# APPLICATION OF SEQUENTIAL EXPERIMENTATION IN A PAPER GYROCOPTER SYSTEM 

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A project report submitted in partial fulfilment of the requirements for the award of Bachelor of Engineering (Hons.) Mechanical Engineering

Faculty of Engineering and Science Universiti Tunku Abdul Rahman

## DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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## APPROVAL FOR SUBMISSION

I certify that this project report entitled "APPLICATION OF SEQUENTIAL EXPERIMENTATION IN A PAPER GYROCOPTER SYSTEM" was prepared by GEE YIH KHOON has met the required standard for submission in partial fulfilment of the requirements for the award of Bachelor of Engineering (Hons.) Mechanical Engineering at Universiti Tunku Abdul Rahman.

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Specially dedicated to my beloved family.

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# APPLICATION OF SEQUENTIAL EXPERIMENTATION IN A PAPER GYROCOPTER SYSTEM 


#### Abstract

Although the design of experiments concept was introduced by Fisher in the early 1920s, the most research on this topic was carried out in the academic environment. One year later, Fisher demonstrated the usefulness of his concept in agricultural experiment; he analysed the optimum water, rain, sunshine, fertilizer, and soil conditions needed to produce the best crop. Taguchi went further with the design of experiment concept by introducing his approach in 1986. The present project that the author is doing can be theoretical or make it to experimental. The knowledge of the project is quite general when the author is study and doing this project. The author should be able to handle and learning different type of experimentation, particularly in engineering type. In the industrial, we can improve their product on process by doing many type of experiment. There are so many factor that have to identified by choosing which factor is the most important one, and which factor should change by increasing, decreasing or maintain it. These are the thing in the project that author is going to learn on how to evaluate the type of experiment. The application of DOE can be either by simulation or experimental. So, the method of experimentation and the data analysis is the key of this project. The objective of the project is more toward learning the method of the experimentation, theory, calculation, and the data analysis. For this project, the author is going to apply the sequential experimentation in a paper gyrocopter system.


## TABLE OF CONTENTS

DECLARATION ..... ii
APPROVAL FOR SUBMISSION ..... iii
ACKNOWLEDGEMENTS ..... vi
ABSTRACT ..... vii
TABLE OF CONTENTS ..... viii
LIST OF TABLES ..... xii
LIST OF FIGURES ..... xiv
LIST OF SYMBOLS / ABBREVIATIONS ..... xvi
CHAPTER
1 INTRODUCTION ..... 1
1.1 Historical Introduction ..... 1
1.2 The Design of Experiments Process ..... 2
1.3 Orthogonal Array Selection and Utilization ..... 2
1.4 Sequential Experimentation ..... 3
1.5 The Paper Gyrocopter ..... 4
1.6 Description of the project ..... 5
1.7 Objective of the project ..... 5
1.8 Scope of the project ..... 6
2 LITERATURE REVIEW ..... 7
2.1 Taguchi's Orthogonal Arrays ..... 7
2.2 Resolution III Designs ..... 10
2.2.1 Constructing Resolution III Designs ..... 10
2.2.2 Fold Over of Resolution III Fractions to Separate
Aliased Effects ..... 12
2.2.3 Single-factor fold-over ..... 12
2.2.4 Full fold-over ..... 14
2.2.5 Advantages of Sequential Experimentation ..... 16
2.3 Bicycle Example ..... 16
2.3.1 First fraction design ..... 17
2.3.2 Second fraction design ..... 18
2.3.3 Linear Contrasts ..... 19
2.3.4 Combined design ..... 20
3 METHODOLOGY ..... 23
3.1 Step of "Design of Experiment" ..... 23
3.1.1 Step 2: Objective ..... 24
3.1.2 Step 3: Problem Statement ..... 24
3.1.3 Step 4: Quality Characteristics ..... 24
3.1.4 Step 5: Current Level of Problem ..... 26
3.1.5 Step 6: Quality Loss Function ..... 26
3.1.6 Step 7: Cause-Effect Diagram ..... 26
3.1.7 Step 8: Parameter-Diagram ..... 28
3.1.8 Step 9: Factors for Study ..... 28
3.1.9 Step 10: Experimental Design ..... 30
3.1.10 Step 11: Response Table ..... 30
3.1.11 Step 12: Analysis of Variance (ANOVA) ..... 30
3.1.12 Step 13: Factor Level Averages ..... 32
3.1.13 Step 14: Response Graphs ..... 32
3.1.14 Step 15: Optimum Factor Selection ..... 32
3.1.15 Step 16: Predicted Values ..... 32
3.1.16 Step 17: Confirmation Experiment ..... 33
3.1.17 Step 18: Full Scale Implementation ..... 33
3.1.18 Step 19: Gain Calculation ..... 33
3.1.19 Step 20: Conclusion ..... 34
3.2 Confidence Intervals ..... 34
3.2.1 Confidence interval for the factor level ..... 34
3.2.2 Confidence Interval for the predicted mean ..... 35
3.2.3 Confidence interval for the confirmation experiment ..... 36
4 RESULTS AND DISCUSSIONS ..... 37
4.1 Current Level of Problem ..... 37
4.2 Quality Loss Function ..... 38
4.3 First fraction design (Part 1) ..... 39
4.3.1 Target Performance Measures and Noise Performance Measures ..... 41
4.3.2 Response Table (Part 1) ..... 42
4.3.3 Analysis of Variance (ANOVA) ..... 44
4.3.4 Confidence Interval for factor levels (Part 1) ..... 48
4.3.5 Factor Level Averages (Part 1) ..... 49
4.3.6 Response Graphs (Part 1) ..... 49
4.4 Second fraction design (Part 2) ..... 51
4.4.1 Response Table (Part 2) ..... 53
4.4.2 Analysis of Variance (Part 2) ..... 54
4.4.3 Confidence Interval for factor levels (Part 2) ..... 55
4.4.4 Factor Level Averages (Part 2) ..... 56
4.4.5 Response Graphs (Part 2) ..... 56
4.5 Combined design (Part $3=$ Part $1+$ Part 2) ..... 58
4.5.1 Response Table (Part 3) ..... 60
4.5.2 Analysis of Variance (Part 3) ..... 61
4.5.3 Significant data ..... 63
4.5.4 Confidence Interval for factor levels (Part 3) ..... 65
4.5.5 Factor Level Averages (Part 3) ..... 65
4.5.6 Response Graphs (Part 3) ..... 67
4.6 Optimum factor selection ..... 69
4.7 Predicted Values ..... 71
4.8 Confirmation Experiment ..... 73
4.9 Full Scale Implementation / Comparison of Before and After 77
4.10 Gain Calculation 79
4.11 Discussion on findings 80
5 CONCLUSION AND RECOMMENDATIONS 83
5.1 CONCLUSIONS 83
5.2 RECOMMENDATION 84
REFERENCES 85

## LIST OF TABLES

## TABLE

TITLE
PAGE
Table 2.1: $\mathrm{L}_{8}\left(\mathbf{2}^{7}\right)$ orthogonal array8
Table 2.3: Single-factor Fold-over of the $2_{I I I}^{7-4}$ Design with the Signs Reversed in the Column for factor $D$ in Table 2.2 ..... 13
Table 2.4: Full Fold-over of the $2_{I I I}^{7-4}$ Design with the Signs Reversed in the Column for all the factors in Table 2.2 ..... 15
Table 2.5: Factors for study ..... 17
Table 2.6: A $2_{I I I}^{7-4}$ Design for the Experiment of Time Bicycle Takes ..... 17
Table 2.7: Fold-over of the $2_{I I I}^{7-4}$ Design in Table 2.6 ..... 19
Table 2.8: De-aliasing all main effect. ..... 21
Table 2.9: Response Table ..... 21
Table 2.10: Analysis of Variance ..... 22
Table 3.1: Step of "Design of Experiment" ..... 23
Table 3.2: Signal-to-Noise Ratio ..... 25
Table 3.3: Factors for study ..... 29
Table 4.1: Current Level of Problem ..... 37
Table 4.2: Quality Loss Function ..... 39
Table 4.3: First fraction design (Part 1) ..... 40
Table 4.4: Response Table for TPM (Part 1) ..... 42
Table 4.5: Response Table for NPM (Part 1) ..... 43
Table 4.6: Analysis of Variance for TPM (Part 1) ..... 44
Table 4.7: Analysis of Variance for NPM (Part 1) ..... 46
Table 4.8: Factor Level Averages (Part 1) ..... 49
Table 4.9: Second fraction design (Part 2) ..... 52
Table 4.10: Response Table for TPM (Part 2) ..... 53
Table 4.11: Response Table for NPM (Part 2) ..... 53
Table 4.12: Analysis of Variance for TPM (Part 2) ..... 54
Table 4.13: Analysis of Variance for NPM (Part 2) ..... 54
Table 4.14: Factor Level Averages (Part 2) ..... 56
Table 4.15: Combined design (Part 3 = Part $1+$ Part 2) ..... 59
Table 4.16: Response Table for TPM (Part 3) ..... 60
Table 4.17: Response Table for NPM (Part 3) ..... 60
Table 4.18: Analysis of Variance for TPM (Part 3) ..... 61
Table 4.19: Analysis of Variance for NPM (Part 3) ..... 62
Table 4.20: Factor Level Averages (Part 3) ..... 66
Table 4.21: Optimum Factor Selection ..... 70
Table 4.22: Confirmation Experiment ..... 75
Table 4.23: Full Scale Implementation ..... 77
Table 4.24: Quality Improvement ..... 79

## LIST OF FIGURES

Figure 1.1: The paper gyrocopter ..... 4
Figure 3.1: The Cause and Effect Diagram ..... 27
Figure 3.2: Parameter Diagram ..... 28
Figure 3.3: ANOVA Formula Sheet ..... 31
Figure 4.1: Current Level of Problem ..... 38
Figure 4.2: Quality Loss Function ..... 39
Figure 4.3: Response Graph for TPM (Part 1) ..... 50
Figure 4.4: Response Graph for NPM (Part 1) ..... 50
Figure 4.5: Response Graph for TPM (Part 2) ..... 57
Figure 4.6: Response Graph for NPM (Part 2) ..... 57
Figure 4.7: Significance Data for TPM show by SSQ ..... 63
Figure 4.8: Significance Data for NPM show by SSQ ..... 63
Figure 4.9: Significance Data for TPM show by percentage of Rho ..... 64
Figure 4.10: Significance Data for NPM show by percentage of Rho ..... 64
Figure 4.11: Response Graph for TPM (Part 3) ..... 67
Figure 4.12: Response Graph for NPM (Part 3) ..... 68
Figure 4.13: Optimum factor selection for TPM ..... 72
Figure 4.14: Optimum factor selection for NPM ..... 72
Figure 4.15: Confirmation Experiment for TPM ..... 76
Figure 4.16: Confirmation Experiment for NPM ..... 76
Figure 4.17: Comparison of Before and After Optimization ..... 78
Figure 4.18: Cost Reduction \$/Piece ..... 80

## LIST OF SYMBOLS / ABBREVIATIONS

| F | F ratio |
| :---: | :---: |
| n | Number of observation |
| $\mathrm{n}_{\text {eff }}$ | Effective number of observation |
| Se | Error sum of squares |
| Sm | Sum of square due to mean |
| ST | Total sum of squares |
| St | Total sum of squares of corrected data |
| v1 | Degree of freedom associated with a mean |
| v2 | Degree of freedom for the pooled error variance |
| Ve | Pooled error variance |
| ve | Degree of freedom of error sum of squares |
| vm | Degree of freedom due to mean |
| vT | Degree of freedom of all data |
| vt | Degree of freedom of corrected data |
| $\alpha$ | Risk |
| $\sigma$ | Standard deviation |
| ANOVA | Analysis of variance |
| CE | Confirmation experiment |
| CI | Confidence interval |
| CV | Current value |
| CE diagram | Cause-and-effect diagram |
| DOE | Design of experiments |
| DOF | Degree of freedom |
| NPM | Noise performance measure |
| OA | Orthogonal array |
| PV | Predicted value |


| P diagram | Parameter diagram |
| :--- | :--- |
| SN ratio | Signal-to-noise ratio |

TPM Target performance measure

## CHAPTER 1

## INTRODUCTION

### 1.1 Historical Introduction

Taguchi methods, like other quality methodologies, have reached Europe from Japan via the USA. At such, there are elements of both Japanese philosophy and American enthusiasm contained in them.

Dr Genichi Taguchi was born on the $1^{\text {st }}$ January 1924, and developed his methods whilst working at the Electrical Communications Laboratory of the Nippon Telephone \& Telegraph Company post 1950. In 1957-8, he published the first version of this two-volume book on Design of Experiments. In 1964, Taguchi became a Professor at Aoyama Gakiun University in Tokyo, a position he held until 1982. In the early 1970s, Taguchi developed the concept of the Quality Loss Function. He published two other books in the 1970s and the third edition of Design of Experiments. At this stage, Taguchi's methods were still essentially unknown in the West, although applications were taking place in Taiwan and India. In this period and throughout the 1970s, most applications of his methods were on production processes, with a shift to product design during the last decade. By the late 1970s, Taguchi had an impressive record in Japan having won the Deming prize several times and was Director of the Japanese Academy of Quality. Following his 1980 visit to the United States, American manufacturers increasingly implemented Taguchi's methodology. There was something of an adverse reaction among American statisticians to the methods, and possibly at the way they were being
marketed. Nonetheless, major US companies became involved in the methods, including Xerox, Ford and ITT (Wilson \& Millar, 1990).

### 1.2 The Design of Experiments Process

The design of an experiment (DOE) is not a simple one-step process but is actually a series of steps which must follow a certain sequence for the experiment to yield an improved understanding of product or process performance. The DOE process is made up of three main phases: the planning phase, the conducting phase, and the analysis/interpretation phase. The steps in the DOE process are generically the same regardless of the experiment design, which is chosen to evaluate factorial effects. The experiment design can be anywhere between a full-factorial experiment and a very small fractional-factorial experiment. Many texts on the subject of designed experiments emphasize the analytical phase of the DOE process; however, positive experimental results are dependent upon the planning of the experiment and not on the analysis (Ross, 1996).

### 1.3 Orthogonal Array Selection and Utilization

A major step in the DOE process is the determination of the combination of factors and levels which will provide the experimenter with the desired information. One approach is to utilize a fractional-factorial approach whenever there are several factors involve, and this may be accomplished with the aid of orthogonal arrays. Orthogonal arrays are introduced from the viewpoint of the pragmatist who is always trying to make product or process improvement decisions with the minimum amount of test data. Using a minimal amount of test data is not necessarily a problem in itself; however, the considerations of what may make up a valid experiment from a risk viewpoint are seldom considered by the typically experimenter (Ross, 1996).

### 1.4 Sequential Experimentation

Sequential experimentation is a very effective strategy when performing experimental design because it provides an element of efficiency which is required for a successful experimentation. Without sequential experimentation many experiment would be unfeasible due to the obvious restrictions in costs or a waste of money and resources if a comprehensive experiment determines that there are no significant factors. Sequential experimentation is executed by combining the runs of two (or more) experiments to assemble a larger design that can estimate factor effects and interactions of interest (Rios, 2008).

Rios (2008) points out that the current literature recommends that the initial design should be a fractional factorial of the highest possible resolution constructed using the minimum aberration criteria. The second fraction is commonly constructed using a standard augmentation technique called foldover. The foldover reverses the signs of one or more factors in the initial design for the follow-up experiment.

Myers and Montgomery (2002) mention that Resolution III Designs are designs in which no main effects are aliased with any other main effect, but main effects are aliased with two-factor interactions and two-factor interactions may be aliased with each other. Box (1993) points out that because of the many uncertainties in choosing an appropriate experimental design, it is best to avoid "all encompassing" experiments which must necessarily be planned when least is known about the system. Instead, where possible, it is best to run smaller sets of experiments in sequence.

According to Myers and Montgomery (2002), using fractional factorial designs often leads to great economy and efficiency in experimentation, particularly if the runs can be made sequentially. For example, suppose that we were investigating $k=4$ factors ( $2^{4}=16$ runs). It is almost always preferable to run a $2_{I V}^{4-1}$ fractional design (8 runs), analyse the results, and then decide on the best set of runs to perform the next. If it is necessary to resolve ambiguities, we can always run the
alternate fraction and complete the $2^{4}$ design. When this method is used to complete the design, both one-half fractions represent blocks of the complete design with the highest-order interaction confounded with blocks would be confounded. Thus, sequential experimentation has the result of losing information only on the highestorder interaction. Alternatively, in many cases we learn enough from the one-half fraction to proceed to the next stage of experimentation, which might involve adding or removing factors, changing responses, or varying some of the factors over new ranges.

