

**OPTIMISATION OF LEAF SPRING DEFLECTION
USING DYNAMIC CHARACTERISTICS**

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**A project report submitted in partial fulfilment of the
requirements for the award of Bachelor of Engineering
(Hons.) Mechanical Engineering**

**Faculty of Engineering and Science
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April 2011

DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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Specially dedicated to
my beloved father and mother

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Praise to the Lord, for He is my good shepherd!

OPTIMISATION OF LEAF SPRING DEFLECTION USING DYNAMIC CHARACTERISTICS

ABSTRACT

Taguchi robust design is an important engineering methodology which renders a product or a process insensitive to the effects of variability and improves the performance at low cost including in static and dynamic characteristics. Dynamic characteristic is also known as the “Signal-Response System”. Dynamic characteristics of an experimental design can be implemented by extending the static characteristics of an experiment with another signal level. Besides that, it involves a two steps optimisation procedure, in which initially the variation around a linear function is minimised, and secondly the sensitivity of the linear function is adjusted to a target value. The goal in a dynamic characteristics experiment is to find the control factors that make the response least sensitive to noise and also the control factors that will give a unit gradient between different signal and the response. In this research, the semi-elliptical leaf spring equation is used to demonstrate the principal of dynamic characteristics by using computer simulation. The deflection of the semi-elliptical leaf spring is optimised over a range of applied load. The sensitivity of the signal-response relationship is adjusted to a target value by an appropriate setting of control factors and their levels. Lastly, control factors that largely affect the sensitivity and variability will be selected as the optimum condition. The result will show an improvement in quality loss after the optimisation.

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LIST OF SYMBOLS / ABBREVIATIONS

| | |
|-----------|--|
| ANOM | Analysis of mean |
| ANOVA | Analysis of Variance |
| CAPD | Computer aided parameter design |
| DOE | Design of experiment |
| NPM | Noise performance measure |
| OA | Orthogonal array |
| P-diagram | Parameter Diagram |
| PM | Performance measure |
| S/N ratio | Signal-to-noise ratio |
| TPM | Target performance measure |
| CI_M | Confidence interval for mean |
| CI_F | Confidence interval for factor level |
| CI_P | Confidence interval for predicted mean |
| CI_C | Confidence interval for confirmation |
| A_o | Loss |
| b | Width of leaf spring |
| D_o | Specification limit |
| E | Young's modulus |
| F | F ratio |
| L | Span of leaf spring |
| L_{avg} | Average quality loss |
| \log | Logarithm base 10 |
| M | Input signal |
| MSE | Mean square error |
| N | Number of graduated-length leaves |
| n | Number of full-length leaves |
| n | Number of observation |

| | |
|----------------------|---|
| n_{eff} | Effective number of observation |
| r_o | Trial number of the outer orthogonal array |
| R | Sensitivity of linear function |
| S | Standard deviation |
| S_{error} | Error sum of squares |
| Sm | Sum of squares due to mean |
| ST | Total sum of squares |
| St | Total sum of squares of corrected data |
| t | Thickness of individual leaf spring |
| $v1$ | Degree of freedom associated with a mean |
| $v2$ | Degree of freedom for the pooled error variance |
| V_{error} | Pooled error variance |
| v_{error} | Degree of freedom of error sum of squares |
| v_m | Degree of freedom due to mean |
| v_T | Degree of freedom of all data |
| v_t | Degree of freedom of corrected data |
| x | Deflection of leaf spring |
| y | Output response |
| \bar{y} | Mean |
| W | Applied Load |
| α | Risk |
| β | Slope of regression line |
| β_t | Target slope |
| $\hat{\beta}$ | Estimated slope |
| s^2 | Estimated mean squares error |
| μ | Mean |
| $\mu_{predicted}$ | Predicted mean |
| $\mu_{confirmation}$ | Mean for confirmation of experiment |
| ρ | Percentage of contribution |
| ε | Error or y-intercept of graph |

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CHAPTER 1

INTRODUCTION

1.1 Background of DOE

Design of experiment (DOE) is a statistical technique first introduced by Sir R. A. Fisher in England in the early 1920s. Sir R. A. Fisher was born in England on February 17, 1890. He was a statistician, evolutionary biologist, eugenicist and geneticist. Fisher's first goal of applying DOE was to produce the best crop by determines the optimum condition of water needed, rain, sunshine, fertilizer, and so on. He laid out all combinations of the factors included in experiment study. The condition were created using matrix. It allowed each factor an equal number of test conditions. When the number of combinations possible became too large, schemes were devised to carry out a fraction of the total possibilities such that all factors would be evenly present.

After Fisher introduced the technique and demonstrated its use in agricultural experiments, much more research and development followed. However, much of the work remained in the academic environment and only few segments of the chemical and fertilizer industries have applied the DOE technique in their production processes. In Fisher's book *The Design of Experiments* (1935), he illustrated the most important ideas of experimental design including comparison, randomisation, replication, blocking, orthogonal, and factorial experiments (Ranjit, 2001).

1.2 Genichi Taguchi and Taguchi Method

Genichi Taguchi is a Japanese engineer and statistician born January 1, 1924, in Tokamachi, Japan. He spent much of his professional life in developing methods of enhancing the quality of manufactured products. The quality engineering method that Taguchi proposed is commonly known as Taguchi methodology or Taguchi techniques. This method has been applied to engineering, biotechnology, marketing and advertising, and so on. Taguchi's concept was adopted by many companies, including Toyota, Fuji Film, Nippon Denso, and other Japanese firms. Taguchi's work includes three principles contributions to statistics which are the Taguchi loss function, the philosophy of off-line quality control, and the innovations in the design of experiments (Ranjit, 2001).

1.3 Design of Experiment via Orthogonal Array

Since the introduction of DOE of by Fisher in the 1920s, Taguchi started to research with it in 1940s. At that time, there was much development with this statistical technique, but its implementation in industry was rare. Thus, Taguchi proposed a way to define quality in general terms to show that DOE could be used not only to improve quality, but also to quantify the improvements made in terms of saving costs. Besides that, to make the technique easier and friendlier to apply, he standardised the application method and created a number of special orthogonal arrays. After that, he introduced a new way to analyse the results. His use of the signal-to-noise ratio for analysis of repeated results helps experimenters easily assure a design that is immune to the influence of uncontrollable factors.

Taguchi's designs aimed to allow greater understanding of variation than did many of the traditional designs from the analysis of variance. The experimental design proposed by Taguchi involves using orthogonal arrays to organise the parameters that affecting the process and the levels at which they should be varied. Compared to the factorial design which includes all possible combinations, Taguchi

methodology tests pair of combinations. This method allows the collection of the necessary data to determine which factors most affect product quality with a minimum amount of experiment, thus saving time and resources.

The orthogonal array designed by Taguchi can be derived or looked up. The small arrays can be drawn out manually while the large arrays can be derived from deterministic algorithms. The selection of arrays based on the number of parameters and the number of levels. Analysis of variance on the collected data from the Taguchi design of experiments can be used to select new parameter values to optimise the performance characteristics. Finally, the data from the arrays can be analysed by plotting the data and performing a visual analysis, ANOVA, or ANOM to test significance.

1.4 Advanced Optimisation Methodology

The methodology of design of experiment has been improved from time to time. Advance optimisation methodology such as Optimisation of Dynamic Characteristic, Sequential Experiment, and Central Composite Design enable different ways of conducting and analysing the experiment. In this research, the optimisation of dynamic characteristics with a particular function is conducted by computer simulation. As compare to the ordinary static optimisation which is only evaluated at one signal level, the optimisation using dynamic characteristic conducted at several signal levels is considered a better optimisation strategy (Belavendram, 2010). For example, the optimisation of a cantilever beam for an applied load of 50kN is considered a static characteristic. When the same system is optimised across a range of applied loads, 30N, 50N and 90N then the optimisation is said to be a dynamic characteristic. Subject to limited resources available for conducting the experiment, parameter optimisation aided by computer simulation is a powerful method of parameter design in advance methodology. Such design is commonly known as the Computer Aided Parameter Design (CAPD).

1.5 Application of DOE in Industry

Today, design of experiment (DOE) is used in many industrial sectors, for instance, in the development and optimisation of manufacturing processes. Typical examples include the production of wafers in the electronics industry, the manufacturing of engines in the car industry, and the synthesis of compounds in the pharmaceutical industry. The areas where DOE is used in industrial research, development and production including optimisation of manufacturing processes, optimisation of analytical instruments, screening and identification of important factors, robustness testing of products and methods, and formulation of experiments.

1.6 Aims and Objectives

In this study entitle “Optimisation of Leaf Spring Deflection using Dynamic Characteristics”, the main aims are:

- To study the advance methodology of experimental design using computer simulation.
- To learn the optimisation of dynamic characteristics for an objective function using L_{18} orthogonal array.

The main objectives are:

- To determine the significant factors those affect the deflection of the semi-elliptical leaf spring.
- To analyse the optimum condition of the semi-elliptical leaf spring design based on the regression analysis.
- To predict the value of deflection of the semi-elliptical leaf spring at the optimum condition over a range of signal factor levels.

CHAPTER 2

LITERATURE REVIEW

2.1 Procedure of Taguchi Method: The 5 Basic Phases

Ranjit (2001) stated that there are five basic phases in applying the Taguchi experimental design technique using orthogonal array to a project and those steps are important to follow closely. These five phases are planning, designing, conducting, analysing, and confirming.

2.1.1 Planning Phase

The planning phase includes all decisions about the project, such as the objective, the quality characteristic, the measurement method, the problems involved (either static or dynamic). Besides that, all factors involved must be identified and their levels must be specified in order to move forward to the designing phase.

The relationship of all factors involved in a function/process with response can be illustrated by Parameter Diagram (P-Diagram). P-Diagram is used to classify the functionality associated with the intended product in the boundary into control factors, noise factors, input signals, and output functions. P-Diagram is a must for every development project and a way of succinctly defining the development scope. In order to construct the P-Diagram, the signal input and the output associated with the design concept are identified. After that, the factors that are beyond the control of

the designer are considered. Those factors are known as noise factors. The examples of noise factors are extreme environmental conditions and manufacturing variability (Phadke, 2010). A P-Diagram is shown in Figure 2.1.

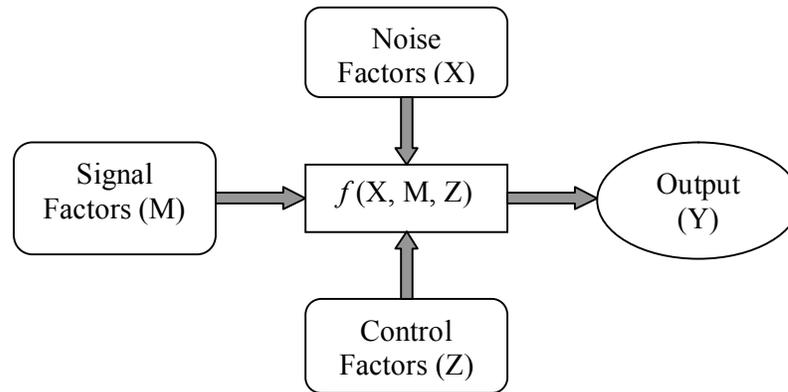


Figure 2.1: Parameter Diagram

2.1.2 Designing Phase

In the following designing phase, an orthogonal array block is constructed based on the factors and levels specified in the planning session. The design block is a combination of inner array and outer array. Normally, the inner array is the combination of control factors setting while the outer array is the combination of noise factors setting. After that, an experiment is designed that readily specifies the number of trials and how each of the trials will be carried out.

2.1.3 Conducting Phase

After the experiments are designed, they are conducted following the exact design combination prescribed and in the order required by the layout statistical validity. Experiments can be conducted either manually or by computer simulation. Parameter design using computer simulation is known as Computer Aided Parameter Design

(CAPD) (Belavendram, 2010). By using computer simulation, it reduces development cost instead of huge expenses on every trial experiment. Experiment conducted in the consideration of the presence of noise, environmental and uncontrollable is known as robust design. The aim of the robust design is to adjust the control factor settings such that the system becomes less sensitive to variations in the noise effects (Tsai, 2009).

2.1.4 Analysing Phase

The results are analysed under standardised procedures to determine information about the project under study. The primary goal of analysis is to obtain information about the new design condition and an estimate of the improvement expected. In dynamic robust design, there are four important analyses to be done, analysis of target performance measure (TPM), analysis of noise performance measure (NPM), analysis of variance (ANOVA), and regression analysis. Besides that, quality loss function is a continuous function that is defined in terms of the deviation of a design parameter from an ideal or target value (Simpson, 2000). Quality loss function is carried out to determine the improvement of loss between the current condition and the optimised condition.

2.1.5 Confirming Phase

The optimum condition predicted by the analysis might not be the one that currently exists or one that has been tested. Verification is done to determine how close the estimate matches actual performance (Ranjit, 2001).

2.1.6 Procedure of Taguchi Analysis: Flow Chart

Tsai et al. (2009) in the research of *Robust Design of SAW Gas Sensors by Taguchi Dynamic Method* proposed the following flow of Taguchi analysis:

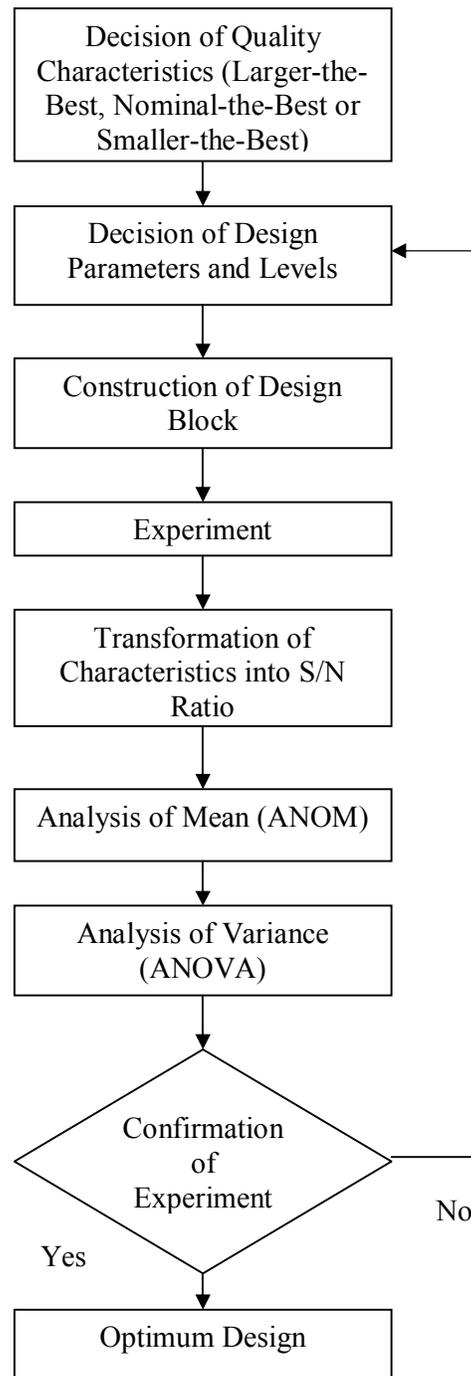


Figure 2.2: Flow Chart of Taguchi Analysis

2.2 Factorial Design vs. Orthogonal Array

The technique of laying out the conditions of experiments involving multiple factors was first introduced by Sir R. A. Fisher, in 1920s (Ranjit, 2001). The method is known as factorial design of experiments and it involves all possible combinations of relevant factors. For example, consider a full factorial design of experiment that consists of three variables with each has two levels. It requires $2^3 = 8$ experiments, as shown in Table 2.1.

Table 2.1: Full Factorial Experiment Table

| Experiments | A | B | C |
|-------------|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 |
| 3 | 1 | 2 | 1 |
| 4 | 1 | 2 | 2 |
| 5 | 2 | 1 | 1 |
| 6 | 2 | 1 | 2 |
| 7 | 2 | 2 | 1 |
| 8 | 2 | 2 | 2 |

In industrial experiments, it may involve a significant number of variables; a full factorial design result that may involve a large number of experiments is not ideally applicable. As compare to full factorial design, an L_4 orthogonal array as shown in Figure 2.3 that consists of three variables only requires 4 experiments.

| Experiments | A | B | C |
|-------------|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 |
| 4 | 2 | 2 | 1 |

Figure 2.3: An $L_4 (2^3)$ Orthogonal Array

Taguchi's orthogonal array is said to be superior to factorial design method since the factorial design method is not efficient in handling large number of factor variables. Besides that, the orthogonal array table allows determination of the contribution of each factor to the output in which factorial design does not. Furthermore, Taguchi method applied on a product design yield similar and

consistent results. Lastly, it allows easy interpretation of experiments with a large number of factors.

Generally, the larger the number of factors, the greater is the reduction from the total possibilities by using orthogonal array. For example, an L_{16} orthogonal array that consists of 15 factors all at two levels requires only 16 experiments instead of 32,768 experiments. The common experimental situations and the corresponding experimental configurations are shown in Table 2.2.

Table 2.2: Various Experimental Situation and Corresponding Experiment Size

| Experimental Situation | Maximum Possible Condition | Size of Taguchi Experiments |
|------------------------|----------------------------|-----------------------------|
| 3 two-level factors | 8 | 4 |
| 7 two-level factors | 128 | 8 |
| 11 two-level factors | 2,048 | 12 |
| 15 two-level factors | 32,768 | 16 |
| 4 three-level factors | 81 | 9 |
| 7 three-level factors | 2,187 | 18 |

2.3 Properties of Orthogonal Arrays

Orthogonal arrays designated by the notation L (L for Latin squares) with a subscript. The subscript refers to the number of rows in the table, which indicates the number of combinations the design will prescribe (Ranjit, 2001).

