RECURRENCE ANALYSIS OF TIME SERIES

By

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A project report submitted in partial fulfilment of the
requirements for the award of Bachelor of Science (Hons.)
Applied Mathematics With Computing

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DECLARATION OF ORIGINALITY

I hereby declare that this project report entitled "**RECURRENCE ANALYSIS OF TIME SERIES**" is my own work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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CHEW KAI YE

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ABSTRACT

Recurrence plot has gradually become a popular and useful tool to analyse data. It allows the visualization of structures in a time series. Besides, it also provides quantification analysis of a time series. Through these, the nonlinearity or deterministic properties of a dynamical system can be determined by its recurrent behaviours.

In this project, the recurrence analysis technique is applied to analyse five sets of electroencephalographic (EEG) time series data of healthy people and epilepsy patients. These data are obtained from University Hospital of Bonn. The EEG data are collected either from electrodes placed on the cortex of the brain or implanted electrodes inside the brain. The analysis methods performed to the EEG data include single, cross and multi-dimensional recurrence plots as well as recurrence quantifications. In different types of methods, the comparisons on recurrence of time series involved are also different. Matlab CRP Toolbox is the tool used for all the plottings and calculations. The patterns inside each recurrence plot and the quantification values acquired can convert to certain meanings to the time series observed.

After the analysis, some conclusions are drawn based on how to distinguish EEG data of normal people and epilepsy patients. A patients' EEG may appear to be periodic in recurrence plot whereas a norm may contain randomness. Most of the recurrence quantification measures may have a greater value on EEG time series of epileptic patients than healthy people. Based on the conclusions, epileptic seizures prediction on a newly received EEG data can be done. The recurrence analysis technique may also be applied to some other applications such as the human-machine interface (HMI).

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CHAPTER 1: INTRODUCTION

1-1 Introduction

1-1-1 Time Series

Time series is a sequence of discrete-time data points which is listed orderly in time. It is often taken at successive equally spaced points. The common examples include the annual Malaysia population data, daily closing stock prices, sales figures, to name a few. A line chart is well-known to be used and represents a time series plotting.

1-1-2 Recurrence and Recurrence Plot

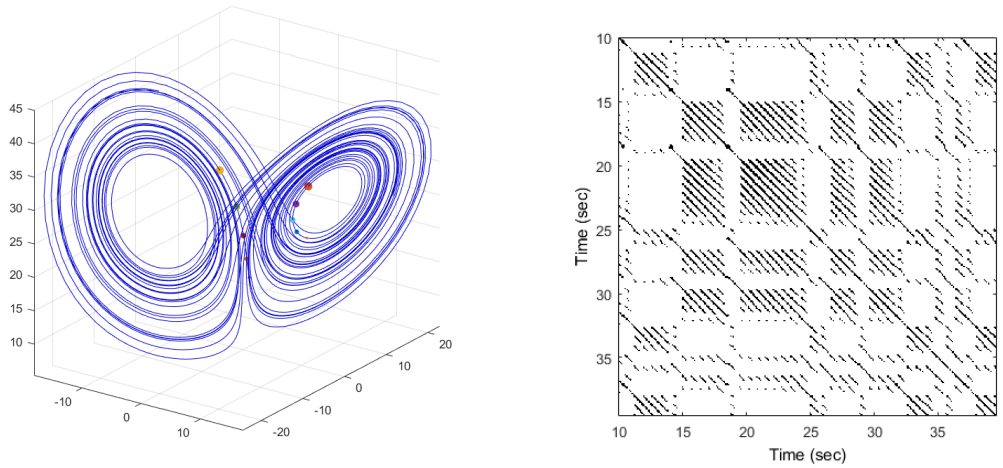
In the real world, there are many distinct recurrent behaviours occur in natural and ordinary processes, for instance music resonance, human heartbeat rates, neurotransmission rates and so on. The recurrence of states in particular means that states are randomly close to the states happened at a certain previous time. In dynamical systems, recurrence is one of the deterministic properties to exhibit nonlinear behaviours or chaotic behaviours. The recurrence of position in a time series means that a given position is randomly similar to another position in another time.

Usually, the recurrence of certain state $\overrightarrow{x_i}$ of a dynamical system in an m-dimensional phase space can be visualised from a recurrence plot (RP), which is introduced by Eckmann et al. (1987). The use of a recurrence plot is to give a more visually perceptible graph for a user to look into high-dimensional dynamical systems. Its ability is to turn recurrences observed in high-dimensional phase space trajectory into two-dimensional representation (Eckmann et al. 1987, Marwan 2008). Recurrence plot is a graph which represents a binary symmetric square matrix where both columns and rows represent the time which a state occurs. In the matrix, the value of each element matches to certain pair of times to indicate whether the states recur, i.e. the value 1 means the state recurs whereas the value 0 means the opposite. On the other hand, the recurrence plot is an $N \times N$ matrix consisted only black and white dots with the features that a black dot depicts a recurrence, along with two time-axes. A recurrence plot is mathematically expressed as the equation below:

$$R_{i,j} = \Theta(\varepsilon_i - \parallel \overrightarrow{x}_i - \overrightarrow{x}_j \parallel), \qquad \overrightarrow{x_i} \epsilon \mathbb{R}^m \qquad i,j = 1,\ldots,N \tag{1.1}$$

where the states x_i considered have an amount of N; ε_i is a threshold distance value (neighbourhood); $\parallel \cdot \parallel$ is a norm; $\Theta(\cdot)$ is the Heaviside function. When the distance between two states i.e. x_i and x_j is smaller than the threshold value ε, a recurrence is

defined.



(a) (b)

Figure 1.1: An example of recurrence plot
(a) phase space trajectory of Lorenz system with parameter r=28, σ=10, b=$\frac{8}{3}$. (b) The corresponding recurrence plot of Lorenz system with threshold value $\varepsilon = 5$.

1-2 Objective

The main objective of this project is to use the recurrence plot tools and apply them to the analysis of real life data sets of time series. The tools provide visualization for the recurrent behaviour of time series and able to quantify them. Thereby, I would like to determine the type of recurrence plots as well as computationally analyse the data by using different measures. Furthermore, discuss the outputs obtained in order to make some conclusions. To achieve the objective, some stages to do are stated as follow:

- To understand the concept of recurrence plot and time series.
- To learn how to use the “CRP toolbox” inside MATLAB application and acquire desired outputs.
- To visually analyse the plotted graphs and perform recurrence quantification analysis.
- To apply recurrence analysis technique on electroencephalogram (EEG) data and discuss the presented outputs.

1-3 Project Scope

Recurrence plot is a nonlinear time series analysing method which is applicable to various types of nonlinear dynamical systems in the real world (Marwan et al. 2007,

Webber & Marwan 2015). Based on recurrence analysis technique on a data set consisting time series, states of nonlinearity can be determined and further by interpreting some quantification measures of a given data, the state of occurrence can be determined in the future time. In this project, the electroencephalogram (EEG) time series data is used. Analysing EEG data through this recurrence analysis technique is the main scope of this subject. By plotting recurrence plot and computing various recurrence quantification measures on human subjects EEG time series, the EEG recording patterns of patients with certain brain diseases (e.g. brain tumour, stroke, epileptic seizures etc.) may be identified. The brain disease concerned in this project is epileptic seizures.

1-4 Methodology and Project Planning

A list of five EEG time series data sets (A-E) is obtained from the website of University Hospital of Bonn. Each data set contains 100 text files and each text file is a recording of single channel EEG segment with time length 23.6 seconds. One text file consists of N=4096 samples of one EEG time series. Sets A and B are surface EEG recordings collected from five healthy volunteers who relaxingly remained awake with opening and closing their eyes respectively. Set C is intracranial EEG recordings from five patients diagnosed for suffering epilepsy during seizure free intervals from outside the seizure generating area whereas set D is from within the seizure generating area. Set E only comprised intracranial EEG recordings of epileptic seizures activity from epilepsy patients (Andrzejak et al. 2001). The electrode placement scheme that consistently used in EEG recordings of sets A and B is illustrated as Fig(1.2). The surgically implanted intracranial electrodes used to record EEG of set C, D and E are shown in Fig(1.3).

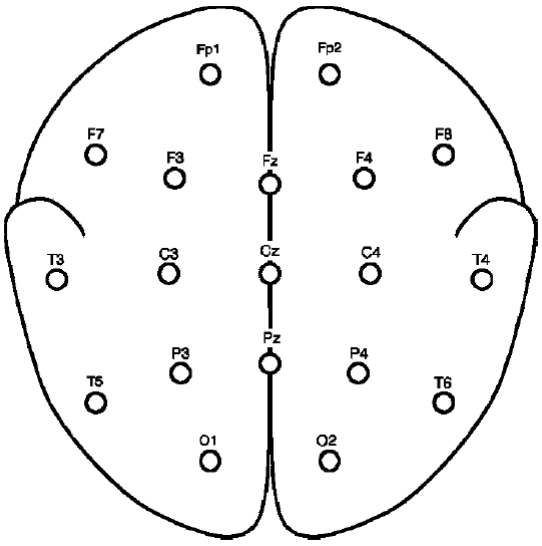


Figure 1.2: Electrode placement scheme of surface EEG

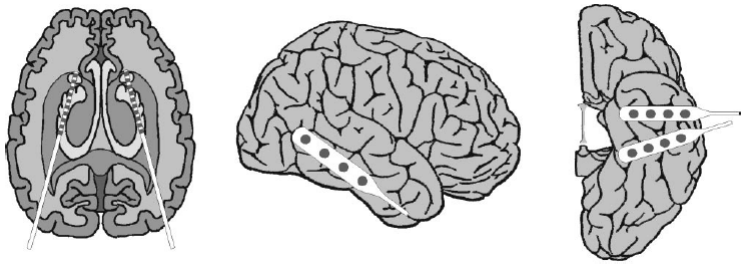
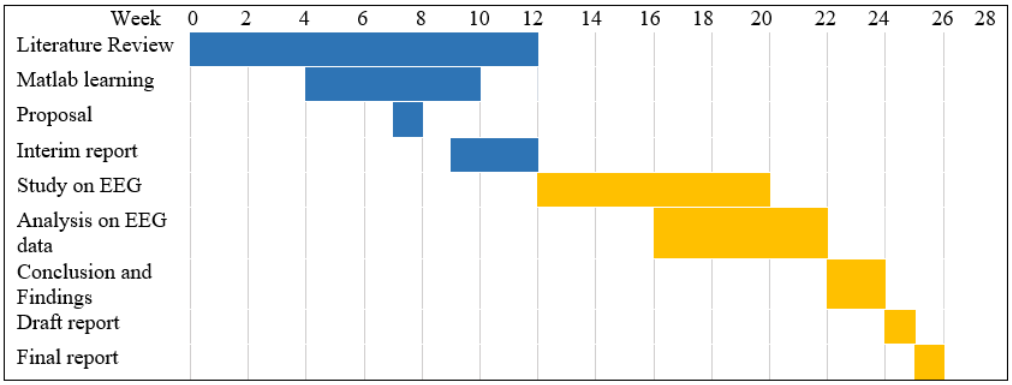


Figure 1.3: Implanted electrodes for intracranial EEG

Several types of recurrence plots are plotted by randomly selecting some time series recordings from EEG set to get the visualization of recurrent behaviours of selected data series. In EEG test, epilepsy may be diagnosed if there is an appearance of certain patterns in the recurrence plot. Next, recurrence quantification analysis (RQA) is implemented to compute the recurrences of EEG data in some measures, hence analyse the outputs attained and sum up a conclusion about the EEG time series recurrent patterns of healthy people and patients. The variety of RQA measures include recurrence rate, determinism, laminarity, trapping time, entropy and others.

In this project, the software application used to plot the RP and calculate all the measures in RQA is Matlab. The "CRP Toolbox 5.5" by Norbert Marwan is an extended feature of Matlab which can be downloaded from website http://tocsy.pik-potsdam.de/CRPtoolbox/. It possesses functions such as to generate cross recurrence plot and joint recurrence plot as well as calculate recurrence quantification.

Project Planning



CHAPTER 2: LITERATURE REVIEW

2-1 Phase Space Trajectory

A phase space is a multidimensional space consisting of all possible states of a particular system (Nolte 2010). Every single state is one-to-one compatible with another special point in the phase space. A system with k state variables

$$x_1(t), x_2(t), \ldots, x_k(t)$$

at time t can form a unit of vector $x(t)$ in a phase space of k-dimensional. Linking the successive phase space vectors together will result in a phase space trajectory. The dynamics of a system can be revealed from the trajectory after a period of time evolution (Webber & Marwan 2015).

Phase space trajectory of a system is usually constructed at the beginning of many approaches used in nonlinear data analysis. The shape of the trajectory depicts some clue and information about the system, e.g. the phase space trajectories of periodic or chaotic system have certain characteristics (Webber & Marwan 2015). A deterministic dynamical system can eventually lead to a possibility to make forecasting to the upcoming states of the system.