### 1.5 The Paper Gyrocopter

The gyrocopter (shown in Figure 1.1) has a number of control factors that have a high potential for interaction. The aim of the gyrocopter experiment is to produce a paper device that has a low terminal velocity, or high air resistance, so that it can floats slowly to the ground. Other requirements are that it be easily built, that it spins in a pleasing way as it falls, and that it has good stability in flight. This last requirement means that it is self-righting or that it orients itself properly for stable flight (Fowlkes \& Creveling, 1995).


Figure 1.1: The paper gyrocopter

For the gyrocopter case, there is nothing simple about the physical analysis. The optimum set points must be found empirically. Nevertheless, the requirements, along with a modicum of knowledge about flight, suggest several fundamental properties that need to be controlled independently. The terminal velocity depends upon the weight of the gyrocopter and the air resistance presented by the wings. The air resistance is assumed to be proportional to the surface area of the wings. The spinning motion is due to the torque provided by the wings. The moment of inertia and the air resistance to spinning determine the angular velocity. Stability depends on the location of the center of gravity. If it is below the point of attachment between the body and wings, then the gyrocopter will be stable in flight (Fowlkes \& Creveling, 1995).

The only factor that controls the torque is the wing width. That is the critical control factor assignment. The resistance to falling is proportional to the wing area. Thus, wing length must be made a sliding level factor (dependent factor) to allow independent control of the area. Paper weight is important because the stiffer the paper, the less the wings can be deflected by the relative motion of the air and rotation of the gyrocopter. However, a simple addition to the design, wing gussets, can stiffen the wing folds independent of paper weight (Fowlkes \& Creveling, 1995).

### 1.6 Description of the project

- To study the methodology for including more factors in an experiment to dealias factor interactions.


### 1.7 Objective of the project

- To apply the method of sequential experimentation.


### 1.8 Scope of the project

- The scope is mainly confined to applying advanced experimental techniques.


## CHAPTER 2

## LITERATURE REVIEW

### 2.1 Taguchi's Orthogonal Arrays

A major part of what has become accepted as Taguchi's methodology is the use of his tabulated sets of orthogonal arrays. Orthogonal array is often referred as Taguchi Method because of this method is popularised by G. Taguchi. An orthogonal array is a type of experiment where the columns for the independent variables are "orthogonal" to one another. It is often employed in industrial experiments to study the effect of several control factors.

Before proceeding further with a closer look, let's look at an example where orthogonal arrays have been employed. A typical tabulation is shown in Table 2.1, where there are seven control factor (A, B, C, D, E, F and G each at two levels) and three noise factors ( $\mathrm{P}, \mathrm{Q}$ and R each at two levels). This is an $\mathrm{L}_{8}\left(2^{7}\right)$ design, the 8 indicting the eight rows, trials or prototypes to be tested, with test characteristics in each case defined by the row of the table. (The L stands for Latin square.) A full factorial experiment would require $2^{7}=128$ experiments. Note that when we conducted a Taguchi experiment with a $\mathrm{L}_{8}\left(2^{7}\right)$ orthogonal array, this design reduces $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2=128$ trials to 8 .

Table 2.1: $\mathrm{L}_{8}\left(\mathbf{2}^{7}\right)$ orthogonal array


The benefits of using orthogonal arrays are; firstly, conclusions valid over the entire region spanned by the control factors and their settings. Second benefit is large saving in the experimental effort. Thirdly, analysis is easy.

To define an orthogonal array, one must identify; firstly, number of factors to be studied. Secondly, levels for each factor. Thirdly, identify the specific 2 -factor interactions to be estimated. Lastly, identify the special difficulties that would be encountered in running the experiment.

We know that with two-level full factorial experiments, we can estimate variable interactions. When two-level fractional designs are used, we begin to confound our interactions, and often lose the ability to obtain unconfused estimates of main and interaction effects. We have also seen that if the generators are chosen carefully then knowledge of lower order interactions can be obtained under that assumption that higher order interactions are negligible.

These designs fundamentally sacrifice information about interactions to reduce testing, although a very limited number of interactions can be induced. The reason why 'by and large' the interactions can be often safety neglected is that industrial responses are, of course, typically faster to evaluate than agricultural ones. Instead of a one year cycle to run a confirmatory trial to try out the predicted optimum, it may only take a few minutes, hours or perhaps days to evaluate one
additional prototype. Thus missing interactions can be identified and if necessary a further trial run carried out (Wilson \& Millar, 1990).

According to Bolboaca \& Jantschi (2007), a triad could better characterize the aim of manufacturing process optimisation which is best quality, less failures and higher productivity. Factorial analysis can be used in order to find the best values for parameters implies in the manufacturing process. Opposite to full factorial analysis, the Taguchi method reduces the number of experimental run to a reasonable one, in terms of cost and time, by using orthogonal arrays. The Taguchi method is used whenever the settings of interest parameters are necessary, not only for manufacturing processes. Therefore, the Taguchi approach is used in many domains such as environmental sciences, agricultural sciences, physics, chemistry, statistics, management and business, medicine, engineering and others.

### 2.2 Resolution III Designs

### 2.2.1 Constructing Resolution III Designs

The sequential use of fractional factorial designs is very useful, often leading to great economy and efficiency in experimentation. These ideas can be illustrate by using the class of resolution III designs. It is possible to construct resolution III designs for investigating up to $k=N-1$ factors in only $N$ runs, where $N$ is a multiple of 4 . These designs are frequently useful in industrial experimentation. Of particular importance are designs requiring " 4 runs for up to 3 factors", " 8 runs for up to 7 factors", and " 16 runs for up to 15 factors". If $k=N-1$, the fractional design is said to be saturated (Montgomery, 2005).

A design for studying up to seven factors in eight runs is the $2_{I I I}^{7-4}$ design. This design is a one-sixteenth fraction of the $2^{7}$. It may be constructed by first writing down as the basic design the plus and minus level for a full $2^{3}$ design in $A, B$, and $C$ and then associating the levels of four additional factors with the interactions of the original three as follows: $D=A B, E=A C, F=B C$, and $G=A B C$. Thus, the generators for this design are $I=A B D, I=A C E, I=B C F$, and $I=A B C G$. The design is shown in Table 2.2.

Table 2.2: The Saturated $2_{I I I}^{7-4}$ Design with the Generators $I=A B D, I=A C E$, $I=B C F$, and $I=A B C G$

|  | LA | LB | LC | LD | LE | LF | LG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | FG | EG | DG | EF | DF | DE | AF |
|  | CE | CF | BF | CG | BG | AG | BE |
|  | BD | AD | AE | AB | AC | BC | CD |
| Exp | A | B | C | D=AB | E=AC | F=BC | G=ABC |
| 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| 2 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 3 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 4 | 1 | 1 | -1 | 1 | -1 | -1 | -1 |
| 5 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 6 | 1 | -1 | 1 | -1 | 1 | -1 | -1 |
| 7 | -1 | 1 | 1 | -1 | -1 | 1 | -1 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

The complete defining relation for this design is obtained by multiplying the four generators $A B D, A C E, B C F$, and $A B C G$ together two at a time, three at a time, and four at a time, yielding

$$
\begin{aligned}
& I=A B D=A C E=B C F=A B C G \\
& 2 @ \text { a time, } I=B C D E=A C D F=C D G=A B E F=B E G=A F G \\
& 3 @ \text { a time, } I=D E F=A D E G=B D F G=C E F G \\
& 4 @ \text { a time, } I=A B C D E F G \\
& \text { So that }
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{I} & =\mathrm{ABD}=\mathrm{ACE}=\mathrm{BCF}=\mathrm{ABCG} \\
& =\mathrm{BCDE}=\mathrm{ACDF}=\mathrm{CDG}=\mathrm{ABEF}=\mathrm{BEG}=\mathrm{AFG} \\
& =\mathrm{DEF}=\mathrm{ADEG}=\mathrm{BDFG}=\mathrm{CEFG} \\
& =\mathrm{ABCDEFG}
\end{aligned}
$$

To find the aliases of any effect, simply multiply the effect by each word in the defining relation. For example, the aliases of A are:

$$
\begin{aligned}
\mathrm{A} & =\mathrm{BD}=\mathrm{CE}=\mathrm{ABCF}=\mathrm{BCG} \\
& =A B C D E=\mathrm{CDF}=\mathrm{ACDG}=\mathrm{BEF}=\mathrm{ABED}=\mathrm{FG} \\
& =A D E F=\mathrm{DEG}=\mathrm{ABDFG}=\mathrm{ACEFG} \\
& =\mathrm{BCDEFG}
\end{aligned}
$$

Taking only the two-factor interactions $\mathrm{A}=\mathrm{BD}=\mathrm{CE}=\mathrm{FG}$ as shown in column A in Table 2.2.

Similarly, for example, the aliases of $B$ are

$$
\begin{aligned}
B & =A D=A B C E=C F=A C G=C D E=A B C D F=B C D G=A E F=E G=A B F G \\
& =B D E F=A B D E G=B C E F G=D F G=A C D E F G
\end{aligned}
$$

And $\mathrm{B}=\mathrm{AD}=\mathrm{CF}=\mathrm{EG}$ as shown in column A in Table 2.2.

The seven degrees of freedom in this design may be used to estimate the seven main effects. Each of these effects has 15 aliases: however, if we assume that three-factor and higher interactions are negligible, then considerable simplification in the alias structure results. Making this assumption, each of the linear combinations associated with the seven main effects in this design actually estimates the main effect and three two-factor interactions:

$$
\begin{align*}
& l_{A} \rightarrow A+B D+C E+F G \\
& l_{B} \rightarrow B+A D+C F+E G \\
& l_{C} \rightarrow C+A E+B F+D G \\
& l_{D} \rightarrow D+A B+C G+E F  \tag{2.1}\\
& l_{E} \rightarrow E+A C+B G+D F \\
& l_{F} \rightarrow F+B C+A G+D E \\
& l_{G} \rightarrow G+C D+B E+A F
\end{align*}
$$

In obtaining these aliases, we have ignored the three-factor and higher-order interactions, assuming that they will be negligible in most practical applications where a design of this type would be considered (Montgomery, 2005).

### 2.2.2 Fold Over of Resolution III Fractions to Separate Aliased Effects

By combining fractional factorial designs in which certain signs are switched, we can systematically isolate effects of potential interest. The alias structure for any fraction with the signs for one or more factors reversed is obtained by making changes of sign on the appropriate factors in the alias structure of the original fraction. This type of sequential experiment is called fold-over of the original design, and it is used in resolution III designs to break the links between main effects and two-factor interactions. There are two types of sequential experimentation:

- Dealiasing any one main effect and all its 2-factor interactions (single-factor fold over),
- Dealiasing all main effects (full fold-over).


### 2.2.3 Single-factor fold-over

In a single-factor fold-over, we add to a fractional factorial design of resolution III a second fraction of the same size with the signs for only one of the factors reversed. In the combined design, we will be able to estimate the main effect of the factor for
which the signs were reversed as well as all two-factor interactions involving that factor. To illustrate, consider the $2_{I I I}^{7-4}$ design in Table 2.2. Suppose that along with this principal fraction a second fractional design with the signs reversed in the column for factor $D$ is also run. That is, the column for $D$ in the second fraction is -++-++- , as shown in Table 2.3.

Table 2.3: Single-factor Fold-over of the $2_{I I I}^{7-4}$ Design with the Signs Reversed in the Column for factor $\boldsymbol{D}$ in Table 2.2

|  | L'A | L'B | L'C | L'D | L'E | L'F | L'G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | FG | EG | -DG | -EF | -DF | -DE | AF |
|  | CE | CF | BF | -CG | BG | AG | BE |
|  | -BD | -AD | AE | -AB | AC | BC | -CD |
| Exp | A | B | C | $D=-A B$ | $\mathrm{E}=\mathrm{AC}$ | $\mathrm{F}=\mathrm{BC}$ | $\mathrm{G}=\mathrm{ABC}$ |
| 9 | -1 | -1 | -1 | -1 | 1 | 1 | -1 |
| 10 | 1 | -1 | -1 | 1 | -1 | 1 | 1 |
| 11 | -1 | 1 | -1 | 1 | 1 | -1 | 1 |
| 12 | 1 | 1 | -1 | -1 | -1 | -1 | -1 |
| 13 | -1 | -1 | 1 | -1 | -1 | -1 | 1 |
| 14 | 1 | -1 | 1 | 1 | 1 | -1 | -1 |
| 15 | -1 | 1 | 1 | 1 | -1 | 1 | -1 |
| 16 | 1 | 1 | 1 | -1 | 1 | 1 | 1 |

The effects that may be estimated from the first fraction are shown in Equation (2.1), and from the second fraction we obtain

$$
\begin{align*}
& l_{A}^{\prime} \rightarrow A-B D+C E+F G \\
& l_{B}^{\prime} \rightarrow B-A D+C F+E G \\
& l_{C} \rightarrow C+A E+B F-D G \\
& l_{D}^{\prime} \rightarrow D-A B-C G-E F  \tag{2.2}\\
& l_{-D}^{\prime} \rightarrow-D+A B+C G+E F \\
& l_{E}^{\prime} \rightarrow E+A C+B G-D F \\
& l_{F}^{\prime} \rightarrow F+B C+A G-D E \\
& l_{G}^{\prime} \rightarrow G-C D+B E+A F
\end{align*}
$$

assuming that three-factor and higher interactions are insignificant. Now from the two linear combinations of effects $1 / 2\left(l_{i}+l^{\prime}{ }_{i}\right)$ and $1 / 2\left(l_{i}-l_{i}{ }_{i}\right)$ we obtain

| $\boldsymbol{i}$ | $1 / 2\left(\boldsymbol{l}_{\boldsymbol{i}}+\boldsymbol{l}_{\boldsymbol{i}}\right)$ | $1^{1 / 2\left(\boldsymbol{l}_{\boldsymbol{i}}-\boldsymbol{l}_{\boldsymbol{i}}\right)}$ |
| :---: | :---: | :---: |
| $A$ | $A+C E+F G$ | $B D$ |
| $B$ | $B+C F+E G$ | $A D$ |
| $C$ | $C+A E+B F$ | $D G$ |
| $D$ | $D$ | $A B+C G+E F$ |
| $E$ | $E+A C+B G$ | $D F$ |
| $F$ | $F+B C+A G$ | $D E$ |
| $G$ | $G+B E+A F$ | $C D$ |

Thus, we have isolated the main effect of $D$ and all of its two-factor interactions. In general, if we add to a fractional design of resolution III or higher a further fraction with the signs of a single factor reversed, then the combined design will provide estimates of the main effect of that factor and its two-factor interactions. This is called a single-factor fold over (Montgomery, 2005).

### 2.2.4 Full fold-over

While the single-factor fold-over strategy illustrated in section 2.2 .3 is occasionally proves helpful, there is another variation of this technique that used frequently. In a full fold-over, we add to a resolution III fractional a second fraction in which the signs for all the factors are reversed. This type of fold-over breaks the alias links between all main effects and their two-factor interactions. That is, we may use the combined design to estimate all the main effects clear of any two-factor interactions. To illustrate, consider the $2_{I I I}^{7-4}$ design in Table 2.2. Suppose that along with this principal fraction a second fractional design in which the signs in the column for all the factors are reversed, as shown in Table 2.4.