2.3.1 An L_8 Orthogonal Array

Generally, an orthogonal array is selected based on the requirement of factors and its levels included in the experiments. For an L_8 (2^7) orthogonal array, it stands for 8 experiments (trials) and 7 factors with each two levels. This array thus has 8 rows and 7 columns. Each row represents an experiment with factor levels indicated by the numbers (1 or 2) in the rows.

| Experiments | Column No. | | | | | | |
|-------------|------------|-------|----|-------|----|----|-----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 |
| | A | B | AB | C | AC | BC | ABC |
| | Grp 1 | Grp 2 | | Grp 3 | | | |

Figure 2.4: An $L_8(2^7)$ Orthogonal Array

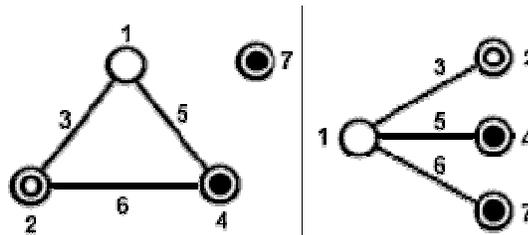
In an L_8 orthogonal array as shown in Figure 2.4, each column contains four levels of ones (1) and four levels of two (2). In such condition, both levels can be combined in four possible ways, such as (1, 1), (1, 2), (2, 1) and (2, 2), with 27 possible combinations of levels. The columns are said to be orthogonal, since the combination of the both levels occurred the same number of times. This array is a good choice for control factors array since it includes more factors for determination of the contribution of each factor to the result. However, it is not a good choice for noise factor array. Furthermore, an L_8 orthogonal array can be used to determine interactions. The term interaction is used to describe a condition in which the influence of one factor upon the result is dependent on the condition of another. For example, column 3 involves the interaction of array A and B. These two factors are said to interact when the effect of changes in level A, determines the influence of B and vice versa. That is, the changes of A and B are not monotonic uniform.

Generally, each orthogonal array has a particular set of linear graphs and a triangular table associated with it. The Triangular Table of Interaction presents information about which columns interact. For an example of triangular table for L_8 orthogonal array as shown in Table 2.3, to find in which column the interaction between columns 4 and 6 will appear, move horizontally across (4) and vertically from (6), the intersection is “2” in the table. That means the interaction effects between columns 4 and 6 will appear at column 2.

Table 2.3: Interaction between Two Columns in an L_8 Orthogonal Array

| Column: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------|---|----------|---------------|--------------------|-------------------------|------------------------------|-----------------------------------|
| | 1 | 3 (2) | 2 1 (3) | 5 6 7 (4) | 4 7 6 1 (5) | 7 4 5 2 3 (6) | 6 5 4 3 2 1 (7) |

For further enhancement of the efficiency of the experiment, Linear Graphs are created based on the triangular tables. Linear graphs are made up of numbers, dots and lines as shown in Figure 2.5 below. A factor is identified as a dot and a number while a connecting line between two dots indicates interaction. The number assigned to the line indicates the column number in which interaction effects will be confounded.

**Figure 2.5: Two Linear Graphs of L_8 Orthogonal Array**

2.3.2 An L_4 Orthogonal Array

An L_4 (2^3) orthogonal array as shown in Figure 2.6 consists of four rows (experiments) and three columns (factors). Similar to L_8 orthogonal array, each of the factors also consists of two levels (1 and 2). This array is not a good choice for control factors since fewer factors involved for the determination of contribution. However, this is a good choice for noise factor array. The interactions of the factors can be determined in this array in which the third column is under the interaction of the first two columns. This can be expressed by a linear graph as Figure 2.7.

| Experiments | Column No. | | |
|-------------|------------|-------|----|
| | 1 | 2 | 3 |
| 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 |
| 3 | 2 | 1 | 2 |
| 4 | 2 | 2 | 1 |
| | A | B | AB |
| | Grp 1 | Grp 2 | |

Figure 2.6: An $L_4 (2^2)$ Orthogonal Array

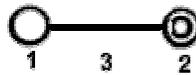


Figure 2.7: A Linear Graph of L_4 Orthogonal Array

2.3.3 Other Orthogonal Arrays

The other orthogonal arrays for number of experiments from 4 to 27 with their descriptions are shown in Table 2.4.

Table 2.4: Properties of each Orthogonal Array

| OA | Descriptions |
|-------------------|--|
| $L_4 (2^3)$ | A full factorial array of 2 test factors and an interaction between them. It has 1 linear graph. |
| $L_8 (2^7)$ | A very flexible array that allows 7 factors and interactions to be studied. It has 2 linear graphs. |
| $L_9 (3^4)$ | It allows for 4 factors or 2 factors and one interaction. It has 1 linear graph. |
| $L_{12} (2^{11})$ | Its uniqueness is that the effects of interactions are distributed evenly across all columns. This minimizes the possibility of confounding effects of factors and interactions, but eliminates the ability to study interactions. |

| | |
|---------------------------|--|
| $L_{16} (2^{15})$ | It allows 15 factors and interactions to be studied. It is often used in place of the $L_{12} (2^{11})$ array when you need to study interactions. It has 18 linear graphs. |
| $L_{18} (2^1 \times 3^7)$ | The first column in this array is 2 level and the other 7 are 3 levels. It is similar to $L_{12} (2^{11})$ in that interactions are distributed evenly across all columns except columns 1 and 2. The interaction between factors assigned to column 1 and column 2 can be studied using a 2x3 response table and does not require use of another column for interaction. It has 1 linear graph. |
| $L_{27} (3^{13})$ | It allows for 13 factors or 7 factors and 3 interactions or 5 factors and 4 interactions. It has 3 linear graphs. |

2.3.4 Inner and Outer Arrays

To implement robust design, Taguchi advocates the use of an “inner array” and “outer array” approach. Thus, an experiment is conducted by extending it with an “outer array” to stimulate the random environment in which the product would function. The “inner array” consists of the OA that contains the control factor settings; the “outer array” consists of the OA that contains the noise factor settings which are under investigation. The combination of the “inner array” and “outer array” constitutes what is called the “product array” or “complete parameter design layout.” This approach provides full information on control-by-noise interactions.

| | | | | | | | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | | | | | | | | R | 1 | 2 | 2 | 1 |
| | | | | | | | | Q | 1 | 2 | 1 | 2 |
| | | | | | | | | P | 1 | 1 | 2 | 2 |
| | | | | | | | | G | 1 | 2 | 3 | 4 |
| 1 | A | B | C | D | E | F | G | 1 | y1,1 | y1,2 | y1,3 | y1,4 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | y2,1 | y2,2 | y2,3 | y2,4 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | y3,1 | y3,2 | y3,3 | y3,4 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | y4,1 | y4,2 | y4,3 | y4,4 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 2 | y5,1 | y5,2 | y5,3 | y5,4 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 1 | y6,1 | y6,2 | y6,3 | y6,4 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | y7,1 | y7,2 | y7,3 | y7,4 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | 2 | y8,1 | y8,2 | y8,3 | y8,4 |

Figure 2.8: A Layout of Combination of Inner and Outer Array

2.4 Taguchi's Robust Parameter Design Methodology

Taguchi methods have been used for improving the quality of Japanese products since 1960 with a great success. Twenty years later, many companies realised that the old methods for ensuring quality they used were not competitive with the Japanese methods. It is because of the old methods they used for quality assurance relied heavily upon inspecting products as they rolled off the production line and rejecting those products that did not fall within a certain acceptance range. Taguchi quickly pointed out that the quality must be designed into a product from the start but not depended on the inspection to improve the quality. After that, Taguchi's robust design approaches were adopted by companies in the United States and Europe in an effort to improve product quality and design robustness (Simpson, 2000).

Robust design is an “engineering methodology for improving productivity during research and development so that high-quality products can be produced quickly and at low cost” (Phadke, 1989). To achieve desirable product quality by design, Taguchi suggests a three-stage process: system design, parameter design, and tolerance design. Simpson et al. (2000) further explained that the system design is the conceptualisation and synthesis of a product or process to be used. The improvement of quality at this level requires innovation and related technical knowledge. Parameter design is related to finding the appropriate design factor levels to make the system less sensitive to variations in uncontrollable noise factors. This design stage is the main thrust of Taguchi's approach and the design system is said to be made of “robust.” Consequently, the product or process performs better, reducing the quality loss. The final step is tolerance design; tolerance design occurs when the tolerances for the product or process are established to minimise the sum of the manufacturing and lifetime costs of the product or process. Indeed, only tolerances of factors that have the largest influence on the variation are adjusted. It is because of tightening the tolerances increases the cost of the product or process due to better requirements of materials, components, or machinery (Simpson, 2000).

The robust parameter design methodology can be found in the book of *System of Experimental Design* (1987) by Taguchi. This method provides a way of improving the quality of a product or process by making it less sensitive to factors

which cause variability. The application for robust parameter design methodology can be divided into two types: “static characteristics” and “dynamic characteristics.” The static characteristic application is also known as a simple response system. It involves the situations where the quality characteristic of interest (response) is a single quantity Y , which has a specified optimal value. For example, the optimisation of a paper gyrocopter at single signal factor (height of release it) such that the response (flight time) is specified at 4 second. On the other hand, the “dynamic characteristics” is also called a signal-response system (A. Miller and C. F. J. Wu, 1996). It describes the situations where the response is required to assume different values as a result of changes in a signal factor M . For example, the optimisations of a paper gyrocopter at several signal levels to obtain different response. The performance of this type of system is evaluated by considering the relationship between the response and the input signal.

2.5 Static Characteristics

Before start designing the experiment, types of the problem (either static or dynamic characteristics) must be identified. All factors included in a process or product might not have the same level of importance; some of it can be neglected. In fact, a process to be optimised would have several main factors that affect that output which is known as the significant factors. In static characteristics, the control factors directly decide the desired value of the output. The noise factor is uncontrolled factor that always presents in the process and causes variation. The input signal is fixed at a specific value without considering the variation that might happen over a range of input signal.

2.5.1 Design Layout for Static Characteristics

The design layout of static characteristics is a combination of inner arrays (control factors) and outer arrays (noise factors). Figure 2.9 is an example of L_8 design layout for static characteristics.

| | A | B | C | D | E | F | G | 1 | 2 | 3 | 4 | TPM | NPM |
|---|---|---|---|---|---|---|---|------|------|------|------|-------|-------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | y1,1 | y1,2 | y1,3 | y1,4 | ln y1 | ln s1 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | y2,1 | y2,2 | y2,3 | y2,4 | ln y2 | ln s2 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | y3,1 | y3,2 | y3,3 | y3,4 | ln y3 | ln s3 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | y4,1 | y4,2 | y4,3 | y4,4 | ln y4 | ln s4 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | y5,1 | y5,2 | y5,3 | y5,4 | ln y5 | ln s5 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | y6,1 | y6,2 | y6,3 | y6,4 | ln y6 | ln s6 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | y7,1 | y7,2 | y7,3 | y7,4 | ln y7 | ln s7 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | y8,1 | y8,2 | y8,3 | y8,4 | ln y8 | ln s8 |

Figure 2.9: A L_8 Design Layout of Static Characteristics

2.5.2 Static S/N Ratio

The Target Performance Measure (TPM) of the static characteristic is normally the mean of the response from each trial. The Noise Performance Measure (NPM) seems to be more complicated which analysed using a log function called Signal-to-Noise ratio (S/N). The log function of desired output is used as objective functions for optimisation. The selecting of the S/N ratio is depended on the quality characteristic desired. There are three standard types of S/N ratios of common interest for optimisation of static problems (Phadke, 1989):

1. Smaller-the-Better (for making the system response as small as possible):

$$S / N_s = -10 \log \left(\frac{1}{n} \sum_{i=1}^n y_i^2 \right) \quad (2.1)$$

2. Larger-the-Better (for making the system response as large as possible):

$$S / N_L = -10 \log \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right) \quad (2.2)$$

3. Nominal-the-Best (for reducing variability around a target):

$$S / N_N = 10 \log \left(\frac{\bar{y}^2}{S^2} \right) \quad (2.3)$$

where

n = observations

y_i = i th response

\bar{y} = mean response

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i \quad (2.4)$$

S = standard deviation

$$S = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}} \quad (2.5)$$

2.6 Dynamic Characteristics

Compare to static characteristics, dynamic characteristics involve several signal levels that directly decide the output with the consideration of all the best control factors levels. Thus, a relationship between the input signal and output response is desirable. The examples involve such relationship are the volume control in audio amplifiers and the accelerator paddle in cars.

2.6.1 Dynamic Robust Design

Tsai et al. (2009) in his research agreed that a robust measurement system has the following capabilities:

- 1) It minimises variability as the input signal changes,
- 2) It provides consistent measurements for the same input,
- 3) It continues to give an accurate reading as the input values changes,
- 4) It adjusts the sensitivity of the design in transforming the input signal into an output, and
- 5) It is robust to noise.

The introduction of noise effect in the system causes the observed value to deviate slightly from the true value. Therefore, when designing the measurement system, it is necessary to develop a robust design with dynamic characteristics by utilising Taguchi's S/N ratio to ensure the optimum design conditions. A robust design is a design that has minimum sensitivity to variations in uncontrollable (noise) factors (Simpson, 2000). The idea behind robust design is to improve the quality of a product/process by minimising the effects of variation without eliminating the causes, since they are uncontrollable.

Tsai et al. (2009) stated that a dynamic study involves a two step optimisation procedure, in which initially the variation around a linear function is minimised, and secondly the sensitivity of the linear function is adjusted to a target value. In order to achieve the desired output range or to meet the target sensitivity, the sensitivity of the response (β) to the input signal value is necessary to be adjusted. This can be done by an appropriate setting of control factors and their levels. Consequently, with the linear relationship between the output response and the input signal, it provides a useful tool to adjust the output by changing the input signal level. Dynamic parameter design is also called signal-response design (Kai Yang & Basem S. El-Haik, 2009). The ideal relationship between signal and output response is illustrated in Figure 2.10.

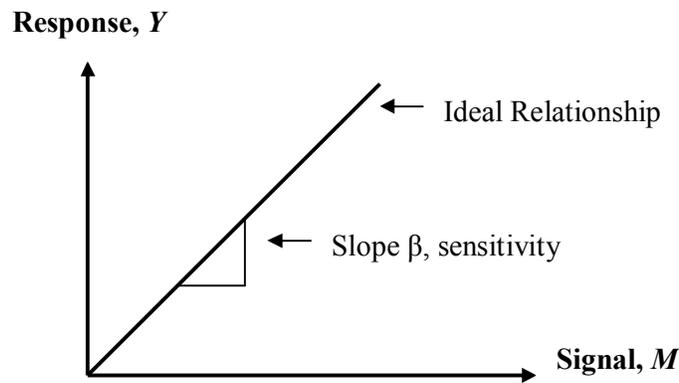


Figure 2.10: Ideal Signal-Response Relationship

2.6.2 Classification of Signal-Response Systems

A. Miller and C. F. J. Wu (1996) in their research *Parameter Design for Signal-Response System: A Different Look at Taguchi's Dynamic Parameter Design* classified the signal-response system into two common types: multiple target systems and measurement systems. They stated that the classification plays an important role in the methodological development as the choice of the performance measure used for parameter design optimisation depends on the type of system. They further defined that the multiple target system is a system whose function requires that the value of a response quantity can be adjusted by changing the level of a signal factors. That means the signal factor is used to adjust the function of the system to accommodate different target values for the response. On the other hand, a measurement system is the process used to obtain an estimate of some quantity of interest for a given unit or sample. Sampling, sample preparation and calibration may be included, as well as the actual measurement process. The true amount of the quantity present can be considered as an input signal M which the system converts into a measured value or response Y . Thus, the precision with which M can be estimated based on Y is determined by the characteristics of the relationship between M and Y .

2.6.3 Design Layout for Dynamic Characteristic

A dynamic problem for an experimental design includes several signal factors (M1, M2...Mk). The signal factor is the “input signal” treated as an experimental factor. In the dynamic characteristics experiment design layout, the control factors are assigned to the inner array and the groups of noise factors with different signal levels are assigned to the outer array. At each signal factor level, several noise factor combinations will be attempted. Besides that, under each signal-noise combination a functional requirement, say, $y_{i,j}$, will be measured. The goal in a dynamic characteristic experiment is to find control factors that make the response least sensitive to noise and also to find control factors that will give a unit gradient (line regression) between input signal and the response. Two examples of using an L_8 orthogonal array are shown in Figure 2.11 and Figure 2.12.

| | | M1 | | | | M2 | | | | | | | | | | | |
|---|---|----|---|---|---|----|---|------|------|------|------|------|------|------|------|-------|-------|
| R | | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | | | | | | | | |
| Q | | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | | | | | | | | |
| P | | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | | | | | | | | |
| | A | B | C | D | E | F | G | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | TPM | NPM |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | y1,1 | y1,2 | y1,3 | y1,4 | y1,1 | y1,2 | y1,3 | y1,4 | ln y1 | ln s1 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | y2,1 | y2,2 | y2,3 | y2,4 | y2,1 | y2,2 | y2,3 | y2,4 | ln y2 | ln s2 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | y3,1 | y3,2 | y3,3 | y3,4 | y3,1 | y3,2 | y3,3 | y3,4 | ln y3 | ln s3 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | y4,1 | y4,2 | y4,3 | y4,4 | y4,1 | y4,2 | y4,3 | y4,4 | ln y4 | ln s4 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | y5,1 | y5,2 | y5,3 | y5,4 | y5,1 | y5,2 | y5,3 | y5,4 | ln y5 | ln s5 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | y6,1 | y6,2 | y6,3 | y6,4 | y6,1 | y6,2 | y6,3 | y6,4 | ln y6 | ln s6 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | y7,1 | y7,2 | y7,3 | y7,4 | y7,1 | y7,2 | y7,3 | y7,4 | ln y7 | ln s7 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | y8,1 | y8,2 | y8,3 | y8,4 | y8,1 | y8,2 | y8,3 | y8,4 | ln y8 | ln s8 |

Figure 2.11: A L_8 Layout of Dynamic Characteristic with Two Signal Levels

| | | M1 | | | | M2 | | | | M3 | | | | | | | |
|---|---|----|---|---|---|----|---|------|------|------|------|------|------|------|------|-------|-------|
| R | | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | | | | |
| Q | | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | | | | |
| P | | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | | | | |
| | A | B | C | D | E | F | G | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | TPM | NPM |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | y1,1 | y1,2 | y1,3 | y1,4 | y1,1 | y1,2 | y1,3 | y1,4 | ln y1 | ln s1 |
| 2 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | y2,1 | y2,2 | y2,3 | y2,4 | y2,1 | y2,2 | y2,3 | y2,4 | ln y2 | ln s2 |
| 3 | 1 | 2 | 2 | 1 | 1 | 2 | 2 | y3,1 | y3,2 | y3,3 | y3,4 | y3,1 | y3,2 | y3,3 | y3,4 | ln y3 | ln s3 |
| 4 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | y4,1 | y4,2 | y4,3 | y4,4 | y4,1 | y4,2 | y4,3 | y4,4 | ln y4 | ln s4 |
| 5 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | y5,1 | y5,2 | y5,3 | y5,4 | y5,1 | y5,2 | y5,3 | y5,4 | ln y5 | ln s5 |
| 6 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | y6,1 | y6,2 | y6,3 | y6,4 | y6,1 | y6,2 | y6,3 | y6,4 | ln y6 | ln s6 |
| 7 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | y7,1 | y7,2 | y7,3 | y7,4 | y7,1 | y7,2 | y7,3 | y7,4 | ln y7 | ln s7 |
| 8 | 2 | 2 | 1 | 2 | 1 | 1 | 2 | y8,1 | y8,2 | y8,3 | y8,4 | y8,1 | y8,2 | y8,3 | y8,4 | ln y8 | ln s8 |

Figure 2.12: An L_8 Layout of Dynamic Characteristic with Three Signal Levels

Generally, the response is expected increase linearly as the signal factor increases. Thus, a typical complete inner-array run of output response data will resemble the scatter plot as the Figure 2.13.