2-2 Typical Dynamical System Examples of Recurrence

2-2-1 Lorenz System

In 1960s, Edward N. Lorenz invented a simple weather model in which small changes at the starting conditions brought about apparent changes in the outcome. It also can be called the butterfly effect. This indicates the impossibility to predict a long range of inaccurate measured system. The notable Lorenz system is a three ordinary differential equations system as follows (Peitgen et al. 2004):

$$\begin{aligned} \dot{x_1} &= -\sigma(x_1 - x_2) \\ \dot{x_2} &= -x_1 x_3 + r x_1 - x_2 \\ \dot{x_3} &= x_1 x_2 - b x_3 \end{aligned} \tag{2.1}$$

This system is associated with the Rayleigh-Bernard convection under rough approximations. The variable x_1 is proportional to the velocity of circulatory fluid parti-

cle; x_2 and x_3 are related to the temperature profile; σ, b, r are the physical parameters of Lorenz system. In Lorenz system (Eq2.1) , $\sigma = 10$, $b = \frac{8}{3}$, $r = 28$ is fixed (Lorenz 1963). With the revolution of today's science, there is abundant literature on chaotic properties of Lorenz system. The meaning of chaos was summarized by Ed Lorenz (Danforth 2013):

> "When the present determines the future, but the approximate present does not approximately determine the future."

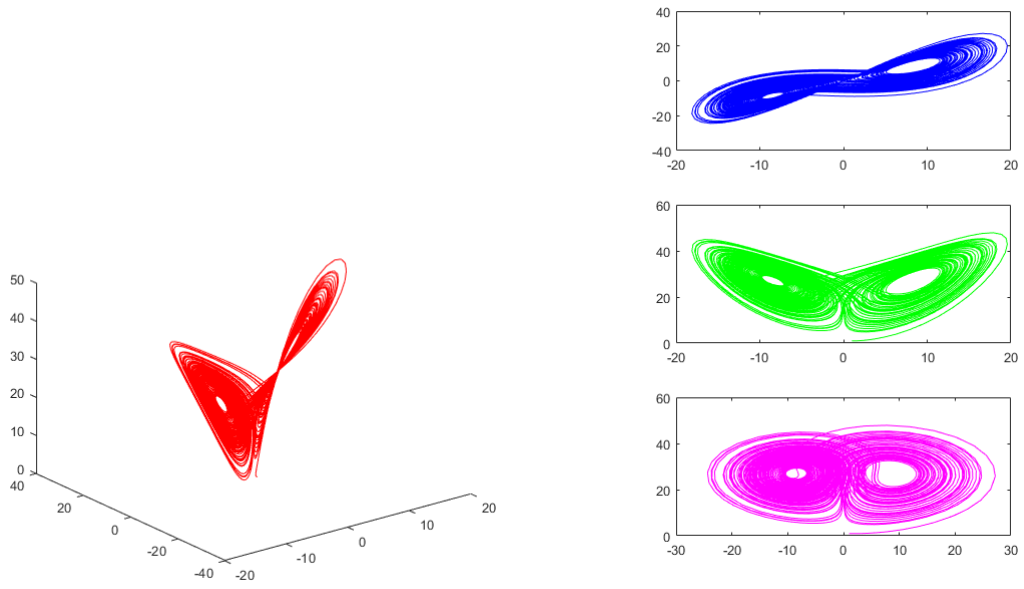


Figure 2.1: The chaotic attractor produced by Lorenz system

The phase space of Lorenz system's attractor and a recurrence plot of Lorenz System is illustrated in Fig(2.1) and Fig(1.1b) respectively.

2-2-2 Rössler System

Rössler System was designed by Otto E. Rössler in 1976. The particular attractor was intended to have identical functions as Lorenz attractor but can be analysed easier (Rössler 1976). The orbit of the attractor (as shown in Fig(2.2)) spirals outwards around an unstable fixed point but remain in a plane near the (x, y)-plane. When the graph spirals enough, the trajectory leaps in the z-dimension. Eventually, the trajectory will land close to the (x, y)-plane again. Chaotic oscillations appear in the orbit within the attractor. The ordinary differential equations of his system are (Peitgen et al. 2004):

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= ay + x \\ \dot{z} &= b + xz - cz \end{aligned} \tag{2.2}$$

where a, b, c are three adjustable constants and Rössler studied with $a = 0.2$, $b = 0.2$, $c = 5.7$ (Rössler 1976).

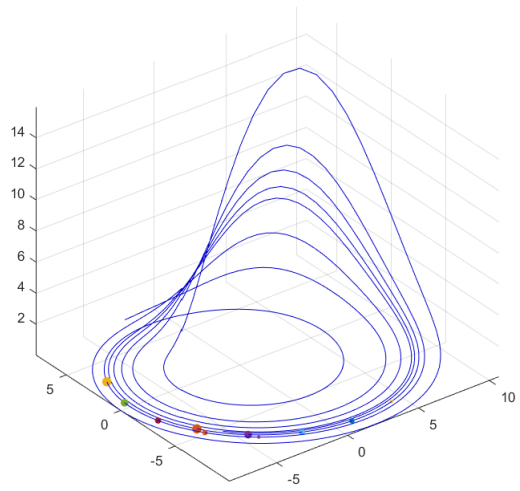


Figure 2.2: The chaotic attractor produced by Rössler System

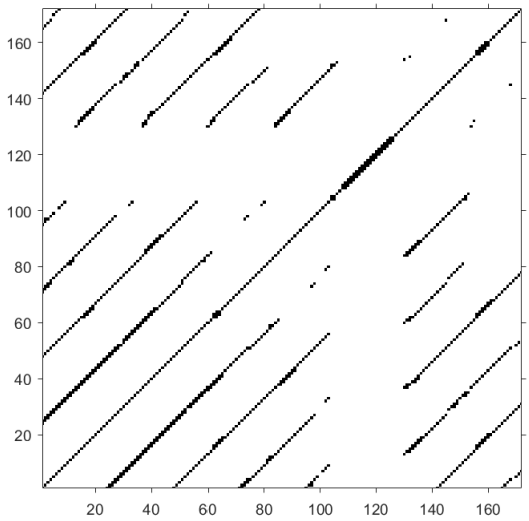


Figure 2.3: A recurrence plot of Rössler System

2-3 Structures in Recurrence Plot

Recurrence plot exhibits patterns based on similarity characteristics. The typology based on formal patterns is categorised into homogeneous, periodic, drift and disrupted (Eckmann et al. 1987, Marwan et al. 2007).

- Homogeneous RPs are uniformly covered in grey in overall despite some existences of texture. A randomly generated time series is an example of homogeneous RP.
- Periodic RPs consist of certain repeating patterns and have more diagonal lines. If there is a difference between diagonal lines distances, it is a quasi-periodic system.
- Recurrence points fading away from the line of identity (LOI), i.e. the upper-left and lower-right corners of RP being brightened along with steady changes over time is called drift.

- An RP is disrupted if there exist abrupt changes in the dynamic and causes white areas or bands.

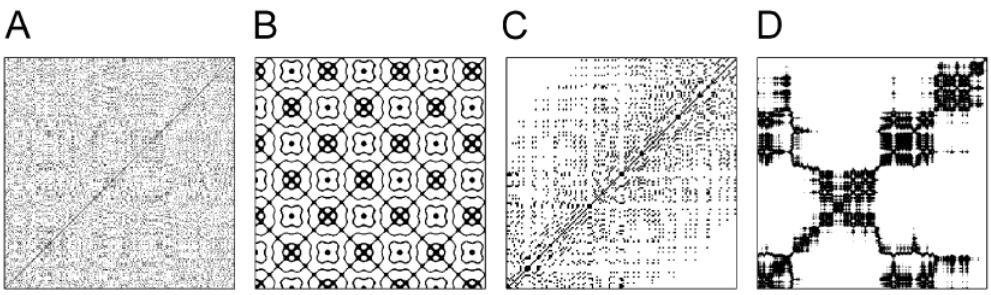


Figure 2.4: Identification of patterns
(A) homogeneous, (B) periodic, (C) drift, (D) disrupted

Turning to small scale patterns i.e. texture, it includes isolated points, diagonal lines as well as horizontal and vertical lines. These are the basis for quantitative analysis of RPs.

- An isolated recurrence point can occur if the state is uncommon, if it does not preserve for any time or it fluctuates heavily.
- A diagonal line $R_{i+k,j+k} = 1$ (for $k = 1, \ldots, l$, l is the diagonal line length) appears when the trajectory visits the same areas in the phase space at different time.
- A vertical (horizontal) line $R_{i,j+k} = 1$ (for $k = 1, \ldots, v$, v is the vertical line length) is formed when a state remains constant or changes very steadily.

To sum up the characteristics mentioned, we can get the interpretations of RPs as below:

1. Homogeneity: more randomness
2. Fading corner lines: nonstationary trend
3. Disruptions: the process is nonstationary; uncommon states exist; transitions within may have occurred
4. Periodic/quasi-periodic patterns: the process contains cyclicities which their length of periods equal to the time distance between repeating periodic patterns; for quasi-periodic process, there will be different distances between every long diagonal line
5. Single isolated points: heavy fluctuation within the process; if an only single isolated point is found, the process may be uncorrelated random.

6. Diagonal lines (parallel to the LOI): the evolution of states occurs similarly at different epochs; the process could be deterministic or else be chaotic if single isolated points occur beside diagonal lines

7. Diagonal lines (orthogonal to the LOI): the evolution of states occurs similarly at different time in a reversing way

8. Vertical and horizontal lines: some states remain unchanged or slowly change during certain periods of time; laminar states is indicated

9. Long bowed line structures: the similar evolution of states exist at different time periods but with distinct velocity; changing dynamics in the system

Experiences are required to have a precise visual interpretation of recurrence plots.

2-4 Recurrence Quantification Analysis (RQA)

Recurrence quantification analysis (RQA) is a nonlinear data analysing approach applied on recurrence plots to quantify the recurrent behaviour of dynamical systems. Several measures to determine the characteristics of different small scale structures in RPs are defined as follows.

Definition:

i. Recurrence Rate, RR

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}$$

is the percentage of recurrence points in an RP where $R_{i,j}$ equal to one or zero as stated in Eq(1.1), N is the number of points on the phase space trajectory. It shows the density of recurrences in a time series.

ii. Determinism, DET

$$DET = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)}$$

is the percentage of recurrence points forming diagonal lines. l is the length of diagonal line and $P(l)$ is histogram value of diagonal lines with length l. l_{min} is the threshold set to exclude diagonal lines which are formed by the tangential motion of phase space trajectory. Normally, l_{min} is set to 2. The larger determinism value indicates the more diagonal line in an RP and hence the stronger predictability of the system.

iii. Ratio, RATIO

$$RATIO = N^2 \frac{\sum_{l=l_{min}}^{N} lP(l)}{(\sum_{l=1}^{N} lP(l))^2} = \frac{DET}{RR}$$

is the ratio between DET and RR. It can help to disclose dynamic transitions; e.g. during certain types of transitions the RR decreases while the DET does not vary.

iv. Average diagonal line length, L

$$L = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=l_{min}}^{N} P(l)}$$

also measures the determinism of a system. The bigger the value L, the smaller the randomness, i.e. easier to determine the behaviour of a system trait.

v. Longest diagonal line, L_{max}

$$L_{max} = max(\{l_i : i = 1, \ldots, N_l\})$$

where N_l is the number of diagonal lines in RP reflects information about the stability of a system. The larger the value, the more stable the system (Yao & Lin 2017).

vi. Divergence, DIV

$$DIV = \frac{1}{L_{max}}$$

is the inverse of L_{max}. It relates to the divergent property of phase space trajectory. The smaller the value of L_{max}, the greater the value DIV and hence the faster the divergence of trajectory segments (Marwan et al. 2007).

vii. Entropy, ENTR

$$ENTR = -\sum_{l=l_{min}}^{N} p(l) ln p(l)$$

where $p(l) = \frac{P(l)}{N_l}$ is the probability distribution of diagonal line lengths. It reveals the variety of diagonal lines as well as the complexity of a system. A large entropy value implies the periodicity of a system while low implies chaoticity (FABRETTI & AUSLOOS 2005). In other words, the larger entropy follows a more complex system.

viii. Longest vertical line, V_{max}

$$V_{max} = max(\{v_i : i = 1, \ldots, N_v\})$$

where N_v is the number of vertical lines, can be considered similar to the standard measure L_{max}.

ix. Laminarity, LAM

$$LAM = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)}$$

is the percent rate of recurrence point that forms vertical lines. v is the length of vertical (horizontal) lines, $P(v)$ is the histogram value of vertical lines with length v. v_{min} is the threshold that usually set to 2 to exclude vertical lines with a certain length. Laminarity calculates the probability that a state will remain for the next time step.

x. Trapping Time, TT

$$TT = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=v_{min}}^{N} P(v)}$$

is the average length of the vertical (horizontal) lines. It indicates the average time of a system staying at each particular state or the length of time that each state is trapped.

xi. Trend, TREND

$$TREND = \frac{\sum_{i=1}^{\widetilde{N}}(i - \frac{\widetilde{N}}{2})(RR_i - \langle RR_i \rangle)}{\sum_{i=1}^{\widetilde{N}}(i - \frac{\widetilde{N}}{2})^2}$$

is the brightening of RP to the direction of its edges. $\widetilde{N}$ is the maximal number of diagonals parallel to the LOI which will be considered for computing TREND, i.e. excluding the edges of RP $(\widetilde{N} < N)$. Trend measures the drift and non-stationarity of a time series. In a homogeneous RP, it is stationary as there are almost the same amount of recurrent points on both sides of the central line. When recurrent points on the RHS is less than that of LHS, trend will get a negative value (FABRETTI & AUSLOOS 2005). A trend value around 0 signifies a quasi-stationary dynamics whereas value far from 0 implies that drift is in the dynamics (Webber & Marwan 2015).

