

**APPLICATION OF CENTRAL COMPOSITE DESIGNS IN A PAPER
GYROCOPTER SYSTEM**

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**A project report submitted in partial fulfilment of the
requirements for the award Bachelor of Engineering
(Hons.) Mechanical Engineering**

**Faculty of Engineering and Science
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June 2010

DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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Specially dedicated to my beloved family members, who have provided me with unconditional love and care.

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APPLICATION OF CENTRAL COMPOSITE DESIGNS IN A PAPER GYROCOPTER SYSTEM

ABSTRACT

Basically, Design of Experiment (DOE) is a statistical technique used in quality control for planning, analysing and interpreting sets of experiments aimed at making wise decisions without incurring a high cost and consuming much time. Amongst the various types of DOE, Central Composite Design (CCD) is one of the methods in which my research design will be focusing on. CCD is intended to study non-linear functions and relationships between factors affecting response. Through the CCD method, I am able to determine the optimum factor which will contribute most to the flight time of a paper gyrocopter system and reduce the variation involve. Ultimately, we could obtain a more precise and better result. Thus, in order to perform the CCD method, we will be assisted by Microsoft Excel in generating outputs based on the variation of different combinations of factors. The paper gyrocopter system experiment is carried out within the university compound. After collecting the data needed, we are able to plot graphs and observe the output so that we could have a clearer understanding of the significance of each factor.

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LIST OF SYMBOLS / ABBREVIATIONS

α	Rotatability
β	Beta-value
σ	Standard deviation
A_0	Loss due to a defective product
Δ_0	Allowed deviation from the target
\bar{y}	MEan
ANOVA	Analysis of Variance
CCD	Central Composite Design
CCC	Circumscribed Central Composite
CCI	Inscribed Central Composite
CCF	Face-Centred Central Composite
Df	Degree of Freedom
DOE	Design of Experiment
DOF	Degree of Freedom
LTB	Larger the Better
NTB	Nominal the Better
QLF	Quality Loss Function
RSM	Response Surface Methodology
STB	Smaller the Better
SN ratio	Signal-to-noise Ratio

CHAPTER 1

INTRODUCTION

1.1 Background of Design of Experiments (DOE)

Design of Experiments (DOE) has portrayed a major contribution in science and technology since the time Sir Ronald Aylmer Fisher introduced this concept in the 1940s (Belavendram, 2011). It was widely used by engineers and scientists for product design and development as well as process development and improvement. The concept of experimental design is to yield the most information from the fewest runs of an experiment. By applying experimental designs in a product cycle, the development lead time and cost could substantially be reduced. Besides, it would also lead the processes and products to perform better and achieve greater reliability.

Since 1950s, George Edward Pelham Box (a statistician) and his collaborators had extensively introduced experimental design methods in chemical and process industry. It was used to investigate and monitor the systems or processes of a particular product. Typical examples are the production of wafers in the electronics industry, the manufacturing of engines in the car industry and the synthesis of compounds in the pharmaceutical industry. Over the past decades, there had been a tremendous increase in the demand and application of experimental design techniques in the industry environment. By applying such experimental designs, the total quality management (TQM) had improved, ultimately increasing the company's revenue.

The method for conducting experimental designs begins with determining the objectives of an experiment and identifying the factors involved for the study. A designed experiment requires establishing a detailed experimental plan in advance of conducting the experiment, which results in a streamlined approach in the data collection stage. Appropriately choosing experimental design maximizes the amount of information that can be obtained for a given amount of experimental effort.

Basically, the design of experiments is divided into three categories, namely One-Factor-at-a-Time-method, Factorial Experimental design and orthogonal array. The illustration will be as follow:

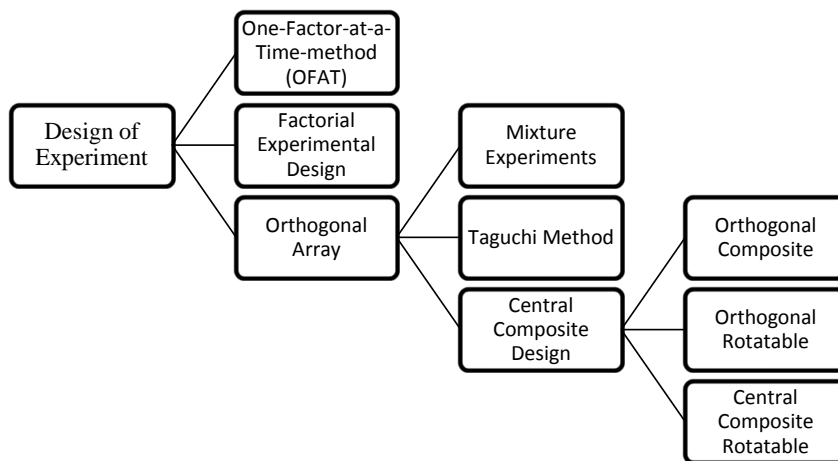


Figure 1.1: The overall view of Design of Experiment

1.1.1 One-Factor-at-a-Time-method (OFAT)

Sir Francis Bacon introduced the one-factor-at-a-time method (OFAT) in experimental design. It was one of the earliest methods which were commonly used in experimental design. As the name suggests, it involves the testing of one factor or cause at a time instead of simultaneously while the other variables remain constant.

The OFAT method could be more advantages compared to factorial experimental design when:

1. The numbers of runs are limited.
2. The primary goal is to attain improvements in the system.
3. The experimental error is not large compared to factor effects, which must be additive and independent of each other.

1.1.2 Factorial Experimental Design

The factorial experimental design method was pioneered by Sir Ronald Aylmer Fisher, a British statistician in the 19th century. This statistical approach underlying design of experiments was largely developed by him while he was working at Rothamsted Experimental Station. This method was further developed to include fractional designs and response surface methods. In 1935, Fisher published "Design of Experiments," a book that emphasize on the features of experimental research design which is still commonly used until today.

Factorial experiment design or factorial design is a more systematic method used to investigate the effect of two or more factors towards the output response of a system. Estimating the effects of various factors on the output of a process with a minimal number of observations is crucial to being able to optimize the output of the process. All the possible combinations of the varying levels of these factors are taken into account for each complete trial of the experiment. Effective factorial design ensures that the least number of experiment runs are conducted to generate the maximum amount of information about how input variables affect the output of a process.

According to (Fisher, 1926), he claimed that "complex" design (such as factorial design) was more efficient than studying one factor at a time. Hence, the

emergence of factorial experimental design was supposed to be an enhancement version of the one-factor-at-a-time method. The reasons stated for favouring the use of factorial design over OFAT are:

- OFAT requires more runs and consumes more time to produce the same precision in effect estimation compared to multi factors design.
- OFAT cannot estimate the interactions between different factors.
- OFAT has the possibility of missing the optimal settings of factors which result in obtaining poor experimental results.

1.1.3 Orthogonal Array

Dr. Genichi Taguchi, a Japanese industrialist, popularised orthogonal array experiments. He had utilized the concept of orthogonal arrays to optimize and determine the best level of control factor which maximized the Signal-to-Noise ratios. Taguchi's orthogonal array method categorized all problems into 2 types, namely Static or Dynamic. In Static problems, the optimization is achieved for a point response by using a Target Performance Measure (TPM) and a Noise Performance Measure (NPM). There are three common Signal-to-Noise ratios - smaller-the-better, larger-the-better and nominal-the-best. On the other hand for Dynamic problems, the optimization is achieved for a line response by using suitable TPM and NPM based on the Slope and Linearity of a function.

1.2 Aim and Objectives

There are numerous methods in experimental design. Nevertheless, the focus of this research would be on Central Composite Design.

The objectives of this research are as follow:

1. To develop a suitable Central Composite Design model using Microsoft Excel to aid in experimental designs
2. To generate an appropriate model
3. To conduct analysis of variance (ANOVA)
4. To determine the optimum condition of an experiment
5. To validate the optimum condition by simulation, which includes control and noise factors
6. To create a spreadsheet for various purposes.