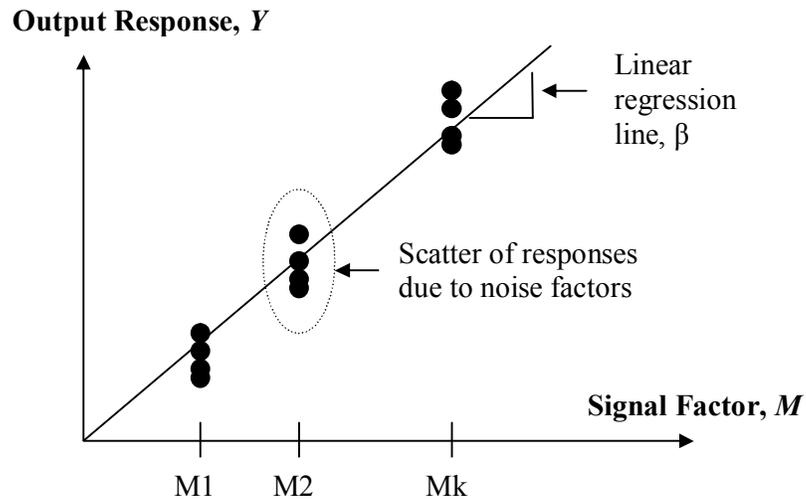


Figure 2.13: Typical Scatter Plot for a Complete Inner-array Run

2.6.4 Performance Measure of Signal-Response System

Performance measure (PM) is an important step in analysing a dynamic characteristic experiment by evaluating the suitability of a given signal-response relationship. Control factors which contribute to the desired engineering objectives (significant factors) can be identified by optimising the chosen measure. One approach to identify a suitable performance measure is to specify an ideal or target signal-response relationship and penalise for deviations from this target function (A. Miller & C. F. J. Wu, 1996). For example, suppose the target function is of the form:

$$E(Y) = \beta_i M \quad (2.6)$$

where

β_t = target slope

M = signal input

The actual signal-response relationship can be represented by:

$$Y = f(M) + \varepsilon \quad (2.7)$$

where

$$E(\varepsilon) = 0$$

$$V(\varepsilon) = \sigma^2$$

A PM can then be generated by averaging the mean square error (MSE) over a specified range for the signal factor (m_a , m_b):

$$PM = \int_{m_a}^{m_b} MSE dm = \int_{m_a}^{m_b} ([f(m) - \beta_t m]^2 + \sigma^2) dm \quad (2.8)$$

If an adjustment factor exists, the PM has to be modified. An adjustment factor affects the system in a well-understood manner so that if the signal-response function is known (or estimated) for any setting of the adjustment factor, then the signal-response function can be reliably deduced for any other level of the adjustment factor. If such a factor exists, then it make sense to evaluate the PM given the adjustment factor is set to its optimal level. Taguchi's dynamic S/N ratio assumes the existence of an adjustment factor which affects the system in the same manner as a change of scale (A. Miller & C. F. J. Wu, 1996).

In practice, suppose a set of observations are made for a fixed set of control factor levels, then the function will be:

$$E(y_{ij}) = \beta M_i \quad (2.9)$$

where

y_{ij} = j th observed response

M_i = i th signal level

Let $\hat{\beta}$ represent the least squares estimate of β and,

$$s^2 = (n-1)^{-1} \sum_i \sum_j \left(y_{ij} - \hat{\beta} M_i \right)^2 \quad (2.10)$$

where

n = total number of observations

So $\hat{\beta}$ represents the estimated slope for the best fitting linear model and s^2 represents the estimated MSE for this model (averaged over the signal levels). Assuming the target function is as equation (2.6), the adjustment factor can then be used to scale the signal-response relationship by a factor of target slope divided by the estimated slope. From this adjustment, the projected MSE would be:

$$\left(\beta_t / \hat{\beta} \right)^2 s^2 \quad (2.11)$$

where

β_t = target slope

$\hat{\beta}$ = estimated slope

s^2 = estimated MSE

Lastly, minimise the MSE will maximise the dynamic S/N ratio as below:

$$\log \left(\hat{\beta}^2 / s^2 \right) \quad (2.12)$$

2.6.5 Dynamic S/N Ratio

As stated by A. Miller and C. F. J. Wu (1996), the approach to identify a suitable performance measure is to specify an ideal or target signal-response relationship and penalise for deviations from this target function. This kind of signal-response relationship can be categorised into two types: the sensitivity of the slope and the linearity.

1. Sensitivity of the slope:

- Treated as Smaller-the-Better when the output is an undesired characteristic. The sensitivity R_S is expressed in mathematical form as follows:

$$R_S = -10 \log(\beta^2) \quad (2.13)$$

- Treated as Larger-the-Better when the output is a desirable characteristic. The sensitivity R_L is expressed in mathematical form as follows:

$$R_L = 10 \log(\beta^2) \quad (2.14)$$

where

β = slope or gradient

However, both R_S and R_L are not suitable implemented in a signal-response system since all the scattered points (response of each signal level) should fall closely to the slope of the graph. That means the response should meet the target signal-response relationship. Thus, the quality characteristic of target performance measure (TPM) for a signal-response system should be Nominal-the-Best, R_N . Its equation is the same as the sensitivity for Larger-the-Better.

2. Linearity:

- Tsai et al. (2009) stated that it is necessary to develop a robust design with dynamic characteristics by utilising Taguchi's signal-to-noise (S/N) ratio in designing the measurement system to ensure the optimum design conditions. Besides that, in considering dynamic relationship, the zero-point proportional equation provides a useful tool to adjust the output by changing the input signal factor. The relationship between the input signal, M , and output response, Y , must be truly linear.

$$Y_{ijk} = \beta_i M_j + \varepsilon_{ijk} \quad (2.15)$$

where

Y = response

M = signal factor

β = slope (gradient) of the graph

ε = error (y-intercept of the graph)

- The dynamic S/N ratio is closely related to the static case and can be expressed conceptually in mathematical as:

$$S / N_i = 10 \log \left(\frac{\beta_i^2}{MSE_i} \right) \quad (2.16)$$

The β_i is the slope as determined by the least squares method (LSM). The LSM minimises the sum of the squares of the data around a best fit and is expressed as follows:

$$\beta_i = \frac{\sum_{j=1}^J \sum_{k=1}^{r_o} y_{ijk} M_j}{r_o \sum_{j=1}^J M_j^2} \quad (2.17)$$

where

y_{ij} = j th characteristic result of the i th experiment

M_j = the j th level input signal

r_o = the experimental trial number of the outer orthogonal array

j = the level setting of the input signal

Lastly, MSE_i is the mean square error (σ^2) for the i th factor and is given by:

$$MSE_i = \frac{1}{r_o J - 1} \sum_{j=1}^J \sum_{k=1}^{r_o} (y_{ijk} - \beta_i M_j)^2 \quad (2.18)$$

2.6.6 Average Quality Loss Function

Taguchi defined a Quality Loss Function to measure the quality. The quality loss function is a continuous function that is defined in terms of the deviation of a design parameter from an ideal or target value. Normally, Taguchi's loss function can be expressed in terms of the quadratic relationship (T. W. Simpson, 2000):

$$L = k(y - m)^2 \quad (2.19)$$

where

L = loss associated with a particular parameter y

y = critical performance parameter value

m = nominal value of the parameter specification

k = constant that depends on the cost at the specification limits

The average quality loss can then be expressed as:

$$L = k[S^2 + (\mu - m)^2] \quad (2.20)$$

where

μ = average value of y for the set of parts

S^2 = variance around the average

However, this average loss function is not applicable for dynamic characteristic since a signal-response system is more complicated and it involves a linear relationship between the signal levels M and the output response Y . Thus, Pisvimol (2010) gives an expression for dynamic average loss function as below:

$$R(\theta) = E_t L(y, t) \quad (2.21)$$

where

θ = setting of controllable factors

E = expectation

L = loss function

y = response quality characteristic

t = target value which is different depending on the signal factor M

The quality loss function can be expressed as:

$$L(y, t) = \sum_{j=1}^J (y_j - t)^2 \quad (2.22)$$

where

y_j = response in the j th experimental number of noise factors

2.6.7 Recommendation for Dynamic Characteristics

A. Miller and C. F. J. Wu (1996) recommended four steps for investigating signal-response system. Firstly, identify a suitable performance measure which reflects the ability of the system to perform its designated function. Besides that, adopt a response function modelling approach. That means modelling the signal-response relationship of both control and noise factors. After that, the identified performance measure is applied to the fitted model to determine preferred settings for the control factors. Furthermore, the experiment should be designed using a two-stage strategy. First, a direct product design array is adopted for the control and noise factors. Then, for each row in this array, the signal factor is varied over a number of levels. Lastly, for each row in the control-noise array, parametric location and dispersion models are fitted for the response. The fitted parameters for these models are then modelled as functions of the control and noise factors. After that, half normal plots and regression analysis are used to identify significant effects and produce a fitted model. The performance measure is then applied to this model and preferred settings of the control factors are identified.

CHAPTER 3

METHODOLOGY

3.1 Planning Experiment

In the planning stage, the research question must be clarified and fully understood at the beginning stage and thus the problems can be identified. In an optimisation study, the current performance is always taken as comparison for improvement. Besides that, the objective of the experiment must be stated out and thus the quality characteristic of the measurement can be determined for analysing. A Cause-and-Effect diagram (fishbone diagram) and Parameter diagram are useful tools to study the factors involved in the problems. After all the relevant factors such as control factors, noise factors, and signal factor were identified, the relationship of these factors can be formed.

3.1.1 Research Question and Problem Statements

The semi-elliptical leaf spring as shown in Figure 3.1 is a very common type of leaf spring used for the suspension in automobiles. It consists of a long curved leaf called the full-length leaf (master leaf) and a certain number of small leaf called graduated leaves. A leaf spring is made of several laminated layers of leaves which one stacked on top of each other. It can also serve locating and to some extent damping as well as springing functions.

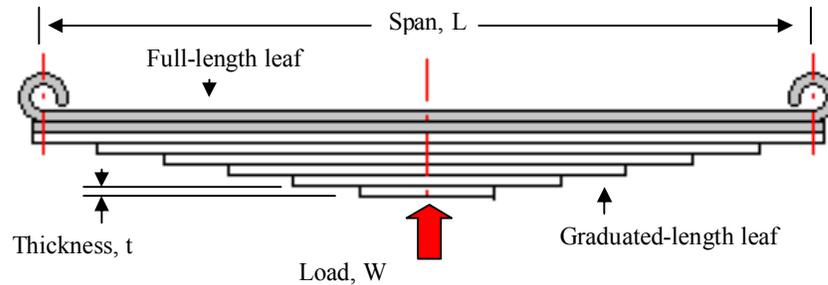


Figure 3.1: A Typical Leaf Spring Arrangement

When the leaf spring is subjected to maximum load, it becomes flat. Thus, the deflection of a leaf spring which subject to the applied load is the main concern when designing the suspension system. Higher deflection is undesired as it will occupy more spaces. Meanwhile, smaller deflection will reduce the shock absorb-ability. Thus, the deflection of the leaf spring has to be optimised over a range of values of a signal factor (applied load).

3.1.2 Objectives and Quality Characteristic

The experiment is carried out by using computer simulation. The objective function is optimised by using an L_{18} orthogonal array with dynamic characteristic. In order to meet the design requirement, the quality characteristic chosen for dynamic characteristic is Nominal-the-Best. The response is optimised to meet the target signal-response relationship where the slope (β) is equal to 1.

3.1.3 Current Level of Problem

In the computer simulation, the current conditions are represented by those 18 trials. However, only one of that is used to be compared with the optimum condition to

show the improvement. In the current condition, if the output response does not meet the target, it causes losses. Those losses are undesired in the performance and it can be calculated using quality loss function.

3.1.4 Cause and Effect Diagram

A Cause-and-Effect diagram is also known as Ishikawa diagram or fishbone diagram used in the brainstorming session to identify the cause-effect between factors related to cause. The main purpose of the Cause-and-Effect diagram is to generate a comprehensive list of possible causes at the first step in problem solving. The Cause-and-Effect diagram for this study is shown in Figure 3.2.

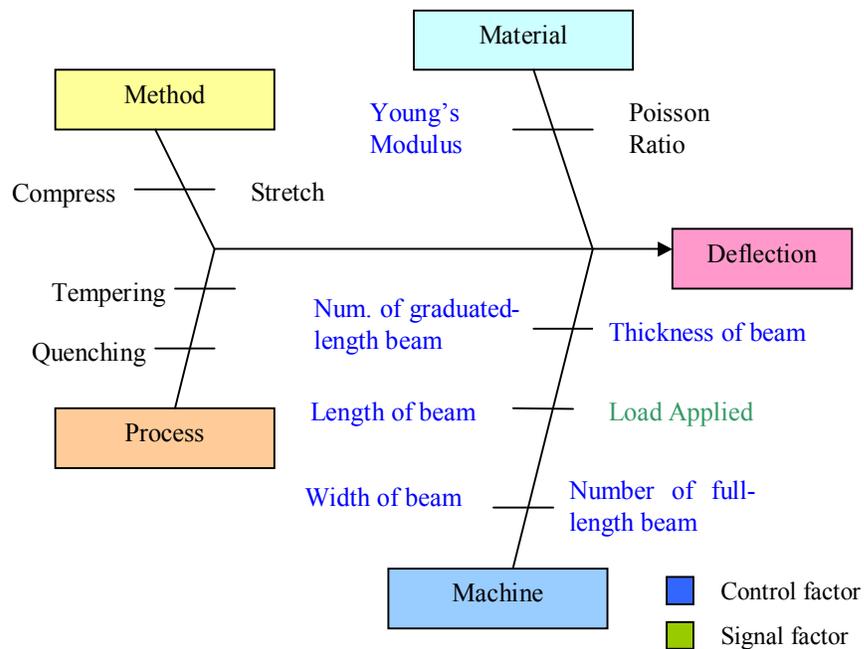


Figure 3.2: A Cause and Effect Diagram

3.1.5 Parameter Diagram

From the brain-storming session, the main causes of the problem are identified. These causes can be categorised into control factors, noise factors, and signal factors. A Parameter diagram studies all those factors which contribute to the output by a graphical way.