2-5 Cross Recurrence Plot (CRP)

Cross recurrence plot (CRP) is an extension of RP in which involved the comparison of two time series. It allows the study of the relationship between two different systems. The dynamical behaviour of both time series are investigated and they are embedded in the phase space at the same time (Marwan & Kurths 2002). The distances between each point of the trajectories $x_i(i = 1, \ldots, N_x)$ and $y_j(j = 1, \ldots, N_y)$ are tested whether they are within the threshold value ε_i. The way that the results supposed to be

acquired is analogous to Eq(1.1). The equation for CRP is shown below:

$$CR_{i,j} = \Theta(\varepsilon_i - \|\overrightarrow{x}_i - \overrightarrow{y}_j\|), \quad \overrightarrow{x}_i, \overrightarrow{y}_j \epsilon \mathbb{R}^m \quad i = 1, \ldots, N,\ j = 1, \ldots, M \quad (2.3)$$

Briefly speaking, CRP reveals all the times that a state of one dynamical system occurs simultaneously in another dynamical system.

In CRP, the length of x_i and y_j are not necessarily needed to be the same. Hence may lead to a non-square matrix. The difference between CRP and RP is that the main diagonal of CRP may not be filled with all black dots as the value of main diagonal $CR_{i,i}$ may not be 1. However, the interpretation to structures of plot mentioned in Sec(2-3) is still applicable. The diagonal lines in CRP represent two trajectories having the same states at a period of time and reflect the similarities between the two dynamical systems.

2-6 Joint Recurrence Plot (JRP)

Joint recurrence plot (JRP) is a multivariate approach invented by Romano et al. (2004). It investigates whether recurrence occurs simultaneously on distinct trajectories. This means: on one trajectory, if a state x occurs at time j is inside the neighbourhood of the previous state at time i which causes recurrence, and meanwhile on another trajectory, it happens that a state y which occurred at time i also recurs at time j, a joint recurrence is found (N. Marwan, M. C. Romano, M. Thiel 2000). It is the element-wise product of single RPs:

$$JR_{i,j} = \Theta(\varepsilon_x - \|\overrightarrow{x}_i - \overrightarrow{x}_j\|) \cdot \Theta(\varepsilon_y - \|\overrightarrow{y}_i - \overrightarrow{y}_j\|), \quad \overrightarrow{x}_i \epsilon \mathbb{R}^m, \overrightarrow{y}_i \epsilon \mathbb{R}^n, \quad i, j = 1, \ldots, N \quad (2.4)$$

In short, JRP depicts all the times that a recurrence happens in one dynamical system simultaneously with a recurrence in another dynamical system.

In JRP, the thresholds ε_x and ε_y can be set to distinct values and it is unnecessary that the recurrence states of both systems should be identical. In addition, the diagonal lines in JRP represent that recurrence occurs continuously in time in two trajectories.

CHAPTER 3: PRELIMINARY RESULTS

3-1 Some Examples of Recurrence Plot

i. Sine and cosine functions can be presented in recurrence plots. Their RPs are periodic graph.

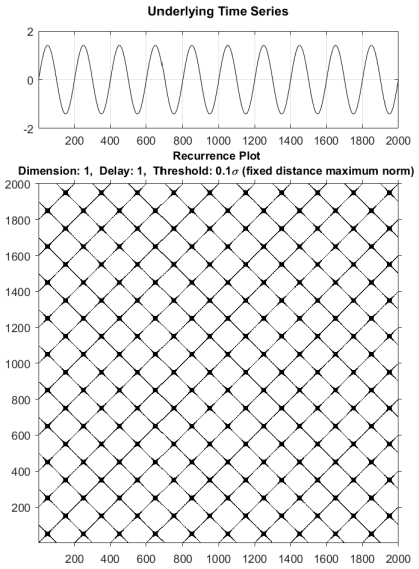


Figure 3.1: Recurrence plot of sine function

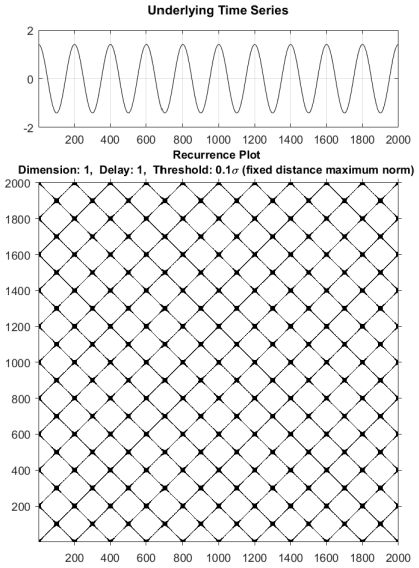


Figure 3.2: Recurrence plot of cosine function

ii. The data set `'cycles.dat'` applied in this example is taken from the website http://www.recurrence-plot.tk/rp-tutorial.php. Here, unthresholded distance matrix is implemented to plot the recurrence plot.

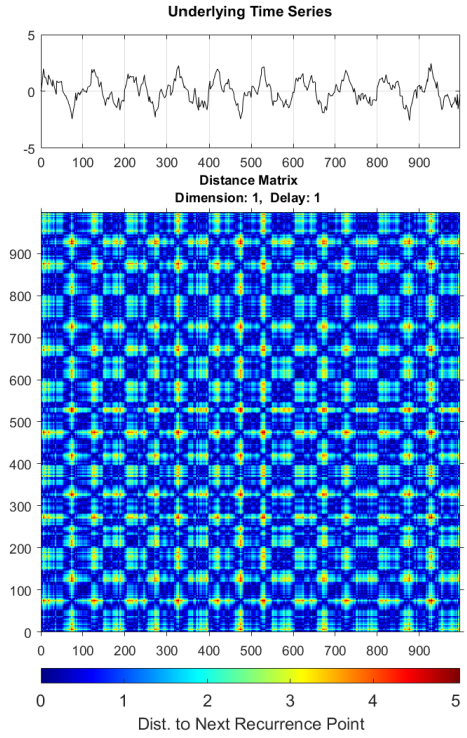


Figure 3.3: Example of periodic RP

From the RP above, we can observe that the RP possesses some periodical patterns. The cyclicities in the time series can be discovered by the distances between periodic patterns. The periodic structures are obviously shown and having 100 and 200 time units. In addition, small substructures with the size of 20 and 40 time units also exist in the RP.

3-2 Example of cross recurrence plot

The graph below is the CRP of harmonic oscillations which involves a comparison of two time series. It is plotted in distance matrix, i.e. unthresholded.

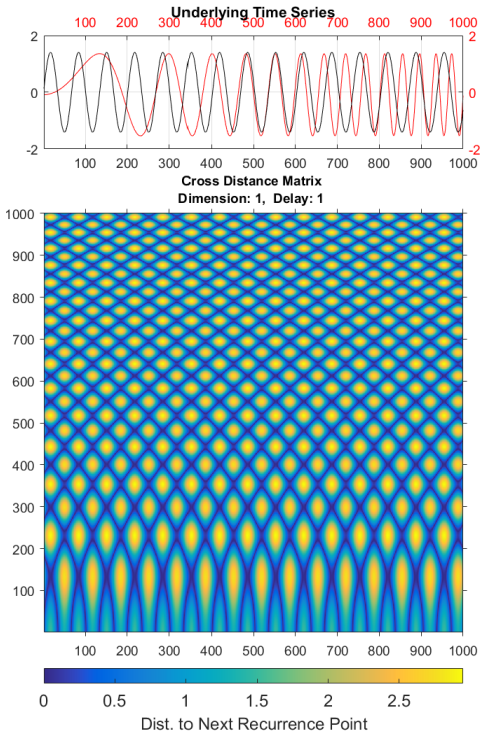


Figure 3.4: Recurrence plot of harmonic oscillations

This is a periodic graph which contains structure with some long bowed lines, implies that similar evolution of states exists in the two time series but with distinct velocity.

3-3 Preliminary Results of EEG Data

To acquire some preliminary results from the given EEG data sets, here we randomly choose two text files, i.e. the 30th and 80th text file, which represent two channels of EEG segments from each of the five data sets. Then, load the text files into CRP toolbox and create a cross recurrence plot. Meanwhile, compute the corresponding recurrence quantification analysis. Fig(3.5) - Fig(3.9) show the CRP obtained and Table(3.1) is the corresponding RQA.