1.3 Problem Statement

Variation in every production process is unavoidable. Therefore, the role of an engineer is to reduce these variations by improving the system performance through identifying the causes of variation and then taking actions to control them to meet respective goals.

The ultimate aim of this research is to apply Central Composite Design (CCD) in an industrial environment with the aid of advanced computer data analysis to reduce such variations. We believe that this model will produce the highest relative accuracy in experimental design. This could be seen when we create the spread sheet using Microsoft Excel. Not to mention that CCD is also more theoretical compared to orthogonal array and mixture components. Therefore, we are able to achieve a greater precision of results while performing any experiment.

CHAPTER 2

LITERATURE REVIEW

2.1 Central Composite Design

Central Composite Design (CCD) would be the highlight of this final year project as we will study in depth regarding this method to perform experiments and to optimize a product or process. CCD is categorized under a type of orthogonal array, so as for the Taguchi method and mixture experiment. Each of these methods has their own significance and it will be further discussed in the subsections later.

A CCD has three groups of design points (Ramlan, May 2008):

1. Two-level factorial or fractional factorial design points
2. Axial points (star points)
3. Center points

CCD's are designed to estimate the coefficient of a quadratic model. All point descriptions will be in terms of coded values of the factors.

2.1.1 Factorial Points

The two-level factorial part of the design consists of all possible combinations of the +1 and -1 levels of the factors. For instance, in a two factor case, there are four design points:

$$(-1, -1) (+1, -1) (-1, +1) (+1, +1)$$

2.1.2 Star or Axial Points

Basically, CCD designs involve plotting a number of 'star points' to form a sphere shape which allows the estimation of curvature. According to (Paul D. Berger, 2002), for every CCD design, it will have k factors and $2k$ star points. It is essential to have twice the amount of star points involved for every factor. Star points represent new extreme low and high values for each factor in the design. Thus, CCD not only has integer values but also decimal numbers. This is to provide a more accurate result.

The star points have all of the factors set to 0, the midpoint, except one factor, which has the value $\pm \alpha$ (α). For a two factor problem, the star points are:

$$(-\alpha, 0) (\alpha, 0) (0, -\alpha) (0, \alpha)$$

A CCD is made rotatable by the choice of α . The value of α for rotatability depends on the number of points in the factorial portion of the design. In fact, $\alpha = (n_f)^{1/4}$ yields a rotatable central composite design where n_f is the number of points used in the factorial portion of the design.

2.1.3 Centre Points

The centre points provide information about the existence of curvature. As the name implies, centre points are points with all levels set to coded level 0 (the midpoint of

each factor range). Centre points are usually repeated 4-6 times to get a good estimation of experimental error (pure error) and the quadratic terms of the model. For example with two factors, the design will be created with five centre points by default.

According to (Montgomery, 2001), sometimes experiment must be conducted in situations where there is little or no prior information about process variability. In such cases, running two or three centre points as the first few runs in the experiment can be very helpful. These runs can provide a preliminary estimate of variability. If the magnitude of the variability seems reasonable, then we may proceed. Otherwise, if larger than anticipated variability is observed, stop the runs.

2.2 The Extension of CCD

Within the compound of CCD, it is then further extended to Orthogonal Composite, Orthogonal Rotatable and Central Composite Rotatable. Central Composite Rotatable is also divided into 3 sub-categories, namely circumscribed (CCC), inscribed (CCI) and face centered (CCF). Most of the time, we are uncertain about the best method to use in a given study. Hence, to make the correct selection, we must be able to distinguish the difference between the region of interest and region of operability.

- Region of interest- a geometric region defined by lower and upper limits on study-variable level setting combinations that are of interest to the experimenter
- Region of operability- a geometric region defined by lower and upper limits on study-variable level setting combinations that can be operationally achieved with acceptable safety and that will output a testable product

CCC is the origin of a CCD. The design and the number of factors determine the location of a star point with a distance of α from the center of the circle. These designs have circular, spherical, or hyper spherical symmetry and require 5 levels for each factor.

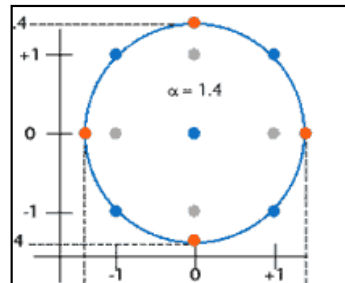


Figure 2.1: Circumscribed Central Composite

For CCI, factor settings are used as the star point to create a factorial or fractional factorial design. As the name inscribed suggest, it is a scaled down version of the CCC design with each factor level of the CCC design divided by α . CCI requires 5 levels for each factor as well.

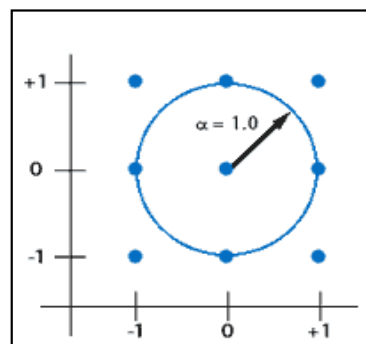


Figure 2.2: Inscribed Central Composite

For CCF, the center of each face of the factorial space is the star point. Hence, this variety require 3 levels of each factor, $\alpha = \pm 1$.

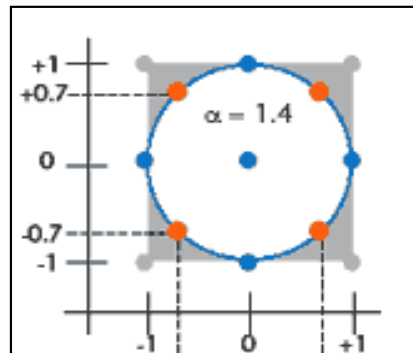


Figure 2.3: Face Centered Central Composite

2.3 CCD leading to Response Surface Methodology (RSM)

Apparently, response surface methodology (RSM) and central composite design (CCD) are more or less interrelated. Both of these methods were pioneered by G. E. P. Box and K. B. Wilson in 1951 in the field of experimental design and analysis. According to (Richard & Verseput, 2000), Response Surface Methodologies is also a branch of Design of Experiment (DOE) which the response of interest is influenced by several variables of the process and the main objective of using this method is to find the optimum condition.

In the RSM, a group of design alternatives is first sampled based on some experimental design methods. Central Composite Design (CCD) is one such method often used and is also selected in this research. The main idea of RSM consists of using a particular set of mathematical and statistical methods experiments to obtain an optimal response. Box and Wilson suggest using a first-degree polynomial model to do this. They acknowledge that this model is only an approximation, but use it because such a model is easy to estimate and apply, even when little is known about the process.

The RSM is selected to explore the capacity decision space in this research because it is able to search in a continuous space with reduced computational cost. Capacity decision space is defined as a continuous space with the range between zero

and maximum of a plant's capacity. Methods such as the Factorial Design or the OFAT are not appropriate in exploring this space as the space must be divided into levels before experiments could be conducted. If the intervals between levels are big, which leads to a low resolution of the exploration, completing all experiments will still leave a large space unexplored. If intervals between levels are small, the cost of conducting experiments will then become very expensive.

On the contrary, for the RSM, although during experiment design phase, the decision space have to be divided to get structured samples in the space, during the second phase optimization of regression model is conducted over the entire space without being constrained in the discrete space. Since the optimization is conducted upon some mathematically simple regression model, it takes less computational time than optimization on the original formulation. An issue though is that optimization is done with a model for which the coefficients are estimated, not known.

Thus, an optimum value may only look optimal for the estimated response model, but be far from the truth because of variability in the coefficients. Nevertheless, when it is not feasible to find an optimal value with a complicated function, RSM provides a way of improving the response with reduced computational cost.