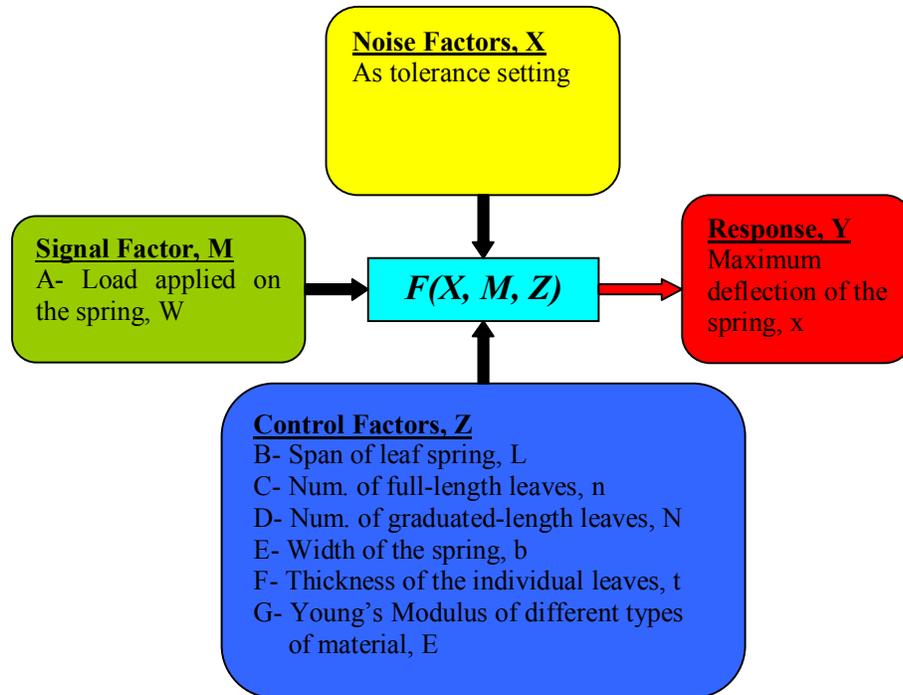


Figure 3.3: A Parameter Diagram

3.1.6 Relationship and Factors for Study

The equation of deflection for the semi-elliptical leaf spring in mathematical form can be expressed as equation (3.1) (C. S. Sharma and Kamlesh Purohit, 2005).

$$x = \frac{12 \times W \times L^3}{E \times b \times t^3 \times (2N + 3n)} \quad (3.1)$$

where

W = applied load on the spring

L = span of the leaf spring

n = number of full-length leaves

N = number of graduated-length leaves

b = width of the spring

t = thickness of the individual spring

E = Young's modulus of the spring material

All factors involved and their levels are shown in Figure 3.4, Figure 3.5, and Figure 3.6.

| Signal | Description | Unit | Level 1 | Level 2 | Level 3 |
|--------|---------------------------------|--------|---------|---------|---------|
| A | Applied load on the spring, W | Newton | 0.2 | 0.4 | 0.8 |

Figure 3.4: Signal Factor and Levels Assignment

| Control | Description | Unit | Level 1 | Level 2 | Level 3 |
|---------|---|-------|---------|---------|---------|
| B | Span of the leaf spring, L | meter | 0.4 | 0.8 | 1.6 |
| C | Number of full-length leaves, n | N/A | 1 | 2 | 4 |
| D | Number of graduated-length leaves, N | N/A | 7 | 9 | 13 |
| E | Width of the spring, b | meter | 0.03 | 0.07 | 0.12 |
| F | Thickness of the individual leaves, t | meter | 0.003 | 0.006 | 0.012 |
| G | Young's Modulus of the spring material, E | Gpa | 100 | 140 | 220 |

Figure 3.5: Control Factors and Levels Assignment

| Noise | Description | Unit | Tolerance, % |
|-------|---|------|--------------|
| B | Span of the leaf spring, L | % | 2 |
| C | Number of full-length leaves, n | % | 2 |
| D | Number of graduated-length leaves, N | % | 2 |
| E | Width of the spring, b | % | 2 |
| F | Thickness of the individual leaves, t | % | 2 |
| G | Young's Modulus of the spring material, E | % | 2 |

Figure 3.6: Noise Factors and Levels Assignment

3.2 Designing Experiment

From the planning session, six control factors and one signal factor were identified. These factors and their respective level settings will determine which orthogonal array is to be used. This shows that the experiment to be adopted is an $L_{18} (2^1 \times 3^7)$ orthogonal array which is known to be less affected by interactions between the various design parameters. However, for the reason of only seven factors will be assigned to the inner arrays, some modification will be made due to the design requirements.

3.2.1 L_{18} Orthogonal Array and Modification

An original $L_{18} (2^1 \times 3^7)$ orthogonal array includes 18 experiments with one 2-level and seven 3-levels factors as shown in Figure 3.7.

| No. Exp. | A | B | C | D | E | F | G | H |
|----------|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 1 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 1 | 2 | 1 | 1 | 2 | 2 | 3 | 3 |
| 5 | 1 | 2 | 2 | 2 | 3 | 3 | 1 | 1 |
| 6 | 1 | 2 | 3 | 3 | 1 | 1 | 2 | 2 |
| 7 | 1 | 3 | 1 | 2 | 1 | 3 | 2 | 3 |
| 8 | 1 | 3 | 2 | 3 | 2 | 1 | 3 | 1 |
| 9 | 1 | 3 | 3 | 1 | 3 | 2 | 1 | 2 |
| 10 | 2 | 1 | 1 | 3 | 3 | 2 | 2 | 1 |
| 11 | 2 | 1 | 2 | 1 | 1 | 3 | 3 | 2 |
| 12 | 2 | 1 | 3 | 2 | 2 | 1 | 1 | 3 |
| 13 | 2 | 2 | 1 | 2 | 3 | 1 | 3 | 2 |
| 14 | 2 | 2 | 2 | 3 | 1 | 2 | 1 | 3 |
| 15 | 2 | 2 | 3 | 1 | 2 | 3 | 2 | 1 |
| 16 | 2 | 3 | 1 | 3 | 2 | 3 | 1 | 2 |
| 17 | 2 | 3 | 2 | 1 | 3 | 1 | 2 | 3 |
| 18 | 2 | 3 | 3 | 2 | 1 | 2 | 3 | 1 |

Figure 3.7: An $L_{18} (2^1 \times 3^7)$ Orthogonal Array

The modification is made to meet the design requirements where only seven 3-levels factors are needed. Thus, the column (A) with only 2-levels is eliminated. Besides that, the sequence of the number of experiment is rearranged and thus the column for signal array can be formed. The new $L_{18}(3^7)$ orthogonal array is shown in Figure 3.8.

| No. Exp. | A | B | C | D | E | F | G |
|----------|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 |

Figure 3.8: A Modified $L_{18}(3^7)$ Orthogonal Array

3.2.2 Design Layout

The design layout for dynamic characteristic is a combination of inner array and outer array. The control factors are assigned to the inner array while the noise factors are assigned to the outer array. Different from the basic static characteristic design layout; the dynamic characteristic design layout consists few groups of outer array over a range of signal levels. In the computer simulation, the noise factor levels are determined by the tolerance setting of the control factors.

| A | A'1 | A'2 | A'3 |
|----|-------|-------|-------|
| A1 | 0.200 | 0.400 | 0.800 |
| A2 | 0.200 | 0.400 | 0.800 |
| A3 | 0.200 | 0.400 | 0.800 |

Figure 3.9: Setting Signal Factor Levels for Factor A

| B | B'1 | B'2 | B'3 |
|----|-------|-------|-------|
| B1 | 0.392 | 0.400 | 0.408 |
| B2 | 0.784 | 0.800 | 0.816 |
| B3 | 1.568 | 1.600 | 1.632 |

Figure 3.10: Setting Control and Noise Factor Levels for Factor B

As shown in the Figure 3.9, the signal levels have no effect in the control and noise factor array. The arrangement is made so thus the response for the first 6 columns take a signal value of A'1, the next 6 columns take the signal value of A'2 and the last 6 columns take the signal value of A'3. For the noise level setting of control factor B, the tolerance setting is 2% (Figure 3.6). Thus, the value of control factor B in level 1 (0.400) is multiplied by a tolerance value of ±2% which contributes to the noise factors B'1 and B'3 respectively. The rest of the control and noise factors follow the similar way. The final dynamic design layout is then formed as Figure 3.11.

| | M1 | | | | | | M2 | | | | | | M3 | | | | | |
|---|----|---|---|---|---|---|----|---|---|---|---|---|----|---|---|---|---|---|
| A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 2 |
| D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 |
| E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 |
| F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 |
| G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 |

| | A | B | C | D | E | F | G |
|----|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 |

Figure 3.11: An L₁₈ (3⁷) Dynamic Characteristic Design Layout

3.3 Conducting Experiment

3.3.1 Computer Simulation

The dynamic parameter design is conducted by computer simulation. The computer simulation was done by using Microsoft Excel instead of any other programming software like Visual Basic or C++ Programming which will include a more complicated and numerous programming codes. Microsoft Excel can simplify all the programming settings at the same time providing better user interface and table construction ability which is very important in this project.

The most difficult part in the computer simulation is to assign the factor levels under noise settings into the signal and control factor arrays. Taking control factor B as example, the level of control and noise factors settings (Figure 3.10) are assigned to the control factor array as shown in Figure 3.12.

| | | M1 | | | | | | M2 | | | | | | M3 | | | | | | | | | | |
|----|-----|-----|-----|---|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A | | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | |
| B | | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | | | | | |
| B | B1 | B2 | B3 | C | | D | | E | | F | | G | | | | | | | | | | | | |
| B1 | 1,1 | 1,2 | 1,3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | | | | | | | | | | | |
| B2 | 2,1 | 2,2 | 2,3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | | | | | | | | | | | |
| B3 | 3,1 | 3,2 | 3,3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | | | | | | | | | | | |
| A | B | C | D | E | F | G | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 | M9 | M10 | M11 | M12 | M13 | M14 | M15 | M16 | M17 | M18 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 | 1,1 | 1,2 | 1,3 |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 | 2,1 | 2,2 | 2,3 |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 | 3,1 | 3,2 | 3,3 |

Figure 3.12: Control Factor Array for Factor B

The values in the control and noise factors setting will be inserted into the design layout according to the row of the inner array and the column of the outer array with respect to factor B. For example, if the level for row (inner array) is 1 and the level for column (outer array) is 2, then the assigned value in the layout will be 1,2 which is the intersect value of B1 and B'2. Similar way is applied to the signal factor and the rest of the control factors. All the values from signal and control array layouts will then be calculated using the semi-elliptical leaf spring deflection equation to form the final design layout. Those calculated values are known as the response (deflection) of the experiment. The response values will then be used for performance measure.

3.4 Analysing Experiment

3.4.1 Regression Analysis

Dynamic characteristic robust design does have a different way in analysing the response data which is known as the regression analysis. The regression analysis provides a signal-response relationship which is expressed in the mathematical form as equation (2.15). The output or called response (Y) is adjustable with respect to the changing of the signal input (M). Besides, the error, ε is ideally to be zero. The system is said to be linear proportionally as the slope of graph is a straight line. The slope (β) of the regression line is known as the sensitivity (R) of the system which will be analysed for the target performance measure (TPM) in this project. The slope of regression line can be calculated using equation (2.17). Larger value of β means higher sensitivity and otherwise. The ideal value of the slope is 1. In such a condition, every unit changed of the input signal will lead to the same unit changed of the output response. A good graphical illustration of regression analysis is shown in Figure 2.13.

3.4.2 Target Performance Measure

In dynamic characteristic, the target performance measure (TPM) is essentially a measure of the sensitivity of the response. It is used to identify control factors that largely affect the sensitivity of the regression line. Those factors are known as significant control factors. As the slope (β) can be determined as equation (2.17), the sensitivity is expressed as the log function of the slope as equation (2.14). The quality characteristic is Nominal-the-Best instead of Larger-the-Better as the responses are desired to fall closely to the target slope.

3.4.3 Noise Performance Measure

The noise performance measure (NPM) is used to measure the variation in the response. Control factors that largely affect the variance can thus be identified. Those factors are known as variability control factors which can be used to minimise the variation. An S/N ratio is used for noise performance measure which depends on the quality characteristics chosen. Available quality characteristics include Smaller-the-Better, Nominal-the-Best, and Larger-the-Better. In this project, Larger-the-Better is used as expressed in equation (2.16).

3.4.4 Analysis of Variance

The Analysis of Variance (ANOVA) is used to analyse the results of an experiment to determine how much variation each factor has contributed. The general trend of the influence factors towards the response can be characterised by studying the main effects of each factor. The contribution of a factor can be controlled to a lower or higher value depending on the pooled error contribution. Thus, the levels of the influencing factors that produce the best results can be predicted. In an ANOVA table, a factor is pooled if it is not considered as a variance influencing factor. The percentage of contribution (Rho %) of each factor can then be determined.

3.4.5 Confidence Intervals

Confidence intervals are used to establish the process average at the predicted condition. Normally, confidence intervals are constructed at a confidence level, such as 95%, determined by the user. That means 95% of the confirmation test results fall within the confidence interval of the predicted mean. There are four different ways of calculating confidence intervals depending on different cases:

- 1) Confidence interval for mean.

$$CI_M = \sqrt{F_{\alpha, v1, v2} \times V_e \times \frac{1}{n}} \quad (3.2)$$

where

$F_{\alpha, v1, v2}$ = tabulated F-ratio

α = risk (1-confidence level)

$v1$ = degree of freedom associated with a mean

$v2$ = degree of freedom for the pooled error variance

V_e = pooled error variance

n = number of observation

- 2) Confidence interval for the factor level (CI_F). The equation is similar to the confidence interval for mean. However, the number of observation (n) is divided by the number of levels of the particular factor.

- 3) Confidence interval for the predicted mean.

$$CI_P = \sqrt{F_{\alpha, v1, v2} \times V_e \times \frac{1}{n_{eff}}} \quad (3.3)$$

where

n_{eff} = effective number of observations,

$$n_{eff} = \frac{n}{n - n_{pool}} \quad (3.4)$$

where

n = number of observations

n_{pool} = degree of freedom for the pooled error variance

4) Confidence interval for the confirmation of experiment.

$$CI_C = \sqrt{F_{\alpha, v1, v2} \times V_e \times \left[\frac{1}{n_{eff}} + \frac{1}{r} \right]} \quad (3.5)$$

where

r = sample size

3.5 Confirming Experiment

In computer simulation, there is no actual experiment to be carried out for confirmation. The optimum condition is compared to the current condition to show improvement. This can be performed by comparing the quality loss between the current condition and optimum condition.

3.5.1 Quality Loss Function

The quality loss function is expressed as equation (2.22). The average loss for dynamic characteristic can be calculated as follows:

$$L_{avg}(y, t) = \frac{k \sum_{j=1}^J (y_j - t)^2}{n} \quad (3.6)$$

where

n = number of observations

k = constant that depends on the cost at the specification limits

$$k = \frac{A_o}{D_o^2} \quad (3.7)$$

where

A_o = costs

D_o = specification limits

Lastly, the percentage of improvement can be calculated as follows:

$$\%improvement = \frac{L_{avg,before} - L_{avg,after}}{L_{avg,before}} \times 100 \quad (3.8)$$

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Data Collection

In this computer simulation, the experiment will run according to an $L_{18} (3^7)$ orthogonal array dynamic characteristic design layout as shown in Figure 3.11. This design layout includes a total number of 18 trials with seven 3-level factors. The input values of signal factor (A), control factors (B, C, D, E, F and G), and noise factors (tolerance of control factors) as shown in Figure 3.4, Figure 3.5, and Figure 3.6 are summarised in Table 4.1.

Table 4.1: Factors and Levels Assignment

| Factors | Description/Unit | Level 1 | Level 2 | Level 3 | Tolerance, % |
|---------|------------------|---------|---------|---------|--------------|
| A | $W, 10^4N$ | 0.2 | 0.4 | 0.8 | NA |
| B | L, m | 0.4 | 0.8 | 1.6 | 2 |
| C | n | 1 | 2 | 4 | 2 |
| D | N | 7 | 9 | 13 | 2 |
| E | b, m | 0.03 | 0.07 | 0.12 | 2 |
| F | t, m | 0.003 | 0.006 | 0.012 | 2 |
| G | E, Gpa | 100 | 140 | 220 | 2 |

These signal and control factors are then be put into the signal and control factors array as Figure 3.12. The final data will then be calculated using the semi-elliptical leaf spring equation to form the final data collection layout for further performance measure.

| | | | | | | | | Trials | | | | | | | | | | | | | | | | | | |
|--------|---|---|---|---|-----|-----|-----|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|
| | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | |
| | | | | A | A'1 | A'2 | A'3 | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | |
| | | | | B | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| | | | | C | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 2 |
| | | | | D | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 |
| | | | | E | | | | E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 |
| | | | | F | | | | F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 |
| | | | | G | | | | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 |
| Trials | A | B | C | D | E | F | G | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.200 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.400 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | 0.800 | |

Figure 4.1: Signal Factor Array (Applied Load, W)

| | | | | | | | Trials | | | | | | | | | | | | | | | | | | | | | | | | | |
|--------|----|-------|-------|-------|---|---|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|---|---|---|---|---|---|
| | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | | | | | | | |
| | | | | | | | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | | |
| | | | | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | | | | |
| | B | B'1 | B'2 | B'3 | | | | | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 3 | 1 | 2 |
| | B1 | 0.392 | 0.400 | 0.408 | | | | | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | | |
| | B2 | 0.784 | 0.800 | 0.816 | | | | | | | | E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 | | |
| | B3 | 1.568 | 1.600 | 1.632 | | | | | | | | F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 | | |
| Trials | A | B | C | D | E | F | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 | | | | | | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | 0.392 | 0.400 | 0.408 | | | | | | | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | 0.784 | 0.800 | 0.816 | | | | | | | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | 1.568 | 1.600 | 1.632 | | | | | | | |

Figure 4.2: Control Factor B Array (Span, L)

| | | | | | | | Trials | | | | | | | | | | | | | | | | | | | | | | | | |
|--------|---|----|-------|-------|-------|---|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|---|
| | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | | | | | | |
| | | | | | | | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | |
| | | | | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | | | |
| | | C | C'1 | C'2 | C'3 | | | | | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 2 |
| | | C1 | 0.980 | 1.000 | 1.020 | | | | | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 |
| | | C2 | 1.960 | 2.000 | 2.040 | | | | | | | | E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 |
| | | C3 | 3.920 | 4.000 | 4.080 | | | | | | | | F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 |
| Trials | A | B | C | D | E | F | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 3.920 | 4.000 | 4.080 | 4.080 | 3.920 | 4.000 | 3.920 | 4.000 | 4.080 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | 4.080 | 3.920 | 4.000 | | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.980 | 1.000 | 1.020 | 1.020 | 0.980 | 1.000 | 0.980 | 1.000 | 1.020 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | 1.020 | 0.980 | 1.000 | | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 1.960 | 2.000 | 2.040 | 2.040 | 1.960 | 2.000 | 1.960 | 2.000 | 2.040 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | 2.040 | 1.960 | 2.000 | | |