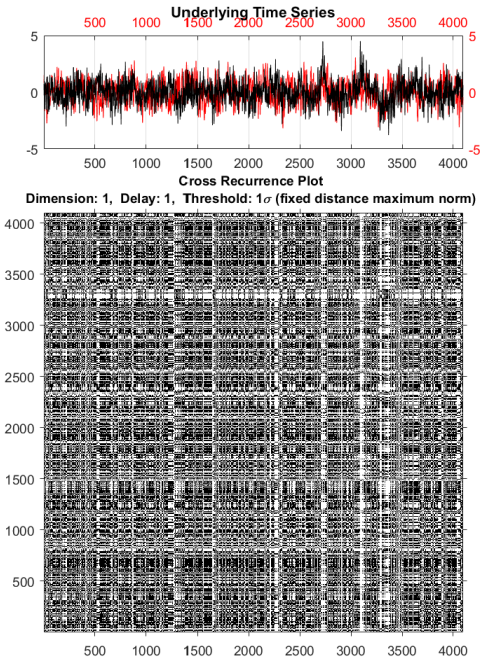


Figure 3.5: CRP of two selected time series from set A

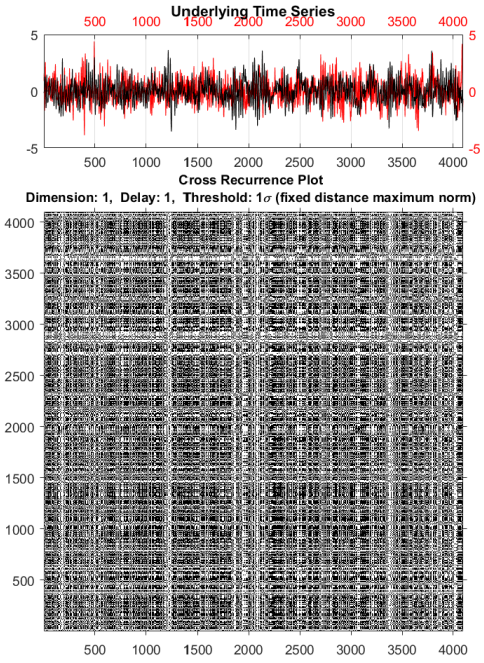


Figure 3.6: CRP of two selected time series from set B

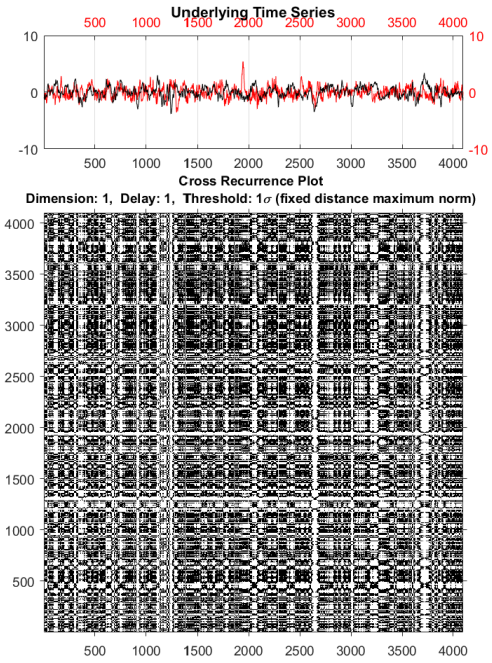


Figure 3.7: CRP of two selected time series from set C

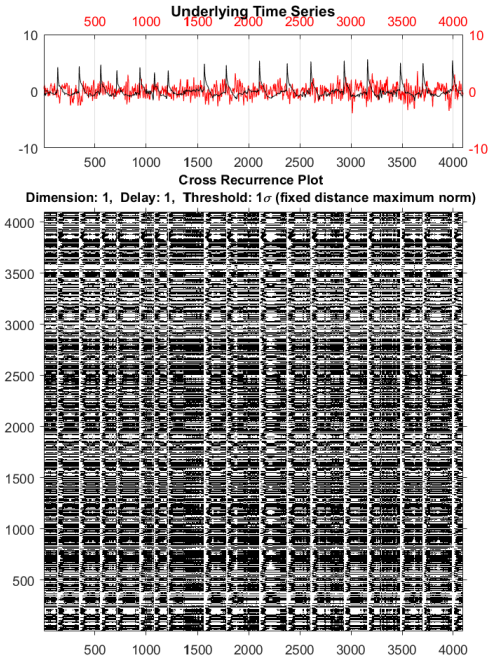


Figure 3.8: CRP of two selected time series from set D

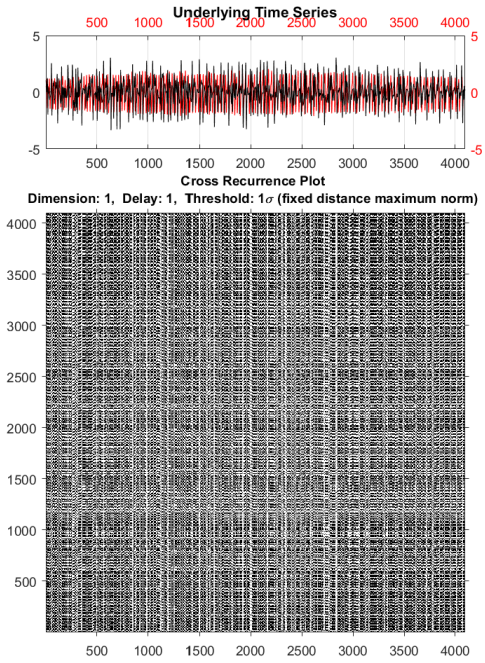


Figure 3.9: CRP of two selected time series from set E

Set	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
A	0.5213	0.9439	4.7398	56	2.1695	0.9897	7.5570	63
B	0.5293	0.9825	5.8277	70	2.4408	0.9914	7.2255	60
C	0.5348	0.9887	9.5978	106	3.0854	0.9889	11.1593	85
D	0.5671	0.9889	10.0761	129	3.1352	0.9891	10.5530	108
E	0.5095	0.9833	6.5493	70	2.5697	0.9997	9.4714	28

Table 3.1: RQA of selected EEG time series

From the graphs above, we may conclude that all graphs tend to be homogenous except for that graph of set D, i.e. Fig(3.8) which is visually quasi-periodic. Turning to the RQA, obviously we can see that the average diagonal line lengths L of CRPs from set C, D, E are greater than that of set A and B. Thus, we may conclude that the time series from set C, D, E is less random and hence more deterministic. Moreover, the value of entropy from the latter 3 sets is also higher than the formal two sets. Hence, they may be dynamically more complex than set A and B. However, the conclusion is not necessarily true as this is just a simple preliminary result. The investigation will be further carried out towards more of the data sets of time series to attain an accurate conclusion.

CHAPTER 4: ELECTROENCEPHALOGRAM (EEG) AND EPILEPTIC SEIZURE

4-1 Electroencephalogram (EEG)

Electroencephalogram (EEG) is a special and precious measure of the electrical function of brain generated by nerve cells of brain cortex. It is a precious clinical tool to diagnose epilepsy diseases and provide treatments (Fu et al. 2015). It is a graph illustrating the recordings different in voltage from left and right sites of brain function over a period of time. Extracranial EEG supplies recording of electrocerebral activity throughout both left and right side of the brain. Intracranial EEG contributes focused EEG recording through surgically implanted electrodes at specifically targeted regions of the brain in a direct manner (Tatum et al. 2015).

The main implementation of EEG is to diagnose epilepsy. This symptom can be assumed if abnormalities are found in the EEG readings. However, EEG can also be utilised to detect sleep disorders, head injuries, brain tumors and etc.

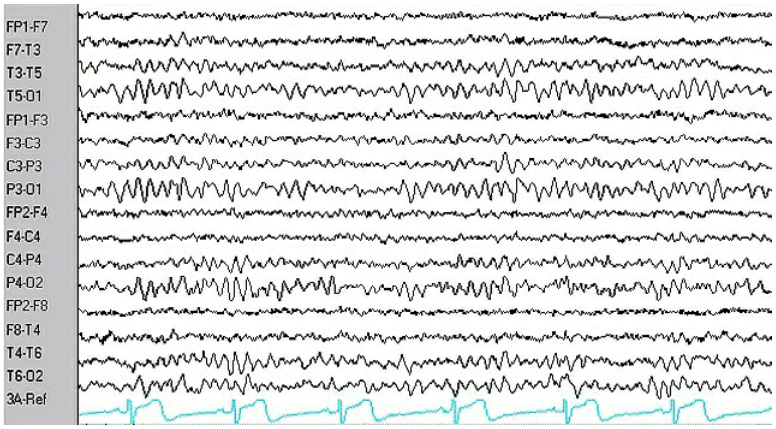


Figure 4.1: Actual recording of normal EEG

4-1-1 Abnormality on EEG

Abnormality on EEG can be distinguished into two categories which are epileptiform and nonepileptiform. If the brain activity is abruptly changed or interrupted from normal, epileptiform abnormality takes place. Interictal epileptiform discharges are a group of special brain waveforms which can be found from epilepsy patients (Tatum et al. 2015). Focal epileptiform discharges can represent the possibility that epileptic seizures happen in an area of brain. Whereas generalized epileptiform discharges normally suggest generalized epilepsy patients.

A general change in the look of brain wave such as abnormal amplitude, frequency, shape manifests a nonepileptiform abnormal activity. Its existence usually refers to the dysfunction of brain (Tatum et al. 2015). The diffuse slowing patterns that appear on EEG exhibit abnormal brain function.

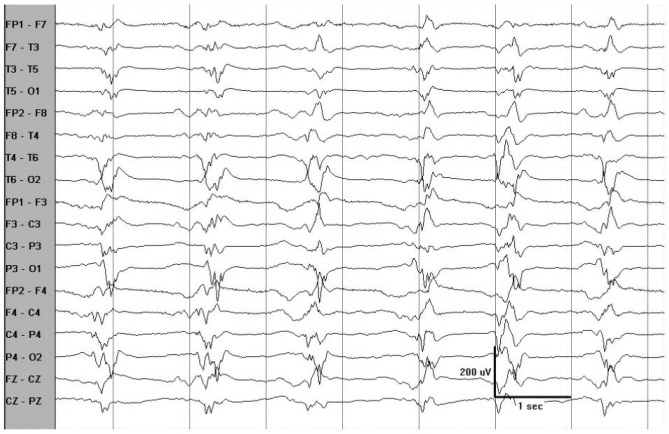


Figure 4.2: Epileptiform abnormality on EEG

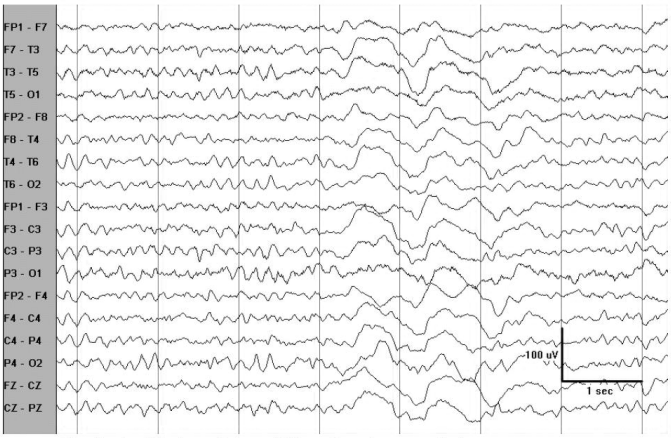


Figure 4.3: Diffuse slowing nonepileptiform abnormality on EEG

4-2 Epileptic Seizure

Epilepsy is a diverse family of brain disorders that leads a patient to the generating of epileptic seizures. By definition, epilepsy is determined only if there is a seizure exists. Furthermore, the brain must have a long-term alteration that may enhance the probability of next seizures to occur. Other than the recurrence of seizures, some conditions for instance the neurobiologic and social disturbances that are suffered by patients are also associated with epilepsy (Fisher et al. 2005). Seizures are stimulated by excessive electrical impulses generated and delivered from an epileptic patients' brain nerve cells (Sree et al. 2011).

An epileptic seizure is a short-lived symptom caused by synchronous or extremely large amount of neuronal activity in the brain (Fisher et al. 2005). There are provoked and unprovoked seizures. Temporary events like fever and low blood sugar can trigger a provoked seizure whereas events like stress or sleep deprivation may bring about unprovoked seizures which happen unawares. When focal seizures occur, solely part of the brain is affected. The brain is affected wholly when generalized seizures take place.

CHAPTER 5: RESULTS AND DISCUSSIONS

For the aim of this project, EEG data sets are analysed through recurrence analysis of time series. The major aspects involved are the inspection through visualisation on several types of recurrence plots constructed by EEG data as well as calculating some of the recurrence quantification measures on different EEG time series data.

5-1 Analysis by Inspection on RP

In this section, there are three types of recurrence plots being constructed for analysis. The first one is the simple recurrence plot, which involves only a single time series i.e. the recurrence is being identified when a state in the time series matches a state occurred in the previous time. Cross recurrence plot is the second type of RPs being discussed and examined here. It compares whether the states in two different time series simultaneously occur to be almost the same. All the cross recurrence plots are made by comparing two time series data that come from the same EEG set. The last one is the multi-dimensional RP, which is plotted by a matrix of data consisting of multiple columns. In the matrix, the first column is the increasing numbers start from one that indicates the time while the rest are two or three time series data obtained from different groups of EEG sets. Here, we group set A and B together as they were acquired from healthy volunteers. Set C, D and E are grouped since they all came from patients with epileptic seizures.

With respect to all the figures plotted, the dimension, delay and threshold values are consistently set to be 1. All the figures shown include an underlying time series line graph and the corresponding recurrence plot. Due to the large number of data files inside each set, we only randomly select several time series from every set for plotting.

5-1-1 Recurrence Plot of EEG

The samples chosen to make simple recurrence plots are the (i) 30^{th} and (ii) 70^{th} time series data of each EEG set.