2.3.1 Optimization of Stationary Points

The contour plots in RSM enable us to optimize the predicted response (Montgomery D. C., 2001). Those points are called the stationary points and they can be obtained by using a general mathematical solution for the location of the stationary point or in other words, the optimal levels. The second-order model in matrix notation would be like the following:

$$\hat{y} = \hat{\beta} + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \hat{\beta}_{11}, & \frac{\hat{\beta}_{12}}{2}, & \dots, & \hat{\beta}_{1k}/2 \\ & \frac{\hat{\beta}_{12}}{2}, & \dots, & \hat{\beta}_{1k}/2 \\ & & \backslash & \\ \text{sym.} & & & \hat{\beta}_{kk} \end{bmatrix}$$

A further explanation would be \mathbf{b} is a $(k \times 1)$ vector of the first-order regression coefficients and \mathbf{B} is a $(k \times k)$ symmetric matrix whose main diagonal elements are the pure quadratic coefficients ($\hat{\beta}_{ii}$) and whose off-diagonal elements are one-half the mixed quadratic coefficients ($\hat{\beta}_{ij}, i \neq j$). The derivative of \hat{y} with respect to the elements of the vector \mathbf{x} equated to $\mathbf{0}$ is

$$\frac{\partial \hat{y}}{\partial \mathbf{x}} = \mathbf{b} + 2\mathbf{B}\mathbf{x} = 0$$

Thus, stationary point is determined by:

$$X_s = -\frac{1}{2} \mathbf{B}^{-1} \mathbf{b}$$

2.4 CCD over Taguchi's orthogonal array method

In this section, we will discuss the reasons we select CCD over Taguchi method as our DOE. Central Composite Design (CCD) and Taguchi's method are both very useful sort of experimental design. However, both of these methods have different kinds of approaches. It is vital to identify the suitable kind of design experiments at the beginning while carrying out a study. This is to ensure that the results obtained are relevant, sufficient and resourceful. In this study, I have chosen CCD over Taguchi's method and further illustration of respective approaches will be as follow:

Taguchi laid a strong emphasis on variability reduction. His method includes two important parts, namely the crossed array designs as the experimental strategy and signal-to-noise ratios (SNR) as the analysis method. Despite criticisms put forward by Western statisticians concerning about his approach, the methods advocated by Taguchi have been successfully applied to many real-life situations.

Taguchi's design strategies include crossed or orthogonal arrays where the inner array that consists of the control variables \mathbf{x} , is crossed with the outer array that contains the noise variables \mathbf{z} . The inner or outer array is a factorial or fractional factorial design, and the outer array is crossed with every combination of the inner array. The orthogonal arrays are denoted by $L_r(m^k)$, where r represents the number of runs in the array, k is the number of variables, and m is the number of levels of each variable.

Taguchi proposes three stages of robust design: system design, parameter design, and tolerance design. The goals of Taguchi's experimental design can be summarized as designing robust products or processes that are insensitive to environmental conditions (external noise factors), developing robust products that are insensitive to component variation (internal noise factors), and minimizing variation around a target value. The most important parts of Taguchi's philosophy are the reduction of variability and minimization of non-conformance cost. They are consistent with the modern continuous quality improvement philosophy.

On the other hand, CCD has a different kind of approach to solve robust design problem and optimize a product or process. It has many advantages over Taguchi's methods of experimental design and data analysis. However, many companies or engineers in the industry world do not fully understand the better approaches coming from the academic field and are still using Taguchi's methods, even though the academy has discussed and reviewed robust design for over 20 years and statisticians have repeatedly compared Taguchi's methods with other new approaches (including RSM). The statisticians should focus on training people working in industry to clarify the misunderstanding and close the gap between academia and industry.

CCD can be used as part of sequential design experiments through RSM. The factorial points contribute toward the estimation of linear or two-factor interaction terms, while the star or axial points are chosen based on the region of interest and region of operability. Not to mention that the centre points could also aid to obtain a good estimation of experimental error (pure error) and the quadratic terms of the model. The dual response surface approach is able to estimate two response models at the same time, one for the process mean and another for the process variance. Thus, CCD involves a combined array design which combines the design and noise variables into one single design, while Taguchi's crossed array includes two designs: one is inner array design consisting of the control variables, and the other is outer array design containing the noise variables.

Plus, CCD method is able to portray a greater precision of results compared to Taguchi's method. This is because CCD takes into account the axial (star) points and centre points while performing the experiment. Hence, it is able to provide a lot of information regarding the experimental variable effects and experimental error within a minimum number of runs. A better and clearer vision of the simulation could also be observed when all the points are plotted. Not to mention that CCD also fulfils most of the company's goal, which is to minimize the cost by running lesser experiments and yet being able to sustain the quality of a product.

2.5 Analysis of Variance (ANOVA)

Generally, Analysis of variance (ANOVA) is used to study sampled-data relationships and analyse the difference between two or more means. This method enables the difference between two or more sample means to be analysed and achieved by subdividing the total sum of squares (Belavendram N. , May 1995). A major aim of analysing data from designed experiments is to quantify and evaluate the importance of possible sources of variation (Kempthorne, 2008). This can be achieved through the ANOVA associated with the underlying linear model, either in its univariate or multivariate. There are a few types of ANOVA, namely the one-way

ANOVA, multifactor ANOVA and General Linear Models. The elaborations will be as follow:

2.5.1 One-way ANOVA

The purpose of one-way ANOVA is to test for significant differences between class means and this is done by analysing the variances. The one-way ANOVA is useful when the experiment involves only one factor. Therefore, we are able to compare in depth the effects of multiple levels of one factor and we could obtain multiple observations at each level. The factor can be either discrete or continuous.

According to (George E.P. Box, 2005), the application of one-way ANOVA will be as follow: suppose that we have a number of observations of a dependent variable Y , and each observation is associated with a categorical factor X that has i different “level” (possible values). Therefore, the ANOVA equation that represents this situation is:

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

Where y_{ij} is the j 'th observation of y_i , the group that has the i 'th level of X , μ is the overall mean of y , α_i is a parameter unique to the i th treatment called the i th treatment effect, and ϵ_{ij} is the random error component that incorporates all other sources of variability in the experiment including measurement, variability arising from uncontrolled factors, differences between the experimental units to which the treatments are applied and the general background noise in the process.

2.5.2 Multifactor ANOVA

Multifactor ANOVA is used when there is more than one categorical factor arranged in a crossed pattern. When factors are crossed, the levels of one factor appear at more than one level of the other factors.

For example, with two independent variables, the model is:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$$

Where $(\alpha\beta)_{ij}$ is the interaction term, representing the additional effect of being in both level i of the first variable and level j of the second variable.

2.5.3 General Linear Model (GLM)

Generalized linear models (GLMs) are used to do regression modelling for non-normal data with a minimum of extra complication compared with normal linear regression. GLMs are flexible enough to include a wide range of common situations, but at the same time allow most of the familiar ideas of normal linear regression to carry over. Besides, GLM is also used whenever there are both crossed and nested factors, when some factors are fixed and some are random, and when both categorical and quantitative factors are present.

The general linear model goes a step beyond the regression model by allowing linear transformations or linear combinations of multiple dependent variables. This extension gives the GLM important advantages over the multiple regression models. One advantage is that multivariate tests of significance can be employed when responses on multiple dependent variables are correlated. Separate univariate tests of significance for correlated dependent variables are not independent and may not be appropriate. Multivariate tests of significance of independent linear combinations of multiple dependent variables also can give insight into which dimensions of the response variables are, and are not, related to the predictor variables.

Another advantage of GLM is the ability to analyse effects of repeated measure factors. Repeated measure designs, or within-subject designs, have

traditionally been analysed using ANOVA techniques. Linear combinations of responses reflecting a repeated measure effect (for example, the difference of responses on a measure under differing conditions) can be constructed and tested for significance using either the univariate or multivariate approach to analyse repeated measures in the general linear model. Hence the GLM allows us to summarize a wide variety of research outcomes.