Figure 4.3: Control Factor C Array (Num. of Full-length Leaf, n)

| | | | | | | | | Trials | | | | | | | | | | | | | | | | | | | | | |
|--------|---|---|---|---|---|---|---|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|---|---|---|
| | | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | | | |
| | | | | | | | | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | | |
| | | | | | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| | | | | | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 2 |
| | | | | | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 |
| | | | | | | | | D1 | 6.86 | 7.00 | 7.14 | | | | | | | | | | | | | | | | | | |
| | | | | | | | | D2 | 8.82 | 9.00 | 9.18 | | | | | | | | | | | | | | | | | | |
| | | | | | | | | D3 | 12.74 | 13.00 | 13.26 | | | | | | | | | | | | | | | | | | |
| Trials | A | B | C | D | E | F | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | | | | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 8.82 | 9.00 | 9.18 | 9.18 | 8.82 | 9.00 | 9.00 | 9.18 | 8.82 | 9.18 | 8.82 | 9.00 | 8.82 | 9.00 | 9.18 | 9.00 | 9.18 | 8.82 | | | | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 12.74 | 13.00 | 13.26 | 13.26 | 12.74 | 13.00 | 13.00 | 13.26 | 12.74 | 13.26 | 12.74 | 13.00 | 12.74 | 13.00 | 13.26 | 13.00 | 13.26 | 12.74 | | | | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 6.86 | 7.00 | 7.14 | 7.14 | 6.86 | 7.00 | 7.00 | 7.14 | 6.86 | 7.14 | 6.86 | 7.00 | 6.86 | 7.00 | 7.14 | 7.00 | 7.14 | 6.86 | | | | |

Figure 4.4: Control Factor D Array (Num. of Graduated-length Leaf, N)

| | | | | | | | Trials | | | | | | | | | | | | | | | | | | | | | | | | | |
|--------|-------|-------|-------|---|---|---|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---|---|---|---|
| | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | | | | | | | |
| | | | | | | | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | | | | | | | |
| | | | | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | | | | |
| E | E'1 | E'2 | E'3 | | | | | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 3 | 1 | 2 | |
| E1 | 0.029 | 0.030 | 0.031 | | | | | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 2 | 3 | 1 | |
| E2 | 0.069 | 0.070 | 0.071 | | | | | | | | E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 |
| E3 | 0.118 | 0.120 | 0.122 | | | | | | | | F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 | 1 | 2 | 3 |
| Trials | A | B | C | D | E | F | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | | | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 0.118 | 0.120 | 0.122 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.118 | 0.120 | 0.122 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | 0.122 | 0.118 | 0.120 | | | | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.029 | 0.030 | 0.031 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.029 | 0.030 | 0.031 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | 0.031 | 0.029 | 0.030 | | | | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 0.069 | 0.070 | 0.071 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | 0.071 | 0.069 | 0.070 | | | | | |

Figure 4.5: Control Factor E Array (Spring Width, b)

| | | | | | | | Trials | | | | | | | | | | | | | | | | | | | |
|--------|--------|--------|--------|---|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---|
| | | | | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | | |
| | | | | | | | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| F | F1 | F2 | F3 | | | | | | | | | | | | | | | | | | | | | | | |
| F1 | 0.0029 | 0.0030 | 0.0031 | | | | | | | | | | | | | | | | | | | | | | | |
| F2 | 0.0059 | 0.0060 | 0.0061 | | | | | | | | | | | | | | | | | | | | | | | |
| F3 | 0.0118 | 0.0120 | 0.0122 | | | | | | | | | | | | | | | | | | | | | | | |
| Trials | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 0.0029 | 0.0030 | 0.0031 | 0.0030 | 0.0031 | 0.0029 | 0.0031 | 0.0029 | 0.0030 | 0.0031 | 0.0029 | 0.0030 | 0.0030 | 0.0031 | 0.0029 | 0.0029 | 0.0030 | 0.0031 | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.0059 | 0.0060 | 0.0061 | 0.0060 | 0.0061 | 0.0059 | 0.0061 | 0.0059 | 0.0060 | 0.0061 | 0.0059 | 0.0060 | 0.0060 | 0.0061 | 0.0059 | 0.0059 | 0.0060 | 0.0061 | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 0.0118 | 0.0120 | 0.0122 | 0.0120 | 0.0122 | 0.0118 | 0.0122 | 0.0118 | 0.0120 | 0.0122 | 0.0118 | 0.0120 | 0.0120 | 0.0122 | 0.0118 | 0.0118 | 0.0120 | 0.0122 | |

Figure 4.6: Control Factor F Array (Thickness of Leaf, t)

| | | | | | | | | M1 | | | | | | M2 | | | | | | M3 | | | | | | |
|---------|---|---|---|---|---|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|--------|--------|--------|----|
| | | | | | | | | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | |
| | | | | | | | | Trial | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| | | | | | | | | A | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 |
| | | | | | | | | B | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| | | | | | | | | C | 1 | 2 | 3 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 3 | 1 | 2 |
| | | | | | | | | D | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 1 | 2 | 3 | 2 | 3 | 1 |
| | | | | | | | | E | 1 | 2 | 3 | 2 | 3 | 1 | 2 | 3 | 1 | 1 | 2 | 3 | 3 | 1 | 2 | 3 | 1 | 2 |
| | | | | | | | | F | 1 | 2 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 3 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 3 |
| Factors | | | | | | | | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 |
| Trial | A | B | C | D | E | F | G | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 | 3 | 1 | 2 | 2 | 3 | 1 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1.185 | 1.115 | 1.051 | 1.050 | 1.052 | 1.258 | 1.947 | 2.333 | 2.447 | 1.986 | 2.354 | 2.377 | 4.104 | 4.362 | 4.967 | 4.359 | 4.407 | 4.629 | |
| 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 0.257 | 0.242 | 0.228 | 0.228 | 0.228 | 0.273 | 0.423 | 0.507 | 0.529 | 0.431 | 0.509 | 0.516 | 0.889 | 0.945 | 1.080 | 0.944 | 0.958 | 1.002 | |
| 3 | 1 | 3 | 3 | 3 | 3 | 3 | 3 | 0.060 | 0.057 | 0.053 | 0.053 | 0.053 | 0.064 | 0.099 | 0.119 | 0.124 | 0.101 | 0.119 | 0.121 | 0.208 | 0.221 | 0.254 | 0.221 | 0.225 | 0.235 | |
| 4 | 1 | 1 | 3 | 3 | 2 | 2 | 1 | 0.028 | 0.027 | 0.025 | 0.025 | 0.025 | 0.030 | 0.047 | 0.056 | 0.058 | 0.048 | 0.056 | 0.057 | 0.098 | 0.104 | 0.120 | 0.104 | 0.106 | 0.111 | |
| 5 | 1 | 2 | 1 | 1 | 3 | 3 | 2 | 0.026 | 0.025 | 0.023 | 0.023 | 0.023 | 0.028 | 0.043 | 0.052 | 0.055 | 0.044 | 0.053 | 0.053 | 0.092 | 0.097 | 0.111 | 0.097 | 0.098 | 0.103 | |
| 6 | 1 | 3 | 2 | 2 | 1 | 1 | 3 | 24.422 | 22.985 | 21.660 | 21.642 | 21.668 | 25.927 | 40.173 | 48.141 | 50.283 | 40.989 | 48.369 | 49.049 | 84.441 | 89.762 | 102.639 | 89.690 | 91.068 | 95.247 | |
| 7 | 2 | 1 | 1 | 2 | 2 | 3 | 3 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.003 | 0.005 | 0.006 | 0.006 | 0.005 | 0.006 | 0.006 | 0.010 | 0.011 | 0.012 | 0.011 | 0.011 | 0.011 | |
| 8 | 2 | 2 | 2 | 3 | 3 | 1 | 1 | 1.259 | 1.185 | 1.117 | 1.116 | 1.117 | 1.337 | 2.069 | 2.479 | 2.599 | 2.111 | 2.500 | 2.526 | 4.360 | 4.634 | 5.279 | 4.630 | 4.684 | 4.917 | |
| 9 | 2 | 3 | 3 | 1 | 1 | 2 | 2 | 4.428 | 4.168 | 3.927 | 3.924 | 3.929 | 4.701 | 7.315 | 8.765 | 9.040 | 7.463 | 8.696 | 8.932 | 15.245 | 16.207 | 18.768 | 16.194 | 16.652 | 17.196 | |
| 10 | 2 | 1 | 2 | 3 | 1 | 3 | 2 | 0.007 | 0.007 | 0.006 | 0.006 | 0.006 | 0.007 | 0.012 | 0.014 | 0.015 | 0.012 | 0.014 | 0.014 | 0.024 | 0.026 | 0.029 | 0.026 | 0.026 | 0.027 | |
| 11 | 2 | 2 | 3 | 1 | 2 | 1 | 3 | 1.208 | 1.137 | 1.071 | 1.070 | 1.072 | 1.282 | 1.995 | 2.391 | 2.465 | 2.035 | 2.372 | 2.436 | 4.158 | 4.420 | 5.118 | 4.417 | 4.541 | 4.690 | |
| 12 | 2 | 3 | 1 | 2 | 3 | 2 | 1 | 1.919 | 1.806 | 1.702 | 1.700 | 1.703 | 2.037 | 3.150 | 3.775 | 3.968 | 3.214 | 3.817 | 3.846 | 6.649 | 7.068 | 8.030 | 7.062 | 7.125 | 7.500 | |
| 13 | 3 | 1 | 2 | 1 | 3 | 2 | 3 | 0.014 | 0.013 | 0.013 | 0.013 | 0.013 | 0.015 | 0.024 | 0.028 | 0.029 | 0.024 | 0.028 | 0.029 | 0.049 | 0.053 | 0.060 | 0.053 | 0.053 | 0.056 | |
| 14 | 3 | 2 | 3 | 2 | 1 | 3 | 1 | 0.084 | 0.079 | 0.074 | 0.074 | 0.074 | 0.089 | 0.139 | 0.166 | 0.172 | 0.141 | 0.165 | 0.169 | 0.289 | 0.308 | 0.355 | 0.307 | 0.315 | 0.326 | |
| 15 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 13.611 | 12.811 | 12.072 | 12.062 | 12.077 | 14.450 | 22.325 | 26.754 | 28.192 | 22.780 | 27.119 | 27.258 | 47.204 | 50.176 | 56.876 | 50.136 | 50.464 | 53.245 | |
| 16 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 0.120 | 0.113 | 0.106 | 0.106 | 0.106 | 0.127 | 0.198 | 0.237 | 0.245 | 0.202 | 0.236 | 0.242 | 0.413 | 0.439 | 0.507 | 0.439 | 0.450 | 0.466 | |
| 17 | 3 | 2 | 1 | 3 | 1 | 2 | 3 | 0.316 | 0.297 | 0.280 | 0.280 | 0.280 | 0.335 | 0.518 | 0.621 | 0.654 | 0.529 | 0.629 | 0.632 | 1.095 | 1.164 | 1.320 | 1.163 | 1.171 | 1.235 | |
| 18 | 3 | 3 | 2 | 1 | 2 | 3 | 1 | 0.432 | 0.406 | 0.383 | 0.383 | 0.383 | 0.458 | 0.711 | 0.852 | 0.887 | 0.725 | 0.853 | 0.868 | 1.491 | 1.585 | 1.818 | 1.584 | 1.613 | 1.682 | |

Figure 4.8: Final Data Computation Layout (Deflection, x)

The final data shown in the Figure 4.8 is calculated using equation (3.1). Taking the data in the first row and first column as example, it is calculated as follows:

$$x = \frac{12 \times W \times L^3}{E \times b \times t^3 \times (2N + 3n)}$$

$$x = \frac{12 \times 0.2(10^4) \times 0.392^3}{98(10^9) \times 0.029 \times 0.0029^3 \times [2(6.86) + 3(0.98)]} = 1.185m$$

The deflection of the leaf spring is 1.185 meter.

4.2 Data Analysis

4.2.1 Current Process Performance

Taking Trial 2 as the current condition, it shows that the output response does not meet the target (signal). The deviation from the target causes quality loss which is undesired in the performance.

Table 4.2: Current Process Performance

| Observation | Signal | Response |
|-------------|--------|----------|
| 1 | 0.2 | 0.257 |
| 2 | 0.2 | 0.242 |
| 3 | 0.2 | 0.228 |
| 4 | 0.2 | 0.228 |
| 5 | 0.2 | 0.228 |
| 6 | 0.2 | 0.273 |
| 7 | 0.4 | 0.423 |
| 8 | 0.4 | 0.507 |
| 9 | 0.4 | 0.529 |
| 10 | 0.4 | 0.431 |
| 11 | 0.4 | 0.509 |
| 12 | 0.4 | 0.516 |
| 13 | 0.8 | 0.889 |
| 14 | 0.8 | 0.945 |
| 15 | 0.8 | 1.080 |
| 16 | 0.8 | 0.944 |
| 17 | 0.8 | 0.958 |
| 18 | 0.8 | 1.002 |

This can also be illustrated by a graph as Figure 4.9.

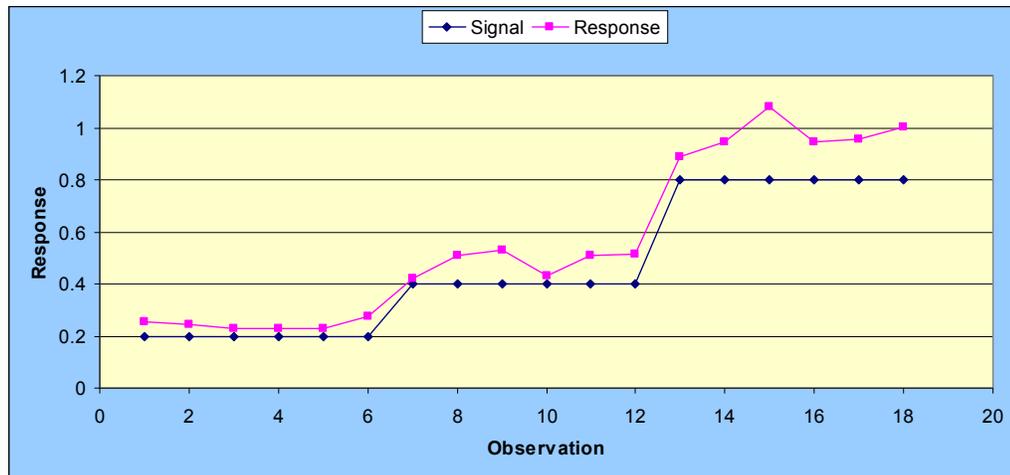


Figure 4.9: Graph Showing the Current Process Performance

4.2.2 Average Quality Loss

The average quality loss of the current condition can be calculated using equation (3.6) and equation (3.7). Assuming the loss A_o is \$20 and specification limit D_o is 0.5 meter, the average quality loss can be established as follows:

$$k = \frac{A_o}{D_o^2} = \frac{20}{0.5^2} = 80$$

$$L_{avg}(y, t) = \frac{k \sum_{j=1}^J (y_j - t)^2}{n} = \frac{80 \times [(0.257 - 0.2)^2 + (0.242 - 0.2)^2 + \dots + (1.002 - 0.8)^2]}{18}$$

$$L_{avg}(y, t) = \$1.162$$

The average quality loss of the current condition is \$1.162

4.2.3 Regression Analysis

Taking trial 2 as the current condition, the slope of the regression line can be calculated using equation (2.17) as follows:

$$\beta_i = \frac{\sum_{j=1}^J \sum_{k=1}^{r_o} y_{ijk} M_j}{r_o \sum_{j=1}^J M_j^2}$$

$$\beta_2 = \frac{[0.257(0.2) + 0.242(0.2) + \dots + 1.002(0.8)]}{6[0.2^2 + 0.4^2 + 0.8^2]} = 1.213$$

The slope of the regression line for trial 2 is 1.21. Figure 4.10 showing a graphical representation of regression analysis.

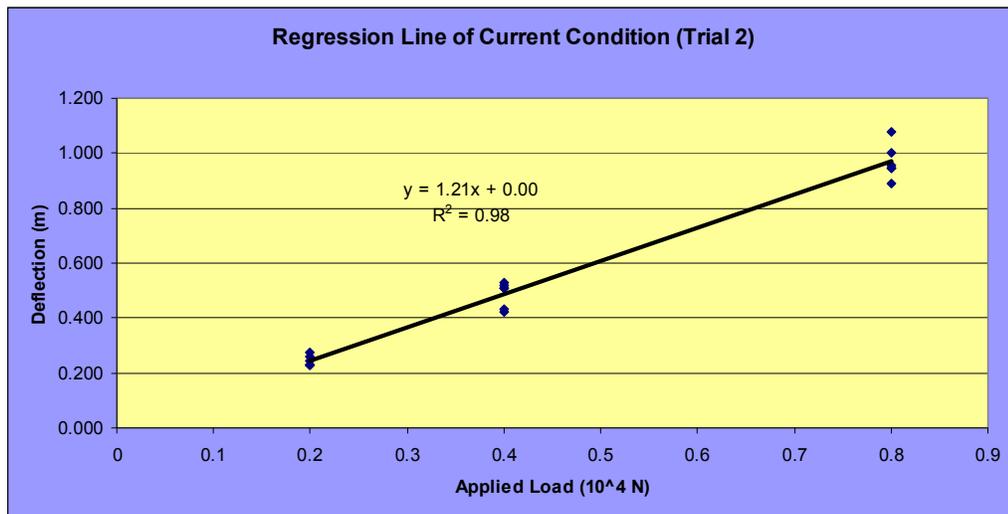


Figure 4.10: Regression Line of Trial 2

The relationship between the input signal (M) and the output response (Y) is expressed in equation (2.15). The target slope is not achieved yet and it should be a target value of one ($\beta=1$) due to design requirement. In this current condition, the error (ϵ) is 0.01. That means the y-intercept of the regression line is slightly above

the origin zero. The deviation is caused by the noise which presents in the system. The ideal function is achieved when the error is reduced to zero.

