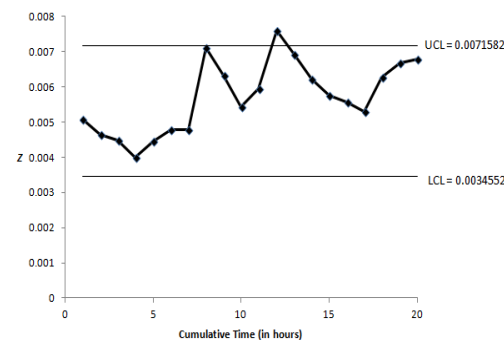
c) VSI CV chart



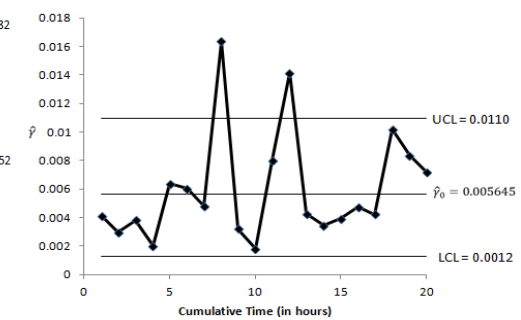
d) VSS CV chart



e) EWMA CV chart



f) Synthetic CV chart



g) Shewhart CV chart

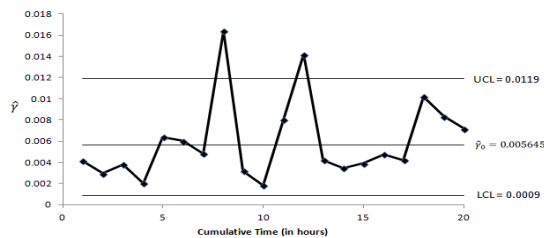


Figure 4.2: (a) VP CV, (b) VSSI CV, (c) VSI CV, (d) VSS CV, (e) EWMA CV, (f) Synthetic CV and (g) Shewhart CV charts are applied to the industrial example (Phase-II with an unknown shift size).

In order to have a fair comparison, the range of shift size, $(\tau_{\min}, \tau_{\max}) = (1, 2)$ is assumed for all the competitor charts. For the VSSI CV chart, the VSI CV chart, the VSS CV chart and the Shewhart CV chart, the optimal chart's parameters to minimize the $EATS_1$ is same with the optimal chart's parameters to minimize the ATS_1 . Thus, the result of the out-of-control samples and the time needed to detect the out-of-control is the same. For instance, the VSSI CV chart detects the 8th, 12th and 18th samples are the out-of-control samples from the chart plotted (Figure 4.2b). The cumulative time for detecting first out-of-control sample is 6.8231 hours (see Appendix B (i)). From the Figure 4.2(c), the VSI CV chart detects the 8th and 12th samples is the out-of-control samples. The VSI CV chart needs take 6.8 hours for detecting first out-of-control sample (see Appendix B (ii)). From the Figure 4.2(d), the VSS CV chart detects the 2nd, 3rd, 4th, 9th, 10th, 11th, 12th and 14th samples are the out-of-control samples. The time required for the VSS CV chart to detect the first out-of-control sample is 2 hours (see Appendix B (iii)). From the Figure 4.2(g), the Shewhart CV chart detect the 8th and 12th samples are the out-of-control samples. The cumulative time for detecting first out-of-control sample is 8 hours (see Appendix B (v)).

For the EWMA CV chart, the optimal chart's parameters, λ and K are 0.2 and 2.882758 respectively to minimize the $EATS_1$. Based on the equation to compute the UCL from the Hong, et al. (2008), the UCL is 0.0071582 and the LCL is 0.0034552. From the Figure 4.2(e), the EWMA CV chart detects the 12th sample are the out-of-control sample. The cumulative time for detecting first out-of-control sample is 12 hours (see Appendix B (vi)).

For the Synthetic CV chart, the optimal chart's parameters, L , LCL and UCL are 40, 0.0012 and 0.0110 respectively. From the Figure 4.2(f), the 8th and 12th samples are falling outside the control limit and only treated as non-conforming samples. For this case, there are two conforming run length which are $CRL_1 = 8$ and $CRL_2 = 4$. As CRL_1 and CRL_2 are less than $L = 40$, the 8th and 12th samples are the out-of-control samples. The Synthetic CV chart takes 8 hours for detecting first out-of-control sample (see Appendix B (v)).

From the Figure 4.1 and Figure 4.2, the VP CV chart outperforms the VSSI CV chart, the VSI CV chart, the VSS CV chart, the EWMA CV chart, the Synthetic CV chart and the Shewhart CV chart in detecting the out-of-control sample. From this industrial example, we can adopt the VP CV chart to monitor the process.

CHAPTER 5

CONCLUSION

5.1 Conclusion

In the context of the Statistical Process Control, monitoring the CV plays an important role in real life. There are many situations where the traditional control chart may not work like the mean is inconstant or variance is the function of the mean. By referring the past reviews, the VP chart showed better performance compare to other charts.