Table 2.4: Full Fold-over of the $2_{I I I}^{7-4}$ Design with the Signs Reversed in the

## Column for all the factors in Table 2.2

|  | L'A | L'B | L'C | L'D | L'E | L'F | L'G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -FG | -EG | -DG | -EF | -DF | -DE | -AF |
|  | -CE | -CF | -BF | -CG | -BG | -AG | -BE |
|  | -BD | -AD | -AE | -AB | -AC | -BC | -CD |
| Exp | A | B | C | D | E | F | G |
| 9 | 1 | 1 | 1 | -1 | -1 | -1 | 1 |
| 10 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| 11 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| 12 | -1 | -1 | 1 | -1 | 1 | 1 | 1 |
| 13 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| 14 | -1 | 1 | -1 | 1 | -1 | 1 | 1 |
| 15 | 1 | -1 | -1 | 1 | 1 | -1 | 1 |
| 16 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |

To separate the main effects and the two-factor interactions, a second fraction is run with all the signs reversed. This fold-over design is shown in Table 2.4. Notice that when we fold over a resolution III design in this manner, we (in effect) change the signs on the generators that have an odd number of letters. The effects estimated by this fraction are

$$
\begin{align*}
& l_{A}^{\prime} \rightarrow A-B D-C E-F G \\
& l_{B}^{\prime} \rightarrow B-A D-C F-E G \\
& l_{C}^{\prime} \rightarrow C-A E-B F-D G \\
& l^{\prime}{ }_{D} \rightarrow D-A B-C G-E F  \tag{2.3}\\
& l^{\prime}{ }_{E} \rightarrow E-A C-B G-D F \\
& l_{F}^{\prime} \rightarrow F-B C-A G-D E \\
& l_{G}{ }_{G} \rightarrow G-C D-B E-A F
\end{align*}
$$

By combining the effect estimates from this second fraction with the effect estimates from the original eight runs, we obtain the following estimates of the effects:

| $\boldsymbol{i}$ | $1 / 2\left(\boldsymbol{l}_{\boldsymbol{i}}+\boldsymbol{l}_{\boldsymbol{i}} \mathbf{j}^{1 / 2\left(\boldsymbol{l}_{\boldsymbol{i}}-\boldsymbol{l}_{\boldsymbol{i}}\right)}\right.$ |  |
| :---: | :---: | :---: |
| $A$ | $A$ | $B D+C E+F G$ |
| $B$ | $B$ | $A D+C F+E G$ |
| $C$ | $C$ | $A E+B F+D G$ |
| $D$ | $D$ | $A B+C G+E F$ |
| $E$ | $E$ | $A C+B G+D F$ |
| $F$ | $F$ | $B C+A G+D E$ |
| $G$ | $G$ | $C D+B E+A F$ |

### 2.2.5 Advantages of Sequential Experimentation

If an experimenter wishes to conduct a $2^{4}$ design involving 16 trials, it is almost always better to conduct a half-fraction containing 8 trials first, analyse the results and conduct the second half-fraction to complete the design if necessary. It may also note that: Firstly, the experimenter should randomise within each fraction. Secondly, if both fractions are run, these fractions will be randomised orthogonal blocks of the complete design. Thirdly, no information is lost except that those people who concerning the interaction which is actually confounded with the block contrast. Lastly, the design run as two randomised fractions eliminates block differences and can therefore give greater precision than the whole design.

### 2.3 Bicycle Example

A bicycle performance analyst is conducting an experiment to study the time takes to pedal up a hill and has built an apparatus in which several factors can be controlled during the test (Updated Spring 2005). The factors he initially regards as important are set (A), dynamo (B), handlebars (C), gear (D), raincoat (E), breakfast (F) and tires (G). Two levels of each factor are considered. He suspects that only a few of these seven factors are of major importance and that high-order interactions between
the factors can be neglected. On the basis of this assumption, the analyst decides to run a screening experiment to identify the most important factors and then to concentrate further study on those. To screen seven factors, he runs the treatment combinations from the $2_{I I I}^{7-4}$ design in Table 2.2 in random order which obtaining the times in minutes, as shown in Table 2.6.

Table 2.5: Factors for study

| Control | Description | $\frac{\text { LeveL }}{(-1)}$ | $\frac{\text { LeveL }}{(1)}$ |
| :---: | :--- | :---: | :---: |
| A | Set | Up | Down |
| B | Dynamo | Off | On |
| C | Handlebars | Up | Down |
| D | Gear | Low | Medium |
| E | Raincoat | On | Off |
| F | Breakfast | Yes | No |
| G | Tires | Hard | Soft |

Table 2.6: A $2_{I I I}^{7-4}$ Design for the Experiment of Time Bicycle Takes

|  | ABCG | LA | LB | LC | LD | LE | LF | LG |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BCF | FG | EG | DG | EF | DF | DE | AF |  |
|  | ACE | CE | CF | BF | CG | BG | AG | BE |  |
|  | ABD | BD | AD | AE | AB | AC | BC | CD |  |
| Exp | I | A | B | C | $\mathrm{D}=\mathrm{AB}$ | $\mathrm{E}=\mathrm{AC}$ | $\mathrm{F}=\mathrm{BC}$ | $\mathrm{G}=\mathrm{ABC}$ | y |
| 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 69 |
| 2 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 52 |
| 3 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 60 |
| 4 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 83 |
| 5 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 71 |
| 6 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 50 |
| 7 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 59 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 88 |

### 2.3.1 First fraction design

Seven main effects and their aliases may be estimated from these data. From Equation (2.1), we see that the effects and their aliases are

$$
\begin{align*}
& l_{A}=3.5 \rightarrow A+B D+C E+F G \\
& l_{B}=12 \rightarrow B+A D+C F+E G \\
& l_{C}=1 \rightarrow C+A E+B F+D G \\
& l_{D}=22.5 \rightarrow D+A B+C G+E F  \tag{2.4}\\
& l_{E}=0.5 \rightarrow E+A C+B G+D F \\
& l_{F}=1 \rightarrow F+B C+A G+D E \\
& l_{G}=2.5 \rightarrow G+C D+B E+A F
\end{align*}
$$

A contrast such as $l_{A}$ is obtained by multiplying a column of signs for factor $A$ by the observation $y$ and dividing by the number of observations for the sign, usually $N / 2$ (for a 2-level factor) where $N$ is the number of observations in $y$ except $l_{I}$ where we divide by $N$ since $I$ is a column + . Also, we note that $l_{I}=\bar{y}$, the overall experimental average.

For example,

$$
l_{A}=1 / 4(-69+52-60+83-71+50-59+88)=3.5
$$

The largest three effects are $l_{A}, l_{B}$, and $l_{D}$. The simplest interpretation of the data is that the main effects of $A, B$, and $D$ are all significant. However, this interpretation is not unique, because one could also logically conclude that ' $A, B$, and the $A B$ interaction', or perhaps ' $B, D$, and the $B D$ interaction', or perhaps ' $A, D$, and the $A D$ interaction' are the true effects. A will be aliased with $\mathrm{BD}, \mathrm{B}$ will be aliased with $A D$, and $D$ will be aliased with $A B$, so the interactions cannot be separated from the main effects.

### 2.3.2 Second fraction design

To separate the main effects and the two-factor interactions, a second fraction is run with all the signs reversed. This fold-over design is shown in Table 2.7 along with the observed responses. Notice that when we fold over a resolution III design in this manner, we (in effect) change the signs on the generators that have an odd number of letters. The effects estimated by this fraction are

$$
\begin{align*}
l_{A}^{\prime} & =2 \rightarrow A-B D-C E-F G \\
l_{B}^{\prime} & =12.5 \rightarrow B-A D-C F-E G \\
l_{C}^{\prime} & =1.5 \rightarrow C-A E-B F-D G \\
l_{D}^{\prime} & =21.5 \rightarrow D-A B-C G-E F  \tag{2.5}\\
l_{E}^{\prime} & =1.5 \rightarrow E-A C-B G-D F \\
l_{F}^{\prime} & =3 \rightarrow F-B C-A G-D E \\
l_{G}^{\prime} & =2 \rightarrow G-C D-B E-A F
\end{align*}
$$

Table 2.7: Fold-over of the $2_{I I I}^{7-4}$ Design in Table 2.6

|  | ABCG | L'A | L'B | L'C | L'D | L'E | L'F | L'G |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -BCF | -FG | -EG | -DG | -EF | -DF | -DE | -AF |  |
|  | -ACE | -CE | -CF | -BF | -CG | -BG | -AG | -BE |  |
|  | -ABD | -BD | -AD | -AE | -AB | -AC | -BC | -CD |  |
| Exp | 1 | A | B | C | D | E | F | G | $y$ |
| 9 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 63 |
| 10 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 82 |
| 11 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | 73 |
| 12 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 53 |
| 13 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 64 |
| 14 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | 84 |
| 15 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | 72 |
| 16 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 45 |

### 2.3.3 Linear Contrasts

We now develop the idea of a linear contrast. For a pair of linear contrasts we can always take their sum or difference and obtain two further linear contrasts. Let us consider the linear contrast of A, which is $l_{A}$ and $l_{A}$ ':

$$
\begin{aligned}
& l_{A} \rightarrow A+B D+C E+F G \\
& l_{A}^{\prime} \rightarrow A-B D-C E-F G
\end{aligned}
$$

Therefore, the sum of the linear contrast is:

$$
\begin{aligned}
& l_{A}+l_{A}^{\prime} \rightarrow 2 A \\
& \text { or } \quad \frac{1}{2}\left(l_{A}+l_{A}^{\prime}\right) \rightarrow A
\end{aligned}
$$

And the difference of the contrast is:

$$
\begin{aligned}
& l_{A}-l_{A}^{\prime} \rightarrow 2(B D+C E+F G) \\
& \text { or } \quad \frac{1}{2}\left(l_{A}-l_{A}^{\prime}\right) \rightarrow B D+C E+F G
\end{aligned}
$$

Using a similar procedure for $\mathrm{A}, \mathrm{B}, \mathrm{C}, \ldots, \mathrm{G}$. By combining the effect estimates from this second fraction with the effect estimates from the original eight runs, we obtain the following estimates of the effects:

| $\boldsymbol{i}$ | $\frac{1}{2}\left(\boldsymbol{l}_{\boldsymbol{i}}+\boldsymbol{l}_{\boldsymbol{i}}^{\prime}\right)$ | $\left(\mathbf{2}\left(\boldsymbol{l}_{\boldsymbol{i}}-\boldsymbol{l}_{\boldsymbol{\prime}}\right)\right.$ |
| :--- | :--- | :--- |
| $A$ | $A=2.75$ | $\boldsymbol{B D}+C E+F G=0.75$ |
| $B$ | $\boldsymbol{B}=12.25$ | $A D+C F+E G=-0.25$ |
| $C$ | $C=1.25$ | $A E+B F+D G=-0.25$ |
| $D$ | $\boldsymbol{D}=22$ | $A B+C G+E F=0.5$ |
| $E$ | $E=1$ | $A C+B G+D F=-0.5$ |
| $F$ | $F=2$ | $B C+A G+D E=-1$ |
| $G$ | $G=2.25$ | $C D+B E+A F=0.25$ |

The largest two effects are $B$ and $D$. Furthermore, the third largest effect is $B D+C E+F G$, so it seems reasonable to attribute this to the $B D$ interaction. The analyst used the two factors dynamo $(B)$ and gear $(D)$ in subsequent experiments with the other factors $A, C, E$, and $F$ at standard settings and verified the results obtained here.

### 2.3.4 Combined design

Sequential experimentation is executed by combining the runs of two (or more) experiments to assemble a larger design that can estimate factor effects and interactions of interest.

Table 2.8: De-aliasing all main effect.

|  |  |  |  |  |  |  |  |  | FG | EG | DG | EF | DF | DE | $C D$ | BCF |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ABCG |  |  |  |  |  |  |  | CE | CF | BF | CG | BG | BC | BE | ACE |  |
| Exp | 1 | A | B | C | D | E | F | G | BD | AD | DG | AB | DF | DE | CD | ABD |  |
| 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 69 |
| 2 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 52 |
| 3 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 60 |
| 4 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 83 |
| 5 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 71 |
| 6 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 50 |
| 7 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 59 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 88 |
| 9 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | 63 |
| 10 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 82 |
| 11 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 73 |
| 12 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | -1 | 53 |
| 13 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 64 |
| 14 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 84 |
| 15 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 72 |
| 16 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | 45 |

Table 2.9: Response Table

| TPM | A | B | C | D | E | F | G | BD | AD | DG | AB | DF | DE | CD | ABD |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| -1 | 65.38 | 60.63 | 66.13 | 55.75 | 66.25 | 65.75 | 65.63 | 66.38 | 66.88 | 66.88 | 66.5 | 67 | 67.25 | 66.63 | 67 |
| 1 | 68.13 | 72.88 | 67.38 | 77.75 | 67.25 | 67.75 | 67.88 | 67.13 | 66.63 | 66.63 | 67 | 66.5 | 66.25 | 66.88 | 66.5 |
| Diff | 2.75 | 12.25 | 1.25 | 22 | 1 | 2 | 2.25 | 0.75 | 0.25 | 0.25 | 0.5 | 0.5 | 1 | 0.25 | 0.5 |
| SSQ | 30.25 | 600.3 | 6.25 | 1936 | 4 | 16 | 20.25 | 2.25 | 0.25 | 0.25 | 1 | 1 | 4 | 0.25 | 1 |
| Rank | 3 | 2 | 6 | 1 | 7 | 5 | 4 | 9 | 12 | 12 | 10 | 10 | 7 | 12 | 10 |
| Opt | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | -1 |

Table 2.10: Analysis of Variance

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| A | 0 | 30.25 | 1 | 30.25 | 14.14 | 28.11 | 1.07 |
| B | 0 | 600.25 | 1 | 600.25 | 280.64 | 598.11 | 22.80 |
| C | 1 | 6.25 | 1 | 6.25 | 2.92 |  |  |
| D | 0 | 1936 | 1 | 1936 | 905.14 | 1933.86 | 73.73 |
| E | 1 | 4 | 1 | 4 | 1.87 |  |  |
| F | 0 | 16 | 1 | 16 | 7.48 | 13.86 | 0.53 |
| G | 0 | 20.25 | 1 | 20.25 | 9.47 | 18.11 | 0.69 |
| BD | 1 | 2.25 | 1 | 2.25 | 1.05 |  |  |
| AD | 1 | 0.25 | 1 | 0.25 | 0.12 |  |  |
| DG | 1 | 0.25 | 1 | 0.25 | 0.12 |  |  |
| AB | 1 | 1 | 1 | 1 | 0.47 |  |  |
| DF | 1 | 1 | 1 | 1 | 0.47 |  |  |
| DE | 1 | 4 | 1 | 4 | 1.87 |  |  |
| CD | 1 | 0.25 | 1 | 0.25 | 0.12 |  |  |
| ABD | 1 | 1 |  |  |  |  |  |
| Error |  |  |  |  |  |  |  |
| Pooled |  | 19.250 | 9 | 2.14 |  | 1 | 30.94 |
| St |  | 2623 | 15 | 174.87 |  | 2623.00 | 100.00 |
| Sm |  | 71289 | 1 |  |  |  |  |
| ST |  | 73912 | 16 |  |  |  |  |

## CHAPTER 3

## METHODOLOGY

### 3.1 Step of "Design of Experiment"

Several steps that are needed for design of experiment. The 20-Step Design of Experiments proposed by Belavendram (2007) is shown below.

Table 3.1: Step of "Design of Experiment"

| Step 1. | Name of Researcher |
| :---: | :--- |
| Step 2. | Objective |
| Step 3. | Problem Statement |
| Step 4. | Quality Characteristics |
| Step 5. | Current Level of Problem |
| Step 6. | Quality Loss Function |
| Step 7. | Cause-Effect Diagram |
| Step 8. | Parameter-Diagram |
| Step 9. | Factors for Study |
| Step 10. | Experimental design |
| Step 11. | Response Table |
| Step 12. | Analysis of variance |
| Step 13. | Factor Level Averages |
| Step 14. | Response Graphs |
| Step 15. | Optimum Factor Selection |
| Step 16. | Predicted Values |


| Step 17. | Confirmation Experiment |
| :--- | :--- |
| Step 18. | Full Scale Implementation |
| Step 19. | Gain Calculation |
| Step 20. | Conclusion |

### 3.1.1 Step 2: Objective

The project objectives are to study the methodology for including more factors in an experiment to dealias factor interactions. In other word, we want to apply the method of sequential experimentation. Since our project title is application of sequential experimentation in a paper gyrocopter system. So, the objective of conducting the experiment is to optimize the flight time of a paper gyrocopter falling from a height of 3 m from 2 second to 3 second.

### 3.1.2 Step 3: Problem Statement

There are several problems encountered in our experiment. First, the preliminary design of gyrocopter has a low flight time, which is about 2 second. Due to this problem, different design of gyrocopter has been fabricated to obtain longer flight duration. However, some of the designs are subjected to unbalance spinning motion which is hard to control. Lastly, the air flow at the outdoor environment is unsteady, which increases the tendency of the gyrocopter to hit the obstacles such as wall, table and chair.

### 3.1.3 Step 4: Quality Characteristics

What is to be measured? This is effectively the response. What is this and how is it to be measured? Quality characteristics can be described as the product features which
can be used to measure the quality of a product that is directly associated to the customer satisfaction. Some of the quality characteristics of a product are the geometry, physical and functional properties. Quality characteristics emphasizes of the use of statistical process control (SPC), which applies statistical method to monitor and control a process so that the process is operating at its optimum condition to produce product with high quality.