4.2.4 Target Performance Measure

The target performance measure (TPM) in dynamic characteristic is a measure of the sensitivity as shown in equation (2.14). The quality characteristic of the target performance measure is Nominal-the-Best. The scattered points of the response are desired to fall closely to the target slope which means reducing the bias. Control factors that largely affect the sensitivity are called significant factors. The sensitivity is calculated as follows:

$$R_N = 10 \log(\beta^2)$$

The slope (β) of trial 2 is 1.213.

$$R_2 = 10 \log(1.213^2) = 1.674 \text{ dB}$$

Obviously, the larger the slope of the regression line then the larger the sensitivity.

4.2.5 Noise Performance Measure

The quality characteristic of the noise performance measure (NPM) is Larger-the-Best. It is used to identify control factors that largely affect the variance. A dynamic S/N ratio as equation (2.16) is used as noise performance measure. The mean square error is calculated as follows:

$$MSE_i = \frac{1}{r_o J - 1} \sum_{j=1}^J \sum_{k=1}^{r_o} (y_{ijk} - \beta_i M_j)^2$$

$$MSE_2 = \frac{1}{6(3) - 1} \left[\{0.257 - 1.213(0.2)\}^2 + \dots + \{0.1002 - 1.213(0.8)\}^2 \right]$$

$$MSE_2 = 0.002$$

Then, the dynamic S/N ratio can be calculated as follows:

$$S / N_i = 10 \log \left(\frac{\beta_i^2}{MSE_i} \right)$$

$$S / N_2 = 10 \log \left(\frac{1.213^2}{0.002} \right) = 28.696dB$$

The high value of dynamic S/N ratio (NPM) is desirable. It is because of greater S/N ratio will result in smaller variance around the target value.

4.2.6 Data Collection of Performance Measure

Table 4.3 shows the calculated values of target performance measure and noise performance measure for the 18 trials.

Table 4.3: Data Collection of Performance Measure

| Trial | TPM | NPM | Beta | MSE |
|-------|---------|--------|---------|--------|
| 1 | 14.951 | 28.771 | 5.592 | 0.041 |
| 2 | 1.674 | 28.696 | 1.213 | 0.002 |
| 3 | -10.925 | 28.623 | 0.284 | 0.000 |
| 4 | -17.457 | 28.623 | 0.134 | 0.000 |
| 5 | -18.075 | 28.771 | 0.125 | 0.000 |
| 6 | 41.231 | 28.696 | 115.226 | 17.925 |
| 7 | -37.216 | 28.803 | 0.014 | 0.000 |
| 8 | 15.478 | 28.761 | 5.941 | 0.047 |
| 9 | 26.401 | 28.442 | 20.895 | 0.625 |
| 10 | -29.589 | 28.761 | 0.033 | 0.000 |
| 11 | 15.115 | 28.442 | 5.699 | 0.046 |
| 12 | 19.136 | 28.803 | 9.053 | 0.108 |
| 13 | -23.412 | 28.641 | 0.068 | 0.000 |
| 14 | -8.044 | 28.522 | 0.396 | 0.000 |
| 15 | 36.153 | 28.838 | 64.220 | 5.389 |
| 16 | -4.946 | 28.522 | 0.566 | 0.000 |
| 17 | 3.463 | 28.838 | 1.490 | 0.003 |
| 18 | 6.180 | 28.641 | 2.037 | 0.006 |

4.2.7 Analysis of Mean

The analysis of mean (ANOM) is an important procedure to see how significant the collection data of performance measure around the overall mean. The ANOM is done for both target and noise performance measure. From Table 4.3, the data of TPM and NPM is used for ANOM.

Table 4.4: Response Table for TPM

| TPM | A | B | C | D | E | F | G |
|-------------|-------|----------|--------|--------|---------|----------|---------|
| Level 1 | 1.900 | -16.278 | 3.069 | 3.527 | 8.069 | 19.664 | 5.041 |
| Level 2 | 1.554 | 1.602 | 1.927 | 1.973 | 0.742 | 1.634 | 1.936 |
| Level 3 | 1.566 | 19.696 | 0.024 | -0.479 | -3.791 | -16.278 | -1.957 |
| Overall AVG | 1.673 | 1.673 | 1.673 | 1.673 | 1.673 | 1.673 | 1.673 |
| Diff | 0.346 | 35.974 | 3.045 | 4.006 | 11.859 | 35.942 | 6.998 |
| Rank | 7 | 1 | 6 | 5 | 3 | 2 | 4 |
| SSQ | 0.462 | 3882.458 | 28.388 | 48.956 | 429.752 | 3875.463 | 147.536 |
| OPT | 3 | 2 | 2 | 2 | 2 | 2 | 2 |

From Table 4.4, The SSQ (S_A) of factor A is calculated as follows:

$$S_A = n_{A1}\bar{y}_{A1}^2 + n_{A2}\bar{y}_{A2}^2 + n_{A3}\bar{y}_{A3}^2 - S_m$$

$$S_m = n\bar{y}^2$$

$$S_m = 18 \times \left[\frac{14.951 + 1.674 + \dots + 6.180}{18} \right]^2 = 50.401$$

$$S_A = 6(1.900)^2 + 6(1.554)^2 + 6(1.566)^2 - 50.401 = 0.462$$

The optimum condition (OPT) is selected based on the quality characteristic. In TPM, the quality characteristic is Nominal-the-Best. That means the average value of the factor levels which is the “nominal” is chosen to be the optimum condition. The response graph for this data is shown in Figure 4.11. Same as the ranking in the response table, factors B and F largely affect the sensitivity.

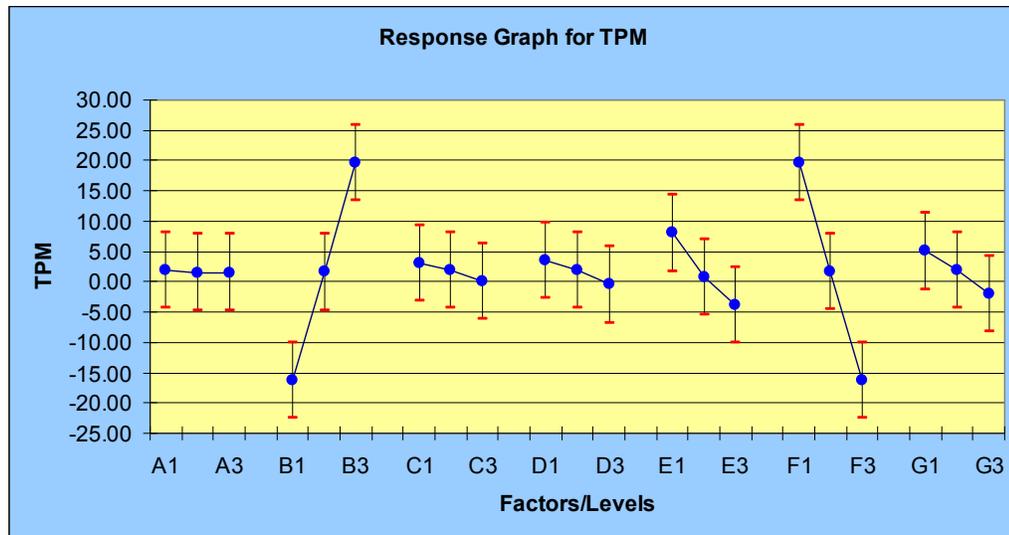


Figure 4.11: Response Graph for TPM

For NPM, the analysis of mean is done as follows:

Table 4.5: Response Table for NPM

| NPM | A | B | C | D | E | F | G |
|-------------|--------|--------|--------|--------|--------|--------|--------|
| Level 1 | 28.697 | 28.687 | 28.804 | 28.618 | 28.672 | 28.672 | 28.687 |
| Level 2 | 28.668 | 28.672 | 28.699 | 28.674 | 28.674 | 28.674 | 28.672 |
| Level 3 | 28.667 | 28.674 | 28.529 | 28.741 | 28.687 | 28.687 | 28.674 |
| Overall AVG | 28.678 | 28.678 | 28.678 | 28.678 | 28.678 | 28.678 | 28.678 |
| Diff | 0.030 | 0.015 | 0.275 | 0.123 | 0.015 | 0.015 | 0.015 |
| Rank | 3 | 4 | 1 | 2 | 4 | 6 | 7 |
| SSQ | 0.003 | 0.001 | 0.232 | 0.045 | 0.001 | 0.001 | 0.001 |
| OPT | 1 | 1 | 1 | 3 | 3 | 3 | 1 |

Similar to TPM, the SSQ (S_A) of factor A in Table 4.5 is calculated as follows:

$$S_A = n_{A1}\bar{y}_{A1}^2 + n_{A2}\bar{y}_{A2}^2 + n_{A3}\bar{y}_{A3}^2 - S_m$$

$$S_m = n\bar{y}^2$$

$$S_m = 18 \times \left[\frac{28.771 + 28.696 + \dots + 28.641}{18} \right]^2 = 14803.208$$

$$S_A = 6(28.697)^2 + 6(28.668)^2 + 6(28.667)^2 - 14803.208 = 0.003$$

The quality characteristic of NPM is Larger-the-Better. The average value of the factor levels which is larger than the others is chosen to be optimum condition. The response graph for this data is shown in Figure 4.12. Clearly, factors C and D largely affect the S/N ratio.

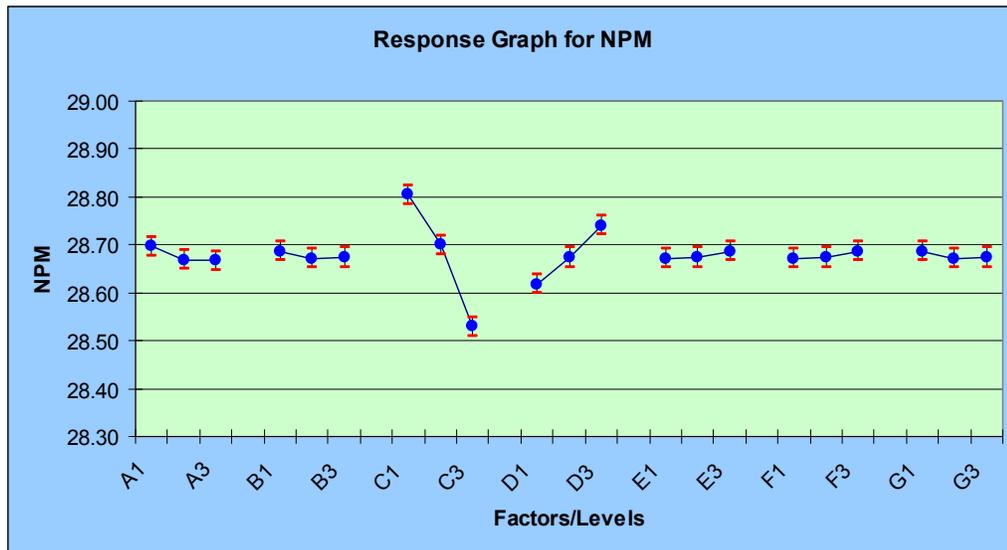


Figure 4.12: Response Graph for NPM

4.2.8 Analysis of Variance

The analysis of variance (ANOVA) is used to find out the significant factors and their percentage of contribution. Similar to ANOM, analysis of variance is also done for both TPM and NPM.

The analysis of variance for TPM is started by pooling the factor with the smallest sum of square (SSQ) which is obtained from the response table of TPM. The number of significant factors required is determined by the user with referring to the response graph to see which factors largely affect the sensitivity (R). Table 4.6 shows the analysis of variance for TPM.

Table 4.6: ANOVA for TPM

| Source | Pool | SSQ | Dof | Variance | SSq' | Rho % |
|--------|------|----------|-----|----------|----------|---------------|
| A | 1 | 0.462 | 2 | 0.231 | | |
| B | 0 | 3882.458 | 2 | 1941.229 | 3781.674 | 44.950 |
| C | 1 | 28.388 | 2 | 14.194 | | |
| D | 1 | 48.956 | 2 | 24.478 | | |
| E | 1 | 429.752 | 2 | 214.876 | | |
| F | 0 | 3875.463 | 2 | 1937.731 | 3774.679 | 44.867 |
| G | 1 | 147.536 | 2 | 73.768 | | |
| Error | 1 | 0.000 | 3 | 0.000 | | |
| epool | | 655.094 | 13 | 50.392 | 856.662 | 10.183 |
| St | | 8413.015 | 17 | 494.883 | 8413.015 | 100.000 |
| Sm | | 50.401 | 1 | | | |
| ST | | 8463.416 | 18 | | | |

The total sum of square (ST) is calculated as follows:

$$ST = \sum y^2 = 14.951^2 + 1.674^2 + \dots + 3.463^2 + 6.180^2 = 8463.416$$

The sum of square due to the mean is calculated as follows:

$$Sm = n\bar{y}^2 = 18 \times \left[\frac{14.951 + 1.674 + \dots + 6.180}{18} \right]^2 = 50.401$$

Thus, St is calculated as follows:

$$St = ST - Sm = 8463.416 - 50.401 = 8413.015$$

The error can then be obtained by:

$$S_{error} = St - (S_A + S_B + \dots + S_G)$$

$$S_{error} = 8413.015 - (0.462 + 3882.458 + \dots + 147.536) = 0.000$$

Regarding to the degree of freedom (v), the calculation included as follows:

$$v_t = v_T - v_m = 18 - 1 = 17$$

$$v_{error} = v_t - (v_A + v_B + \dots + v_G) = 17 - (7 \times 2) = 3$$

Regarding to the variance (V), the calculation included as follows:

$$V_t = \frac{St}{v_t} = \frac{8413.015}{17} = 494.883$$

$$V_{error} = \frac{S_{error}}{v_{error}} = \frac{0.000}{3} = 0.000$$

For factor B, its variance (V_B) can be calculated as follows:

$$V_B = \frac{S_B}{v_B} = \frac{3882.458}{2} = 1941.229$$

Since factor B is chosen as significant factor, then its pure sum of square (S'_B) is calculated as:

$$S'_B = S_B - v_B V_{pool} = 3882.458 - 2(50.392) = 3781.674$$

From the ANOVA table for TPM, factors B and F are significant factors. The percentage of contribution of factors B and F can be calculated as:

$$\rho_B = \frac{S'_B}{St} \times 100\% = \frac{3781.674}{8413.015} \times 100\% = 44.950\%$$

$$\rho_F = \frac{S'_F}{St} \times 100\% = \frac{3774.679}{8413.015} \times 100\% = 44.867\%$$

Lastly, the percentage of the error in the system is calculated as follows:

$$\rho_{error} = 100 - (\rho_B + \rho_F) = 100 - (44.950 + 44.867) = 10.183\%$$

The percentage of the error is always the smaller the better. Figure 4.13 shows the percentage of contribution of the significant factors by a graphical way.

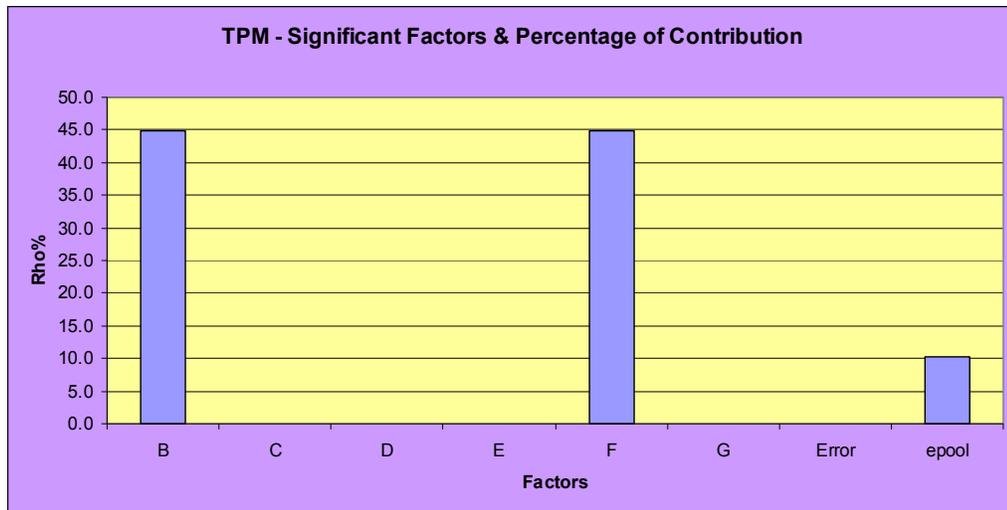


Figure 4.13: Percentage of Contribution for Significant Factors (TPM)

The analysis of variance for NPM is similar to the analysis of variance for TPM. Table 4.7 shows the analysis of variance for NPM.