2.6 Quality Loss Function

According to (Sharma, Cudney, Ragsdell, & Paryani, 2007), the quality loss function is developed by Genichi Taguchi. It is a continuous function which defines losses as a design parameter that deviates from an ideal or targeted value. Basically, Taguchi's philosophy takes into account three cases to evaluate the relationship between quality and variability, namely nominal-the-best (NTB), smaller-the-better (STB) and larger-the-better (LTB). To start off (Montgomery D. C., 1998), Taguchi quantifies the deviation for NTB and STB from the requirements in terms of monetary units by using a quadratic loss function given by:

$$L(y) = k(Y - \tau)^2 ; k = \frac{A_0}{\Delta_0^2} \text{ --- Equation (1)}$$

Where Y is the quality characteristic, τ is the targeted value, k as the cost-related numerical constant, Δ_0 as the point of intolerance and A_0 as the cost of a corrective action.

2.6.1 Nominal-the-best (NTB)

For nominal-the-best (NTB), the best characteristic of a target value is the median of the specified upper and lower limits while the losses owing to deviance from the targeted value rise proportionally to the extent of deviance on either side of the mean.

As the quality is evaluated with respect to a nominal value τ , the loss function for NTB will be as follow:

$$L(y) = \frac{A_0}{\Delta_0^2} (Y - \tau^2)$$

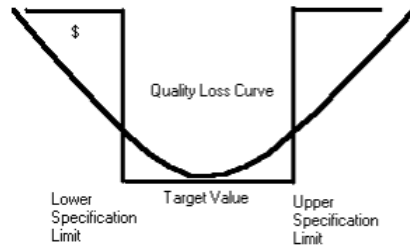


Figure 2.4: Nominal-the-best (NTB)

2.6.2 Smaller-the-better (STB)

For smaller-the-better (STB), the characteristics involve are only non-negative variables with target $\tau = 0$. If the actual value becomes higher, the quality loss would also be greater. Thus, its loss function is given by as follow:

$$L(Y) = \frac{A_0}{\Delta_0^2} Y^2$$



Figure 2.5: Smaller-the-better (STB)

2.6.3 Larger-the-better (LTB)

For larger-the-better (LTB), the characteristic should also be non-negative and its most desirable value is infinity. Thus, if the actual value becomes higher, greater quality could be obtained and quality loss would be lesser. In this case, we could not replace $\tau = \infty$ into equation (1). As an alternative, Taguchi proposes that if Y is LTB, and then $1/Y$ should be STB and therefore its loss function is given as follow:

$$\begin{aligned} L(Y) &= \frac{\frac{k}{y_1^2} + \frac{k}{y_2^2} + \dots + \frac{k}{y_n^2}}{n} \\ &= kn \left(\frac{k}{y_1^2} + \frac{k}{y_2^2} + \dots + \frac{k}{y_n^2} \right) \\ &= A_0 \Delta_0^2 \left[\frac{1}{\bar{y}^2} \left(1 + \frac{3\sigma^2}{\bar{y}^2} \right) \right] \end{aligned}$$



Figure 2.6: Larger-the-better (LTB)

CHAPTER 3

METHODOLOGY

3.1 Research Design

The research design for our study is to determine the optimal factor which affects the flight time of a paper gyrocopter system using the central composite design (CCD) method. Through the CCD method, we believe that it is able to provide us with the most information in order to obtain a greater precision of results.

We begin this methodology section by designing a template on Microsoft Excel for three different types of CCD, namely CCC, CCI and CCF. With the aid of Microsoft Excel, we are able to perform numerous applications by entering different formulas at the formula bar. For example, formulas such as CHOOSE, SUM, IF, CONCATENATE, SUMPRODUCT and MMULT are used extensively whenever needed at the Microsoft Excel formula bar.

3.2 Designing an array based on suitable amount of factors and levels

Basically, the amount of factors involved in an experiment will ultimately determines the design of an array. For this experiment, we have chosen 3 factors (X1, X2, and X3), namely wing length (X1), body width (X2) and shoulder length (X3) as the three significant factors in which we think will affect the flight time of the paper gyrocopter system. Based on CCD, these 3 factors also known as the independent

factors will then be combined to form 6 other combinations. The aim is to have more combinations of factors so that it covers most part of the region of interest to produce a more precise result.

After identifying the three factors, we are able to set the number of levels to each factor. The number of levels also depends mainly on the type of CCD. For both CCC and CCI, they have 5 levels for every factor and for CCF, 3 levels are assigned to each factor. For example, CCC has 5 levels for each factor consisting of -1, 1.414, 1, 0, 1, and 1.414 while CCF has only 3 levels, which are -1, 0 and 1. Besides, the value of each level also consists of the coded and uncoded value.

Table 3.1: CCC with three independent factors

	x0	x1	x2	x3	y
1	1	-1	-1	-1	2.83
2	1	-1	-1	1	3.03
3	1	-1	1	-1	3.00
4	1	-1	1	1	3.03
5	1	1	-1	-1	3.43
6	1	1	-1	1	3.43
7	1	1	1	-1	3.23
8	1	1	1	1	3.77
9	1	-1.414	0	0	2.63
10	1	1.414	0	0	3.57
11	1	0	-1.414	0	3.70
12	1	0	1.414	0	3.83
13	1	0	0	-1.414	3.87
14	1	0	0	1.414	4.03
15	1	0	0	0	3.40
16	1	0	0	0	3.50
17	1	0	0	0	3.60
18	1	0	0	0	3.40

The array shown above is a central composite design array which consists of a total runs of 18 experiment and involves three independent factors (X1, X2, and X3) with each factor consists of five different levels (-1.414, -1, 0, 1, 1.414). Not to mention that the X0 row is the identity factor. From the array, you will notice that it is categorized into three different parts. This is to distinguish the Taguchi and the central composite design method. The array in red is known as the orthogonal array while the arrays in green and blue represent the axial points and the centre points respectively.

As mentioned earlier on, additional factors will be added on to the existing ones so that it covers most part of the region of interest to produce a more precise result. These additional factors are the quadratic factors ($X1^2$, $X2^2$ and $X3^2$) and the interaction factors ($X1X2$, $X1X3$ and $X2X3$).

The different combinations of all the factors will then produce a dependent variable Y, which is indicated on the right column. This Y values corresponds to the flight time of the gyrocopter in which later will also be used in performing other analysis.

3.3 Correlation Check

To perform the correlation check, the main formula involved is $\beta = (X^T X)^{-1} X^T Y$, where β is the regression coefficient. In order to do so, we will have to make use of the array shown previously to determine the correlation between the independent and dependent factors.

Step One

Table 3.2: X array multiply with the transpose of X array

	x0	x1	x2	x3	x1x2	x1x3	x2x3	x1^2	x2^2	x3^2
x0	18	0	0	0	0	0	0	0	0	0
x1	0	12	0	0	0	0	0	0	0	0
x2	0	0	12	0	0	0	0	0	0	0
x3	0	0	0	12	0	0	0	0	0	0
x1x2	0	0	0	0	8	0	0	0	0	0
x1x3	0	0	0	0	0	8	0	0	0	0
x2x3	0	0	0	0	0	0	8	0	0	0
x1^2	0	0	0	0	0	0	0	8	0	0
x2^2	0	0	0	0	0	0	0	0	8	0
x3^2	0	0	0	0	0	0	0	0	0	8

Based on the formula, the first step is to multiply the X array with the transpose of X array. The formula involve to produce the array above can be obtained by just typing the command of “=MMULT(TRANPOSE(X array),X array)” into the Microsoft Excel formula bar. The table above shows the result of after performing the formula.