The key components of quality characteristics are Target Performance Measure (TPM) and Noise Performance Measure (NPM). The function of TPM is to adjust to target while NPM is an objective measure to reduce variability.

Generally, quality characteristics are classified into five categories which are nominal the best, smaller the better, larger the better, signed target and yield. The first three types of quality characteristics are widely applied for the industrial processes due to their ease of understanding as well as for comparison purposes. The type of characteristics determines the objective target intended. Beside, the equation of Target Performance Measure (TPM) and Noise Performance Measure (NPM) are provided for each quality characteristic as shown in Table 3.2. So, much of the data analysis later in a design of experiments is determined by the type quality characteristic.

Table 3.2: Signal-to-Noise Ratio

| Characteristic | TPM | NPM |
| :---: | :---: | :---: |
| Nominal the best | $\bar{y}$ | $-10 \log \left(\frac{\sigma^{2}}{\bar{y}}\right)$ |
| Smaller the better | $\bar{y}$ | $-10 \log \left(\sigma^{2}+\bar{y}^{2}\right)$ |
| Larger the better | $\bar{y}$ | $-10 \log \left(\frac{1}{\bar{y}^{2}}\left(1+\frac{3 \sigma^{2}}{\bar{y}^{2}}\right)\right)$ |
| Signed target | $\bar{y}$ | $-10 \log \sigma^{2}$ |
| Yield | P | $-10 \log \left(\frac{1}{P}-1\right)$ |

Where $\bar{y}$ is the mean and $\sigma$ is the standard deviation.

In the experiment, the intended target is to capture the flight time response of the gyrocopter. The unit of measure used for the flight time is in seconds. Therefore, Larger-the-Better is used in this experiment since we wish to obtain a longer flight period of the gyrocopter to meet the requirements.

### 3.1.4 Step 5: Current Level of Problem

The current level of problem is an indication of the status quo. This should clearly show the current performance of the response being measured. In particular, the target, process mean and variation should be evident from a suitable graphical representation.

### 3.1.5 Step 6: Quality Loss Function

The quality loss function is often attributed to Taguchi although there is evidence that it may be used earlier without attribution. The quadratic loss function is an approach to quantifying the "average quality loss" based on its deviation from the target. This is often done on a "loss per piece" basis. Many statistics are not at ease with this method. If this is the case, the stakeholder should identify a suitable method of quantifying the loss. After all, the practical need for this loss function quantification is for a comparison of cost savings. Any other reasonable method may be used.

### 3.1.6 Step 7: Cause-Effect Diagram

The cause-effect diagram step allows us to brainstorm for factors affecting the response. It is not unusual that an optimization be done by an individual. Even in that case, the cause-effect diagram forces us to look for all likely causes that affect the
response. It is commonly used during product design and quality defect prevention stages.


Figure 3.1: The Cause and Effect Diagram

According to our experiment, there are three major categories of factors which can affect the outcome or in this case, flight time of the gyrocopter. First of all, control factors usually consist of two or more levels and can be controlled by designers. Moreover, control factors contribute significant effect to the flight time of the gyrocopter. Meanwhile, the noise factors would cause variation in the results. Noise factors usually arise as a result of different measuring apparatus, people, environment and methods are used during the experiment. Lastly, signal factors are usually fixed throughout the entire experiment. For instance, the dropping height is fixed to 3 m in our experiment.

### 3.1.7 Step 8: Parameter-Diagram

After identified the likely causes, the next step is to build a Parameter Diagram. This diagram is essentially a short list of factors that can be classified according to its role in the experiment.


Figure 3.2: Parameter Diagram

Parameter diagram creates relationship between inputs and outputs of a system through mathematical model. Parameter diagram takes all the noise factors into consideration, fix the signal factors throughout the whole process and finally choose the appropriate control factors in order to achieve optimum outcome. Indeed, parameter diagram is usually used when we want to minimize the design sensitivity and variation in the outcomes.

### 3.1.8 Step 9: Factors for Study

Control factors are factors that are easy and inexpensive to control in the design of the product. Noise factors are factors that may affect the response of interest but
which are difficult to control when the product is being manufactured or being used by the customer (although they can be controllable for purposes of a test). These noise factors are often functions of environmental conditions, for example, humidity, properties of raw materials, temperature, etc. Signal factors change the overall effect of the response. They are often included for study in an experiment and no optimum condition is selectable. Instead, the signal value is set according to the user's condition.

In this step therefore, the factor level settings (current and proposed levels) are specified for subsequent experimentation. By factor levels it is meant whether a factor is studied 2-levels, 3-levels, etc. Although more levels may appear better, since experimental trials increase by a power function, often 2-level factors are suitable for screening experiments and for most practical reasons 3-level factors are sufficient to detect curvature (Belavendram, 2007).

Table 3.3: Factors for study

| Control | Description | Level (-1) | Level (1) | Units |
| :---: | :--- | :---: | :---: | :--- |
| A | Paper type | manila kad | A4/80 | gsm |
| B | Body width | 2 | 3 | cm |
| C | Body Length | 6 | 10 | cm |
| D | Wing Length | 8 | 12 | cm |
| E | Shoulder | 2 | 4 | cm |
| F | Wing width | 2 | 4 | cm |
| G | Clip | 1 | 2 | pcs |


| Noise | Description | Level (-1) | Level (1) |
| :---: | :--- | :---: | :---: |
| P | Thrower | Gee Yih Khoon | Ooi Lip Khun |
| Q | Location | Outdoor | Indoor |
| R | Time piece | Mobile Phone | Watch |


| Signal | Description | Level 1 | Units |
| :---: | :---: | :---: | :--- |
| H | Height of Throw | 3 | m |

### 3.1.9 Step 10: Experimental Design

The Experimental Design step is where the experimental recipes are created. This forms the "table of factor settings" that allow us to construct the experimental trials. In most cases, a good design should include a direct product design consisting of a control factor array and noise factor array. Data collection then follows the factor settings. Data for experimental trials can be logically and easily entered into a two way array.

### 3.1.10 Step 11: Response Table

Once data is entered, a response table of values indicating the mean effects of the performance indicators, such as the Target Performance Measure (TPM) and Noise Performance Measures (NPM), are calculated. Response tables show some information regarding factor importance; however, they do not show the factor effects relative to the experimental error (Belavendram, 2007).

### 3.1.11 Step 12: Analysis of Variance (ANOVA)

The analysis of variance is where the significance of the factor effects is calculated relative to the experimental error. Pooling of factors should be done intuitively based on the factors that can be pooled. A dynamic approach would be ideal as users can see the effects real-time and make decision easily (Belavendram, 2007).

To perform the analysis of variance, some of the value we should get it from the data. Below is the formula using in analysis of variance.

$$
\begin{array}{|ll|}
\hline S_{T}=\sum_{i=1, j=1}^{c, r} y_{i, j}^{2} & v_{T}=c \times r \\
S_{m}=n \bar{y}^{2} & v_{m}=1 \\
S_{t}=S_{T}-S_{m} & v_{t}=v_{T}-v_{m} \\
S_{A}=n_{A 1} \bar{y}_{A 1}^{2}+n_{A 2} \bar{y}_{A 2}^{2}-S_{m} & v_{A}=1 \\
S_{e}=S_{t}-\left(S_{A}+S_{B}+\cdots+S_{G}\right) & v_{e}=v_{t}-\left(v_{A}+v_{B}+\cdots+v_{G}\right) \\
V_{A}=\frac{S_{A}}{v_{A}} ; V_{e}=\frac{S_{e}}{v_{e}} ; V_{\text {Pool }}=\frac{S_{\text {Pool }}}{v_{\text {Pool }}} & \\
S_{A}^{\prime}=S_{A}-v_{A} \times V_{\text {Pool }} & \\
S_{\text {Pool }}^{\prime}=S_{t}-\left(S_{A}^{\prime}+S_{B}^{\prime}+\cdots+S_{G}^{\prime}\right) & \\
\rho_{A}=\frac{S_{A}^{\prime}}{S_{t}} \times 100 \% & \\
\hline
\end{array}
$$

Figure 3.3: ANOVA Formula Sheet
where
$S \quad=\mathrm{SSQ}$
$S_{T} \quad=$ total sum of squares
$S_{m} \quad=$ sum of squares of the mean
$S_{e} \quad=$ error sum of square
$V \quad=$ degree of freedom, DOF
$v_{T} \quad=$ total degree of freedom
$v_{e} \quad=$ degree of freedom for error
$V \quad=$ variance (VAR)
$S^{\prime}=\mathrm{SSq}{ }^{\prime}$
$\rho \quad=$ percentage of variance, Rho

### 3.1.12 Step 13: Factor Level Averages

Information from Steps 11 and 12 are combined to generate these tables which show the mean factor levels for the performance measures.

### 3.1.13 Step 14: Response Graphs

Data from the response tables are not very easily appreciated unless shown as a graphical display. The response graphs display factor effects very clearly. So that most of us will now begin to "feel" the processes we studied.

### 3.1.14 Step 15: Optimum Factor Selection

Once the analysis of variance is completed and unimportant factors pooled to error, the remaining factors can be considered important towards the response that is being studied.

Given the response type in Step 4, optimum conditions can be routinely calculated. Likewise, since the current condition is known from step 9, it is also possible to include comparisons of the current condition. With a dynamic model, we can evaluate other optimum conditions as required and immediately compare its performance with the recommended optimum condition. Most commercial Design of Experiments software does not come anywhere close to this.

### 3.1.15 Step 16: Predicted Values

Even if we conducted full factorial experiments, it is necessary to calculate the response performance at the predicted value (PV), i.e. the optimum condition.

### 3.1.16 Step 17: Confirmation Experiment

Following the estimation of the predicted value, a confirmation experiment (CE) must be conducted with factor level settings as in the optimum condition. In any fractional experiment, there will be many interactions that are not included in the experimental model. If all the important factor effects including any interactions account for a large contribution (i.e. small experimental error), then the confirmation experiment (CE) must be approximately equal to the predicted value (PV). That is, if (all other effects) $\neq 0$, then the model has not adequately captured (important factors) and the experiment is said to be not successful. So, need to go back to the drawing board again.

### 3.1.17 Step 18: Full Scale Implementation

If the Confirmation Experiments verifies the Predicted Value, then a control implementation can be done under operating conditions. A series of observations can then be done at the optimum condition. A comparison of "Before" and "After" graphs is drawn to provide a visual impression of the optimization.

### 3.1.18 Step 19: Gain Calculation

A quantitative gain calculation also needs to be performed, so that there is a one-toone objective or manager's bottom-line comparison in monetary units. Even a $50 \%$ improvement may fail to pleased by the senior manager, if the value of this improvement is only, say RM10 per unit, compared to another improvement of only $10 \%$ but with a cost gain of RM100 per unit. Thus, the manager has to know both the
percentage gain as well as the monetary gain. Monetary gain is best displayed in a bar chart on a $10-100 \%$ bar height.

### 3.1.19 Step 20: Conclusion

Finally, reach to the conclusion. Here, we can summarize any implementation action, lesson learnt and cost savings achieved.

### 3.2 Confidence Intervals

Confidence intervals are used to establish the process average at the predicted condition. This prediction is usually a point estimate. To improve the situation I have to know for instance that $95 \%$ (confidence level) of the confirmation test results must be within $\pm \mathrm{x}$ units (confidence interval) of the predicted mean. There are three cases where the confidence intervals have to calculate:

1. for the factor level
2. for the predicted mean
3. for the confirmation experiment

### 3.2.1 Confidence interval for the factor level

The method of calculating the confidence interval for a factor level is use the formula:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V_{e} \times\left[\frac{1}{n}\right]}
$$

where
$F_{\alpha, v l, v 2}=$ the tabulated F-ratio
$\alpha \quad=$ risk. The confidence level $=1$ - risk
v1 = the degree of freedom for the numerator associated with a mean and is always 1 for a confidence interval
$v 2=$ the degrees of freedom for the denominator associated with the degrees of freedom for the pooled error variance
$V_{e} \quad=$ is the pooled error variance
$n \quad=$ number of observations used to calculate the mean

Hence, if the true mean is $\mu_{\overline{A 1}}$,

$$
\begin{gathered}
\mu_{\overline{A 1}}=\overline{A 1} \pm C I \\
\overline{A 1}-C I \leq \mu_{\overline{A 1}} \leq \overline{A 1}+C I
\end{gathered}
$$

### 3.2.2 Confidence Interval for the predicted mean

To calculate the confidence interval for the predicted optimum process mean, we use the following formula:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V_{e} \times\left[\frac{1}{n_{e f f}}\right]}
$$

Where $\mathrm{n}_{\text {eff }}$ is the effective number of observations,

$$
n_{\text {eff }}=\frac{\text { total number of experiment }}{\text { sum of degrees of freedom used in estimate of mean }}
$$

For the effective number of observation must include the degrees of freedom for the overall mean. Note that $\mathrm{n}_{\text {eff }}$ depends on the number of degrees of freedom used to calculate the predicted optimum process mean and does not depend on which factor levels is used. Thus, all factors and interaction terms used in calculating the predicted mean must be included in the degrees of freedom for calculating $\mathrm{n}_{\text {eff }}$. The confidence interval for this optimum process mean therefore:

$$
\mu_{\text {Predicted }}-C I \leq \mu_{\text {Predicted }} \leq \mu_{\text {predicted }}+C I
$$

### 3.2.3 Confidence interval for the confirmation experiment

The confirmation experiment is used to verify that the predicted mean for the factors and level chosen from an orthogonal array experiment is valid. If too few samples are taken, then it should be difficult to establish the validity of the predicted mean. Hence, we shall provide a formula to calculate the confidence intervals for a confirmation experiment as follow:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V_{e} \times\left[\frac{1}{n_{e f f}}+\frac{1}{r}\right]}
$$

where $r$ is the sample size (number of replicates) for the confirmation experiment. If the r approached a very large number (infinity), then $1 / \mathrm{r}$ approaches zero and the formula is reduced to that of the confidence interval around a predicted mean. As r become smaller, $1 / \mathrm{r}$ become larger and the confidence interval increase. Of course, r cannot be less than one. The confidence interval is therefore

$$
\mu_{\text {confirmation }}-C I \leq \mu_{\text {confirmation }} \leq \mu_{\text {confirmation }}+C I
$$

## CHAPTER 4

## RESULTS AND DISCUSSIONS

### 4.1 Current Level of Problem

The response table below shows the flight time records of the preliminary gyrocopter which was dropped from height of 3 m . Based on the table, the mean time of the paper gyrocopter is 2.30 second. In this experiment, we want to increase the flight time of paper gyrocopter to 3 seconds or above by using sequential experiment.

Table 4.1: Current Level of Problem

| Observation | Response | Spec |
| :---: | :---: | :---: |
| 1 | 2.25 | 3.00 |
| 2 | 2.09 | 3.00 |
| 3 | 2.38 | 3.00 |
| 4 | 2.22 | 3.00 |
| 5 | 2.22 | 3.00 |
| 6 | 2.34 | 3.00 |
| 7 | 2.59 | 3.00 |
| 8 | 2.32 | 3.00 |
| 9 | 2.41 | 3.00 |
| 10 | 2.25 | 3.00 |
| 11 | 2.22 | 3.00 |
| 12 | 2.25 | 3.00 |
| 13 | 2.22 | 3.00 |
| 14 | 2.34 | 3.00 |
| 15 | 2.38 | 3.00 |

The response graph below depicts the flight time response of the preliminary gyrocopter falling based on 15 observations in our experiment. From the graph, it is
apparent that there is no presence of outlier during the trials. With that, we intend to increase the current flight time from approximately 2 seconds to the specification of 3 seconds or above as indicated by the red line in the graph.


Figure 4.1: Current Level of Problem

### 4.2 Quality Loss Function

Assume that the loss for the specification of 3 seconds is $\$ 1$. So the k coefficient is 9 . The quality loss function can be established as follow:

$$
\begin{aligned}
& L(y)=\frac{k}{y^{2}} \\
& \therefore L(y)=\frac{9}{y^{2}}
\end{aligned}
$$

Table 4.2: Quality Loss Function

| Spec, $\mathbf{y}$ | Loss, $\mathbf{L}(\mathbf{y})$ |
| :---: | :---: |
| 1.0 | 9.00 |
| 1.5 | 4.00 |
| 2.0 | 2.25 |
| 2.5 | 1.44 |
| 3.0 | 1.00 |
| 3.5 | 0.73 |
| 4.0 | 0.56 |
| 4.5 | 0.44 |
| 5.0 | 0.36 |
| 5.5 | 0.30 |
| 6.0 | 0.25 |
| 6.5 | 0.21 |



Figure 4.2: Quality Loss Function

### 4.3 First fraction design (Part 1)

First fraction design is the initial orthogonal array which shown in table below.