Table 4.7: ANOVA for NPM

| Source | Pool | SSQ | Dof | Variance | SSq' | Rho % |
|--------|------|-----------|-----|----------|-------|---------------|
| A | 1 | 0.003 | 2 | 0.002 | | |
| B | 1 | 0.001 | 2 | 0.000 | | |
| C | 0 | 0.232 | 2 | 0.116 | 0.115 | 40.504 |
| D | 0 | 0.045 | 2 | 0.023 | 0.022 | 7.602 |
| E | 1 | 0.001 | 2 | 0.000 | | |
| F | 1 | 0.001 | 2 | 0.000 | | |
| G | 1 | 0.001 | 2 | 0.000 | | |
| Error | 1 | 0.000 | 3 | 0.000 | | |
| epool | | 0.007 | 13 | 0.001 | 0.147 | 51.895 |
| St | | 0.284 | 17 | 0.017 | 0.284 | 100.000 |
| Sm | | 14803.208 | 1 | | | |
| ST | | 14803.492 | 18 | | | |

The total sum of square (*ST*) is calculated as follows:

$$ST = \sum y^2 = 28.771^2 + 28.696^2 + \dots + 28.838^2 + 28.641^2 = 14803.492$$

The sum of square due to the mean is calculated as follows:

$$Sm = n\bar{y}^2 = 18 \times \left[\frac{28.771 + 28.696 + \dots + 28.641}{18} \right]^2 = 14803.208$$

Thus, St is calculated as follows:

$$St = ST - Sm = 14803.492 - 14803.208 = 0.284$$

The error can then be obtained by:

$$S_{error} = St - (S_A + S_B + \dots + S_G)$$

$$S_{error} = 0.284 - (0.003 + 0.001 + \dots + 0.001) = 0.000$$

Regarding to the degree of freedom (v), the calculation included as follows:

$$v_t = v_T - v_m = 18 - 1 = 17$$

$$v_{error} = v_t - (v_A + v_B + \dots + v_G) = 17 - (7 \times 2) = 3$$

Regarding to the variance (V), the calculation included as follows:

$$V_t = \frac{St}{v_t} = \frac{0.284}{17} = 0.017$$

$$V_{error} = \frac{S_{error}}{v_{error}} = \frac{0.000}{3} = 0.000$$

For factor C, its variance (V_C) can be calculated as follows:

$$V_C = \frac{S_C}{v_C} = \frac{0.232}{2} = 0.116$$

Since factor C is chosen as significant factor, then its pure sum of square (S'_C) is calculated as:

$$S'_C = S_C - v_C V_{pool} = 0.232 - 2(0.001) = 0.115$$

From the ANOVA table for NPM, factors C and D are significant factors. The percentage of contribution of factors C and D can be calculated as:

$$\rho_C = \frac{S'_C}{St} \times 100\% = \frac{0.115}{0.284} \times 100\% = 40.504\%$$

$$\rho_D = \frac{S'_D}{St} \times 100\% = \frac{0.022}{0.284} \times 100\% = 7.602\%$$

Lastly, the percentage of the error in the system is calculated as follows:

$$\rho_{error} = 100 - (\rho_C + \rho_D) = 100 - (40.504 + 7.602) = 51.895\%$$

From the analysis of variance for NPM, the percentage of error is too large. This is undesired in the system where the noise is not much reduced. Foresee that the bias of the scattered points of the response around the target slope is not much reduced. Figure 4.14 shows the percentage of contribution of the significant factors by a graphical way.

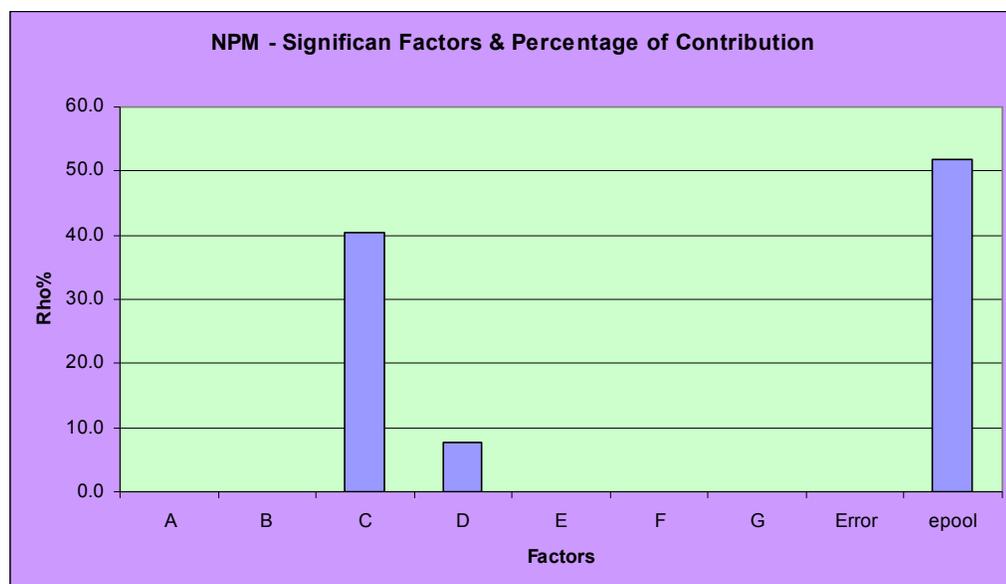


Figure 4.14: Percentage of Contribution for Significant Factors (NPM)

4.2.9 Confidence Interval for Factor Level

The confidence interval for a factor level is calculated using equation (3.2). For TPM, the confidence interval for a factor level is calculated as follows:

$$CI_F = \sqrt{F_{\alpha, v_1, v_2} \times V_e \times \frac{1}{n}}$$

$$CI_F = \sqrt{F_{0.05, 1, 13} \times 50.39 \times \frac{1}{6}} = \sqrt{4.667 \times 50.39 \times \frac{1}{6}} = \pm 6.261$$

Taking A1 as example, the confidence interval for factor A1 is as follows:

$$\mu_{A1} - CI_F \leq \mu_{A1} \leq \mu_{A1} + CI_F$$

$$1.90 - 6.261 \leq 1.90 \leq 1.90 + 6.261$$

$$-4.36 \leq \mu_{A1} \leq 8.16$$

Table 4.8: Confidence Interval for Factor Level (TPM)

| Factor/Level | Mean+CI _F | Mean | Mean-CI _F |
|--------------|----------------------|--------|----------------------|
| A1 | 8.16 | 1.90 | -4.36 |
| A2 | 7.81 | 1.55 | -4.71 |
| A3 | 7.83 | 1.57 | -4.69 |
| B1 | -10.02 | -16.28 | -22.54 |
| B2 | 7.86 | 1.60 | -4.66 |
| B3 | 25.96 | 19.70 | 13.44 |
| C1 | 9.33 | 3.07 | -3.19 |
| C2 | 8.19 | 1.93 | -4.33 |
| C3 | 6.29 | 0.02 | -6.24 |
| D1 | 9.79 | 3.53 | -2.73 |
| D2 | 8.23 | 1.97 | -4.29 |
| D3 | 5.78 | -0.48 | -6.74 |
| E1 | 14.33 | 8.07 | 1.81 |
| E2 | 7.00 | 0.74 | -5.52 |
| E3 | 2.47 | -3.79 | -10.05 |
| F1 | 25.92 | 19.66 | 13.40 |
| F2 | 7.90 | 1.63 | -4.63 |
| F3 | -10.02 | -16.28 | -22.54 |
| G1 | 11.30 | 5.04 | -1.22 |
| G2 | 8.20 | 1.94 | -4.32 |
| G3 | 4.30 | -1.96 | -8.22 |

Table 4.8 shows the confidence interval for factor levels which is used in the TPM. The results in the table are used to construct the response graph as shown in Figure 4.11. For NPM, the confidence interval for a factor level is calculated as follows:

$$CI_F = \sqrt{F_{\alpha, v1, v2} \times V_e \times \frac{1}{n}}$$

$$CI_F = \sqrt{F_{0.05, 1, 13} \times 0.001 \times \frac{1}{6}} = \sqrt{4.667 \times 0.001 \times \frac{1}{6}} = \pm 0.020$$

Taking A1 as example,

$$\mu_{A1} - CI_F \leq \mu_{A1} \leq \mu_{A1} + CI_F$$

$$28.70 - 0.020 \leq 28.70 \leq 28.70 + 0.020$$

$$28.68 \leq \mu_{A1} \leq 28.72$$

Table 4.9: Confidence Interval for Factor Level (NPM)

| Factor/Level | Mean+CI _F | Mean | Mean-CI _F |
|--------------|----------------------|-------|----------------------|
| A1 | 28.72 | 28.70 | 28.68 |
| A2 | 28.69 | 28.67 | 28.65 |
| A3 | 28.69 | 28.67 | 28.65 |
| B1 | 28.71 | 28.69 | 28.67 |
| B2 | 28.69 | 28.67 | 28.65 |
| B3 | 28.69 | 28.67 | 28.65 |
| C1 | 28.82 | 28.80 | 28.78 |
| C2 | 28.72 | 28.70 | 28.68 |
| C3 | 28.55 | 28.53 | 28.51 |
| D1 | 28.64 | 28.62 | 28.60 |
| D2 | 28.69 | 28.67 | 28.65 |
| D3 | 28.76 | 28.74 | 28.72 |
| E1 | 28.69 | 28.67 | 28.65 |
| E2 | 28.69 | 28.67 | 28.65 |
| E3 | 28.71 | 28.69 | 28.67 |
| F1 | 28.69 | 28.67 | 28.65 |
| F2 | 28.69 | 28.67 | 28.65 |
| F3 | 28.71 | 28.69 | 28.67 |
| G1 | 28.71 | 28.69 | 28.67 |
| G2 | 28.69 | 28.67 | 28.65 |
| G3 | 28.69 | 28.67 | 28.65 |

The results of Table 4.9 are used to construct the response graph for NPM as shown in Figure 4.12.

4.3 Optimum Condition Selection

According to the analysis of variance for both TPM and NPM, the significant factors selected from each are different. For TPM, the significant factors are factors B and F. On the other hand, factors C and D are selected as significant factors for NPM. The optimum condition is determined by integrating the ranking and optimum levels from both TPM and NPM as shown in Table 4.10.

Table 4.10: Optimum Factors and Levels Selection

| Factor | TPM | | | NPM | | | Decision | TPM | NPM |
|-----------------|-----|------------------|-------|-----|------|-------|----------|--------|--------|
| | OPT | RANK | Rho | OPT | RANK | Rho | | | |
| A | 3 | 7 | | 1 | 3 | | 1 | | |
| B | 2 | 1 | 44.95 | 1 | 4 | | 2 | 1.602 | |
| C | 2 | 6 | | 1 | 1 | 40.50 | 1 | | 28.804 |
| D | 2 | 5 | | 3 | 2 | 7.60 | 3 | | 28.741 |
| E | 2 | 3 | | 3 | 5 | | 2 | | |
| F | 2 | 2 | 44.87 | 3 | 6 | | 2 | 1.634 | |
| G | 2 | 4 | | 1 | 7 | | 2 | | |
| Process μ | | | | | | | | 1.673 | 28.678 |
| Predicted μ | | | | | | | | 1.563 | 28.867 |
| Predicted μ | | -Cl _P | | | | | | -6.520 | 28.842 |
| Predicted μ | | +Cl _P | | | | | | 9.646 | 28.893 |

The optimum level selection and the factors ranking for both TPM and NPM are obtained from the response table of each. The significant factors are obviously the first two high ranking factors. Hence, decision is made whereby the level of the significant factors (B, C, D, and F) is selected to be the optimum level condition. Besides that, for those factors which are not that “significant” (A, E, and G), their optimum level condition is determined by comparing the factor ranking between TPM and NPM. Taking factor E as example, the factor ranking at the side of TPM is higher than the side of NPM, thus level 2 is chosen to be the optimum condition. As summary, the final decision for the optimum condition is A1, B2, C1, D3, E2, F2, and G2. The optimum condition is called OP C1.

4.3.1 Prediction of Optimum Condition

Significant factors B2 and F2 are used for the prediction of TPM.

$$\mu_{predicted} = \bar{y} + (\overline{B2} - \bar{y}) + (\overline{F2} - \bar{y})$$

$$\mu_{predicted} = \overline{B2} + \overline{F2} - \bar{y}$$

$$\mu_{predicted} = 1.602 + 1.634 - 1.673 = 1.563dB$$

Significant factors C1 and D3 are used for the prediction of NPM.

$$\mu_{predicted} = \bar{y} + (\overline{C1} - \bar{y}) + (\overline{D3} - \bar{y})$$

$$\mu_{predicted} = \overline{C1} + \overline{D3} - \bar{y}$$

$$\mu_{predicted} = 28.804 + 28.741 - 28.678 = 28.867dB$$

4.3.2 Confidence Interval for Predicted Mean

The confidence interval for predicted optimum process mean is calculated using equation (3.3).

$$CI_P = \sqrt{F_{\alpha, v1, v2} \times V_e \times \frac{1}{n_{eff}}}$$

The effective number of observation for both TPM and NPM is calculated using equation (3.4) as follows:

$$n_{eff} = \frac{n}{n - n_{pool}} = \frac{18}{18 - 13} = 3.6$$

The confidence interval for predicted mean for TPM is calculated as follows:

$$CI_p = \sqrt{F_{0.05,1,13} \times 50.39 \times \frac{1}{3.6}}$$

$$CI_p = \sqrt{4.667 \times 50.39 \times \frac{1}{3.6}} = \pm 8.083$$

Thus,

$$\mu_{\text{predicted}} - CI_p \leq \mu_{\text{predicted}} \leq \mu_{\text{predicted}} + CI_p$$

$$1.563 - 8.083 \leq \mu_{\text{predicted}} \leq 1.563 + 8.083$$

$$-6.520 \leq \mu_{\text{predicted}} \leq 9.646$$

For NPM, the confidence interval for predicted mean is calculated as follows:

$$CI_p = \sqrt{F_{0.05,1,13} \times 0.001 \times \frac{1}{3.6}}$$

$$CI_p = \sqrt{4.667 \times 0.001 \times \frac{1}{3.6}} = \pm 0.026$$

Thus,

$$\mu_{\text{predicted}} - CI_p \leq \mu_{\text{predicted}} \leq \mu_{\text{predicted}} + CI_p$$

$$28.867 - 0.026 \leq \mu_{\text{predicted}} \leq 28.867 + 0.026$$

$$28.842 \leq \mu_{\text{predicted}} \leq 28.893$$

4.4 Confirmation of Experiment

The confirmation of experiment in the computer simulation is done by showing the improvement after optimisation. From the previous section, the optimum condition is determined as A1, B2, C1, D3, E2, F2, and G2. This optimum factor level setting (OP C1) and another group of subjective factor level setting (OP C2) are simulated for the output response. Table 4.11 shows the output response data of the optimum condition. The target and noise performance measure of OP C1 and OP C2 are

calculated following the steps what have done in the previous section. The output responses of OP C1 and OP C2 are shown in Table 4.12.

Table 4.11: Output Response of Optimum Condition

| Observation | Signal | Before | After | OP C2 |
|-------------|--------|--------|-------|-------|
| 1 | 0.2 | 0.257 | 0.213 | 0.298 |
| 2 | 0.2 | 0.242 | 0.200 | 0.280 |
| 3 | 0.2 | 0.228 | 0.189 | 0.264 |
| 4 | 0.2 | 0.228 | 0.188 | 0.264 |
| 5 | 0.2 | 0.228 | 0.189 | 0.264 |
| 6 | 0.2 | 0.273 | 0.226 | 0.316 |
| 7 | 0.4 | 0.423 | 0.349 | 0.488 |
| 8 | 0.4 | 0.507 | 0.418 | 0.585 |
| 9 | 0.4 | 0.529 | 0.441 | 0.617 |
| 10 | 0.4 | 0.431 | 0.356 | 0.498 |
| 11 | 0.4 | 0.509 | 0.424 | 0.593 |
| 12 | 0.4 | 0.516 | 0.426 | 0.596 |
| 13 | 0.8 | 0.889 | 0.738 | 1.033 |
| 14 | 0.8 | 0.945 | 0.784 | 1.098 |
| 15 | 0.8 | 1.080 | 0.889 | 1.244 |
| 16 | 0.8 | 0.944 | 0.783 | 1.097 |
| 17 | 0.8 | 0.958 | 0.789 | 1.104 |
| 18 | 0.8 | 1.002 | 0.832 | 1.165 |

Table 4.12: TPM and NPM for Optimum Condition

| | TPM | NPM | BETA | MSE |
|-------------------------|-------|-------|-------|-------|
| Trial 2 (Before) | 1.674 | 8.696 | 1.213 | 0.002 |
| OP C1 (After) | 0.030 | 8.838 | 1.003 | 0.001 |
| OP C2 | 2.952 | 8.838 | 1.405 | 0.003 |

4.4.1 Confidence Interval for Confirmation Experiment

In the computer simulation, the result of the optimum condition is always the same no matter how many times it is run for confirmation. That means, if assuming the confirmation experiment is run for 18 times ($r=18$), then the mean of confirmation ($\mu_{\text{confirmation}}$) for TPM is still equal to 0.030dB since there is no variation of the response for the same value of input. Thus, the confidence interval for confirmation of experiment can be calculated as follows using equation (3.5).

$$CI_C = \sqrt{F_{\alpha, v_1, v_2} \times V_e \times \left[\frac{1}{n_{eff}} + \frac{1}{r} \right]}$$

$$CI_C = \sqrt{4.667 \times 50.39 \times \left[\frac{1}{3.6} + \frac{1}{18} \right]} = \pm 8.854$$

The mean of the confirmation experiment for TPM falls in the range as follows:

$$\mu_{\text{confirmation}} - CI_C \leq \mu_{\text{confirmation}} \leq \mu_{\text{confirmation}} + CI_C$$

$$0.030 - 8.854 \leq \mu_{\text{confirmation}} \leq 0.030 + 8.854$$

$$-8.824 \leq \mu_{\text{confirmation}} \leq 8.884$$

4.4.2 Comparison of Before and After

The result from Table 4.11 and Table 4.12 can be illustrated by a graphical expression as shown in Figure 4.15 and Figure 4.16. In Figure 4.15, the response after the optimisation falls closely to the target (meet the target). Beside that, the slope of the regression line after the optimisation has been optimized to 1 as shown in Figure 4.16.