Step Two

The next step is to perform an inverse multiplication of the table above. The formula involved in this step is “=MINVERSE($X^T X$ array)”. Ultimately, you will get a new table as shown below.

Table 3.3: Inverse of the ($X^T X$) array

	x0	x1	x2	x3	x1x2	x1x3	x2x3	x1^2	x2^2	x3^2
x0	0.056	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x1	0.000	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x2	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x3	0.000	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.000	0.000
x1x2	0.000	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.000	0.000
x1x3	0.000	0.000	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.000
x2x3	0.000	0.000	0.000	0.000	0.000	0.000	0.125	0.000	0.000	0.000
x1^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.125	0.000	0.000
x2^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.125	0.000
x3^2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.125

Step Three

After doing the inverse multiplication, the inversed matrix above is then multiplied with the transpose of the original array. The Excel function performed is “=MMULT($X^T X$ array), TRANSPOSE(X array)”. The result of this step will be as follow.

Table 3.4: Multiplication of ($X^T X$)⁻¹ with the X transpose array

	x0	x1	x2	x3	x1x2	x1x3	...	
x0	0.056	0.056	0.056	0.056	0.056	0.056	...	0.056
x1	-0.083	-0.083	-0.083	-0.083	0.083	0.083	...	0.000
x2	-0.083	-0.083	0.083	0.083	-0.083	-0.083	...	0.000
x3	-0.083	0.083	-0.083	0.083	-0.083	0.083	...	0.000
x1x2	0.125	0.125	-0.125	-0.125	-0.125	-0.125	...	0.000
x1x3	0.125	-0.125	0.125	-0.125	-0.125	0.125	...	0.000
x2x3	0.125	-0.125	-0.125	0.125	0.125	-0.125	...	0.000
x1^2	0.042	0.042	0.042	0.042	0.042	0.042	...	-0.083
x2^2	0.042	0.042	0.042	0.042	0.042	0.042	...	-0.083
x3^2	0.042	0.042	0.042	0.042	0.042	0.042	...	-0.083

Note that there will be an additional of 8 columns after the last quadratic factor. This is to allow the current array to be able to multiply with the Y array, which consist of 18 rows.

Step Four

Lastly, use the table above to multiply with the dependent variable Y to obtain the regression coefficient, β value. With these β values, we can then proceed with the analysis of variance, ANOVA. The β values can be obtained from the ANOVA table (Table 3-6).

3.4 Contours of Constant Variance

Contour plots play a very vital role in the study of the response surface model. According to (Montgomery, 2001), by generating contour plots for response surface analysis, we are able to characterize the shape of the surface and locate the optimum with reasonable precision. For this experiment, we have used a range of -1.25 to 1.25 to determine the correlation contours of the constant variance.

Table 3.5: Correlation Table

0.722	-1.250	-1.000	-0.750	-0.500	-0.250	0.000	0.250	0.500	0.750	1.000	1.250
1.250	1.310	0.996	0.807	0.701	0.649	0.634	0.649	0.701	0.807	0.996	1.310
1.000	0.996	0.722	0.564	0.480	0.441	0.431	0.441	0.480	0.564	0.722	0.996
0.750	0.807	0.564	0.429	0.363	0.334	0.327	0.334	0.363	0.429	0.564	0.807
0.500	0.701	0.480	0.363	0.308	0.287	0.282	0.287	0.308	0.363	0.480	0.701
0.250	0.649	0.441	0.334	0.287	0.271	0.267	0.271	0.287	0.334	0.441	0.649
0.000	0.634	0.431	0.327	0.282	0.267	0.264	0.267	0.282	0.327	0.431	0.634
-0.250	0.649	0.441	0.334	0.287	0.271	0.267	0.271	0.287	0.334	0.441	0.649
-0.500	0.701	0.480	0.363	0.308	0.287	0.282	0.287	0.308	0.363	0.480	0.701
-0.750	0.807	0.564	0.429	0.363	0.334	0.327	0.334	0.363	0.429	0.564	0.807
-1.000	0.996	0.722	0.564	0.480	0.441	0.431	0.441	0.480	0.564	0.722	0.996
-1.250	1.310	0.996	0.807	0.701	0.649	0.634	0.649	0.701	0.807	0.996	1.310

From the data above, we are able to plot the response surface both in 2D and 3D. These graphs are also known as response surface methodology.

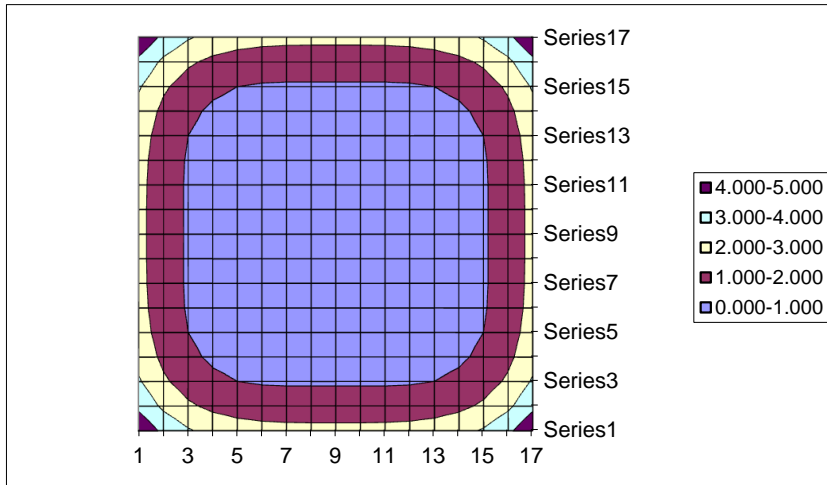


Figure 3.1: 2D Response Surface Methodology

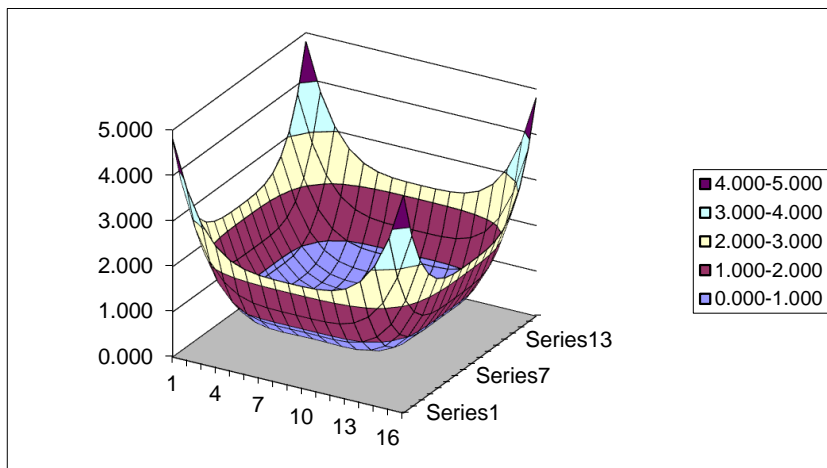


Figure 3.2: 3D Response Surface Methodology

From the 2D graph, you will notice a circular shape of the diagram in which it corresponds to a Face Centred central composite design. When the range of the correlation table is extended further with smaller differences, the graphs would be smoother to portray a more circular 2D shape.

3.5 Analysis of Variance (ANOVA)

The regression coefficients for the second order polynomial equations and results for the linear, quadratic and interaction term are calculated and presented in an ANOVA table as follows.