In this project, the VP CV chart is introduced. The VP CV chart varies four parameters which are the sample size, the sampling interval, the warning limits and the control limits. To the best of our knowledge, there is no one using VP strategy to monitor the CV, so the VP CV chart can be known as a valuable contribution to any fields. The T_i statistic is used instead of using the CV for the implementation of the VP CV chart, because the T_i statistic approximately follows a normal distribution. The VP CV chart is demonstrated with one-sided upward chart. The operation of the VP CV chart strategy is demonstrated too. The formula for compute the small control limit coefficient (K_S), the small warning limit coefficient (W_S), large warning limit coefficient (W_L), ATS_1 and the $EATS_1$ values are discussed in this project. To compute the ATS_1 , Markov chain approach is used. Apart from that, the Gauss-Legendre quadrature is used to approximate $EATS_1$ values. The optimization algorithms which minimize the ATS_1 and the $EATS_1$ values subject to constraints are proposed. In order to build the implementation of VP CV chart, the optimal parameters

$(n_S^*, n_L^*, h_S^*, h_L^*, W_S^*, W_L^*, K_S^*, K_L^*)$ are obtained by minimizing ATS_1 and the $EATS_1$ values. All the optimization procedure is written by using the Scicoslab software.

The value of ATS_1 and $SDTS_1$ decrease when the sample size increases. Next, there is a slightly different in term of ATS_1 when the $h_S = 0.01$ and $h_S = 0.10$. Different value of γ_0 have small impact on the ATS_1 and $SDTS_1$ values of the VP CV chart. In terms of the variability of the time to signal, the VP CV chart generally shows smaller variability than the VSSI, VSI, VSS, Synthetic and Shewhart CV charts for small and moderate shifts, while the variability in the time to signal of the VP CV chart is larger than that of the VSSI, VSI, VSS, Synthetic and Shewhart CV charts for large shifts. Compared to the EWMA CV^2 chart, the VP CV chart generally shows larger variability. For the $EATS_1$ value, there is a slightly different in term of $EATS_1$ when the $h_S = 0.01$ and $h_S = 0.10$. By comparing the different charts based on the $EATS_1$, when the shift size is small, the EWMA CV^2 chart performs better than the VP CV chart. For the $EATS_1$ comparison, the VP CV chart shows had the smallest $EATS_1$ value for $(\tau_{\min}, \tau_{\max}) = (1.25, 2.00)$ and $(\tau_{\min}, \tau_{\max}) = (1.50, 2.00)$. However, the EWMA CV^2 chart outperforms the other existing CV charts for $(\tau_{\min}, \tau_{\max}) = (1.00, 2.00)$ and the second better performance chart is VP CV chart. By referring to the result showed in the Table 4.11 and Table 4.12, the VP CV chart performs better than other charts when the different between the shift size is small.

The VP CV chart is then demonstrated through an illustrative example. The VP CV types of charts with the fixed sampling rate (i.e. VSS CV chart, Shewhart

CV chart, EWMA CV chart and Synthetic CV control) are compared to the variable sampling rate types of charts (i.e. VSI CV chart and VSSI CV chart). We found that VP CV chart surpasses all the competitor charts since the VP CV chart consumes the shortest time to detect the out-of-control sample than the competitor charts.

5.2 Recommendations for Future Research

Last but not least, the VP strategy can be extended to other charts to improve the performance like EWMA CV chart. Next, the T_i statistic is recommended rather than use the CV to implement the chart. The T_i statistic can be incorporated with other charts such as VSSI CV, EWMA CV. Apart from that, if the practitioner is willing to vary the sample size and/or sampling interval, it is recommended that the practitioner also varies the warning and control limits, since it does not incur any additional cost to vary these limits. Instead, it results in a faster detection of an out-of-control condition. Hence, faster corrective actions can be taken and this will result in cost savings, as less defective products are produced.

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APPENDICES

APPENDIX A

EQUATION DERIVATION

To show $F_{\hat{\gamma}}(x|n, \gamma) = 1 - F_t\left(\frac{\sqrt{n}}{x} | n-1, \frac{\sqrt{n}}{\gamma}\right)$

$$\hat{\gamma} = \frac{S}{\bar{X}}, \text{ where } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \text{ and } S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

$$\bar{X} = N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$(n-1) \frac{S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$\text{Thus, } \frac{\bar{X} / \frac{\sigma}{\sqrt{n}}}{\sqrt{S^2 / \sigma^2}} = \sqrt{n} \frac{\bar{X}}{S}$$