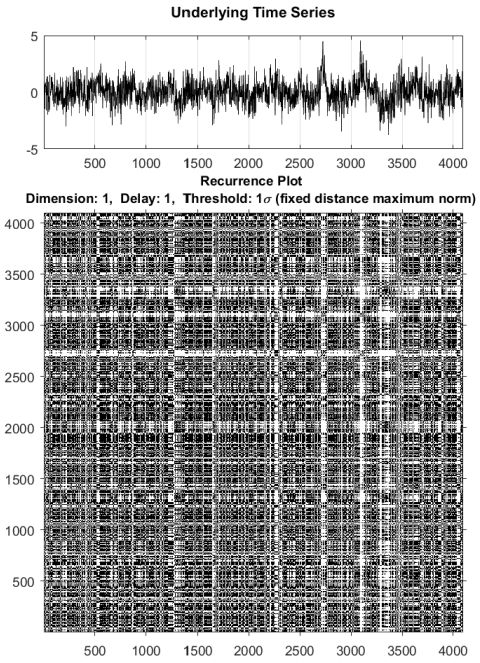


Figure 5.1: Recurrence plot of sample (i) data in set A

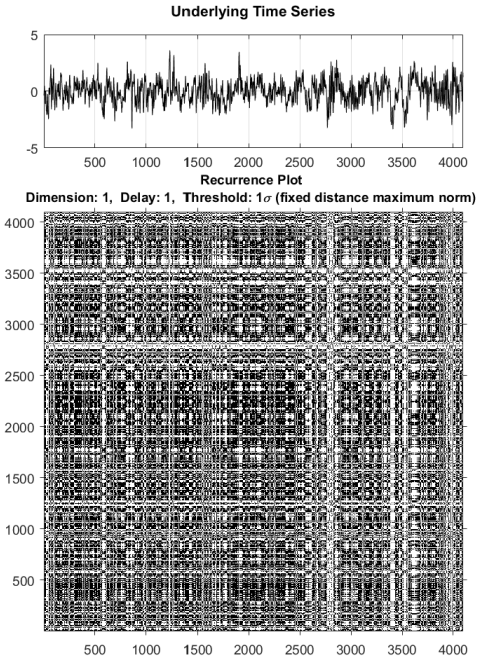


Figure 5.2: Recurrence plot of sample (ii) data in set A

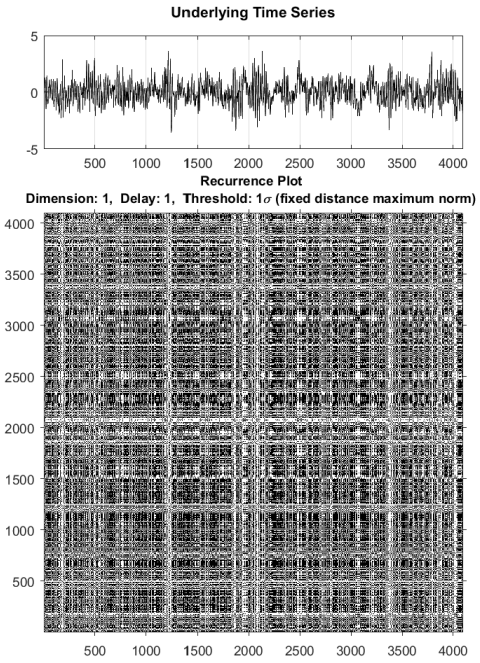


Figure 5.3: Recurrence plot of sample (i) data in set B

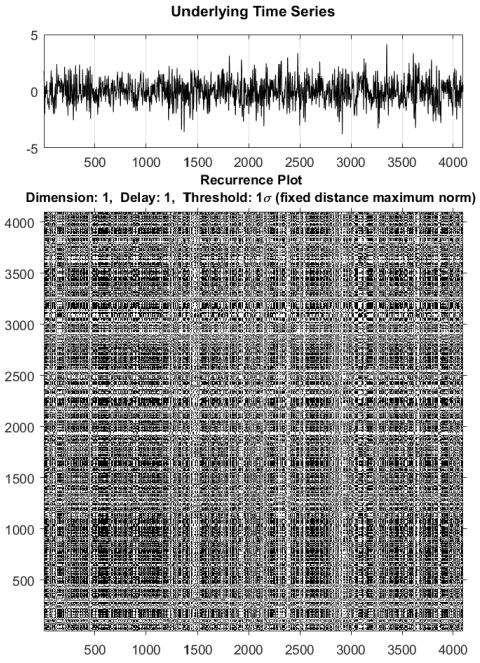


Figure 5.4: Recurrence plot of sample (ii) data in set B

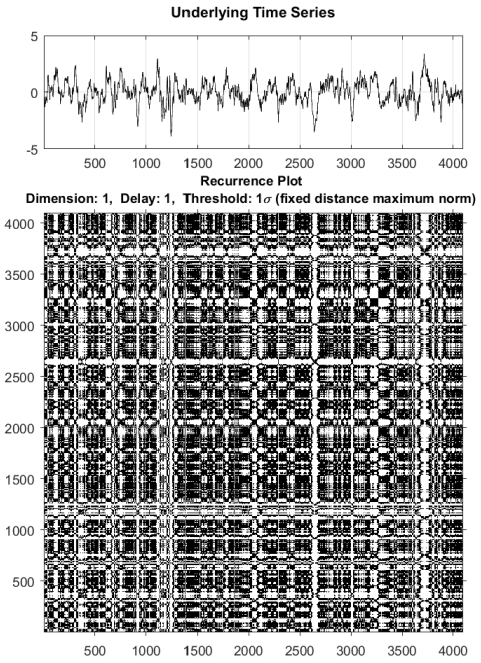


Figure 5.5: Recurrence plot of sample (i) data in set C

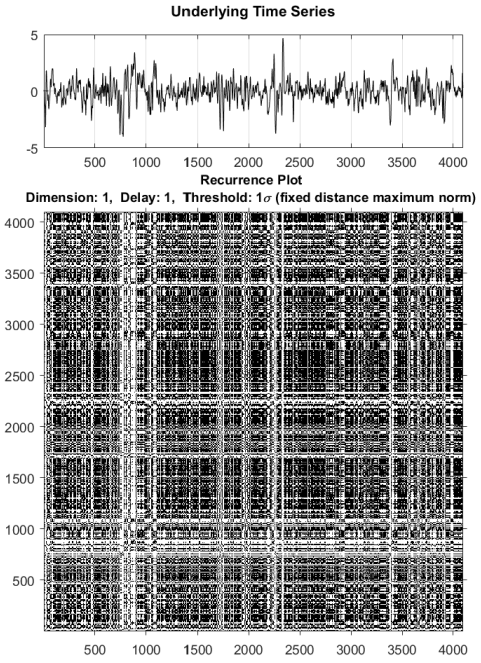


Figure 5.6: Recurrence plot of sample (ii) data in set C

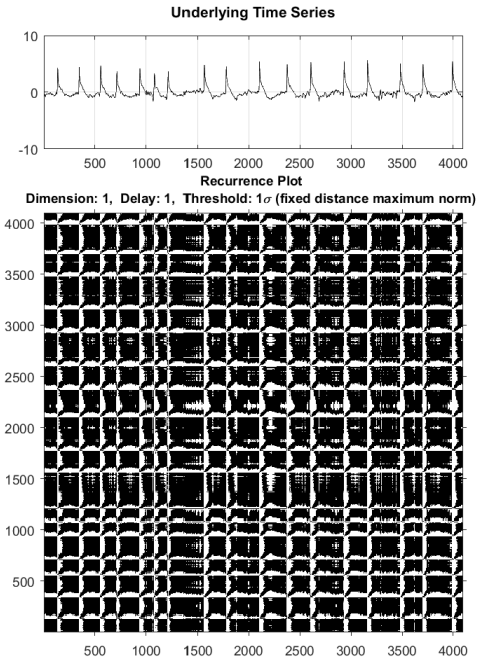


Figure 5.7: Recurrence plot of sample (i) data in set D

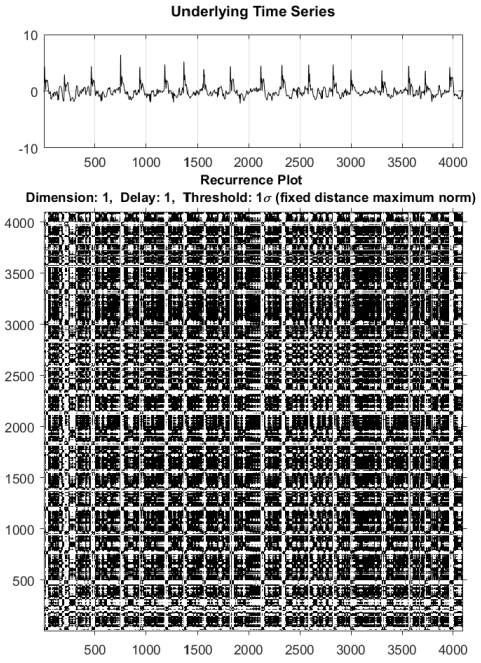


Figure 5.8: Recurrence plot of sample (ii) data in set D

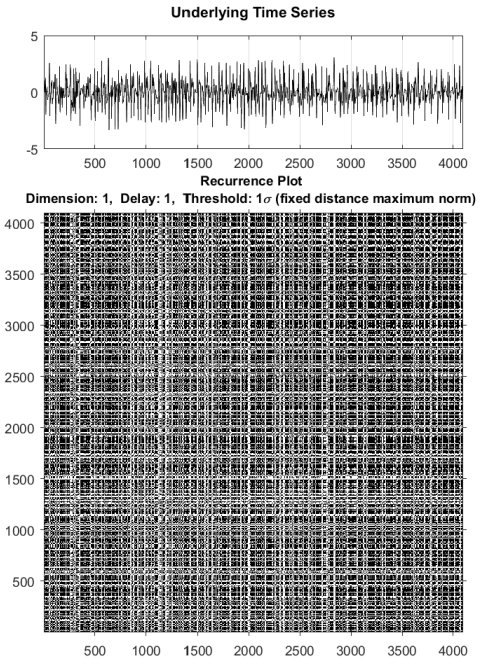


Figure 5.9: Recurrence plot of sample (i) data in set E

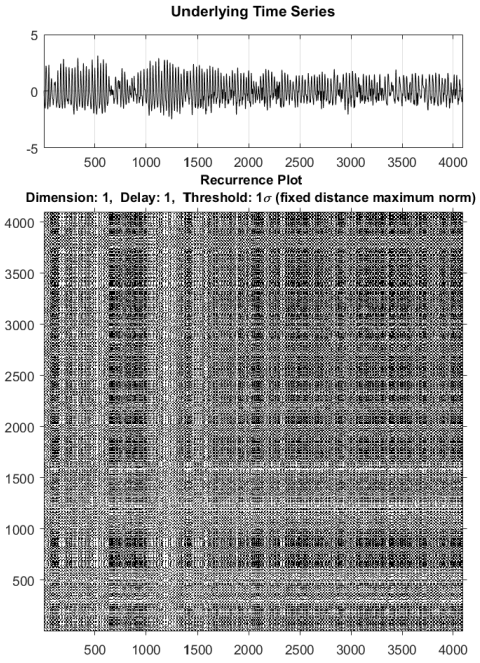


Figure 5.10: Recurrence plot of sample (ii) data in set E

From Fig(5.1) to Fig(5.4), we can see that the underlying time series of these data appear to be randomly fluctuating. The corresponding recurrence plots of them also

present in almost covered in grey. It means that it matches the statement that these time series contain more randomness. However, it is hard to determine the differences between recurrence plots of data from set A and set B.

On the other hand, from Fig(5.5) to Fig(5.10), all plotted by data received from epilepsy patients, we can observe a periodic pattern on most of them. The periodicity can be most obviously seen from the plot of data from set D, which is the EEG recordings from within the seizure generating area during seizure free interval. The underlying time series of them have depicted the existence of cyclicities within the time series. The typologies of the matching recurrence plots also show that the data is periodic. The recurrence plots consist of a lot of diagonal lines as well as vertical and horizontal lines. It conveys to the meaning that the EEG recordings could be deterministic and have laminar states. The underlying time series of data from set E exhibit heavy fluctuations. Their recurrence plots also appear to be periodic but with small structures.

5-1-2 Cross Recurrence Plot of EEG

To plot cross recurrence plots, we randomly select two data from the same EEG set. Thereby, make comparisons to the selected data. Here, we have chosen (i) $2^{nd}\&28^{th}$ and (ii) $40^{th}\&93^{rd}$ data from each EEG set as the sample data.