Table 3.6: ANOVA Table

Main effects Source	Beta	XY	Pool	Sum	df	V	F-ratio	S	Rho
x0	3.598	64.8	0	233.04					
x1	0.395	2.4	0	0.934	1	0.934	35.134	0.907	49.943
x2	0.059	0.4	1	0.021	1	0.021			
x3	0.190	1.1	0	0.217	1	0.217	8.156	0.190	10.469
x1x2	0.150	0.3	1	0.045	1	0.045			
x1x3	0.083	0.2	1	0.014	1	0.014			
x2x3	-0.383	-0.8	0	0.294	1	0.294	11.057	0.267	14.715
x1 ²	-0.044	-0.1	1	0.004	1	0.004			
x2 ²	0.172	0.3	1	0.059	1	0.059			
x3 ²	0.172	0.3	1	0.059	1	0.059			
error				1	0.028	1	0.028	1.035	
LoF				1	0.142	7	0.020	0.766	
Pool				0.372	14	0.027	1.000	0.452	24.873
St				1.817	17	0.107		1.817	100.000
Sm				233.04	1				
ST				234.86	18				

Starting from the β column, the values are obtained by performing the formula $\beta = (X^T X)^{-1} X^T Y$, in which it is already mentioned in the correlation check section (Chapter 3.3).

Moving on, pool is obtained by checking the ranking of variance. The pool value of “0” shows that it is significant and “1” is the factor that is insignificant. Based on the table above, we have pooled three main effects, hence it means that only three factors with the highest significant/weightage are chosen. This could be done with the help of the Excel function of “IF” and “RANK”. The “RANK” function will rank the values of each variance and then display the values based on the number of main effects selected.

Based on the significant factors which are not pooled, it will then have their SSq and Rho values calculated, as can be seen in the last two columns of the ANOVA table. The Rho values will indicate the significant of that particular factor

using percentage. On the other hand, the SSq values can be calculated by having the SSQ of the factor subtract the multiplication of the corresponding degree of freedom and the pool's variance.

Finally, a graph of Rho against the factors is plotted to show the importance of that particular factor towards the output response of the experiment. A high percentage of pool will indicate that the experiment is ineffective. For our case, the pool percentage is still fairly acceptable. Despite adding more factors will reduce the pool percentage, the ideal amount of factors would range from one to four only. The graph which corresponds to the example's ANOVA table can be seen in the following:

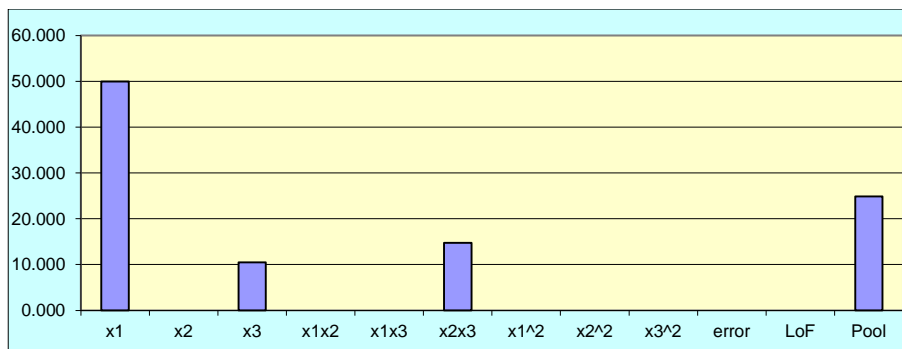


Figure 3.3: Graph of Rho against factor

Based on the graph and simulations done by the Microsoft Excel software, we are able to analyze the results to determine the optimal factor which contributes most to the flight time of a paper gyrocopter system. Besides, we could also implement recommendations to further enhance the performance and discuss problems which cause the ineffective of that particular experiment.

3.6 Determining the optimum condition value

In order to determine the optimum condition value (X_s), we will have to utilize the formula as follow:

$$X_s = -\frac{1}{2}B^{-1}b$$

where b is a vector of the first order regression coefficients, β for X_1 , X_2 and X_3 while B is the beta value of the quadratic factors along the diagonal while the off-diagonal are interaction factors. (Montgomery D. C., 2001)

After obtaining the optimum values for each factor, the paper gyrocopter is constructed based on these new dimensions and the experiment is carried out again to produce a new set of results. Then, the flight time for before (blue line) and after (purple line) optimization are compared to see whether CCF has made any improvement towards the experiment. Not to mention that a targeted value (red line) will also be set as a benchmark to show if the optimized results reaches our expectations or not. By plotting the before and after optimization results together, the outcome will be as the following figure 3.4.

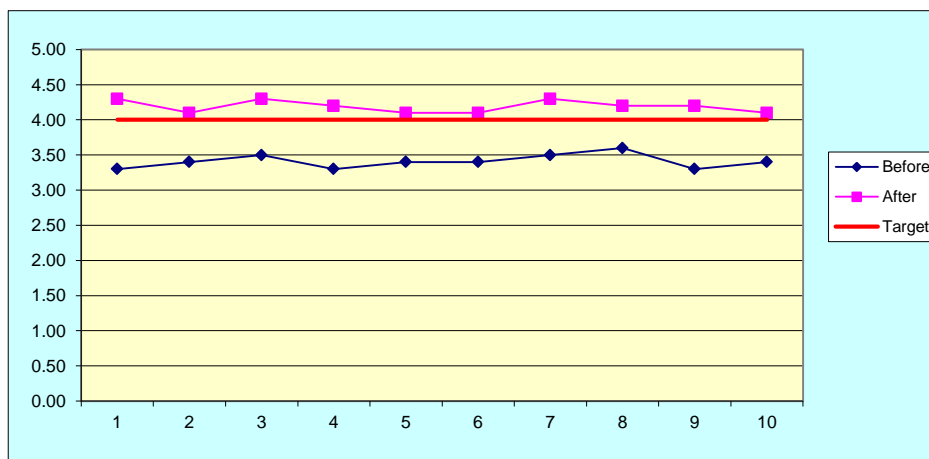


Figure 3.4: Graph of before and after optimization

3.7 Quantity Loss Function (QLF)

Last but not least, a quality loss function is performed to determine whether CCF has made any improvements in terms of cost towards the experiment. In our case, we will be using the Larger-the better (LTB) approach as we would want to increase the flight time of the paper gyrocopter while reducing the cost involved. The formula for LTB would be:

$$\begin{aligned}
 L(Y) &= \frac{\frac{k}{y_1^2} + \frac{k}{y_2^2} + \dots + \frac{k}{y_n^2}}{n} \\
 &= kn \left(\frac{k}{y_1^2} + \frac{k}{y_2^2} + \dots + \frac{k}{y_n^2} \right) \\
 &= A_0 \Delta_0^2 \left[\frac{1}{\bar{y}^2} \left(1 + \frac{3\sigma^2}{\bar{y}^2} \right) \right]
 \end{aligned}$$

Therefore, once again, the graph which shows the reduction of cost in terms of percentage will be as the following figure 3.5.

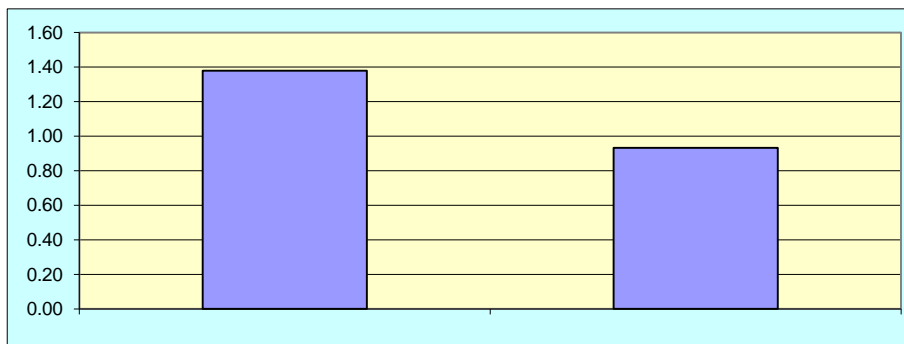


Figure 3.5: Graph which shows the reduction of cost

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Experimental Data

As mentioned earlier on, we will be taking three factors into account for this paper gyrocopter system experiment. The three factors involve are the wing length (X1), body width (X2) and shoulder length (X3). At the starting point, we have set the range for X1 to be 6 cm to 10 cm while the range for both X2 and X3 are 2 cm to 3 cm. In terms of the coded value, the higher and lower value will correspond to level 1 and level -1 respectively. Not to mention that level 0 will be the average between the high and low value.