$= \frac{\sqrt{n}}{\hat{\gamma}}$ will follow a noncentral t distribution with $(n-1)$ degree of freedom and

$$\text{non-centrality parameter } E\left[\frac{\bar{X}}{\sigma/\sqrt{n}}\right] = \frac{\sqrt{n}}{\sigma} E[\bar{X}]$$

$$= \sqrt{n} \frac{\mu}{\sigma}$$

$$= \frac{\sqrt{n}}{\gamma}, \text{ where } \gamma = \frac{\sigma}{\mu}$$

Since $\frac{\sqrt{n}}{\hat{\gamma}} \sim \text{non-central } t\left(n-1, \frac{\sqrt{n}}{\gamma}\right)$,

$$F_{\hat{\gamma}}(x) = P(\hat{\gamma} \leq x)$$

$$= P\left(\frac{1}{\hat{\gamma}} \geq \frac{1}{x}\right)$$

$$= P\left(\frac{\sqrt{n}}{\hat{\gamma}} \geq \frac{\sqrt{n}}{x}\right)$$

$$= 1 - F_t\left(\frac{\sqrt{n}}{x} | n-1, \frac{\sqrt{n}}{\gamma}\right)$$

$$\text{To show } F_{\hat{\gamma}}^{-1}(\alpha | n, \gamma) = \frac{\sqrt{n}}{F_{\hat{\gamma}}^{-1}\left(1 - \alpha | n - 1, \frac{\sqrt{n}}{\gamma}\right)}$$

$$\text{Let } F_{\hat{\gamma}}^{-1}(\alpha) = y$$

$$F_{\hat{\gamma}}(y) = \alpha$$

$$1 - F_t\left(\frac{\sqrt{n}}{y}\right) = \alpha$$

$$F_t\left(\frac{\sqrt{n}}{y}\right) = 1 - \alpha$$

$$\frac{\sqrt{n}}{y} = F_t^{-1}(1 - \alpha)$$

$$\frac{y}{\sqrt{n}} = \frac{1}{F_t^{-1}(1 - \alpha)}$$

$$y = \frac{\sqrt{n}}{F_t^{-1}(1 - \alpha)}$$

$$\text{Thus, } F_{\hat{\gamma}}^{-1}(\alpha | n, \gamma) = \frac{\sqrt{n}}{F_{\hat{\gamma}}^{-1}\left(1 - \alpha | n - 1, \frac{\sqrt{n}}{\gamma}\right)}$$

The derivation to obtain the K_S :

$$P[|T_i| > K_L]p_0 + P[|T_i| > K_S](1-p_0) = P[|T_i| > K_0]$$

and let $P[|T_i| > K_0] = \alpha$,

$$\begin{aligned} P[|T_i| > K_L] &= P[T_i > K_L] + P[T_i < -K_L] \\ &= 1 - P[T_i < K_L] + P[T_i < -K_L] \end{aligned}$$

$$\begin{aligned} &p_0 \left[1 - P \left[\hat{\gamma}_i < \exp \left(\frac{K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] + P \left[\hat{\gamma}_i < \exp \left(\frac{-K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] \right] \\ &+ (1-p_0) \left[1 - P \left[\hat{\gamma}_i < \exp \left(\frac{K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] + P \left[\hat{\gamma}_i < \exp \left(\frac{-K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] \right] \\ &= \alpha \end{aligned}$$

$$\begin{aligned} &p_0 - p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] + p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{-K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] + 1 \\ &- F_{\hat{\gamma}} \left[\exp \left(\frac{K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] + F_{\hat{\gamma}} \left[\exp \left(\frac{-K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] - p_0 \\ &+ p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] - p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{-K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] = \alpha \end{aligned}$$

$$\begin{aligned} &1 - \alpha - p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] + p_0 F_{\hat{\gamma}} \left[\exp \left(\frac{-K_L - a}{b} \right) + c \mid n_S, \gamma_0 \right] + \\ &(p_0 - 1) F_{\hat{\gamma}} \left[\exp \left(\frac{K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] + (1 - p_0) F_{\hat{\gamma}} \left[\exp \left(\frac{-K_S - a}{b} \right) + c \mid n_L, \gamma_0 \right] = 0 \end{aligned}$$

The derivation to obtain the W_S / W_L :

$$\begin{aligned}
p_0 &= p[T_i < W_i \mid T_i < K_i] \\
&= \frac{p[T_i < W_i]}{p[T_i < K_i]} \\
&= \frac{F_{\hat{\gamma}} \left[\exp\left(\frac{W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right]}{F_{\hat{\gamma}} \left[\exp\left(\frac{K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right]}
\end{aligned}$$

$$\begin{aligned}
&F_{\hat{\gamma}} \left[\exp\left(\frac{W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - \\
p_0 &\left[F_{\hat{\gamma}} \left[\exp\left(\frac{K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] \right] = 0
\end{aligned}$$

$$\text{Let } p_0 = \frac{n_0 - n_L}{n_S - n_L},$$

$$\begin{aligned}
&F_{\hat{\gamma}} \left[\exp\left(\frac{W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-W_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - \\
\frac{n_0 - n_L}{n_S - n_L} &\left[F_{\hat{\gamma}} \left[\exp\left(\frac{K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] - F_{\hat{\gamma}} \left[\exp\left(\frac{-K_i - a}{b}\right) + c \mid n_i, \gamma_0 \right] \right] = 0
\end{aligned}$$