Table 4.3: First fraction design (Part 1)

|  | ABCG | LA | LB | LC | LD | LE | LF | LG |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BCF | FG | EG | DG | EF | DF | DE | AF | $\stackrel{\square}{-}$ | $\rightarrow$ | $\rightarrow$ | $\stackrel{\square}{\square}$ | 0 |  |
|  | ACE | CE | CF | BF | CG | BG | AG | BE | $\stackrel{\text { - }}{ }$ | $\rightarrow$ | $\stackrel{\square}{\square}$ | $\rightarrow$ | $\bigcirc$ |  |
|  | ABD | BD | AD | AE | AB | AC | BC | CD | $\stackrel{\text { 土 }}{ }$ | $\stackrel{\square}{4}$ | $\rightarrow$ | $\rightarrow$ | ग |  |
| Exp | I | A | B | C | $D=A B$ | $\mathrm{E}=\mathrm{AC}$ | $\mathrm{F}=\mathrm{BC}$ | $\mathrm{G}=\mathrm{ABC}$ | 1 | 2 | 3 | 4 | TPM | NPM |
| 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 2.02 | 2.50 | 2.03 | 2.13 | 2.170 | 6.591 |
| 2 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 2.61 | 3.16 | 3.06 | 2.75 | 2.895 | 9.131 |
| 3 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1.79 | 1.83 | 2.16 | 1.72 | 1.875 | 5.321 |
| 4 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 3.66 | 3.83 | 3.85 | 3.97 | 3.828 | 11.644 |
| 5 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1.93 | 2.16 | 2.09 | 1.94 | 2.030 | 6.109 |
| 6 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 2.80 | 3.13 | 3.19 | 2.82 | 2.985 | 9.439 |
| 7 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1.70 | 2.33 | 2.19 | 2.75 | 2.243 | 6.554 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2.49 | 3.55 | 2.94 | 2.59 | 2.893 | 8.882 |

### 4.3.1 Target Performance Measures and Noise Performance Measures

Taguchi has suggested two performance measures:

1. Target Performance Measure (TPM)
2. Noise Performance Measure (NPM)

### 4.3.1.1 Target Performance Measure (TPM)

The target performance measure is essentially a measure of the mean response. It is used to identify control factor that largely affect the mean (and not the variability). TPM is an objective measure to adjust to target. It is used to evaluate performance measurement data by averaging the four reading for each trial. This is essential to identify which factors can mostly affect the mean.

For instance, TPM calculates the means of Experiment 1's readings by dividing the sum of readings by four, that is, $(2.02+2.50+2.03+2.13) / 4=2.170$. This procedure is repeated for the succeeding trials. A detailed method of analysis is discussed in Belavendram (1995).

### 4.3.1.2 Noise Performance Measure (NPM)

For this final year project, to obtain a longer flight period of the gyrocopter, I used larger-the-better characteristics to analyze the noise performance measures. Therefore,

$$
N P M=-10 \log \left(\frac{1}{\bar{y}^{2}}\left(1+\frac{3 \sigma^{2}}{\bar{y}^{2}}\right)\right)
$$

The noise performance measure uses the signal-to-noise (SN) ratio to measure the sensitivity of the quality investigated to those uncontrollable factors (error) in the experiment. The high value of SN ratio is desirable because greater SN ratio will result in smaller product variance around the target value.

### 4.3.2 Response Table (Part 1)

A response table is a method which displays the responses from the experiment based on various combinations of levels and factors.

### 4.3.2.1 Response Table for TPM (Part 1)

The response table for TPM (Part 1) is shown in table below.

Table 4.4: Response Table for TPM (Part 1)

| TPM | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 2.079 | 2.520 | 2.692 | 2.499 | 2.749 | 2.679 | 2.806 |
| 1 | 3.150 | 2.709 | 2.538 | 2.730 | 2.481 | 2.550 | 2.423 |
| Diff | 1.071 | 0.189 | 0.154 | 0.231 | 0.268 | 0.129 | 0.383 |
| SSQ | 9.170 | 0.287 | 0.191 | 0.426 | 0.575 | 0.134 | 1.174 |
| Rank | 1 | 5 | 6 | 4 | 3 | 7 | 2 |
| Opt | 1 | 1 | -1 | 1 | -1 | -1 | -1 |

The sample calculation for the response table for TPM is shown below.

The average flight time of control factor A in Level -1, 2.079 can be calculated by taking the average of TPM based on the trial tested under Level -1 of factor A , that is, $(2.170+1.875+2.030+2.243) / 4=2.079$. This step is repeated for the subsequent control factors under both levels to calculate the average flight duration in Level -1 and Level 1, on TPM.

Next, the difference between the level -1 and level 1, denoted as "Diff", is also introduced in the response table. Also, the determination of Sum of Squares, SSQ for the response table requires the values of $\mathrm{S}_{\mathrm{m}}$ and $\bar{y}^{2}$. For that, $\bar{y}^{2}$ can be defined as the average of TPM for the eight trials or experiments. For instance, SSQ
for control factor A can be obtained by applying the equation of $S S Q=$ $\left(n_{A-1} \bar{y}_{A-1}^{2}\right)+\left(n_{A 1} \bar{y}_{A 1}^{2}\right)-S_{m}$, where $S_{m}=n \bar{y}^{2}, \mathrm{n}=32$. So, SSQ for control factor A is $\left(16 \times 2.079^{2}\right)+\left(16 \times 3.150^{2}\right)-\left(32 \times 2.615^{2}\right)=9.170$. The same way applies to the following control factors to measure their respective SSQ values.

In addition, response table also exhibits the rank of the factors. In this case, the rank of factors is determined based on the different values between Level -1 and Level 1, which signifies the larger the difference, the more important the factor is.

From Table 4.4, it is observed that factor A , factor G and factor E play a vital role in influencing the flight time of gyrocopter. In contrast, factor F and factor C are the most insignificant factors which are unlikely to alter the flight duration in a great extent.

Moreover, the optimum factor level, denoted as "Opt" in the response table, is established based on the higher TPM values of Level -1 and Level 1. In this case, we intend to obtain a higher flight time of gyrocopter on the TPM, which means "Larger the Better".

### 4.3.2.2 Response Table for NPM (Part 1)

The response table for NPM (Part 1) is shown in table below.

Table 4.5: Response Table for NPM (Part 1)

| NPM | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 6.144 | 7.817 | 8.172 | 7.611 | 8.360 | 8.128 | 8.557 |
| 1 | 9.774 | 8.100 | 7.746 | 8.307 | 7.558 | 7.789 | 7.361 |
| Diff | 3.630 | 0.283 | 0.425 | 0.695 | 0.801 | 0.339 | 1.196 |
| SSQ | 26.354 | 0.160 | 0.362 | 0.967 | 1.285 | 0.229 | 2.861 |
| Rank | 1 | 7 | 5 | 4 | 3 | 6 | 2 |
| Opt | 1 | 1 | -1 | 1 | -1 | -1 | -1 |

The concept and the calculation for the response table for NPM are quite similar with the response table for TPM. The only difference between both responses tables is the n for NPM is no longer is 32 . The n for Table 4.5 is 8 . So, SSQ for control factor A is $\left(4 \times 6.144^{2}\right)+\left(4 \times 9.774^{2}\right)-\left(8 \times 7.959^{2}\right)=26.354$. The same way applies to the following control factors to measure their respective SSQ values.

### 4.3.3 Analysis of Variance (ANOVA)

The analysis of variance is used to find out the significant factor. There are two types of analysis of variance that I did:

1. Analysis of Variance for TPM
2. Analysis of Variance for NPM

### 4.3.3.1 Analysis of Variance for TPM (Part 1)

The ANOVA for TPM (Part 1) is shown in table below.

Table 4.6: Analysis of Variance for TPM (Part 1)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 9.170 | 1 | 9.170 | 86.564 | 9.064 | 65.277 |
| B | 1 | 0.287 | 1 | 0.287 | 2.708 |  |  |
| C | 1 | 0.191 | 1 | 0.191 | 1.800 |  |  |
| D | 1 | 0.426 | 1 | 0.426 | 4.017 |  |  |
| E | 0 | 0.575 | 1 | 0.575 | 5.429 | 0.469 | 3.379 |
| F | 1 | 0.134 | 1 | 0.134 | 1.264 |  |  |
| G | 0 | 1.174 | 1 | 1.174 | 11.085 | 1.068 | 7.694 |
| Error | 1 | 1.929 | 24 | 0.080 | 0.759 |  |  |
| Pooled |  | 2.966 | 28 | 0.106 | 1.000 | 3.284 | 23.650 |
| St |  | 13.885 | 31 | 0.448 |  | 13.885 | 100.000 |
| Sm |  | 218.771 | 1 |  |  |  |  |
| ST |  | 232.656 | 32 |  |  |  |  |

Pool 3 Factors are included in model.

In the analysis of variance, the insignificance factors had been pool, the pooling was started by pooling the factor with the smallest sum of square. From the

Table 4.6, there are three significant factor which is factor A, E and G, which have higher value of SSQ among other factors. Hence, four insignificant factors are pooled.

The sample calculation for the ANOVA for TPM (Part 1) is shown below.

## SSQ

1. The values of SSQ for seven factors can be getting from the Response Table.
2. $\mathrm{S}_{\mathrm{e}}=13.885-(9.170+0.287+0.191+0.426+0.575+0.134+1.174)=1.929$
3. $\mathrm{SSQ}_{\text {Pooled }}=0+(1 * 0.287)+(1 * 0.191)+(1 * 0.426)+0+(1 * 0.134)+0+(1 * 1.929)$ $=2.966$

DOF

1. $\mathrm{DOF}_{\mathrm{ST}}=8 * 4=32$
2. $\mathrm{DOF}_{\mathrm{Sm}}=1$
3. $\mathrm{DOF}_{\mathrm{St}}=\mathrm{DOF}_{\mathrm{ST}}-\mathrm{DOF}_{\mathrm{Sm}}=32-1=31$

Variance

1. $\mathrm{VAR}=\mathrm{SSQ} / \mathrm{DOF} \mathrm{Eg}: \mathrm{V}_{\mathrm{A}}=\mathrm{S}_{\mathrm{A}} / \mathrm{v}_{\mathrm{A}}=9.170 / 1=9.170$
2. $\mathrm{V}_{\mathrm{e}}=1.929 / 24=0.080$
3. $\mathrm{V}_{\text {pool }}=2.966 / 28=0.106$

## Ftest

$\mathrm{F}_{\text {test }}=\operatorname{Var}_{\mathrm{i}} / \operatorname{Var}_{\text {pool }}$
Eg: $\mathrm{F}_{\text {test }(\mathrm{A})}=9.170 / 0.106=86.564$

SSq'

1. $\mathrm{SSq}^{\prime}$ is calculated based on the three significant factors which are factor $\mathrm{A}, \mathrm{E}$ and G.
2. Samples of calculation are shown in below:

- $\mathrm{SSq}^{\prime}{ }_{\mathrm{A}}=\mathrm{SSQ}-\mathrm{DOF}^{*} \mathrm{~V}_{\text {pool }}=9.170-(1 * 0.106)=9.064$
- $\mathrm{SSq}^{\prime}{ }_{\text {pool }}=\mathrm{SSq}{ }^{\prime}{ }^{\text {st }}-\left(\mathrm{SSq}^{\prime}{ }_{\mathrm{A}}+\mathrm{SSq}{ }^{\prime}{ }_{\mathrm{E}}+\mathrm{SSq}^{\prime}{ }_{\mathrm{G}}\right)$

$$
\mathrm{SSq}_{\text {pool }}=13.885-(9.064+0.469+1.068)=3.284
$$

Rho
Rho $=\left(\mathrm{SSq}^{\prime} / \mathrm{SSq}{ }^{\prime} \mathrm{st}\right)^{*} 100 \%$
Rho $_{\mathrm{A}}=(9.064 / 13.885) * 100 \%=65.277 \%$

From the calculated Rho, factor A has the highest percentage ( $65.277 \%$ ), followed by factor $\mathrm{G}(7.694 \%)$ and $\mathrm{E}(3.379 \%)$. These three factors are factors that can affect the flight time significantly.

### 4.3.3.2 Analysis of Variance for NPM (Part 1)

The ANOVA for NPM (Part 1) is shown in table below.

Table 4.7: Analysis of Variance for NPM (Part 1)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 26.354 | 1 | 26.354 | 61.338 | 25.925 | 80.464 |
| B | 1 | 0.160 | 1 | 0.160 | 0.373 |  |  |
| C | 1 | 0.362 | 1 | 0.362 | 0.842 |  |  |
| D | 1 | 0.967 | 1 | 0.967 | 2.251 |  |  |
| E | 0 | 1.285 | 1 | 1.285 | 2.990 | 0.855 | 2.654 |
| F | 1 | 0.229 | 1 | 0.229 | 0.534 |  |  |
| G | 0 | 2.861 | 1 | 2.861 | 6.659 | 2.432 | 7.547 |
| Error |  |  |  |  |  |  |  |
| Pooled |  | 1.719 | 4 | 0.430 | 1.000 | 3.008 | 9.335 |
| St |  | 32.219 | 7 | 4.603 |  | 32.219 | 100.000 |
| Sm |  | 506.745 | 1 |  |  |  |  |
| ST |  | 538.963 | 8 |  |  |  |  |

Pool 3 Factors are included in model.

In the analysis of variance, the insignificance factors had been pool, the pooling was started by pooling the factor with the smallest sum of square. From the Table 4.7, there are three significant factor which is factor A, E and G, which have higher value of SSQ among other factors. Hence, four insignificant factors are pooled. In the ANOVA for NPM, there is no error because of there is no degree of freedom for error. $\mathrm{v}_{\mathrm{e}}=\mathrm{v}_{\mathrm{t}}-\left(\mathrm{v}_{\mathrm{A}}+\mathrm{v}_{\mathrm{B}}+\mathrm{v}_{\mathrm{C}}+\mathrm{v}_{\mathrm{D}}+\mathrm{v}_{\mathrm{E}}+\mathrm{v}_{\mathrm{F}}+\mathrm{v}_{\mathrm{G}}\right)=7-(1+1+1+1+1+1+1)=0$

The sample calculation for the ANOVA for NPM (Part 1) is shown below.

## SSQ

1. The values of $\operatorname{SSQ}$ for seven factors can be getting from the Response Table.
2. $\mathrm{SSQ}_{\text {Pooled }}=0+(1 * 0.160)+(1 * 0.362)+(1 * 0.967)+0+(1 * 0.229)+0=1.719$

## DOF

1. $\mathrm{DOF}_{\mathrm{ST}}=8$
2. $\mathrm{DOF}_{\mathrm{Sm}}=1$
3. $\mathrm{DOF}_{\mathrm{St}}=\mathrm{DOF}_{\mathrm{ST}}-\mathrm{DOF}_{\mathrm{Sm}}=8-1=7$

Variance

1. $\mathrm{VAR}=\mathrm{SSQ} / \mathrm{DOF} \mathrm{Eg}: \mathrm{V}_{\mathrm{A}}=\mathrm{S}_{\mathrm{A}} / \mathrm{v}_{\mathrm{A}}=26.354 / 1=26.354$
2. $\mathrm{V}_{\text {pool }}=1.719 / 4=0.430$

Ftest
$\mathrm{F}_{\text {test }}=\operatorname{Var}_{\mathrm{i}} / \operatorname{Var}_{\text {pool }}$
Eg: $\mathrm{F}_{\text {test }(\mathrm{A})}=26.354 / 0.430=61.338$

SSq'

1. SSq ' is calculated based on the three significant factors which are factor $\mathrm{A}, \mathrm{E}$ and G .
2. Samples of calculation are shown in below:

- $\mathrm{SSq}^{\prime}{ }_{\mathrm{A}}=\mathrm{SSQ}-\mathrm{DOF}^{*} \mathrm{~V}_{\text {pool }}=26.354-(1 * 0.430)=25.925$
- $\mathrm{SSq}^{\prime}{ }_{\text {pool }}=\mathrm{SSq}{ }^{\prime}{ }^{\text {st }}-\left(\mathrm{SSq}^{\prime}{ }_{\mathrm{A}}+\mathrm{SSq}{ }^{\prime}{ }_{\mathrm{E}}+\mathrm{SSq}^{\prime}{ }_{\mathrm{G}}\right)$
$\mathrm{SSq}^{\prime}{ }_{\text {pool }}=32.219-(25.925+0.855+2.432)=3.008$

Rho
Rho $=\left(\text { SSq' }^{\prime} / \text { SSq'st }^{\prime}\right)^{*} 100 \%$
Rho $_{\mathrm{A}}=(25.925 / 32.219) * 100 \%=80.464 \%$

From the calculated Rho, factor A has the highest percentage (80.464\%), followed by factor $\mathrm{G}(7.547 \%)$ and $\mathrm{E}(2.654 \%)$. These three factors are factors that can affect the flight time significantly.