Table 4.1: Coded and uncoded value for X1, X2 and X3

Wing length x1	UncVar	5.172	6	8	10	10.828
	CodVar	-1.414	-1	0	1	1.414
Body width x2	UncVar	1.793	2	3	3	3.207
	CodVar	-1.414	-1	0	1	1.414
Shoulder length x3	UncVar	1.793	2	3	3	3.207
	CodVar	-1.414	-1	0	1	1.414

For both CCC and CCI, there are 5 levels with 2 extra levels being values of ± 1.414 and ± 0.7071 respectively. The real value (uncoded) of ± 1.414 and ± 0.7071 is then calculated using ratio from levels -1, 0 and 1.

4.2 Central Composite Face Centred (CCF)

Table 4.2: Coded array for CCF

	x0	x1	x2	x3	x1x2	x1x3	x2x3	x1^2	x2^2	x3^2
1	1	-1	-1	-1	1	1	1	0.444	0.444	0.444
2	1	-1	-1	1	1	-1	-1	0.444	0.444	0.444
3	1	-1	1	-1	-1	1	-1	0.444	0.444	0.444
4	1	-1	1	1	-1	-1	1	0.444	0.444	0.444
5	1	1	-1	-1	-1	-1	1	0.444	0.444	0.444
6	1	1	-1	1	-1	1	-1	0.444	0.444	0.444
7	1	1	1	-1	1	-1	-1	0.444	0.444	0.444
8	1	1	1	1	1	1	1	0.444	0.444	0.444
9	1	-1.000	0	0	0	0	0	0.444	-0.556	-0.556
10	1	1.000	0	0	0	0	0	0.444	-0.556	-0.556
11	1	0	-1.000	0	0	0	0	-0.556	0.444	-0.556
12	1	0	1.000	0	0	0	0	-0.556	0.444	-0.556
13	1	0	0	-1.000	0	0	0	-0.556	-0.556	0.444
14	1	0	0	1.000	0	0	0	-0.556	-0.556	0.444
15	1	0	0	0	0	0	0	-0.556	-0.556	-0.556
16	1	0	0	0	0	0	0	-0.556	-0.556	-0.556
17	1	0	0	0	0	0	0	-0.556	-0.556	-0.556
18	1	0	0	0	0	0	0	-0.556	-0.556	-0.556

The array shown above is the actual experiment data for Face-centred Central Composite (CCF). It consists of a total runs of 18 experiments and involves three independent factors (X1, X2, and X3), namely wing length (X1), body width (X2) and shoulder length (X3). It is then further multiplied with each other to obtain additional factors, namely the interaction factors (X1X2, X1X3 and X2X3) and the quadratic factors (X1^2, X2^2 and X3^2).

Each factor consists of three different levels (-1, 0 and 1). From the array, you will notice that it is categorized into three different parts. This is to distinguish the Taguchi and the central composite design method. The array in red is the orthogonal array for three factors CCF while the arrays in green and blue represent the axial points and the centre points respectively.

Table 4.3: Coded and uncoded value for CCF factors

Wing length x1	Uncoded	6	8	10
	Coded	-1	0	1
Body width x2	Uncoded	2	2.5	3
	Coded	-1	0	1
Shoulder length x3	Uncoded	2	2.5	3
	Coded	-1	0	1

For CCF, there isn't star points involve, thus the response surface model will displays a square (2D) or cube (3D) shape shown in Figure 4.1 and 4.2. Unlike CCC (circumscribed) and CCI (inscribed), there are star points involve which enables the response surface model to become a circle (2D) or sphere (3D) shape, hence covering most part of the region of interest to produce a more precise result.

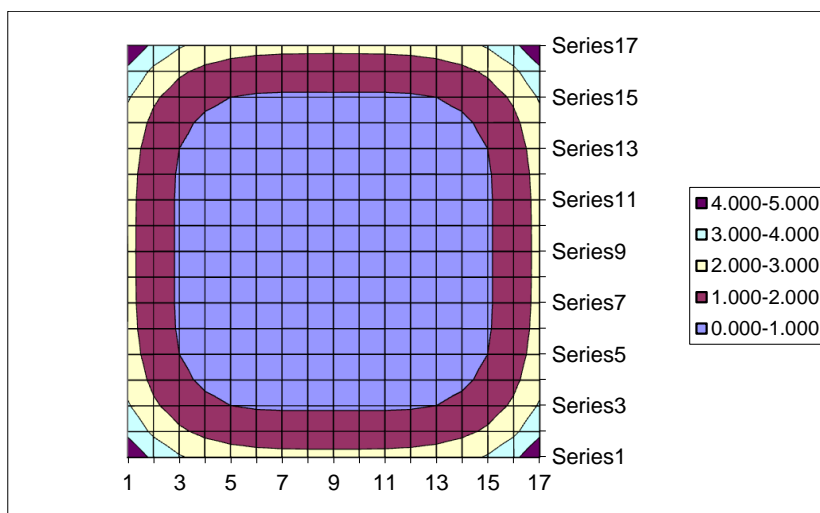


Figure 4.1: CCF 2D response surface graph

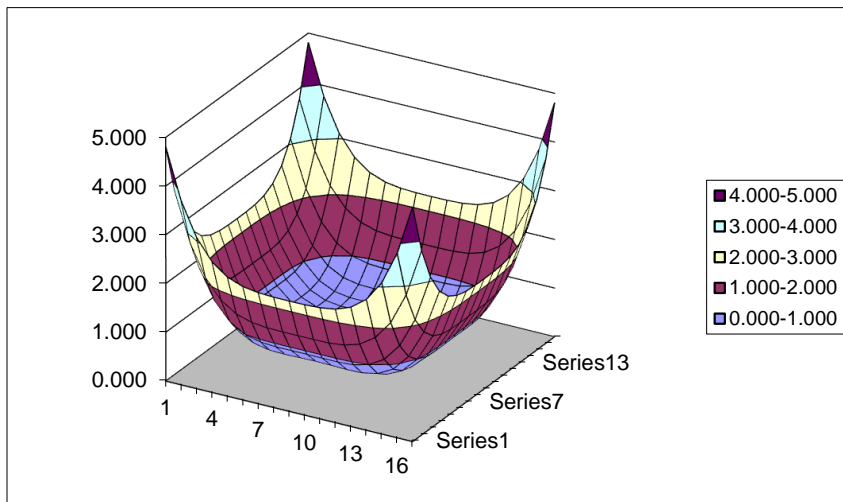


Figure 4.2: CCF 3D response surface graph

Based on the different combinations of all the factors, we will then be able to run the experiment and produce a dependent variable Y result as shown in the table below. This Y values corresponds to the flight time of the gyrocopter and each experiment was run three times to obtain an average value of the duration.

Table 4.4: Average value of Y value

Gyro No.	Run1	Run2	Run3	Y average
1	2.6	2.8	3.1	2.83
2	3.0	2.8	3.3	3.03
3	3.0	3.1	2.9	3.00
4	3.1	3.0	3.0	3.03
5	3.4	3.7	3.2	3.43
6	3.3	3.4	3.6	3.43
7	3.1	3.6	3.0	3.23
8	4.2	3.8	3.3	3.77
9	3.1	2.7	2.9	2.90
10	3.6	3.7	3.6	3.63
11	3.1	3.6	3.4	3.37
12	3.5	3.3	3.6	3.47
13	3.4	3.8	3.6	3.60
14	3.5	3.3	3.6	3.47
15	3.4	3.5	3.3	3.40
16	3.5	3.6	3.4	3.50
17	3.6	3.7	3.5	3.60
18	3.4	3.3	3.5	3.40

4.2.1 Data Analysis based on ANOVA table

For this data analysis section, we have decided to take into account only three main factors, which is X_1 (57.0%), X_1^2 (18.4%) and X_2^2 (3.54%). According to the ANOVA table shown below, the pool value is 21.05%, which is still within the acceptable range. A lower pool value would be desirable as it indicates that the experiment was a success in the sense that the factors which are not pooled were highly significant.