APPENDIX B

PHASE-II DATASETS FOR OTHER CHARTS

Table B.i: Phase-II datasets for VSSI CV chart.

| | \bar{X}_i | S_i | n_i | h_i | $\hat{\gamma}_i$ |
|----|-------------|-------|-------|----------|------------------|
| 1 | 50.86 | 0.210 | 31 | 0.1 | 0.004129 |
| 2 | 50.18 | 0.148 | 2 | 1.103846 | 0.002949 |
| 3 | 50.05 | 0.191 | 2 | 1.103846 | 0.003816 |
| 4 | 50.06 | 0.101 | 2 | 1.103846 | 0.002018 |
| 5 | 50.21 | 0.321 | 31 | 0.1 | 0.006393 |
| 6 | 50.22 | 0.302 | 2 | 1.103846 | 0.006014 |
| 7 | 50.86 | 0.245 | 2 | 1.103846 | 0.004817 |
| 8 | 50.64 | 0.829 | 2 | 1.103846 | 0.01637 |
| 9 | 50.19 | 0.161 | 31 | 0.1 | 0.003208 |
| 10 | 49.86 | 0.091 | 2 | 1.103846 | 0.001825 |
| 11 | 50.39 | 0.414 | 31 | 0.1 | 0.008216 |
| 12 | 50.43 | 0.715 | 2 | 1.103846 | 0.014178 |
| 13 | 49.84 | 0.211 | 31 | 0.1 | 0.004234 |
| 14 | 50.12 | 0.173 | 2 | 1.103846 | 0.003452 |
| 15 | 51.09 | 0.200 | 2 | 1.103846 | 0.003915 |
| 16 | 50.23 | 0.239 | 2 | 1.103846 | 0.004758 |
| 17 | 50.43 | 0.213 | 2 | 1.103846 | 0.004224 |
| 18 | 50.34 | 0.513 | 2 | 1.103846 | 0.010191 |
| 19 | 50.02 | 0.417 | 31 | 0.1 | 0.008337 |
| 20 | 50.43 | 0.362 | 2 | 1.103846 | 0.007178 |

Table B.ii: Phase-II datasets for VSI CV chart.

| | \bar{X}_i | S_i | h_i | $\hat{\gamma}_i$ |
|----|-------------|-------|-------|------------------|
| 1 | 50.86 | 0.210 | 0.1 | 0.004129 |
| 2 | 50.18 | 0.148 | 1.1 | 0.002949 |
| 3 | 50.05 | 0.191 | 1.1 | 0.003816 |
| 4 | 50.06 | 0.101 | 1.1 | 0.002018 |
| 5 | 50.21 | 0.321 | 0.1 | 0.006393 |
| 6 | 50.22 | 0.302 | 1.1 | 0.006014 |
| 7 | 50.86 | 0.245 | 1.1 | 0.004817 |
| 8 | 50.64 | 0.829 | 1.1 | 0.016370 |
| 9 | 50.19 | 0.161 | 0.1 | 0.003208 |
| 10 | 49.86 | 0.091 | 1.1 | 0.001825 |
| 11 | 50.39 | 0.414 | 0.1 | 0.008216 |
| 12 | 50.43 | 0.715 | 1.1 | 0.014178 |
| 13 | 49.84 | 0.211 | 0.1 | 0.004234 |
| 14 | 50.12 | 0.173 | 1.1 | 0.003452 |
| 15 | 51.09 | 0.20 | 1.1 | 0.003915 |
| 16 | 50.23 | 0.239 | 1.1 | 0.004758 |
| 17 | 50.43 | 0.213 | 1.1 | 0.004224 |
| 18 | 50.34 | 0.513 | 1.1 | 0.010191 |
| 19 | 50.02 | 0.417 | 0.1 | 0.008337 |
| 20 | 50.43 | 0.362 | 1.1 | 0.007178 |

Table B.iii: Phase-II datasets for VSS CV chart.