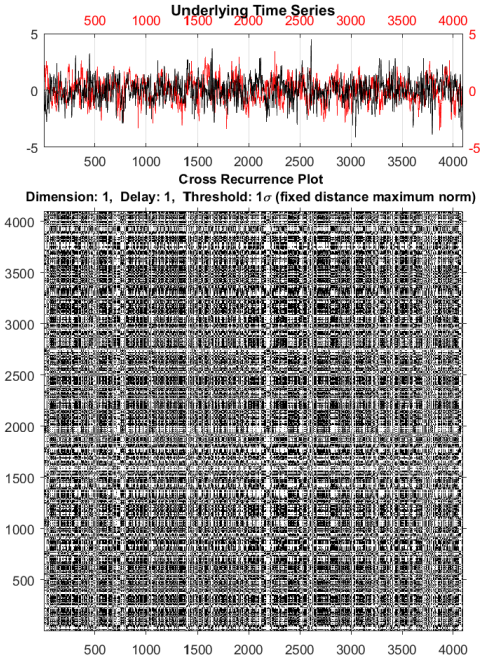


Figure 5.11: Cross recurrence plot of sample (i) data in set A

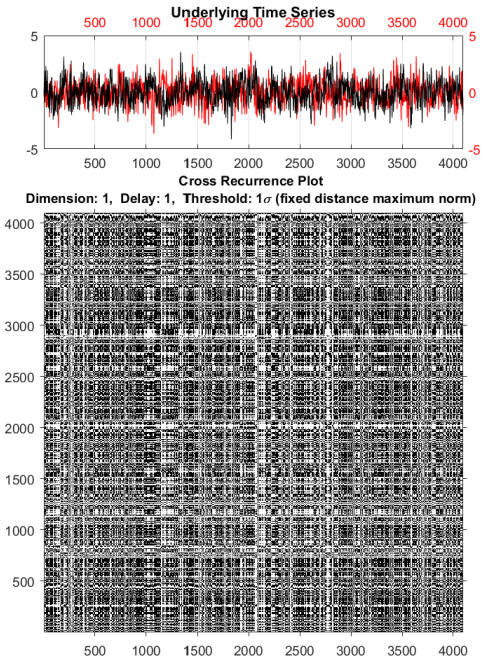


Figure 5.12: Cross recurrence plot of sample (ii) data in set A

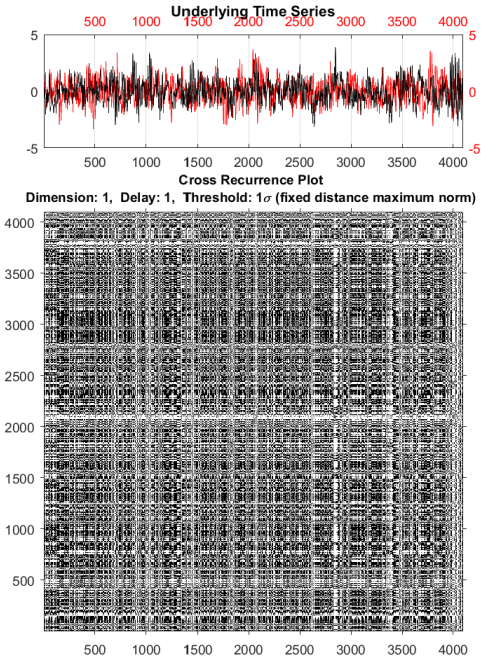


Figure 5.13: Cross recurrence plot of sample (i) data in set B

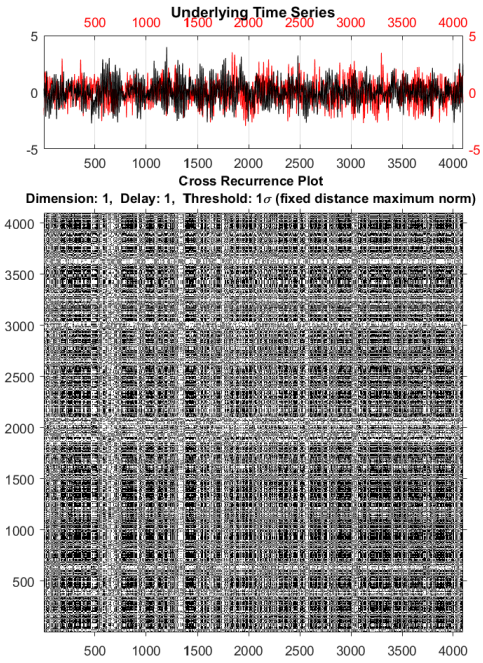


Figure 5.14: Cross recurrence plot of sample (ii) data in set B

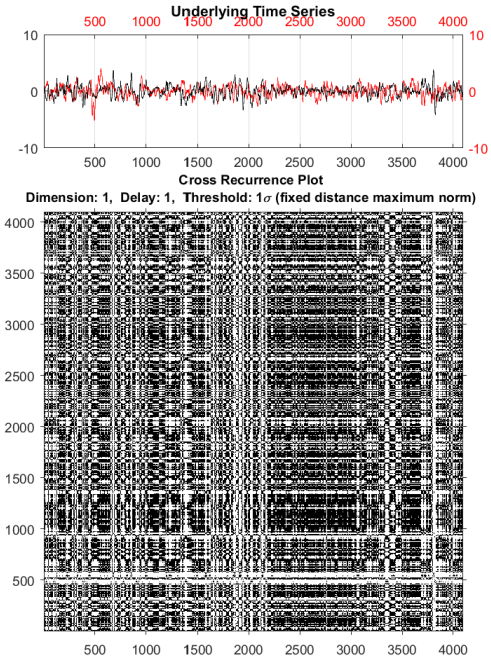


Figure 5.15: Cross recurrence plot of sample (i) data in set C

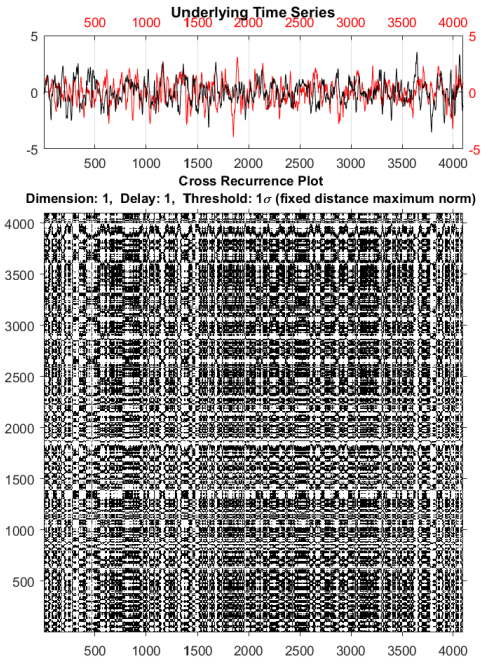


Figure 5.16: Cross recurrence plot of sample (ii) data in set C

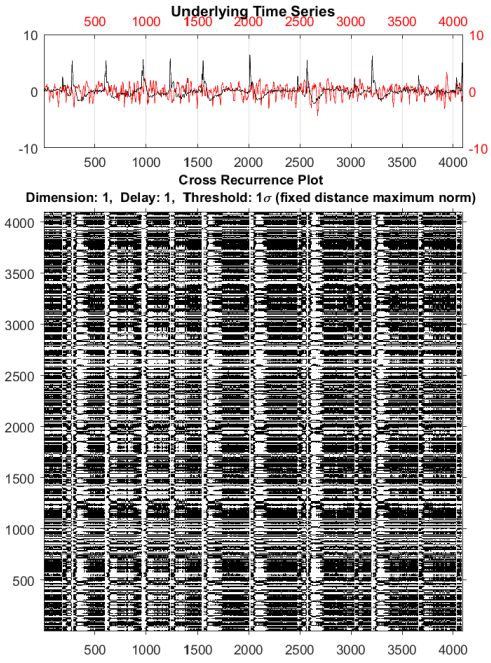


Figure 5.17: Cross recurrence plot of sample (i) data in set D

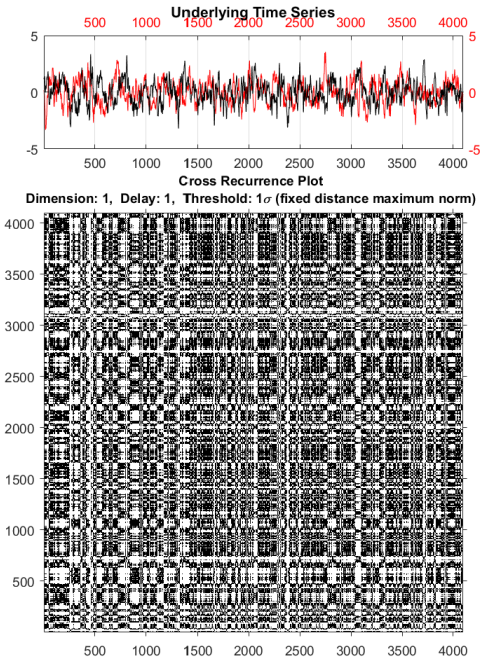


Figure 5.18: Cross recurrence plot of sample (ii) data in set D

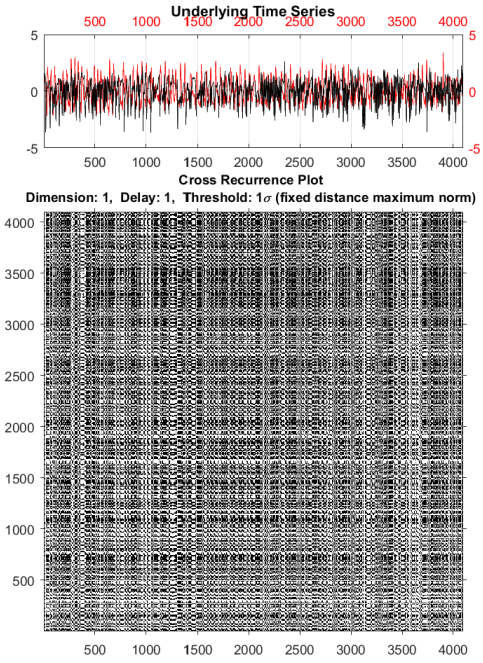


Figure 5.19: Cross recurrence plot of sample (i) data in set E

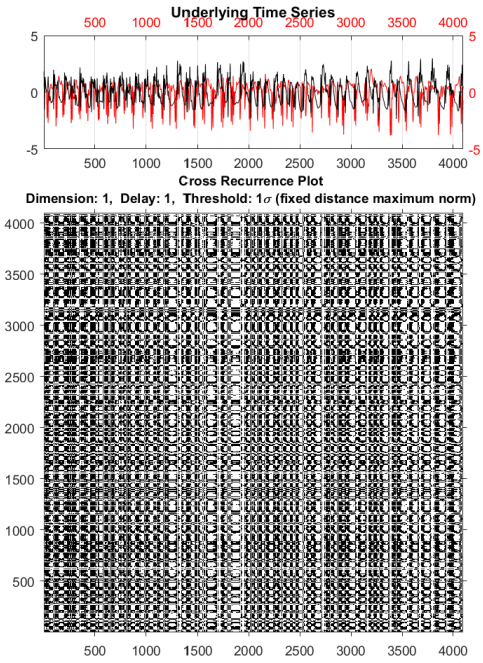


Figure 5.20: Cross recurrence plot of sample (ii) data in set E

In this section, the cross recurrence plots obtained from data of sets A and B (Fig 5.11 - Fig 5.14) present in homogeneous typology which is similar to their simple recurrence plots since the corresponding underlying time series also possess random fluctuations.

Figures of set C do not exhibit any significant patterns however Fig(5.17) plotted from data of set D has some noticeable periodic patterns. The cross recurrence plot of set E data which underlying time series varies in a repeating same pattern occur to be small structures periodic.

5-1-3 Multi-dimensional Recurrence Plot of EEG

We let sets A, B and sets C, D, E be two respective groups. Then, we plot the multi-dimensional recurrence plot by a matrix containing values of data from sets within each of the group. The (i) 1^{st} and (ii) 95^{th} data from each set is chosen and grouped for plotting. To provide a clear understanding on multi-dimension, a 2D or 3D plot is also graphed below accordingly.

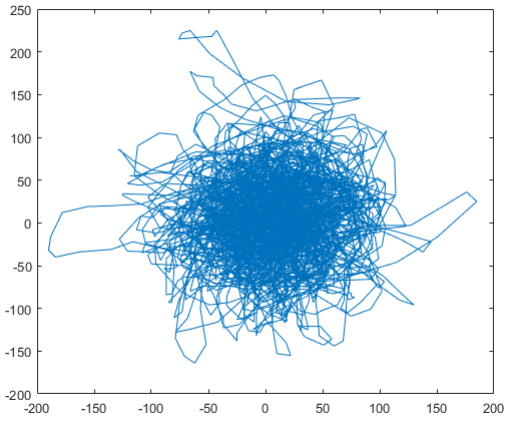


Figure 5.21: 2D visualisation plot of sample (i) data from sets A&B

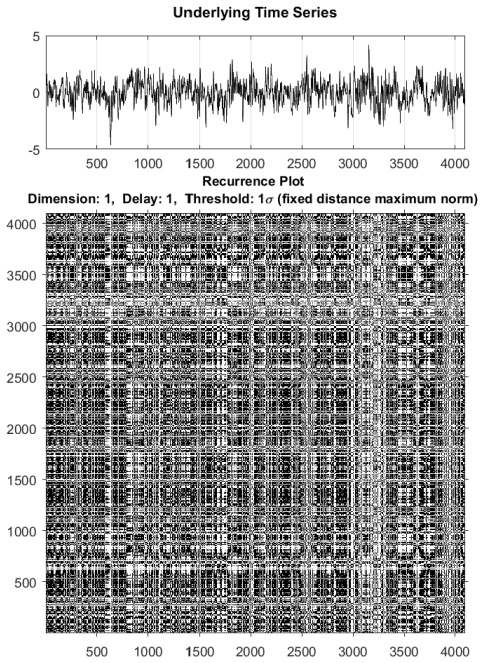


Figure 5.22: Multi-dimensional recurrence plot involving sample (i) data from sets A&B

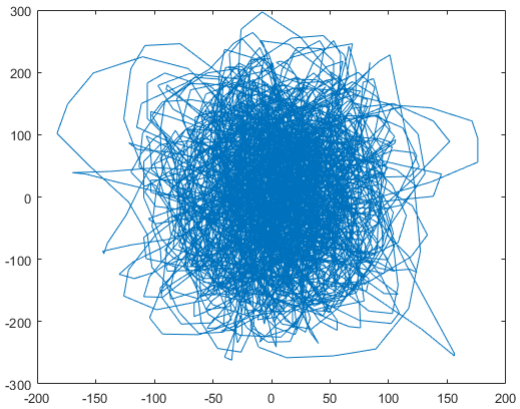


Figure 5.23: 2D visualisation plot of sample (ii) data from sets A&B

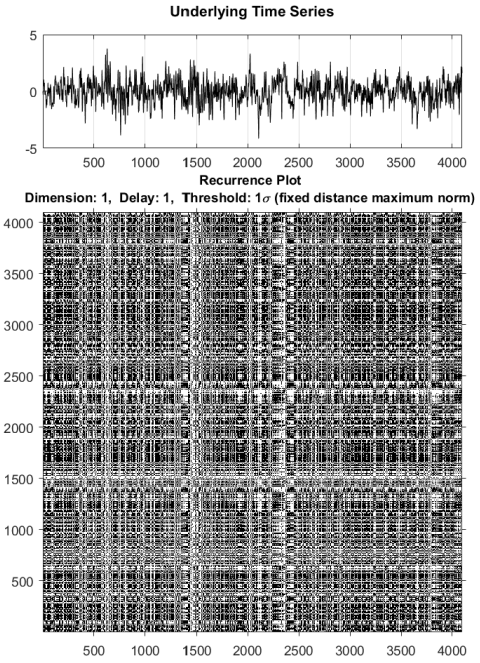


Figure 5.24: Multi-dimensional recurrence plot involving sample (ii) data from sets A&B