### 4.3.4 Confidence Interval for factor levels (Part 1)

The formula below is provided to calculate the confidence interval for factor levels:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V e \times \frac{1}{n}}
$$

Where,

$$
\begin{array}{ll}
F & =\text { tabulated F-ratio } \\
\alpha & =\text { risk } \\
v 1 & =\text { DOF associated with a mean } \\
v 2 & =\text { DOF for the pooled error variance } \\
V e & =\text { pooled error variance } \\
n & =\text { number of observations }
\end{array}
$$

For TPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,28}=4.196 \\
& C I_{T P M}=\sqrt{4.196 \times 0.106 \times \frac{1}{32}}=0.117857
\end{aligned}
$$

For NPM,

$$
\mathrm{F}_{0.05,1,4}=7.71
$$

$$
C I_{N P M}=\sqrt{7.71 \times 0.430 \times \frac{1}{8}}=0.643436
$$

The risk taken is assumed at 0.05 . DOF of mean and pooled error variance, and pooled error variance are obtained in ANOVA in Table 4.6 and Table 4.7.

### 4.3.5 Factor Level Averages (Part 1)

After obtained the values of confidence interval for factor levels, then the factor level averages for TPM and NPM has been shown in table below.

Table 4.8: Factor Level Averages (Part 1)

| TPM | Mean | Mean+CI | Mean-Cl |
| :---: | :---: | :---: | :---: |
| A1 | 2.079 | 2.197 | 1.962 |
| A2 | 3.150 | 3.268 | 3.032 |
|  |  |  |  |
| B1 | 2.520 | 2.638 | 2.402 |
| B2 | 2.709 | 2.827 | 2.592 |
|  |  |  |  |
| C1 | 2.692 | 2.810 | 2.574 |
| C2 | 2.538 | 2.655 | 2.420 |
|  |  |  |  |
| D1 | 2.499 | 2.617 | 2.382 |
| D2 | 2.730 | 2.848 | 2.612 |
|  |  |  |  |
| E1 | 2.749 | 2.867 | 2.631 |
| E2 | 2.481 | 2.598 | 2.363 |
|  |  |  |  |
| F1 | 2.679 | 2.797 | 2.562 |
| F2 | 2.550 | 2.668 | 2.432 |
|  |  |  |  |
| G1 | 2.806 | 2.924 | 2.688 |
| G2 | 2.423 | 2.541 | 2.305 |


| NPM | Mean | Mean+Cl | Mean-Cl |
| :---: | :---: | :---: | :---: |
| A1 | 6.144 | 6.787 | 5.500 |
| A2 | 9.774 | 10.417 | 9.130 |
|  |  |  |  |
| B1 | 7.817 | 8.461 | 7.174 |
| B2 | 8.100 | 8.744 | 7.457 |
|  |  |  |  |
| C1 | 8.172 | 8.815 | 7.528 |
| C2 | 7.746 | 8.390 | 7.103 |
|  |  |  |  |
| D1 | 7.611 | 8.255 | 6.968 |
| D2 | 8.307 | 8.950 | 7.663 |
|  |  |  |  |
| E1 | 8.360 | 9.003 | 7.716 |
| E2 | 7.558 | 8.202 | 6.915 |
|  |  |  |  |
| F1 | 8.128 | 8.772 | 7.485 |
| F2 | 7.789 | 8.433 | 7.146 |
|  |  |  |  |
| G1 | 8.557 | 9.200 | 7.913 |
| G2 | 7.361 | 8.004 | 6.717 |

### 4.3.6 Response Graphs (Part 1)

After the table of factor level average is generate, then the response graphs has been plotted in figure below.


Figure 4.3: Response Graph for TPM (Part 1)


Figure 4.4: Response Graph for NPM (Part 1)

From the response graph above, we can observe that there are differences between level 1 and level 2 for various control factors. The aim of this experiment is to enhance the gyrocopter's flight time or we want the flight time to be "the larger, the better" Therefore, levels with higher value of TPM and NPM are desirable and
chosen for as the level for each control factor. For instance, level 1 is chosen for control factors like C, E, F and G while level 2 is selected for control factors such as A, B and D.

We can observe that the control factor A and G show the largest difference between level 1 and 2. The differences in level 1 and 2 for control factors B, C, D, E and F are moderate. This indicates that not all the control factors are critical in influencing the flight time of gyrocopter.

### 4.4 Second fraction design (Part 2)

A second fraction design is run with the signs for only one of the factors reversed. For this final year project, a second fractional design with the signs reversed in the column for factor D is run. This is called as a single-factor fold-over.

Table 4.9: Second fraction design (Part 2)

|  | ABCG | L'A | L'B | L'C | L'D | L'E | L'F | L'G |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BCF | FG | EG | -DG | -EF | -DF | -DE | AF | $\pm$ | $\rightarrow$ | $\rightarrow$ | $\stackrel{\square}{4}$ | 0 |  |
|  | ACE | CE | CF | BF | -CG | BG | AG | BE | $\stackrel{-}{\square}$ | $\rightarrow$ | $\stackrel{-}{\square}$ | $\rightarrow$ | $\bigcirc$ |  |
|  | -ABD | -BD | -AD | AE | -AB | AC | BC | -CD | $\stackrel{-}{\square}$ | $\stackrel{-}{\square}$ | $\rightarrow$ | $\rightarrow$ | ס |  |
| Exp | 1 | A | B | C | $D=-A B$ | $E=A C$ | $\mathrm{F}=\mathrm{BC}$ | $\mathrm{G}=\mathrm{ABC}$ | 1 | 2 | 3 | 4 | TPM | NPM |
| 9 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 2.19 | 2.29 | 2.1 | 2.03 | 2.153 | 6.623 |
| 10 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 2.9 | 4.08 | 3.47 | 3.47 | 3.480 | 10.589 |
| 11 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1.85 | 2.21 | 2.21 | 1.75 | 2.005 | 5.859 |
| 12 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | 3.3 | 3.32 | 3.72 | 3.22 | 3.390 | 10.547 |
| 13 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1.44 | 2.4 | 2.34 | 2 | 2.045 | 5.649 |
| 14 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 2.08 | 2.83 | 2.57 | 1.97 | 2.363 | 7.098 |
| 15 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 2.48 | 2.49 | 2.22 | 2.06 | 2.313 | 7.176 |
| 16 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | 2.4 | 2.81 | 2.47 | 2.44 | 2.530 | 7.990 |

### 4.4.1 Response Table (Part 2)

The response table for TPM and NPM is shown in table below.

Table 4.10: Response Table for TPM (Part 2)

| TPM | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 2.129 | 2.510 | 2.757 | 2.529 | 2.807 | 2.451 | 2.554 |
| 1 | 2.941 | 2.559 | 2.313 | 2.540 | 2.263 | 2.619 | 2.515 |
| Diff | 0.812 | 0.049 | 0.444 | 0.011 | 0.544 | 0.168 | 0.039 |
| SSQ | 5.273 | 0.020 | 1.580 | 0.001 | 2.371 | 0.226 | 0.012 |
| Rank | 1 | 5 | 3 | 7 | 2 | 4 | 6 |
| Opt | 1 | 1 | -1 | 1 | -1 | 1 | -1 |

Table 4.11: Response Table for NPM (Part 2)

| NPM | A | B | C | D | E | F | G |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 6.327 | 7.490 | 8.405 | 7.702 | 8.490 | 7.288 | 7.861 |
| 1 | 9.056 | 7.893 | 6.978 | 7.680 | 6.893 | 8.095 | 7.522 |
| Diff | 2.729 | 0.403 | 1.426 | 0.022 | 1.597 | 0.806 | 0.339 |
| SSQ | 14.899 | 0.326 | 4.069 | 0.001 | 5.104 | 1.300 | 0.230 |
| Rank | 1 | 5 | 3 | 7 | 2 | 4 | 6 |
| Opt | 1 | 1 | -1 | -1 | -1 | 1 | -1 |

The calculation method for the response table can be referring back to Chapter 4.3.2 Response Table (Part 1).

From the response table for TPM and NPM, it is observed that factor A, factor E and factor C play a vital role in influencing the flight time of gyrocopter. In contrast, factor G and factor D are the most insignificant factors which are unlikely to alter the flight duration in a great extent. Apart from that, the optimum level show in both response tables for control factor D is also different. The optimum level for control factor D in the response table for TPM is level 1, while the optimum level for control factor D in the response table for NPM is level -1. The difference in level for control factor D inside both the response table may be due to the signs reversed in the column for factor D in second fraction design. Since control factor D is one of the most insignificant factors, hence we not need to worry about it.

### 4.4.2 Analysis of Variance (Part 2)

The ANOVA for TPM and NPM is shown in table below.

Table 4.12: Analysis of Variance for TPM (Part 2)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 5.273 | 1 | 5.273 | 56.077 | 5.179 | 43.681 |
| B | 1 | 0.020 | 1 | 0.020 | 0.207 |  |  |
| C | 0 | 1.580 | 1 | 1.580 | 16.800 | 1.486 | 12.531 |
| D | 1 | 0.001 | 1 | 0.001 | 0.010 |  |  |
| E | 0 | 2.371 | 1 | 2.371 | 25.212 | 2.277 | 19.202 |
| F | 1 | 0.226 | 1 | 0.226 | 2.405 |  |  |
| G | 1 | 0.012 | 1 | 0.012 | 0.132 |  |  |
| Error | 1 | 2.374 | 24 | 0.099 | 1.052 |  |  |
| Pooled |  | 2.633 | 28 | 0.094 | 1.000 | 2.915 | 24.586 |
| St |  | 11.857 | 31 | 0.382 |  | 11.857 | 100.000 |
| Sm |  | 205.589 | 1 |  |  |  |  |
| ST |  | 217.445 | 32 |  |  |  |  |

Pool 3 Factors are included in model.

Table 4.13: Analysis of Variance for NPM (Part 2)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 14.899 | 1 | 14.899 | 32.095 | 14.434 | 55.671 |
| B | 1 | 0.326 | 1 | 0.326 | 0.701 |  |  |
| C | 0 | 4.069 | 1 | 4.069 | 8.766 | 3.605 | 13.903 |
| D | 1 | 0.001 | 1 | 0.001 | 0.002 |  |  |
| E | 0 | 5.104 | 1 | 5.104 | 10.994 | 4.639 | 17.893 |
| F | 1 | 1.300 | 1 | 1.300 | 2.800 |  |  |
| G | 1 | 0.230 | 1 | 0.230 | 0.496 |  |  |
| Error |  |  |  |  |  |  |  |
| Pooled |  | 1.857 | 4 | 0.464 | 1.000 | 3.249 | 12.533 |
| St |  | 25.928 | 7 | 3.704 |  | 25.928 | 100.000 |
| Sm |  | 473.269 | 1 |  |  |  |  |
| ST |  | 499.197 | 8 |  |  |  |  |

Pool 3 Factors are included in model.

In the analysis of variance, the insignificance factors had been pool, the pooling was started by pooling the factor with the smallest sum of square. From the table above, there are three significant factors which is factor $\mathrm{A}, \mathrm{C}$ and E , which have
higher value of SSQ among other factors. Hence, four insignificant factors are pooled.

The calculation for the ANOVA can be referring back to the sample calculation for the Chapter 4.3.3 Analysis of Variance (ANOVA).

### 4.4.3 Confidence Interval for factor levels (Part 2)

The formula below is provided to calculate the confidence interval for factor levels:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V e \times \frac{1}{n}}
$$

Where,

$$
\begin{array}{ll}
F & =\text { tabulated F-ratio } \\
\alpha & =\text { risk } \\
v 1 & =\text { DOF associated with a mean } \\
v 2 & =\text { DOF for the pooled error variance } \\
V e & =\text { pooled error variance } \\
n & =\text { number of observations }
\end{array}
$$

For TPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,28}=4.196 \\
& C I_{\text {TPM }}=\sqrt{4.196 \times 0.094 \times \frac{1}{32}}=0.111041
\end{aligned}
$$

For NPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,4}=7.71 \\
& C I_{N P M}=\sqrt{7.71 \times 0.464 \times \frac{1}{8}}=0.668807
\end{aligned}
$$

The risk taken is assumed at 0.05 . DOF of mean and pooled error variance, and pooled error variance are obtained in ANOVA in Table 4.12 and Table 4.13.

### 4.4.4 Factor Level Averages (Part 2)

After obtained the values of confidence interval for factor levels, then the factor level averages for TPM and NPM has been shown in table below.

Table 4.14: Factor Level Averages (Part 2)

| TPM | Mean | Mean+CI | Mean-CI |
| :---: | :---: | :---: | :---: |
| A1 | 2.129 | 2.240 | 2.018 |
| A2 | 2.941 | 3.052 | 2.830 |
|  |  |  |  |
| B1 | 2.510 | 2.621 | 2.399 |
| B2 | 2.559 | 2.670 | 2.448 |
|  |  |  |  |
| C1 | 2.757 | 2.868 | 2.646 |
| C2 | 2.313 | 2.424 | 2.201 |
|  |  |  |  |
| D1 | 2.529 | 2.640 | 2.418 |
| D2 | 2.540 | 2.651 | 2.429 |
|  |  |  |  |
| E1 | 2.807 | 2.918 | 2.696 |
| E2 | 2.263 | 2.374 | 2.151 |
|  |  |  |  |
| F1 | 2.451 | 2.562 | 2.340 |
| F2 | 2.619 | 2.730 | 2.508 |
|  |  |  |  |
| G1 | 2.554 | 2.665 | 2.443 |
| G2 | 2.515 | 2.626 | 2.404 |


| NPM | Mean | Mean+Cl | Mean-CI |
| :---: | :---: | :---: | :---: |
| A1 | 6.327 | 6.996 | 5.658 |
| A2 | 9.056 | 9.725 | 8.387 |
|  |  |  |  |
| B1 | 7.490 | 8.159 | 6.821 |
| B2 | 7.893 | 8.562 | 7.224 |
|  |  |  |  |
| C1 | 8.405 | 9.073 | 7.736 |
| C2 | 6.978 | 7.647 | 6.309 |
|  |  |  |  |
| D1 | 7.702 | 8.371 | 7.034 |
| D2 | 7.680 | 8.349 | 7.012 |
|  |  |  |  |
| E1 | 8.490 | 9.159 | 7.821 |
| E2 | 6.893 | 7.562 | 6.224 |
|  |  |  |  |
| F1 | 7.288 | 7.957 | 6.620 |
| F2 | 8.095 | 8.763 | 7.426 |
|  |  |  |  |
| G1 | 7.861 | 8.530 | 7.192 |
| G2 | 7.522 | 8.191 | 6.853 |

### 4.4.5 Response Graphs (Part 2)

After the table of factor level average is generate, then the response graphs has been plotted in figure below.


Figure 4.5: Response Graph for TPM (Part 2)


Figure 4.6: Response Graph for NPM (Part 2)

From the response graph above, we can observe that there are differences between level 1 and level 2 for various control factors. The aim of this experiment is to enhance the gyrocopter's flight time or we want the flight time to be "the larger, the better" Therefore, levels with higher value of TPM and NPM are desirable and
chosen for as the level for each control factor. For instance, level 1 is chosen for control factors like C, E and G while level 2 is selected for control factors such as A, $B$ and $F$.

We can observe that the difference in level 1 and 2 for control factor $D$ is considerably small while control factor $\mathrm{A}, \mathrm{C}$ and E show the largest difference between level 1 and 2. The differences in level 1 and 2 for control factors B, D, F and G are moderate. This indicates that not all the control factors are critical in influencing the flight time of gyrocopter.

### 4.5 Combined design (Part 3 = Part $1+$ Part 2)

Sequential experimentation is executed by combining the runs of two (or more) experiments to assemble a larger design that can estimate factor effects and interactions of interest.