| | \bar{X}_i | S_i | n_i | h_i | $\hat{\gamma}_i$ | T_i |
|----|-------------|-------|-------|-------|------------------|------------------|
| 1 | 50.86 | 0.21 | 31 | 1 | 0.004129 | -2.098782 |
| 2 | 50.18 | 0.148 | 31 | 1 | 0.002949 | -3.958081 |
| 3 | 50.05 | 0.191 | 31 | 1 | 0.003816 | -2.577145 |
| 4 | 50.06 | 0.101 | 31 | 1 | 0.002018 | -5.542898 |
| 5 | 50.21 | 0.321 | 31 | 1 | 0.006393 | 1.091942 |
| 6 | 50.22 | 0.302 | 2 | 1 | 0.006014 | 0.639643 |
| 7 | 50.86 | 0.245 | 2 | 1 | 0.004817 | 0.315770 |
| 8 | 50.64 | 0.829 | 2 | 1 | 0.01637 | 2.363416 |
| 9 | 50.19 | 0.161 | 31 | 1 | 0.003208 | -3.537492 |
| 10 | 49.86 | 0.091 | 31 | 1 | 0.001825 | -5.884532 |
| 11 | 50.39 | 0.414 | 31 | 1 | 0.008216 | 3.132411 |
| 12 | 50.43 | 0.715 | 31 | 1 | 0.014178 | 9.536489 |
| 13 | 49.84 | 0.211 | 31 | 1 | 0.004234 | -1.941095 |
| 14 | 50.12 | 0.173 | 31 | 1 | 0.003452 | -3.147574 |
| 15 | 51.09 | 0.2 | 31 | 1 | 0.003915 | -2.425452 |
| 16 | 50.23 | 0.239 | 31 | 1 | 0.004758 | -1.166192 |
| 17 | 50.43 | 0.213 | 2 | 1 | 0.004224 | 0.135646 |
| 18 | 50.34 | 0.513 | 2 | 1 | 0.010191 | 1.500445 |
| 19 | 50.02 | 0.417 | 2 | 1 | 0.008337 | 1.158825 |
| 20 | 50.43 | 0.362 | 2 | 1 | 0.007178 | 0.915114 |

Table B.iv: Phase-II datasets with the shift size, $\tau = 1.1$ for EWMA CV chart.

| | \bar{X}_i | S_i | h_i | $\hat{\gamma}_i$ | Z_i |
|----|-------------|-------|-------|------------------|----------|
| 1 | 50.86 | 0.210 | 1 | 0.0041 | 0.005231 |
| 2 | 50.18 | 0.148 | 1 | 0.0029 | 0.005083 |
| 3 | 50.05 | 0.191 | 1 | 0.0038 | 0.005002 |
| 4 | 50.06 | 0.101 | 1 | 0.0020 | 0.004809 |
| 5 | 50.21 | 0.321 | 1 | 0.0064 | 0.004911 |
| 6 | 50.22 | 0.302 | 1 | 0.0060 | 0.004982 |
| 7 | 50.86 | 0.245 | 1 | 0.0048 | 0.004972 |
| 8 | 50.64 | 0.829 | 1 | 0.0164 | 0.005708 |
| 9 | 50.19 | 0.161 | 1 | 0.0032 | 0.005546 |
| 10 | 49.86 | 0.091 | 1 | 0.0018 | 0.005306 |
| 11 | 50.39 | 0.404 | 1 | 0.0080 | 0.005481 |
| 12 | 50.43 | 0.715 | 1 | 0.0142 | 0.006042 |
| 13 | 49.84 | 0.211 | 1 | 0.0042 | 0.005926 |
| 14 | 50.12 | 0.173 | 1 | 0.0035 | 0.005766 |
| 15 | 51.09 | 0.200 | 1 | 0.0039 | 0.005646 |
| 16 | 50.23 | 0.239 | 1 | 0.0048 | 0.005589 |
| 17 | 50.43 | 0.213 | 1 | 0.0042 | 0.005501 |
| 18 | 50.34 | 0.513 | 1 | 0.0102 | 0.005804 |
| 19 | 50.02 | 0.417 | 1 | 0.0083 | 0.005967 |
| 20 | 50.43 | 0.362 | 1 | 0.0072 | 0.006045 |

Table B.v: Phase-II datasets for Synthetic CV chart and Shewhart chart.

| | \bar{X}_i | S_i | h_i | $\hat{\gamma}_i$ |
|----|-------------|-------|-------|------------------|
| 1 | 50.86 | 0.210 | 1 | 0.004129 |
| 2 | 50.18 | 0.148 | 1 | 0.002949 |
| 3 | 50.05 | 0.191 | 1 | 0.003816 |
| 4 | 50.06 | 0.101 | 1 | 0.002018 |
| 5 | 50.21 | 0.321 | 1 | 0.006393 |
| 6 | 50.22 | 0.302 | 1 | 0.006014 |
| 7 | 50.86 | 0.245 | 1 | 0.004817 |
| 8 | 50.64 | 0.829 | 1 | 0.016370 |
| 9 | 50.19 | 0.161 | 1 | 0.003208 |
| 10 | 49.86 | 0.091 | 1 | 0.001825 |
| 11 | 50.39 | 0.414 | 1 | 0.008216 |
| 12 | 50.43 | 0.715 | 1 | 0.014178 |
| 13 | 49.84 | 0.211 | 1 | 0.004234 |
| 14 | 50.12 | 0.173 | 1 | 0.003452 |
| 15 | 51.09 | 0.20 | 1 | 0.003915 |
| 16 | 50.23 | 0.239 | 1 | 0.004758 |
| 17 | 50.43 | 0.213 | 1 | 0.004224 |
| 18 | 50.34 | 0.513 | 1 | 0.010191 |
| 19 | 50.02 | 0.417 | 1 | 0.008337 |
| 20 | 50.43 | 0.362 | 1 | 0.007178 |