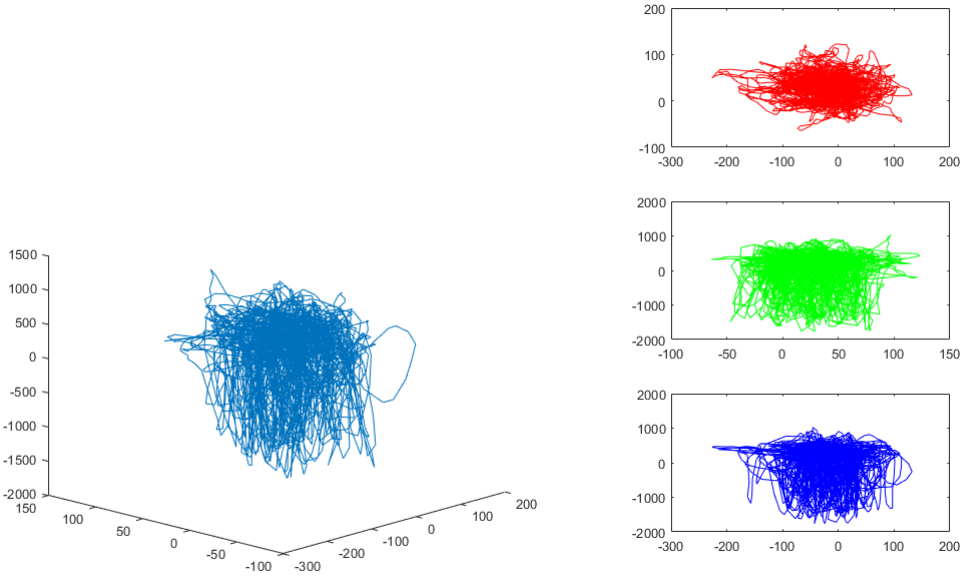


Figure 5.25: 3D visualisation plot of sample (i) data from sets C&D&E

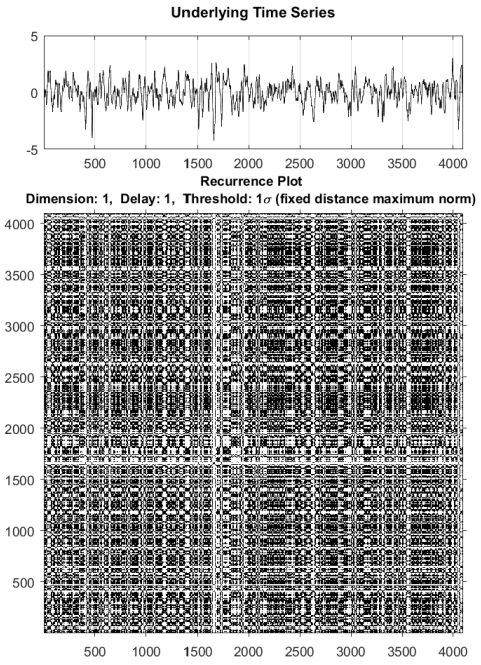


Figure 5.26: Multi-dimensional recurrence plot involving sample (i) data from sets C&D&E

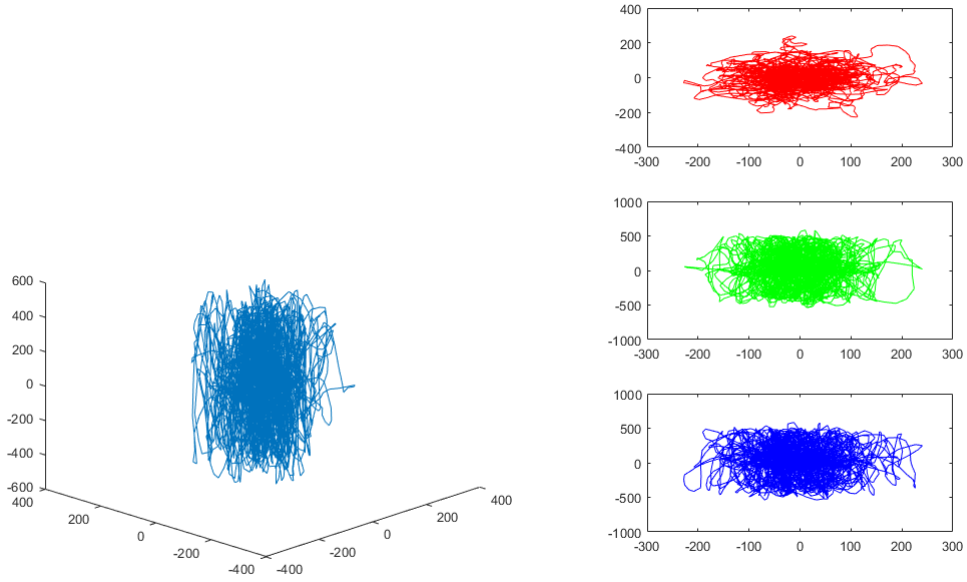


Figure 5.27: 3D visualisation plot of sample (ii) data from sets C&D&E

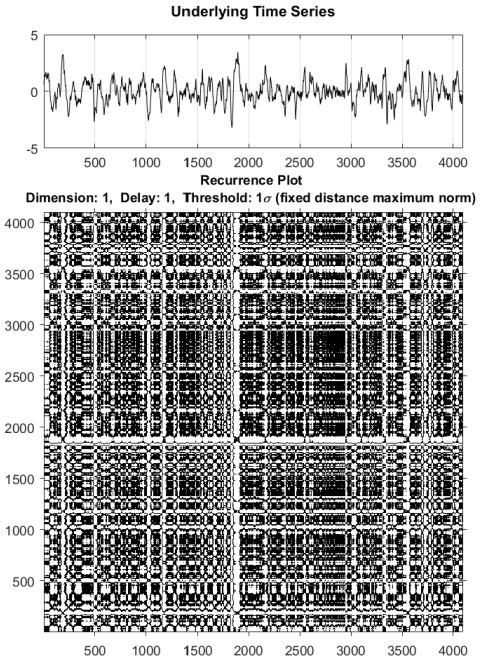


Figure 5.28: Multi-dimensional recurrence plot involving sample (ii) data from sets C&D&E

Expectedly, the multi-dimensional recurrence plots of the first group, i.e. sets A and B, are observed to be in homogeneity despite having some inapparent textures. This recurrence plot interpretation is the same as in single recurrence and cross recurrence plot sections.

In the group including sets C, D and E, the multi-dimensional recurrence plots have some insignificant periodic patterns. Moreover, Fig(5.28) even contains some white bands which may indicate that there exist abrupt changes in the data series.

5-2 Analysis on RQA Measures

To analyse the EEG data in a more detailed manner, we further carry out a series of recurrence quantification analysis. In this section, tables containing values of various RQA measures are constructed. Five samples are arbitrarily selected from each EEG data set or group for calculations of different types of RQA. The RQA measures included in this analysis are recurrence rate (RR), determinism (DET), average diagonal line length (L), longest diagonal line (L_{max}), entropy ($ENTR$), laminarity (LAM), trapping time (TT) and longest vertical length (V_{max}). In all the calculations, we set the dimension and delay to be 1 and the threshold to be 0.1.

5-2-1 Recurrence Quantification Analysis of EEG

The random samples selected from every EEG data set for calculating RQA measures are the 6^{th}, 19^{th}, 30^{th}, 67^{th} and 89^{th} data. There is only a single time series involved in one calculation.

Set A	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
6^{th}	0.0567	0.2844	2.3109	13	0.7179	0.3858	2.4516	9
19^{th}	0.0645	0.3734	2.3849	10	0.8184	0.4972	2.5194	10
30^{th}	0.0604	0.1884	2.1191	6	0.3793	0.2426	2.1686	6
67^{th}	0.0550	0.2926	2.2963	10	0.6967	0.3925	2.4297	8
89^{th}	0.0551	0.2807	2.3105	9	0.7174	0.3793	2.4086	9
Mean	0.0583	0.2839	2.2843	9.6	0.6659	0.3795	2.3956	8.4

Table 5.1: RQA of randomly selected samples data from set A

Set B	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
6^{th}	0.0620	0.2827	2.2912	9	0.6893	0.3817	2.4153	8
19^{th}	0.0586	0.2723	2.5884	16	1.0469	0.3699	2.6476	8
30^{th}	0.0532	0.2663	2.4299	13	0.8741	0.3520	2.5505	10
67^{th}	0.0612	0.2496	2.2164	8	0.5695	0.3269	2.2764	6
89^{th}	0.0610	0.2455	2.2372	7	0.6046	0.3362	2.3163	7
Mean	0.0592	0.2633	2.3526	10.6	0.7569	0.3533	2.4412	7.8

Table 5.2: RQA of randomly selected samples data from set B

Set C	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
6^{th}	0.0619	0.5413	2.8081	19	1.2419	0.6971	3.0831	20
19^{th}	0.0565	0.5159	2.5896	13	1.0479	0.6581	2.8106	14
30^{th}	0.0623	0.5577	2.6515	15	1.1075	0.7027	2.9368	12
67^{th}	0.0493	0.3985	2.3416	14	0.7610	0.5140	2.5125	10
89^{th}	0.0579	0.6705	3.0394	35	1.4064	0.8126	3.3979	19
Mean	0.0576	0.5368	2.6860	19.2	1.1129	0.6769	2.9482	15

Table 5.3: RQA of randomly selected samples data from set C

Set D	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
6^{th}	0.0622	0.4362	2.4069	11	0.8458	0.5678	2.6366	12
19^{th}	0.0713	0.5122	2.5646	15	1.0221	0.6609	2.8186	14
30^{th}	0.0982	0.8949	4.1637	130	1.9660	0.9479	5.3780	38
67^{th}	0.0540	0.5015	2.6630	15	1.1175	0.6480	2.9329	14
89^{th}	0.0640	0.4090	2.4168	13	0.8582	0.5309	2.6158	9
Mean	0.0699	0.5508	2.8430	36.8	1.1619	0.6711	3.2764	17.4

Table 5.4: RQA of randomly selected samples data from set D

Set E	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
6^{th}	0.0558	0.3831	2.7418	19	1.1863	0.5275	2.9983	12
19^{th}	0.0697	0.5327	3.0543	51	1.4193	0.6691	3.4670	28
30^{th}	0.0634	0.3826	2.8685	21	1.2820	0.5639	3.3107	18
67^{th}	0.0643	0.4110	2.8295	27	1.2547	0.5671	3.2173	17
89^{th}	0.0562	0.5609	3.5068	86	1.6143	0.7186	3.8709	47
Mean	0.0619	0.4541	3.0002	40.8	1.3513	0.6092	3.3728	24.4

Table 5.5: RQA of randomly selected samples data from set E

From the five tables above, we can see that the average DET values of data from sets A and B are slightly lower than those from sets C, D and E although the recurrence rate values of all are almost the same. This may indicate that the data from the latter 3 sets have stronger predictability than the formal 2 sets. On the measure of average diagonal line length, epileptic patients' EEG have smaller randomness than normal people as L values of sets C, D and E are smaller than sets A and B. The means of longest diagonal and vertical line of sets A, B data are around 10 whereas the means of sets C, D, E are excessively larger than 10. It means that the EEG data from epileptic patients are more stable than normal people. The entropy values means of sets A and B are around 0.7 whereas those of sets C, D and E are greater than 1. It implies that EEG data of healthy people and epileptic patients possess chaotic and periodic behaviours respectively. On average, the laminarity values of the latter 3 sets are higher than the formal 2 sets around 0.3. Trapping time of data from patients are also averagely higher

than normal people on a small scale. This shows that more laminar states exist in the EEG of patients.

5-2-2 Cross Recurrence Quantification Analysis of EEG

We randomly selected the $2^{nd}\&28^{th}$, $15^{th}\&58^{th}$, $36^{th}\&90^{th}$, $45^{th}\&70^{th}$ and $50^{th}\&100^{th}$ data from each of the five sets EEG data to make a cross recurrence quantification analysis. It analyses the recurrent behaviours between two distinct time series data from the same set.