Table 4.15: Combined design (Part $3=$ Part $1+$ Part 2)

|  | ABCG |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\pm$ | $\rightarrow$ | $\rightarrow$ | $\pm$ | $\bigcirc$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BCF | FG | EG | DG |  | DF | DE | $C D$ |  |  |  | EF |  |  |  |  | $\stackrel{\square}{\square}$ | $\rightarrow$ | $\stackrel{-}{\square}$ | $\rightarrow$ | $\bigcirc$ |  |
|  | ACE | CE | CF | BF |  | BG | BC | BE |  |  |  | CG |  |  |  |  | $\stackrel{\square}{-}$ | $\stackrel{\square}{-}$ | $\rightarrow$ | $\rightarrow$ | ग |  |
| Exp | 1 | A | B | C | D | E | F | G | BD | AD | DG | $A B$ | DF | DE | CD | ABD | 1 | 2 | 3 | 4 | TPM | NPM |
| 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 2.02 | 2.5 | 2.03 | 2.13 | 2.170 | 6.591 |
| 2 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 2.61 | 3.16 | 3.06 | 2.75 | 2.895 | 9.131 |
| 3 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | 1.79 | 1.83 | 2.16 | 1.72 | 1.875 | 5.321 |
| 4 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | 3.66 | 3.83 | 3.85 | 3.97 | 3.828 | 11.644 |
| 5 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 1.93 | 2.16 | 2.09 | 1.94 | 2.030 | 6.109 |
| 6 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | 2.8 | 3.13 | 3.19 | 2.82 | 2.985 | 9.439 |
| 7 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1.7 | 2.33 | 2.19 | 2.75 | 2.243 | 6.554 |
| 8 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 2.49 | 3.55 | 2.94 | 2.59 | 2.893 | 8.882 |
| 9 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | -1 | 2.19 | 2.29 | 2.1 | 2.03 | 2.153 | 6.623 |
| 10 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | -1 | 2.9 | 4.08 | 3.47 | 3.47 | 3.480 | 10.589 |
| 11 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | -1 | 1.85 | 2.21 | 2.21 | 1.75 | 2.005 | 5.859 |
| 12 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | 3.3 | 3.32 | 3.72 | 3.22 | 3.390 | 10.547 |
| 13 | 1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1.44 | 2.4 | 2.34 | 2 | 2.045 | 5.649 |
| 14 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | 2.08 | 2.83 | 2.57 | 1.97 | 2.363 | 7.098 |
| 15 | 1 | -1 | 1 | 1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | -1 | 2.48 | 2.49 | 2.22 | 2.06 | 2.313 | 7.176 |
| 16 | 1 | 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | -1 | -1 | -1 | 2.4 | 2.81 | 2.47 | 2.44 | 2.530 | 7.990 |

### 4.5.1 Response Table (Part 3)

The response table for TPM and NPM is shown in table below.

Table 4.16: Response Table for TPM (Part 3)

| TPM | A | B | C | D | E | F | G | BD | AD | DG | AB | DF | DE | CD | ABD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 2.104 | 2.515 | 2.724 | 2.514 | 2.778 | 2.565 | 2.680 | 2.510 | 2.540 | 2.502 | 2.520 | 2.506 | 2.649 | 2.661 | 2.535 |
| 1 | 3.045 | 2.634 | 2.425 | 2.635 | 2.372 | 2.584 | 2.469 | 2.639 | 2.610 | 2.647 | 2.630 | 2.644 | 2.500 | 2.489 | 2.615 |
| Diff | 0.941 | 0.119 | 0.299 | 0.121 | 0.406 | 0.019 | 0.211 | 0.129 | 0.070 | 0.145 | 0.110 | 0.138 | 0.149 | 0.172 | 0.080 |
| SSQ | 14.175 | 0.228 | 1.434 | 0.233 | 2.641 | 0.006 | 0.714 | 0.268 | 0.078 | 0.336 | 0.194 | 0.305 | 0.354 | 0.473 | 0.102 |
| Rank | 1 | 11 | 3 | 10 | 2 | 15 | 4 | 9 | 14 | 7 | 12 | 8 | 6 | 5 | 13 |
| Opt | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 |

Table 4.17: Response Table for NPM (Part 3)

| NPM | A | B | C | D | E | F | G | BD | AD | DG | AB | DF | DE | CD | ABD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 | 6.235 | 7.654 | 8.288 | 7.657 | 8.425 | 7.708 | 8.209 | 7.600 | 7.855 | 7.575 | 7.646 | 7.626 | 8.111 | 8.039 | 7.691 |
| 1 | 9.415 | 7.997 | 7.362 | 7.993 | 7.225 | 7.942 | 7.441 | 8.050 | 7.795 | 8.075 | 8.005 | 8.024 | 7.539 | 7.611 | 7.959 |
| Diff | 3.180 | 0.343 | 0.926 | 0.337 | 1.199 | 0.234 | 0.768 | 0.450 | 0.060 | 0.501 | 0.359 | 0.398 | 0.572 | 0.428 | 0.267 |
| SSQ | 40.442 | 0.471 | 3.429 | 0.453 | 5.755 | 0.219 | 2.358 | 0.811 | 0.015 | 1.002 | 0.515 | 0.634 | 1.311 | 0.734 | 0.286 |
| Rank | 1 | 11 | 3 | 12 | 2 | 14 | 4 | 7 | 15 | 6 | 10 | 9 | 5 | 8 | 13 |
| Opt | 1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | 1 |

The calculation method for the response table can be referring back to Chapter 4.3.2 Response Table (Part 1).

### 4.5.2 Analysis of Variance (Part 3)

The ANOVA for TPM and NPM is shown in table below.

Table 4.18: Analysis of Variance for TPM (Part 3)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 14.175 | 1 | 14.175 | 111.990 | 14.049 | 54.359 |
| B | 1 | 0.228 | 1 | 0.228 | 1.801 |  |  |
| C | 0 | 1.434 | 1 | 1.434 | 11.329 | 1.307 | 5.059 |
| D | 1 | 0.233 | 1 | 0.233 | 1.839 |  |  |
| E | 0 | 2.641 | 1 | 2.641 | 20.862 | 2.514 | 9.728 |
| F | 1 | 0.006 | 1 | 0.006 | 0.047 |  |  |
| G | 1 | 0.714 | 1 | 0.714 | 5.641 |  |  |
| BD | 1 | 0.268 | 1 | 0.268 | 2.116 |  |  |
| AD | 1 | 0.078 | 1 | 0.078 | 0.619 |  |  |
| DG | 1 | 0.336 | 1 | 0.336 | 2.658 |  |  |
| AB | 1 | 0.194 | 1 | 0.194 | 1.530 |  |  |
| DF | 1 | 0.305 | 1 | 0.305 | 2.412 |  |  |
| DE | 1 | 0.354 | 1 | 0.354 | 2.797 |  |  |
| CD | 1 | 0.473 | 1 | 0.473 | 3.734 |  |  |
| ABD | 1 | 0.102 | 1 | 0.102 | 0.809 |  |  |
| Error | 1 | 4.303 | 48 | 0.090 | 0.708 |  |  |
| Pooled |  | 7.595 | 60 | 0.127 | 1.000 | 7.974 | 30.855 |
| St |  | 25.844 | 63 | 0.410 |  | 25.844 | 100.000 |
| Sm |  | 424.257 | 1 |  |  |  |  |
| ST |  | 450.101 | 64 |  |  |  |  |

Pool 3 Factors are included in model.

Table 4.19: Analysis of Variance for NPM (Part 3)

| Source | Pool | SSQ | DOF | VAR | Ftest | Ssq' | Rho |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 40.442 | 1 | 40.442 | 55.101 | 39.708 | 67.954 |
| B | 1 | 0.471 | 1 | 0.471 | 0.642 |  |  |
| C | 0 | 3.429 | 1 | 3.429 | 4.672 | 2.695 | 4.612 |
| D | 1 | 0.453 | 1 | 0.453 | 0.618 |  |  |
| E | 0 | 5.755 | 1 | 5.755 | 7.841 | 5.021 | 8.593 |
| F | 1 | 0.219 | 1 | 0.219 | 0.298 |  |  |
| G | 1 | 2.358 | 1 | 2.358 | 3.212 |  |  |
| BD | 1 | 0.811 | 1 | 0.811 | 1.105 |  |  |
| AD | 1 | 0.015 | 1 | 0.015 | 0.020 |  |  |
| DG | 1 | 1.002 | 1 | 1.002 | 1.365 |  |  |
| AB | 1 | 0.515 | 1 | 0.515 | 0.701 |  |  |
| DF | 1 | 0.634 | 1 | 0.634 | 0.863 |  |  |
| DE | 1 | 1.311 | 1 | 1.311 | 1.786 |  |  |
| CD | 1 | 0.734 | 1 | 0.734 | 1.000 |  |  |
| ABD | 1 | 0.286 | 1 | 0.286 | 0.390 |  |  |
| Error |  |  |  |  |  |  |  |
| Pooled |  | 8.808 | 12 | 0.734 | 1.000 | 11.009 | 18.841 |
| St |  | 58.433 | 15 | 3.896 |  | 58.433 | 100.000 |
| Sm |  | 979.728 | 1 |  |  |  |  |
| ST |  | 1038.160 | 16 |  |  |  |  |

Pool 3 Factors are included in model.

In the analysis of variance, the insignificance factors had been pool, the pooling was started by pooling the factor with the smallest sum of square. From the table above, there are three significant factors which is factor $\mathrm{A}, \mathrm{C}$ and E , which have higher value of SSQ among other factors. Hence, four insignificant factors and all the two-factor interactions are pooled.

The calculation for the ANOVA can be referring back to the sample calculation for the Chapter 4.3.3 Analysis of Variance (ANOVA).

### 4.5.3 Significant data

The significant data for TPM and NPM taken from ANOVA is shown in figure below.


Figure 4.7: Significance Data for TPM show by SSQ


Figure 4.8: Significance Data for NPM show by SSQ


Figure 4.9: Significance Data for TPM show by percentage of Rho


Figure 4.10: Significance Data for NPM show by percentage of Rho

The figure (bar chart and pie chart) indicates that we only need to consider three significant data that own higher percentage of Rho, which are factor A, factor C and factor E. Nevertheless, other factors and all the two-factor interactions are not significant factors, which do not affect much on the flight time.

### 4.5.4 Confidence Interval for factor levels (Part 3)

The formula below is provided to calculate the confidence interval for factor levels:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V e \times \frac{1}{n}}
$$

Where,

$$
\begin{array}{ll}
F & =\text { tabulated F-ratio } \\
\alpha & =\text { risk } \\
v 1 & =\text { DOF associated with a mean } \\
v 2 & =\text { DOF for the pooled error variance } \\
V e & =\text { pooled error variance } \\
n & =\text { number of observations }
\end{array}
$$

For TPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,60}=4 \\
& C I_{T P M}=\sqrt{4 \times 0.127 \times \frac{1}{64}}=0.088957
\end{aligned}
$$

For NPM,

$$
\begin{aligned}
\mathrm{F}_{0.05,1,12} & =4.75 \\
C I_{N P M} & =\sqrt{4.75 \times 0.734 \times \frac{1}{16}}=0.466655
\end{aligned}
$$

The risk taken is assumed at 0.05 . DOF of mean and pooled error variance, and pooled error variance are obtained in ANOVA in Table 4.18 and Table 4.19.

### 4.5.5 Factor Level Averages (Part 3)

After obtained the values of confidence interval for factor levels, then the factor level averages for TPM and NPM has been shown in table below.

Table 4.20: Factor Level Averages (Part 3)

| TPM | Mean | Mean+CI | Mean-CI |
| :---: | :---: | :---: | :---: |
| A1 | 2.104 | 2.193 | 2.015 |
| A2 | 3.045 | 3.134 | 2.956 |
| B1 | 2.515 | 2.604 | 2.426 |
| B2 | 2.634 | 2.723 | 2.545 |
| C1 | 2.724 | 2.813 | 2.635 |
| C2 | 2.425 | 2.514 | 2.336 |
| D1 | 2.514 | 2.603 | 2.425 |
| D2 | 2.635 | 2.724 | 2.546 |
| E1 | 2.778 | 2.867 | 2.689 |
| E2 | 2.372 | 2.461 | 2.283 |
| F1 | 2.565 | 2.654 | 2.476 |
| F2 | 2.584 | 2.673 | 2.495 |
| G1 | 2.680 | 2.769 | 2.591 |
| G2 | 2.469 | 2.558 | 2.380 |
| BD1 | 2.510 | 2.599 | 2.421 |
| BD2 | 2.639 | 2.728 | 2.550 |
| AD1 | 2.540 | 2.629 | 2.451 |
| AD2 | 2.610 | 2.699 | 2.521 |
| DG1 | 2.502 | 2.591 | 2.413 |
| DG2 | 2.647 | 2.736 | 2.558 |
| AB1 | 2.520 | 2.609 | 2.431 |
| AB2 | 2.630 | 2.719 | 2.541 |
| DF1 | 2.506 | 2.595 | 2.417 |
| DF2 | 2.644 | 2.733 | 2.555 |
| DE1 | 2.649 | 2.738 | 2.560 |
| DE2 | 2.500 | 2.589 | 2.411 |
| CD1 | 2.661 | 2.750 | 2.572 |
| CD2 | 2.489 | 2.578 | 2.400 |
| ABD1 | 2.535 | 2.624 | 2.446 |
| ABD2 | 2.615 | 2.704 | 2.526 |


| NPM | Mean | Mean+CI | Mean-CI |
| :---: | :---: | :---: | :---: |
| A1 | 6.235 | 6.702 | 5.769 |
| A2 | 9.415 | 9.882 | 8.948 |
|  |  |  |  |
| B1 | 7.654 | 8.120 | 7.187 |
| B2 | 7.997 | 8.463 | 7.530 |
|  |  |  |  |
| C1 | 8.288 | 8.755 | 7.821 |
| C2 | 7.362 | 7.829 | 6.896 |
|  |  |  |  |
| D1 | 7.657 | 8.123 | 7.190 |
| D2 | 7.993 | 8.460 | 7.527 |
|  |  |  |  |
| E1 | 8.425 | 8.892 | 7.958 |
| E2 | 7.225 | 7.692 | 6.759 |
|  |  |  |  |
| F1 | 7.708 | 8.175 | 7.242 |
| F2 | 7.942 | 8.409 | 7.475 |
|  |  |  |  |
| G1 | 8.209 | 8.676 | 7.742 |
| G2 | 7.441 | 7.908 | 6.975 |
|  |  |  |  |
| BD1 | 7.600 | 8.067 | 7.133 |
| BD2 | 8.050 | 8.517 | 7.584 |
|  |  |  |  |
| AD1 | 7.855 | 8.322 | 7.389 |
| AD2 | 7.795 | 8.262 | 7.328 |
|  |  |  |  |
| DG1 | 7.575 | 8.042 | 7.108 |
| DG2 | 8.075 | 8.542 | 7.609 |
|  |  |  |  |
| AB1 | 7.646 | 8.112 | 7.179 |
| AB2 | 8.005 | 8.471 | 7.538 |
|  |  |  |  |
| DF1 | 7.626 | 8.093 | 7.160 |
| DF2 | 8.024 | 8.491 | 7.557 |
|  |  |  |  |
| DE1 | 8.111 | 8.578 | 7.645 |
| DE2 | 7.539 | 8.006 | 7.072 |
|  |  |  |  |
| CD1 | 8.039 | 8.506 | 7.573 |
| CD2 | 7.611 | 8.078 | 7.144 |
| ABD1 | 7.691 | 8.158 | 7.225 |
| ABD2 | 7.959 | 8.425 | 7.492 |
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|  |  |  |  |

After the table of factor level average is generate, then the response graphs has been plotted in figure below.


Figure 4.11: Response Graph for TPM (Part 3)


Figure 4.12: Response Graph for NPM (Part 3)

From the response graph above, we can observe that there are differences between level 1 and level 2 for various control factors and two-factor interactions. The aim of this experiment is to enhance the gyrocopter's flight time or we want the flight time to be "the larger, the better" Therefore, levels with higher value of TPM and NPM are desirable and chosen for as the level for each control factor. For instance, level 1 is chosen for control factors like $\mathrm{C}, \mathrm{E}$ and G while level 2 is selected for control factors such as A, B, D and F.

We can observe that the difference in level 1 and 2 for two-factor interactions is considerably small while control factor $\mathrm{A}, \mathrm{C}$ and E show the largest difference between level 1 and 2. The differences in level 1 and 2 for control factors B, D, F and G are moderate. This indicates that not all the control factors and two-factor interactions are critical in influencing the flight time of gyrocopter.