Table B.vi: Phase-II datasets with unknown shift size for EWMA CV chart.

| | \bar{X}_i | S_i | h_i | $\hat{\gamma}_i$ | Z_i |
|----|-------------|-------|-------|------------------|-----------------|
| 1 | 50.86 | 0.210 | 1 | 0.0041 | 0.005071 |
| 2 | 50.18 | 0.148 | 1 | 0.0029 | 0.004647 |
| 3 | 50.05 | 0.191 | 1 | 0.0038 | 0.004481 |
| 4 | 50.06 | 0.101 | 1 | 0.0020 | 0.003988 |
| 5 | 50.21 | 0.321 | 1 | 0.0064 | 0.004469 |
| 6 | 50.22 | 0.302 | 1 | 0.0060 | 0.004778 |
| 7 | 50.86 | 0.245 | 1 | 0.0048 | 0.004786 |
| 8 | 50.64 | 0.829 | 1 | 0.0164 | 0.007103 |
| 9 | 50.19 | 0.161 | 1 | 0.0032 | 0.006324 |
| 10 | 49.86 | 0.091 | 1 | 0.0018 | 0.005424 |
| 11 | 50.39 | 0.404 | 1 | 0.0080 | 0.005943 |
| 12 | 50.43 | 0.715 | 1 | 0.0142 | 0.007590 |
| 13 | 49.84 | 0.211 | 1 | 0.0042 | 0.006919 |
| 14 | 50.12 | 0.173 | 1 | 0.0035 | 0.006225 |
| 15 | 51.09 | 0.200 | 1 | 0.0039 | 0.005763 |
| 16 | 50.23 | 0.239 | 1 | 0.0048 | 0.005562 |
| 17 | 50.43 | 0.213 | 1 | 0.0042 | 0.005294 |
| 18 | 50.34 | 0.513 | 1 | 0.0102 | 0.006274 |
| 19 | 50.02 | 0.417 | 1 | 0.0083 | 0.006686 |
| 20 | 50.43 | 0.362 | 1 | 0.0072 | 0.006785 |

APPENDIX C

PROGRAM FOR VARIABLE PARAMETERS CHART

This program is used to compute the optimal chart parameters for the VP CV chart.

```
//-----  
function [a,b,c]=standard(n,cv0)  
//-----  
r=0.05  
x1=idfcv(r,n,cv0)//xr  
x2=idfcv(0.5,n,cv0)//xmedian  
x3=idfcv(1-r,n,cv0)//x1-r  
b=idfnormal(r)/log((x2-x1)/(x3-x2))  
a=-b*log((x2-x1)/(1-exp(idfnormal(r)/b)))  
c=x2-exp(-a/b)  
endfunction  
  
//-----  
function [Q,q]=qvpcv(nS,nL,WnS,WnL,KnS,KnL,delta,cv0)  
//-----  
cv1=delta*cv0  
[a1,b3,c1]=standard(nS,cv0)  
[a2,b4,c2]=standard(nL,cv0)  
pSnS1=cdfcv(exp((WnS-a1)/b3)+c1,nS,cv1)-cdfcv(exp((-WnS-  
a1)/b3)+c1,nS,cv1)  
pSnL1=cdfcv(exp((WnL-a2)/b4)+c2,nL,cv1)-cdfcv(exp((-WnL-  
a2)/b4)+c2,nL,cv1)  
pLnS1=cdfcv(exp((KnS-a1)/b3)+c1,nS,cv1)-cdfcv(exp((WnS-  
a1)/b3)+c1,nS,cv1)+cdfcv(exp((-WnS-a1)/b3)+c1,nS,cv1)-cdfcv(exp((-KnS-  
a1)/b3)+c1,nS,cv1)
```

```

pLnL1=cdfcv(exp((KnL-a2)/b4)+c2,nL,cv1)-cdfcv(exp((WnL-
a2)/b4)+c2,nL,cv1)+cdfcv(exp((-WnL-a2)/b4)+c2,nL,cv1)-cdfcv(exp((-KnL-
a2)/b4)+c2,nL,cv1)
Q=[pSnS1,pLnS1;pSnL1,pLnL1]
pSnS=cdfcv(exp((WnS-a1)/b3)+c1,nS,cv0)-cdfcv(exp((-WnS-
a1)/b3)+c1,nS,cv0)
pK=cdfcv(exp((KnS-a1)/b3)+c1,nS,cv0)-cdfcv(exp((-KnS-a1)/b3)+c1,nS,cv0)
b1=pSnS/pK
b2=1-b1
q=[b1;b2]
endfunction

//-----
function [zt1,e1,e2]=randomfunc(Q,q,tS,tL)
//-----

[argout,argin]=argn()
if argin~=4
    error("incorrect number of arguments")
end
q=q(:)'
W=inv(eye(Q)-Q)
z1=q*W
t=[tL;tS]
zt1=z1*t
e1=z1*t-q*t//modified
Dd=[tL,0;0,tS]
a1=W*t
a2=2*Dd*a1
a3=a2-(t.^2)
e2=q*(W*a3)
endfunction