Set A	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
$2^{nd}\&28^{th}$	0.0574	0.2934	2.3046	11	0.7090	0.4370	2.4780	7
$15^{th}\&58^{th}$	0.0572	0.2626	2.1894	8	0.5215	0.3083	2.2460	6
$36^{th}\&90^{th}$	0.0565	0.2616	2.2465	11	0.6198	0.3241	2.3180	8
$45^{th}\&70^{th}$	0.0569	0.2894	2.2702	10	0.6573	0.4099	2.4007	9
$50^{th}\&100^{th}$	0.0573	0.2854	2.2683	9	0.6545	0.3551	2.3388	7
Mean	0.0571	0.2785	2.2558	9.8	0.6324	0.3669	2.3563	7.4

Table 5.6: CRQA of randomly selected samples data with each consisting two time series from set A

Set B	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
$2^{nd}\&28^{th}$	0.0569	0.2814	2.3351	11	0.7522	0.3632	2.4313	10
$15^{th}\&58^{th}$	0.0563	0.2365	2.2631	9	0.6463	0.3071	2.2730	6
$36^{th}\&90^{th}$	0.0569	0.2289	2.1568	7	0.4586	0.2819	2.1736	5
$45^{th}\&70^{th}$	0.0577	0.2492	2.2754	9	0.6654	0.3097	2.2814	7
$50^{th}\&100^{th}$	0.0563	0.2434	2.4419	12	0.8886	0.2838	2.5865	11
Mean	0.0568	0.2479	2.2945	9.6	0.6822	0.3091	2.3492	7.8

Table 5.7: CRQA of randomly selected samples data with each consisting two time series from set B

Set C	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
$2^{nd}\&28^{th}$	0.0592	0.4782	2.4650	13	0.9149	0.5694	2.6071	11
$15^{th}\&58^{th}$	0.0611	0.4344	2.3803	10	0.8125	0.7437	3.1893	20
$36^{th}\&90^{th}$	0.0586	0.5192	2.5727	14	1.0313	0.5900	2.6755	11
$45^{th}\&70^{th}$	0.0609	0.4630	2.5825	14	1.0408	0.5817	2.8457	13
$50^{th}\&100^{th}$	0.0592	0.4238	2.3450	12	0.7656	0.6689	2.9400	15
Mean	0.0598	0.4637	2.4691	12.6	0.9130	0.6307	2.8515	14

Table 5.8: CRQA of randomly selected samples data with each consisting two time series from set C

Set D	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
2^{nd}&28^{th}	0.0625	0.5534	2.7512	17	1.1922	0.5767	2.7874	13
15^{th}&58^{th}	0.0567	0.4239	2.4821	11	0.9344	0.5129	2.5910	10
36^{th}&90^{th}	0.0577	0.5036	2.7578	18	1.1998	0.7008	3.1241	20
45^{th}&70^{th}	0.0625	0.4499	2.4572	14	0.9064	0.8179	3.6873	21
50^{th}&100^{th}	0.0586	0.4912	2.5457	15	1.0034	0.5485	2.6034	12
Mean	0.0596	0.4844	2.5988	15	1.0472	0.6314	2.9586	15.2

Table 5.9: CRQA of randomly selected samples data with each consisting two time series from set D

Set E	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
2^{nd}&28^{th}	0.0524	0.2830	2.4938	13	0.9465	0.4904	2.9021	20
15^{th}&58^{th}	0.0583	0.4536	2.8631	26	1.2784	0.5541	3.0436	21
36^{th}&90^{th}	0.0572	0.2993	2.3759	13	0.8013	0.3648	2.4591	13
45^{th}&70^{th}	0.0617	0.3976	2.6753	15	1.1270	0.4079	2.6783	10
50^{th}&100^{th}	0.0587	0.3822	2.7635	16	1.2052	0.4752	2.8466	14
Mean	0.0577	0.3631	2.6343	16.6	1.0717	0.4585	2.7859	15.6

Table 5.10: CRQA of randomly selected samples data with each consisting two time series from set E

In cross recurrence quantification analysis, the obtained results are almost identical with the previous section. Despite that EEG of epileptic patients are indistinguishable in recurrence rate measure, all the other involved measures indicate noticeable differences in value. The determinism and average diagonal line length are higher in patients' EEG than in healthy people's EEG. L_{max} and V_{max} are again obviously larger in EEG of patients which means they are less in randomness. Entropy means of the latter 3 sets are still larger than the formal 2 sets although here the average entropy in set C does not exceed 1. Laminar states and the average time each state is trapped are also more in patients' EEG than the norms' as LAM and TT values are greater at the formal in this CRQA.

5-2-3 Multi-dimensional Recurrence Quantification Analysis of EEG

To analyse the EEG data, here we separate the five sets of data into two groups, i.e. sets A and B in one group, sets C, D and E in one group. The 1^{st}, 25^{th}, 55^{th}, 75^{th} and 95^{th} data are the selected random samples from each sets to calculate the multi-dimensional recurrence quantification. The multi-dimensional time series are as stated in Sec(5-1-3).

Sets A&B	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
1^{st}	0.0611	0.3125	2.3652	11	0.7928	0.4192	2.5291	12
25^{th}	0.0609	0.3282	2.3513	9	0.7741	0.4430	2.4944	10
55^{th}	0.0568	0.2379	2.2398	8	0.6078	0.3222	2.3198	8
75^{th}	0.0584	0.3075	2.3394	9	0.7580	0.4021	2.4790	13
95^{th}	0.0577	0.2751	2.3070	11	0.7124	0.3651	2.4160	9
Mean	0.0590	0.2922	2.3205	9.6	0.7292	0.3903	2.4477	10.4

Table 5.11: Multi-dimensional RQA of randomly selected samples data from both sets A&B

Sets C&D&E	RR	DET	L	L_{max}	$ENTR$	LAM	TT	V_{max}
1^{st}	0.0532	0.4696	2.6433	15	1.0994	0.6034	2.2882	13
25^{th}	0.0603	0.4683	2.6511	16	1.1061	0.6016	2.8472	14
55^{th}	0.0514	0.4194	2.5031	17	0.9578	0.5504	2.7364	10
75^{th}	0.0572	0.4471	2.4305	11	0.8748	0.5868	2.6585	10
95^{th}	0.0633	0.6099	2.8024	16	1.2382	0.7568	3.1025	15
Mean	0.0571	0.4829	2.6061	15	1.0553	0.6198	2.8453	12.4

Table 5.12: Multi-dimensional RQA of randomly selected samples data from all sets C&D&E

From the above two tables, we can see that all the RQA measures have greater value on the EEG data of epileptic patients than healthy people except the recurrence rate. However, there are some very significantly different in value measures such as entropy, longest diagonal line and longest vertical line. The entropy values of the sample data from patients group are around 1 whereas those from normal people group are only approximately 0.7. This suggests that the unpredictability of fluctuation is higher in the normal people's EEG time series. L_{max} and V_{max} values of normal EEG are about 10 but those of patients' EEG are larger. The acquired results are similar to the previous two sections, which means we may conclude that if a person's EEG data have abnormally high value compared to ordinary individuals in most of the RQA measures, the person is potentially suffered from epilepsy which may cause epileptic seizures.

CHAPTER 6: CONCLUSION

In conclusion, recurrence analysis technique is capable of providing some indications on whether one possibly suffers epilepsy. By reviewing various type of recurrence plots plotted on Sec(5-1), we are able to make some inferences on commonly how to define an epileptic patient when we own the EEG data of the patient. Recurrence plots that appear to be periodic have a greater possibility to represent that the particular person possesses epilepsy than to represent a normal person. Furthermore, the EEG plot of patients may exist longer vertical and diagonal lines. On the other hand, homogenous EEG RPs usually indicate a healthy person. This implies that EEG of a normal person consists more randomness than epileptic patients as the brain of patients may generate and send more signal with periodic patterns as a normal person will do. This also corresponds to that epileptic patients' brain produce more epileptical signals from their brain nerve cells than the normal. The effects brought about are that their brains will abnormally work faster and they will feel more tensed than normal people.

Turning to recurrence quantification analysis on EEG, measures including determinism, longest diagonal line, entropy, laminarity, trapping time, and longest vertical line can be different in EEG of epileptic patients and healthy people. As shown in Sec(5-2), we may conclude that the higher the values of these measures, the larger the probability that epileptic seizures may occur to a particular individual. This also indicates that the EEG of epileptic patients can be recognized as a periodic time series whereas the normal EEG has less predictability. Other than that, more laminarities will exist in patient's EEG. Hence, to determine a new EEG data series, we may assure that a patient will be diagnosed as suffering epilepsy if the recurrence quantification analysis on his/her EEG data meets the conditions mentioned.

The prospective of this project is to apply the methodology used to other applications, for instance, the human-machine interface (HMI) based on EEG, EEG game control and dream recorder. This methodology may help to convert human brain signals to computer signals so that a controller based on the brain current flows can be developed.

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APPENDIX A: MATLAB CODES

Code of Fig(1.1a) and Fig(1.1b)

```
x = load('lorenz.dat');
a = x(1:4000,2); b = x(1:4000,3); c = x(1:4000,4);
X = crp(y(:,1),3,4,5,'euc','nonorm');
phasespace(a,b,c)
```

Code of Fig(2.1)

```
sigma = 10; beta = 8/3; rho = 28;
f = @(t,a) [-sigma*a(1) + sigma*a(2); ...
            ...rho*a(1) - a(2) - a(1)*a(3);...
            ...-beta*a(3) + a(1)*a(2)];
[t,a] = ode45(f,[0 100],[1 1 1]);
plot3(a(:,1),a(:,2),a(:,3),'red')
figure;
subplot(3,1,1); plot(a(:,1),a(:,2),'blue')
subplot(3,1,2); plot(a(:,1),a(:,3),'green')
subplot(3,1,3); plot(a(:,2),a(:,3),'magenta')
```

Code of Fig(2.2) and Fig(2.3)

```
x=load('roessler.dat');
t = x(1:5:900,1); y = x(1:5:900,2);
crp(y(:,1),3,4,2,'euc','nonorm');
a = x(1:902,2);  b = x(1:902,3);  c = x(1:902,4);
phasespace(a,b,c)
```

Code of Fig(3.1)

```
x=sin(2*pi*linspace(1,11,2000));crp(x)
```

Code of Fig(3.2)

```
x=cos(2*pi*linspace(1,11,2000));crp(x)
```

Code of Fig(3.3)

```
x=load('cycles.dat'); crp(x,'distance')
```

Code of Fig(3.4)

```
a = sin((1:1000) * 2 * pi/67);
b = sin(.01 * ([1:1000] * 2 * pi/67) .^ 2);
crp(a,b,'distance')
```

Code of Fig(3.5) and first row results in Table(3.1)

```
a=load('Z030.txt'); b=load('Z080.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y,1,1,1)
```

Code of Fig(3.6) and second row results in Table(3.1)

```
a=load('O030.txt'); b=load('O080.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y,1,1,1)
```

Code of Fig(3.7) and third row results in Table(3.1)

```
a=load('N030.txt'); b=load('N080.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y,1,1,1)
```

Code of Fig(3.8) and fourth row results in Table(3.1)

```
a=load('F030.txt'); b=load('F080.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y,1,1,1)
```

Code of Fig(3.9) and last row results in Table(3.1)

```
a=load('S030.txt'); b=load('S080.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y,1,1,1)
```

Sample code of RPs in Sec(5-1-1) and RQA results in Sec(5-2-1)

```
a=load('S030.txt');
x=a(1:4096,1);
crp(x,1,1,1)
crqa(x)
```

Sample code of CRPs in Sec(5-1-2) and CRQA results in Sec(5-2-2)

```
a=load('S002.txt'); b=load('S028.txt');
x=a(1:4096,1); y=b(1:4096,1);
crp(x,y,1,1,1)
crqa(x,y)
```

Sample code of 2D visualisation plots in Sec(5-1-3)

```
a=load('Z095.txt');  b=load('O095.txt');
x=a(1:4096,1); y=b(1:4096,1);
plot(x,y)
```

Sample code of 3D visualisation plots in Sec(5-1-3)

```
a=load('N095.txt');  b=load('F095.txt');
c=load('S095.txt');
x=a(1:4096,1); y=b(1:4096,1); z=c(1:4096,1);
plot3(x,y,z)
figure;
subplot(3,1,1); plot(x,y,'red')
subplot(3,1,2); plot(y,z,'green')
subplot(3,1,3); plot(x,z,'blue')
```

Sample code of multi-dimensional RPs in Sec(5-1-3) and multi-dimensional RQA results in Sec(5-2-3)

```
a=load('N095.txt'); b=load('F095.txt');
c=load('S095.txt');
x=a(1:4096,1); y=b(1:4096,1); z=c(1:4096,1);
t=[1:4096]'; m=[t,x,y,z];
crp(m,1,1,1)
crqa(m)
```