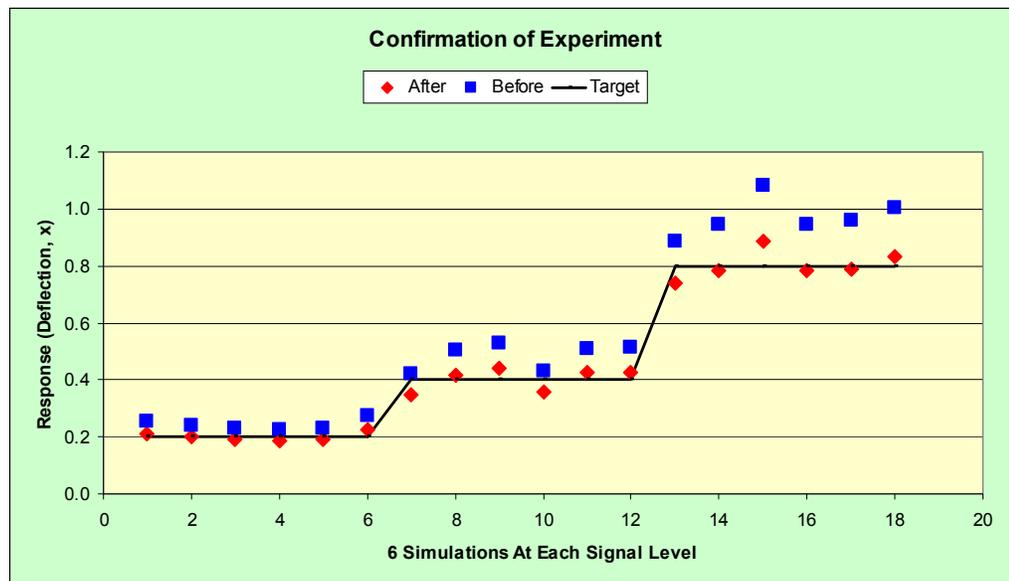


Figure 4.15: Comparison of Responses Before and After Optimisation

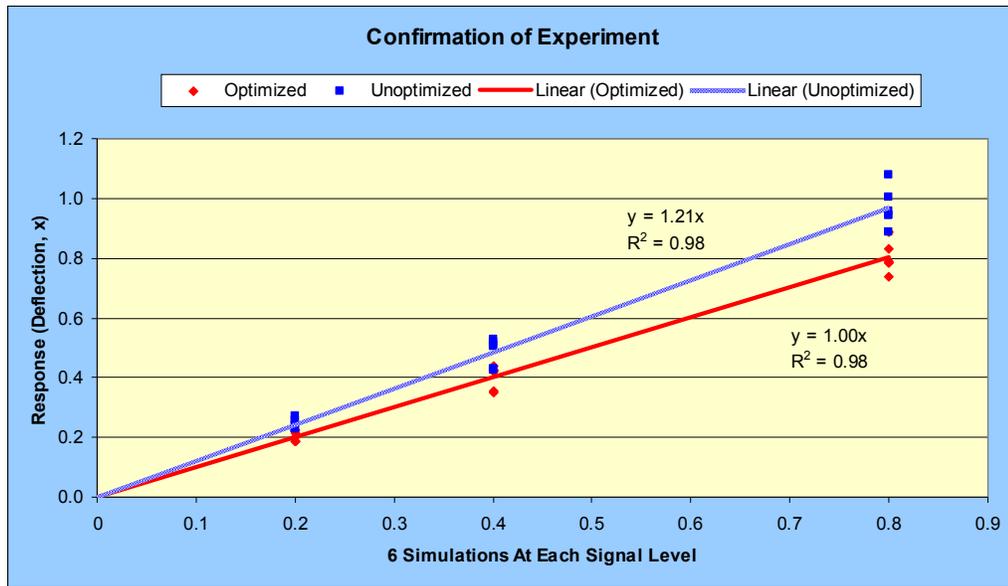


Figure 4.16: Result of Confirmation Experiment

4.4.3 Comparison of OP C1 and OP C2

OP C2 is the modification of OP C1 in which the factor level is determined by the user. OP C2 is not the optimum condition but a subjective decision which the user feels that it might be better than the optimum condition. OP C2 in this case is different from OP C1 where the level of factor G is changed to 1 as shown in Table 4.13.

Table 4.13: Level Setting of OP C1 and OP C2

| | A | B | C | D | E | F | G |
|-------|---|---|---|---|---|---|---|
| OP C1 | 1 | 2 | 1 | 3 | 2 | 2 | 2 |
| OP C2 | 1 | 2 | 1 | 3 | 2 | 2 | 1 |

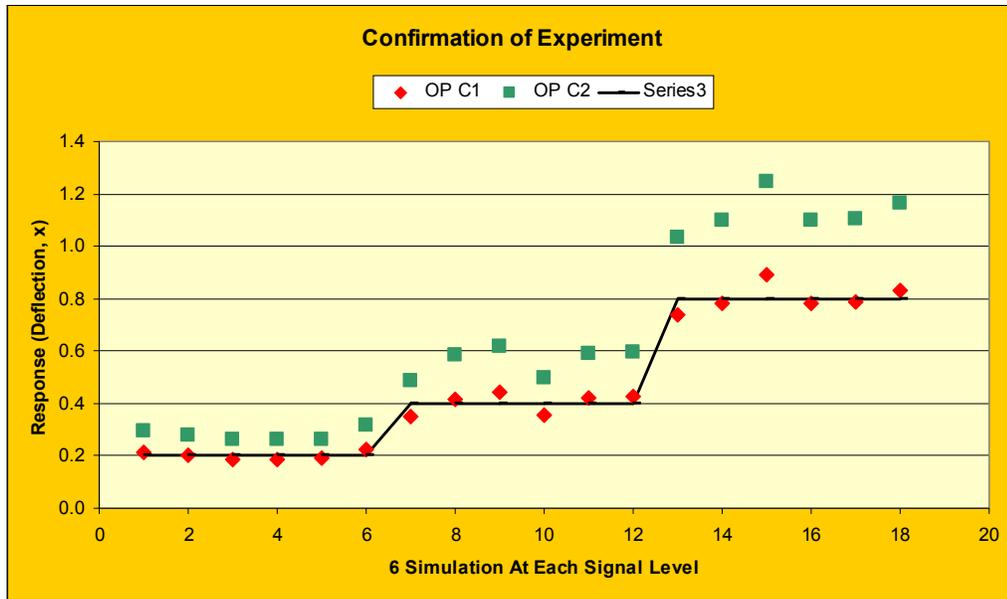


Figure 4.17: Comparison of Response of OP C1 and OP C2

From the Figure 4.17, the response of OP C2 is far away from the target as compared to OP C1. This proves that the factor level of the optimum condition does not improve the performance compared to OP C1. The difference between OP C1 and OP C2 is illustrated in Figure 4.18 by comparing the slope of each. Clearly, the result of OP C2 does not meet the target.

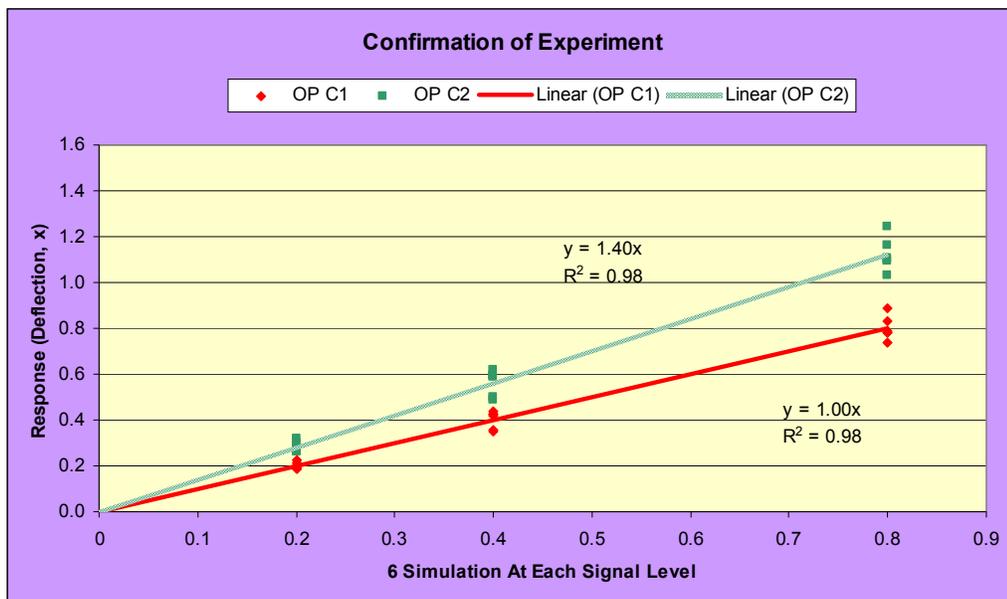


Figure 4.18: The Comparison of OP C1 and OP C2

4.4.4 Average Quality Loss after Optimisation

The average quality loss for OP C1 and OP C2 can be calculated using equation (3.6). Let the k constant be the same (80, same as trial 2), the result of the average quality loss will show how much there is the improvement.

For OP C1, the average quality loss is calculated as follows:

$$L_{avg}(y, t) = \frac{k \sum_{j=1}^J (y_j - t)^2}{n} = \frac{80 \times [(0.213 - 0.2)^2 + (0.200 - 0.2)^2 + \dots + (0.832 - 0.8)^2]}{18}$$

$$L_{avg}(y, t) = \$0.10$$

For OP C2, the average quality loss is calculated as follows:

$$L_{avg}(y, t) = \frac{k \sum_{j=1}^J (y_j - t)^2}{n} = \frac{80 \times [(0.298 - 0.2)^2 + (0.280 - 0.2)^2 + \dots + (1.165 - 0.8)^2]}{18}$$

$$L_{avg}(y, t) = \$3.866$$

The average quality loss of Trial 2 (before), OP C1 (after) and OP C2 is summarised in Table 4.14.

Table 4.14: Average Quality Loss of Before, After, and OP C2

| | Before | After(OP C1) | OP C2 | |
|---------------|--------|--------------|----------|-------------------|
| Loss Ao | 20 | 20 | 20 | \$ |
| Spec Do | 0.500 | 0.500 | 0.500 | s |
| k | 80.000 | 80.000 | 80.000 | \$/s ² |
| Avg Loss | 1.162 | 0.100 | 3.866 | \$/Piece |
| % Improvement | | 91.420 | -232.757 | % |

After the optimisation, the average quality loss has been reduced to \$0.10. The percentage of improvement after the optimisation is calculated using equation (3.8):

$$\%improvement = \frac{L_{avg,before} - L_{avg,after}}{L_{avg,before}} \times 100\%$$

$$\%improvement = \frac{1.162 - 0.10}{1.162} \times 100\% = 91.42\%$$

The improvement shows 91.42% after optimisation. The average quality loss of OP C2 in this case is much more than the current condition before optimisation.

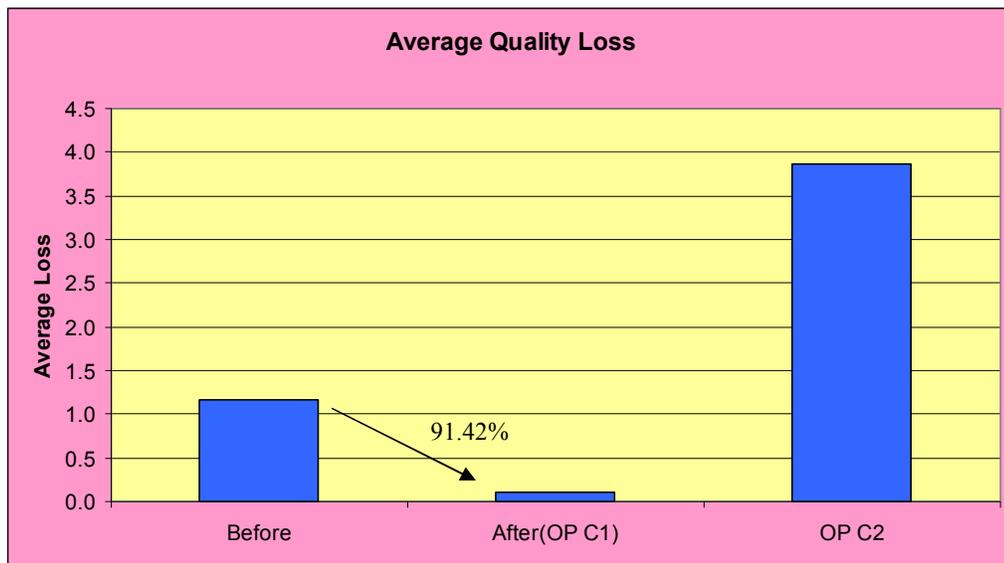


Figure 4.19: The Improvement after Optimisation

4.5 Discussions

The major difference between dynamic characteristic and static characteristic is the signal-response relationship. In such a linear relationship, the input signal is transformed into a desired output by adjusting the sensitivity of the linear function to a target value. The sensitivity of the linear function is determined by the slope (β) of the regression line. A target slope can be achieved by an appropriate setting of control factors and their levels. But, the presence of noise effect in the system causes deviation of the actual output. Therefore, the optimisation of dynamic characteristic is in objective to first minimise the variation around the linear function, and second adjust the slope of the regression line to a target value. Hence, the optimisation is done by finding out all significant factors and their levels that largely affect the

sensitivity and variability of the system. The target function as equation (2.9) is exactly a linear graph function. In this study, the target value of the slope is optimised to 1 according to design requirement. In such condition, the sensitivity required is neither too much nor too less. Therefore, the quality characteristic for this function is called Nominal-the-Best.

By going through the optimisation of dynamic characteristic, the optimum condition is found to be A1, B2, C1, D3, E2, F2, and G2 (A1 is signal factor). According to the analysis of variance for TPM, control factors B (span of the leaf spring, L) and F (thickness of the individual leaves, t) significantly affect the sensitivity of the signal-response relationship by a percentage of contribution of 44.950% and 44.867%. From the analysis of variance for NPM, control factors C (number of full-length leaves, n) and D (number of graduated-length leaves, N) significantly affect the variability by a percentage of contribution of 40.504% and 7.602%. The predicted mean of TPM is 1.563dB, while the predicted mean of NPM is 28.867dB. The predicted mean of TPM falls in the range of -6.520dB to 9.646dB according to the confidence interval of ± 8.083 , while the predicted mean of NPM is between 28.842dB to 28.893dB according to the confidence interval of ± 0.026 .

After the optimisation, the result clearly shows a reduction of average quality loss from \$1.162 to \$0.10, an improvement of 91.42%. Unlike the actual experiment in where the optimum condition can be tested by several runs for confirmation, the confidence interval for confirmation experiment which using simulated result in this study provides a range from -8.824dB to 8.884dB. As compared to the confidence interval for predicted mean which ranged from -6.520dB to 9.646dB, it shows that there is a good overlap. The overlap of the confidence intervals for both prediction and confirmation means that the result is additive and acceptable.

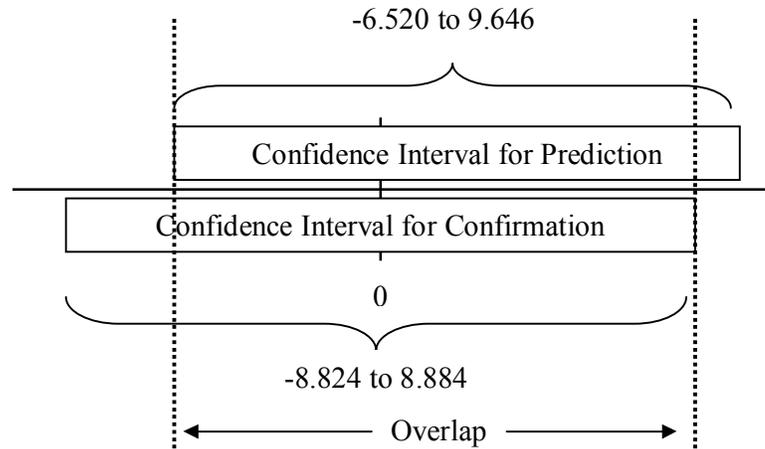


Figure 4.20: Overlap of Confidence Interval for Prediction and Confirmation

Form the result of the optimisation, the slope of the linear function clearly has been optimised to the target value which is equal to 1. However, the variation of the response around the linear function has not much been minimised. This can be explained by the analysis of variance for NPM. The percentage of contribution for significant factors (C and D) is a total of 48.106%, less than a half of total. Thus, not much percentage of variation has been reduced. The high percentage of pooling insignificant factors in this study is undesired and the reason of it should be investigated.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this final year project, optimisation of dynamic characteristic was done for optimising the deflection of semi-elliptical leaf spring using computer simulation. Dynamic characteristic is said to be better than static characteristic since it allows the objective function to be optimised over a range of signal factor levels. After optimisation of dynamic characteristic, the variation around the linear function was minimised while the slope of the linear function was adjusted to the target value. The Taguchi method of analysis showed that the optimum condition of control factors levels is B2, C1, D3, E2, F2, and G2. Control factors B (span of the leaf spring) and F (thickness of the individual leaves) significantly affect the sensitivity of linear function while C (number of full-length leaves) and D (number of graduated-length leaves) largely affect the variability. The confirmation of experiment showed the average quality loss has been much reduced with an improvement of 91.42%. Thus, the conclusions can be made as follows:

1. Dynamic characteristic is better than static characteristic as it allows the objective function to be optimised over a range of signal factor levels.
2. Those significant factors that largely affect the sensitivity of the linear function were found to be the span of the leaf spring (factor B) and the thickness of the individual leaves (factor F). Factors that significantly affect the variability are the number of the full-length leaves (factor C) and the number of the graduated-length leaves (factor D).

3. The regression analysis showed that the target slope of the optimum condition has been achieved by an appropriate setting of control factor levels.
4. In the optimum condition, the deflection of the semi-elliptical leaf spring does change according to the optimised linear function ($\beta = 1$) over a range of input signal.

5.2 Recommendation

After completing this final year project, it is recommended to conduct the computer simulation for dynamic characteristic optimisation using other orthogonal arrays like L_{36} or L_{27} and other objective functions for deeper understanding of Taguchi dynamic robust design.

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APPENDICES

APPENDIX A: F Distribution Table ($\alpha = 0.05$)

| v2/v1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|--------------|----------|----------|----------|----------|----------|----------|----------|
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 |