```

```

//-----
function
[ATS,SDTS,ASS,ASI]=rlvpcv(nS,nL,tS,tL,WnS,WnL,KnS,KnL,delta,cv0)
//-----

[Q,q]=qvpcv(nS,nL,WnS,WnL,KnS,KnL,delta,cv0)
    q1=q(1,:)
    q2=q(2,:)
    ASS=nS*q1+nL*q2
    ASI=tL*q1+tS*q2

    [zt1,e1,e2]=randomfunc(Q,q,tS,tL)//modified
    ATS=e1
    SDTS=sqrt(e2-zt1^2)//modified
endfunction

//-----
function tL=long_int(n,nS,nL,tS,t)
//-----

tL=(t*(nL-nS)-tS*(n-nS))/(nL-n)
endfunction

//-----
function dif=optKnL(KnL,n,nS,nL,KnS,cv0)
//-----

if (KnL<=0)|(KnS<=0)|(KnL>KnS)
    dif=%inf
else
    [a0,b0,c0]=standard(n,cv0)
    [a1,b1,c1]=standard(nS,cv0)
    [a2,b2,c2]=standard(nL,cv0)
    p0=(n-nL)/(nS-nL)

```

```

alpha=0.0027
dif=1-alpha-p0*cdfcv(exp((KnS-a1)/b1)+c1,nS,cv0)+p0*cdfcv(exp((-KnS-
a1)/b1)+c1,nS,cv0)+(p0-1)*cdfcv(exp((KnL-a2)/b2)+c2,nL,cv0)+(1-
p0)*cdfcv(exp((-KnL-a2)/b2)+c2,nL,cv0)
end
endfunction

//-----
function dif=optWnS(WnS,n,nS,nL,KnS,cv0)
//-----

if (WnS<=0)|(KnS<=0)|(WnS>KnS)
    dif=%inf
else
    [a,b,c]=standard(nS,cv0)
    p0=(n-nL)/(nS-nL)
    dif=cdfcv(exp((WnS-a)/b)+c,nS,cv0)-cdfcv(exp((-WnS-a)/b)+c,nS,cv0)-
p0*(cdfcv(exp((KnS-a)/b)+c,nS,cv0)-cdfcv(exp((-KnS-a)/b)+c,nS,cv0))
end
endfunction

//-----
function dif=optWnL(WnL,n,nS,nL,KnL,cv0)
//-----

if (WnL<=0)|(KnL<=0)|(WnL>KnL)
    dif=%inf
else
    [a,b,c]=standard(nL,cv0)
    p0=(n-nL)/(nS-nL)
    dif=cdfcv(exp((WnL-a)/b)+c,nL,cv0)-cdfcv(exp((-WnL-a)/b)+c,nL,cv0)-
p0*(cdfcv(exp((KnL-a)/b)+c,nL,cv0)-cdfcv(exp((-KnL-a)/b)+c,nL,cv0))
end
endfunction

```

```

//-----
function [KnL,WnS,WnL]=limit(n,nS,nL,KnS,cv0)
//-----

K10=1;
W10=0.5;
W20=0.5;
KnL=simplexolve(K10,optKnL,list(n,nS,nL,KnS,cv0),tol=1e-6)
WnS=simplexolve(W10,optWnS,list(n,nS,nL,KnS,cv0),tol=1e-6)
WnL=simplexolve(W20,optWnL,list(n,nS,nL,KnL,cv0),tol=1e-6)
endfunction

//-----
function
[nS,nL,tS,tL,WnS,WnL,KnS,KnL,ASS,ASI,ATS,SDTS]=optvpcv(n,delta,cv0,i
tS,t)
//-----
ATS=%inf
for inS=2:n-1
    for inL=n+1:31
        itL=long_int(n,inS,inL,itS,t)
        for KnSa=288:309
            iKnS=KnSa*0.01
            [KnL1,WnS1,WnL1]=limit(n,inS,inL,iKnS,cv0)

[ATS1,SDTS1,ASS1,ASI1]=rlvpcv(inS,inL,itS,itL,WnS1,WnL1,iKnS,KnL1,d
elta,cv0)
            if (ATS1<=ATS)
                ASS=ASS1
                ASI=ASI1
                ATS=ATS1
                SDTS=SDTS1
                KnS=iKnS
                KnL=KnL1