### 4.6 Optimum factor selection

By using the response graph or comparing the values of TPM and NPM between level -1 and level 1, we can easily identify the optimum level for each factor. With that, an optimum set of design factors can be attained. The optimum factor selection is shown in Table 4.21. Among these factors, we found that $\mathrm{A}, \mathrm{C}$ and E are the main factors which bring significant effects on the overall duration for the gyrocopter to spinning down to the ground from a height of 3 m .

Table 4.21: Optimum Factor Selection

|  | TPM |  |  | NPM |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Source | Opt | Rank | Rho | Opt | Rank | Rho | Comments | Decision | TPM | NPM |
| A | 1 | 1 | 54.359 | 1 | 1 | 67.95 | 80 gsm A4 paper is the optimum paper type. | 1 | 3.045 | 9.41 |
| B | 1 | 11 |  | 1 | 11 |  | Body width of 3 cm leads to optimum result. | 1 |  |  |
| C | -1 | 3 | 5.059 | -1 | 3 | 4.61 | Body length of 6 cm leads to optimum result. | -1 | 2.724 | 8.29 |
| D | 1 | 10 |  | 1 | 12 |  | Wing length of 12 cm give rise to optimum results. | 1 |  |  |
| E | -1 | 2 | 9.728 | -1 | 2 | 8.59 | Shoulder height of 2 cm gives rise to optimum result. | -1 | 2.778 | 8.42 |
| F | 1 | 15 |  | 1 | 14 |  | Wing width of 4 cm give rise to optimum result. | 1 |  |  |
| G | -1 | 4 |  | -1 | 4 |  | 1 piece of clip contributes to optimum result. | -1 |  |  |
|  |  |  |  |  |  |  | Process $\mu(\mathrm{TPM})=2.57$ |  |  |  |
|  |  |  |  |  |  |  | Process $\mu(\mathrm{NPM})=7.83$ |  |  |  |
|  |  |  |  |  |  |  | Predicted $\mu(\mathrm{TPM})=3.40$ |  |  |  |
|  |  |  |  |  |  |  | redicted $\mu($ NPM $)=10.01$ |  |  |  |

### 4.7 Predicted Values

The sample calculation for the predicted values is shown below.

## Predicted values for TPM

$\mathrm{PV}_{\text {TPM }}=\bar{y}+$ Main effects + Interaction effects
$P V_{T P M}=\bar{y}+\left(\overline{A_{1}}-\bar{y}\right)+\left(\overline{C_{-1}}-\bar{y}\right)+\left(\overline{E_{-1}}-\bar{y}\right)+$ Interaction effects
$P V_{T P M}=\bar{y}+\left(\overline{A_{1}}-\bar{y}\right)+\left(\overline{C_{-1}}-\bar{y}\right)+\left(\overline{E_{-1}}-\bar{y}\right)+0$
$\mathrm{PV}_{\mathrm{TPM}}=2.57+(3.045-2.57)+(2.724-2.57)+(2.778-2.57)$
$P V_{\text {TPM }}=3.40$

## Predicted values for NPM

$\mathrm{PV}_{\mathrm{NPM}}=\bar{y}+$ Main effects + Interaction effects
$P V_{N P M}=\bar{y}+\left(\overline{A_{1}}-\bar{y}\right)+\left(\overline{C_{-1}}-\bar{y}\right)+\left(\overline{E_{-1}}-\bar{y}\right)+$ Interaction effects
$P V_{N P M}=\bar{y}+\left(\overline{A_{1}}-\bar{y}\right)+\left(\overline{C_{-1}}-\bar{y}\right)+\left(\overline{E_{-1}}-\bar{y}\right)+0$
$\mathrm{PV}_{\mathrm{NPM}}=7.83+(9.41-7.83)+(8.29-7.83)+(8.42-7.83)$
$\mathrm{PV}_{\mathrm{NPM}}=10.48$

## Confidence Interval for a predicted means

The formula below is provided to calculate the confidence interval for factor levels:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V e \times\left[\frac{1}{n_{e f f}}\right]}
$$

Where $n_{\text {eff }}$ is the effective number of observation

For TPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,60}=4 \\
& C I_{T P M}=\sqrt{4 \times 0.127 \times\left[\frac{1}{3+1}\right]}= \pm 0.3564
\end{aligned}
$$

For NPM,

$$
\mathrm{F}_{0.05,1,12}=4.75
$$

$$
C I_{N P M}=\sqrt{4.75 \times 0.734 \times\left[\frac{1}{3+1}\right]}= \pm 0.9336
$$

The risk taken is assumed at 0.05 . DOF of mean and pooled error variance, and pooled error variance are obtained in ANOVA in Table 4.18 and Table 4.19.

Predicted Value (TPM) must lie between [3.0436, 3.7564]
Predicted Value (NPM) must lie between [9.5464, 11.4136]


Figure 4.13: Optimum factor selection for TPM


Figure 4.14: Optimum factor selection for NPM

By comparing the values between process $\mu$ and predicted $\mu$, we found that the predicted $\mu$ is larger than the process $\mu$. This indicates that all design factors of the gyrocopter have been optimized and this can lead to significant increase in the flight time. From the above sample calculation, we can clearly observe that not every single factor is used to calculate the predicted flight time for the gyrocopter with optimum set of design factors. This is because we want to ensure that the predicted flight time would be conservative and avoid over estimation. As a result, the actual flight time for the gyrocopter with optimum set of design factors would be longer than the predicted flight time, given that the condition of longer flight time is desirable.

### 4.8 Confirmation Experiment

After we have determined an optimum set of design factors, we need to conduct a confirmation test under the same noise factors conditions in order to verify the accuracy and validity of the experiment. Op C 1 represents the optimum set of design factors obtained through the experiment while Op C 2 and Op C 3 stands for set of design factors that are set by us based on our personal perception that this set design factors might give rise to better results.

## Confidence Interval for a confirmation experiment

The formula below is provided to calculate the confidence interval for factor levels:

$$
C I=\sqrt{F_{\alpha, v 1, v 2} \times V e \times\left[\frac{1}{n_{e f f}}+\frac{1}{r}\right]}
$$

Where $\mathrm{r}=15$,

For TPM,

$$
\begin{aligned}
& \mathrm{F}_{0.05,1,60}=4 \\
& C I_{T P M}=\sqrt{4 \times 0.127 \times\left[\frac{1}{3+1}+\frac{1}{15}\right]}= \pm 0.40
\end{aligned}
$$

For NPM,

$$
\begin{aligned}
\mathrm{F}_{0.05,1,12} & =4.75 \\
C I_{N P M} & =\sqrt{4.75 \times 0.734 \times\left[\frac{1}{3+1}+\frac{1}{15}\right]}= \pm 1.05
\end{aligned}
$$

The risk taken is assumed at 0.05 . DOF of mean and pooled error variance, and pooled error variance are obtained in ANOVA in Table 4.18 and Table 4.19.

Table 4.22: Confirmation Experiment



Figure 4.15: Confirmation Experiment for TPM


Figure 4.16: Confirmation Experiment for NPM

By comparing the figures of TPM and NPM, we notice that Op C 1 has longer flight time than Op C 2 and Op C 3. Thus, this shows that the optimum set of design factors obtained through experiment gives us the optimum flight time.

### 4.9 Full Scale Implementation / Comparison of Before and After

The table below shows the comparison of flight time between the gyrocopters from initial stage and optimum condition 1. At first, the flight time obtained from the initial stage gyrocopter are only ranging from 2.09 sec to 2.59 sec with none of them meet the specification flight time of 3 sec .

Table 4.23: Full Scale Implementation

| No | Before | After | Spec |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 2.25 | 4.03 | 3.00 |
| $\mathbf{2}$ | 2.09 | 4.18 | 3.00 |
| $\mathbf{3}$ | 2.38 | 4.50 | 3.00 |
| $\mathbf{4}$ | 2.22 | 4.41 | 3.00 |
| $\mathbf{5}$ | 2.22 | 4.25 | 3.00 |
| $\mathbf{6}$ | 2.34 | 4.44 | 3.00 |
| $\mathbf{7}$ | 2.59 | 4.35 | 3.00 |
| $\mathbf{8}$ | 2.32 | 4.15 | 3.00 |
| $\mathbf{9}$ | 2.41 | 4.47 | 3.00 |
| $\mathbf{1 0}$ | 2.25 | 4.28 | 3.00 |
| $\mathbf{1 1}$ | 2.22 | 4.09 | 3.00 |
| $\mathbf{1 2}$ | 2.25 | 4.03 | 3.00 |
| $\mathbf{1 3}$ | 2.22 | 4.00 | 3.00 |
| $\mathbf{1 4}$ | 2.34 | 4.10 | 3.00 |
| $\mathbf{1 5}$ | 2.38 | 4.04 | 3.00 |

After the optimization, the gyrocopter has a significant improvement in the flight time. In fact, the flight times obtained from the optimized gyrocopter are ranging from 4 sec to 4.50 sec with all of them exceed the specification flight time.

## Comparison of Before and After



Figure 4.17: Comparison of Before and After Optimization

The above graph is plotted after obtaining 15 readings from the initial stage gyrocopters and the optimized gyrocopter respectively. In fact, the graph clearly indicates that flight times obtained from the initial stage gyrocopter (Blue Line) are below the specification of 3 sec (Red Line). After the optimization, all of the flight times of the optimized gyrocopter (Pink Line) are above the specification.

### 4.10 Gain Calculation

Table 4.24: Quality Improvement

|  |  |  |  |  | Loss |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | Before | After | Spec | Before | After |  |
| 1 | 2.25 | 4.03 | 3.00 | 1.78 | 0.55 |  |
| 2 | 2.09 | 4.18 | 3.00 | 2.06 | 0.52 |  |
| 3 | 2.38 | 4.50 | 3.00 | 1.59 | 0.44 |  |
| 4 | 2.22 | 4.41 | 3.00 | 1.83 | 0.46 |  |
| 5 | 2.22 | 4.25 | 3.00 | 1.83 | 0.50 |  |
| 6 | 2.34 | 4.44 | 3.00 | 1.64 | 0.46 |  |
| 7 | 2.59 | 4.35 | 3.00 | 1.34 | 0.48 |  |
| 8 | 2.32 | 4.15 | 3.00 | 1.67 | 0.52 |  |
| 9 | 2.41 | 4.47 | 3.00 | 1.55 | 0.45 |  |
| 10 | 2.25 | 4.28 | 3.00 | 1.78 | 0.49 |  |
| 11 | 2.22 | 4.09 | 3.00 | 1.83 | 0.54 |  |
| 12 | 2.25 | 4.03 | 3.00 | 1.78 | 0.55 |  |
| 13 | 2.22 | 4.00 | 3.00 | 1.83 | 0.56 |  |
| 14 | 2.34 | 4.10 | 3.00 | 1.64 | 0.54 |  |
| 15 | 2.38 | 4.04 | 3.00 | 1.59 | 0.55 |  |

## Gain Calculation

|  | Before | After |  |
| :--- | :---: | :---: | :--- |
| Loss | 1.00 | 1.00 | $\$$ |
| Target | 0.00 | 0.00 | s |
| Spec | 3.00 | 3.00 | s |
| k | 9.00 | 9.00 | $\$ / \mathrm{s} 2$ |
| Variability | NA | NA | s 2 |
| Bias | NA | NA | s 2 |
| Mixture | 1.72 | 0.55 | s 2 |
| Total | 1.72 | 0.55 | \$ /Piece |
| Cost |  | 67.82 | \% Improvement in Cost |



Figure 4.18: Cost Reduction \$/Piece

From the Figure 4.18, there is total $67.82 \%$ of improvement in cost reduction.

### 4.11 Discussion on findings

My project title is application of sequential experimentation in a paper gyrocopter system. In the discussion, I will not discussing about paper gyrocopter. I am more focus on one of the "design of experiment" approach, which is sequential experimentation. In this final year project, sequential experimentation was carried out by using Taguchi method approach. Taguchi method technique was used to design the experiments and analysis of variance for the analysis of results.

There are two types of sequential experimentation, which are single-factor fold-over and full fold-over. This project details the single factor fold-over. Singlefactor fold over is to dealiasing any one main effect and all its two-factor interactions.

Firstly, I would like to explain why sequential experimental is so useful. This final year project concerns improvement in the flight duration of paper gyrocopter.

To build a paper gyrocopter, we need to determine the seven factors. Each of the seven factors has two possible selections, which amounts to a total of 128 possible combinations. We encode each factor as letters and the corresponding selection as -1 and +1 . A full factorial experiment would require $2^{7}=128$ experiments. Taguchi experiment with a $\mathrm{L}_{8}\left(2^{7}\right)$ orthogonal array reduces 128 experiments to 8 experiments. However, if the experiment duration and resources are very limited, then the experiments can be conducted sequentially. Since the experiments are conducted sequentially, at the end of each experiment, we are able to locate the optimal levels of the factors, and determine whether to run next experiment. So, there is a large saving in the experimental effort. If we want, we can always build towards the full experiment. Sequential experimental can save up to $50 \%$ of sampling time.

Secondly, I would like to explain on how to conduct sequential experimental. In this final year project, we divide the Taguchi experiment into two part, which is part 1 and part 2 . Each part is carry with a $\mathrm{L}_{8}\left(2^{7}\right)$ orthogonal array. After getting data for this two part, we combined these two $\mathrm{L}_{8}\left(2^{7}\right)$ together to form a $\mathrm{L}_{16}\left(2^{15}\right)$ orthogonal array. $\mathrm{L}_{8}\left(2^{7}\right)+\mathrm{L}_{8}\left(2^{7}\right) \rightarrow \mathrm{L}_{16}\left(2^{15}\right)$. If we want to continue to conduct the experiment s sequentially, we can combined two $\mathrm{L}_{16}\left(2^{15}\right)$ together to form a $\mathrm{L}_{32}\left(2^{31}\right)$ orthogonal array. This has to depend on previous results to decide whether continue or stop. If the result is inconclusive, then conduct further experiments. So, there is no necessary that we must stop until $\mathrm{L}_{16}\left(2^{15}\right)$ orthogonal array, we can further conduct experiment to $L_{32}\left(2^{31}\right)$ or $L_{64}\left(2^{63}\right)$ orthogonal array.

The Taguchi method approach got 20 steps which are shown by step by step. So, it is better for us to follow the step given by this approach. Optimum factors and level were identified objectively.

According to the analysis of TPM, NPM, response table and ANOVA, it is found that the most significant factors are the main effects. So, two-factor interaction doesn't affect on the control factors. This is because all the two-factor interaction is pooling already in the ANOVA table.

After conducted the confirmation experiment using the predicted optimum condition, comparison of the experimental result with the expected performance is
done and it is found that there is an improvement. Before and after graph shows a breakthrough improvement.

## CHAPTER 5

## CONCLUSION AND RECOMMENDATIONS

### 5.1 CONCLUSIONS

In this final year project, sequential experimental analysis was carried out by the Taguchi method technique. There are two types of sequential experimentation, which are single-factor fold over and full fold-over. Single-factor fold over is to dealiasing any one main effect and all its two-factor interaction. While full fold-over is to dealising all main effects. In other words, a second fraction design with the signs for only one of the factors reversed is called as single-factor fold over. While a second fraction design in which the signs for all the factors are reversed is called as full foldover.

Large experiments are costly and time consuming. Moreover, when we are going out to work especially in R\&D department, the experiment duration and resources are very limited. Under these conditions, sequential experimentation is one of the good options for design of experiment approach. Meanwhile, some of the experiments have to depending on the budget and schedule. Since the experiments are conducted sequentially, first we have to see whether the experiment can give us the desired data. Then, we started to determine whether to run next experiment or not

So, sequential experimental provides an easier, faster, and better design of experiment approach.

### 5.2 RECOMMENDATION

After completing this final year project, there are some suggestions and recommendations for the sequential experimentation:

1. Sequential experimentation is an exciting area of research to achieve better improvement. The Taguchi method approach presented in this thesis had much to explore in the Design of Experiment field, in which the updated information could be used to determine the next stage of experiment. We may consider extending sequential experimental to dynamic characteristics in which both experimental approaches are related.
2. It is not necessary for us to use Taguchi approach to calculate NPM. NPM is not only can apply by Taguchi approach, but also can apply in other area, such as coefficient of variation. So, the choice of NPM is entire on us.
3. Sequential experimental can be used in many domain such as manufacturing processes, environment sciences, agricultural sciences, physics, chemistry, statistics, medicine, engineering and others.

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