```



```

WnS=WnS1
WnL=WnL1
nS=inS
nL=inL
tS=itS
tL=itL
end
end
end
endfunction

//-----
function EATS=eatsvpcv(n,n1,n2,h1,h2,W1,W2,K1,K2,cv0)
//-----

[xi,wi]=quadlegendre(15,1.25,2.0)
EATS=0
for il=1:15
    xil=xi(il)
    wil=wi(il)
    cv1=xil*cv0
    ATS=rlvpcv(n1,n2,h1,h2,W1,W2,K1,K2,cv1/cv0,cv0)
    EATS=EATS+ATS.*wil
end

EATS=EATS/(2-1.25)
endfunction

//-----
function [nS,nL,tS,tL,WnS,WnL,KnS,KnL,EATS]=opteatsvpcv(n,cv0,itS,t)
//-----

EATS=%inf
for inS=2:n-1

```

```

for inL=n+1:31
    itL=long_int(n,inS,inL,itS,t)
    for KnSa=321:339
        iKnS=KnSa*0.01
        [KnL1,WnS1,WnL1]=limit(n,inS,inL,iKnS,cv0)
        EATS1=eatsvpcv(n, inS, inL,itS,itL,WnS1,WnL1,iKnS,KnL1,cv0)
        if (EATS1<=EATS)
            EATS=EATS1
            KnS=iKnS
            KnL=KnL1
            WnS=WnS1
            WnL=WnL1
            nS=inS
            nL=inL
            tS=itS
            tL=itL
        end
    end
end
end
end
endfunction

//-----
function dif=optK(K,n,cv0)
//-----

[a,b,c]=standard(n,cv0)
alpha=0.0027
dif=1-alpha-cdfcv(exp((K-a)/b)+c,n,cv0)+cdfcv(exp((-K-a)/b)+c,n,cv0)
endfunction

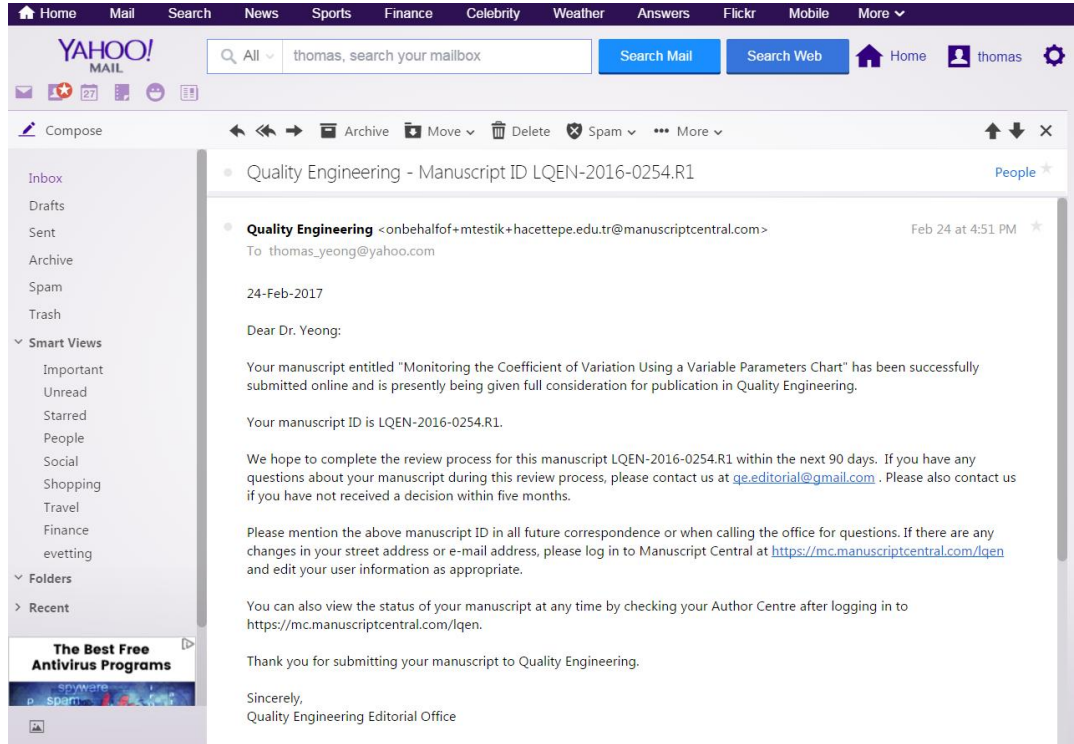
```

```
//-----  
function K=limit2(n,cv0)  
//-----  
  
K0=1;  
K=simplexolve(K0,optK,list(n,cv0),tol=1e-6)  
endfunction
```

APPENDIX D

ACCEPTANCE EMAIL FOR PUBLICATION IN QUALITY

ENGINEERING



APPENDIX E

TURNITIN REPORT

Turnitin Originality Report

Monitoring the Coefficient of Variation through a Variable Parameters Chart
Yang Lim

by Chuen



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