Besides, it is also best that we only choose three factors or less when determining the significant factors involved. This is because when we were to select more factors, the Rho value for these additional factors will either be less than 1% (insignificant) or negative value.

Table 4.5: ANOVA table for CCF

Main Effect	3									
Source	Beta	X ² Y	Pool	Sum	df	V	F-ratio	S	Rho	
x0	3.339	60.1	0	200.67						
x1	0.270	2.7	0	0.729	1	0.729	47.030	0.713	57.004	
x2	0.040	0.4	1	0.016	1	0.016				
x3	0.063	0.6	1	0.040	1	0.040				
x1x2	-0.004	0.0	1	0.000	1	0.000				
x1x3	0.038	0.3	1	0.011	1	0.011				
x2x3	0.046	0.4	1	0.017	1	0.017				
x1 ²	-0.226	-1.1	0	0.246	1	0.246	15.861	0.230	18.405	
x2 ²	-0.076	-0.8	0	0.060	1	0.060	3.857	0.044	3.539	
x3 ²	0.041	-0.6	1	-0.023	1	-0.023				
error			1	0.028	1	0.028	1.774			
LoF			1	0.128	7	0.018	1.179			
Pool				0.217	14	0.016	1.000	0.264	21.053	
St				1.252	17	0.074		1.252	100.000	
Sm				200.67	1					
ST				201.92	18					

Finally, a bar chart is then constructed to portray an even clearer picture of the percentages (Rho).

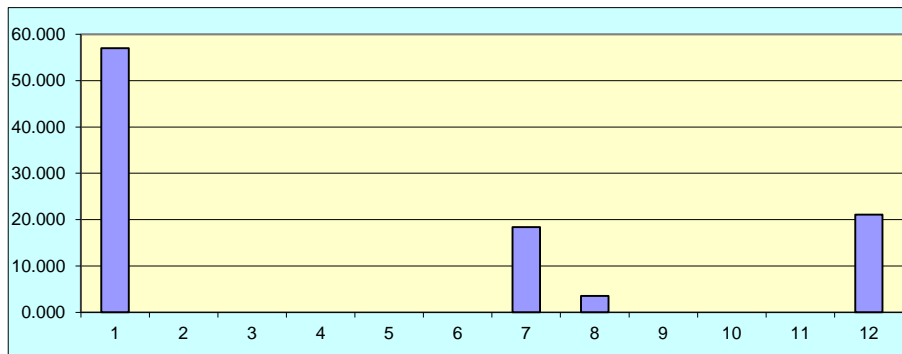


Figure 4.3: Graph of Rho (%) against Factors

4.2.2 Data Interpretation based on response surface model

From the graph above, the centre point of the light blue circle indicates the factor which contributes most to the highest or longest flight time. We are able to see that the longest flight time falls within the time range of 3.50 seconds to 3.70 seconds.

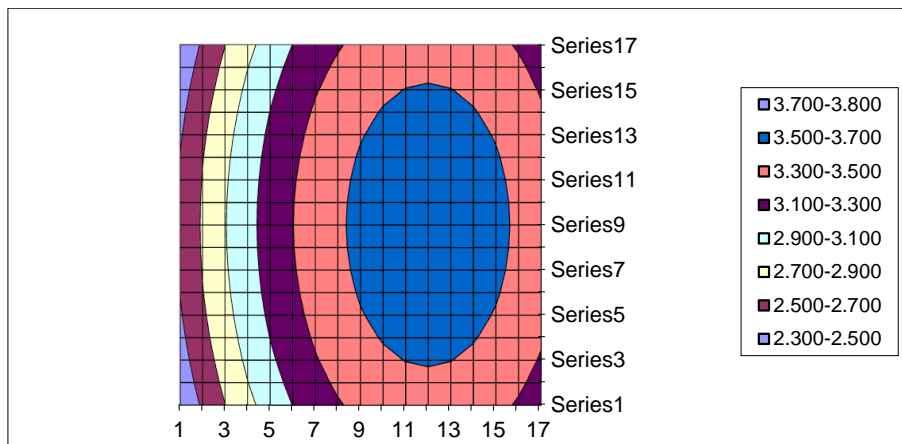


Figure 4.4: Response Surface for X1 and X2

When the response surface model for X1 and X3 is plotted as shown in the figure above, a linear graph is obtained. In other words, it shows that X3 is certainly not one of the significant factors and certainly does not portray any correlation with the X1 factor. Plus, this may also be due to the choice when selecting the parameter for X3. Apparently, the interval for minimum and maximum parameter of X3, which is 2

cm and 3 cm respectively for the wing length, is too close. Thus, this will also affect the final outcome.

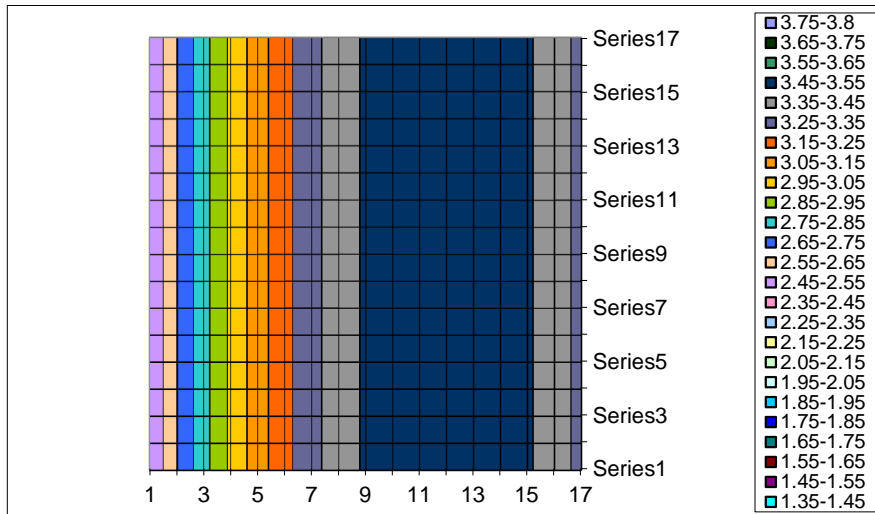


Figure 4.5: Response Surface for X1 and X3

4.2.3 Optimum condition value

In order to determine the optimum condition value, we will have to utilize the formula as follow:

$$X_s = -\frac{1}{2}B^{-1}b$$

	b	B			Xs
X1	3.339				0.458
X2	0.270	-0.226	-0.004	0.038	-0.289
X3	0.063	-0.004	-0.076	0.046	-0.872
		0.038	0.046	0.041	

Figure 4.6: Optimum values for CCF

The X_s column of values indicates the optimum dimension for factor X1 (0.458), X2 (-0.289) and X3 (-0.872). However, these values are in the coded form.

Hence, we will have to perform simple ratio calculations based on the initial conditions to determine the coded value of X_5 .

Optimum Values						
Wing length x1	Uncoded	7.08	6	8	10	8.92
	Coded	-0.46	-1	0	1	0.458
Body width x2	Uncoded	2.36	2	2.5	3	2.64
	Coded	-0.289	-1	0	1	0.29
Shoulder length x3	Uncoded	2	2.06	2.5	2.94	3
	Coded	-1	-0.872	0	0.872	1

Figure 4.7: Uncoded optimum values for CCF

From the diagram above, we have already derived the uncoded CCF optimum values for X1, X2 and X3 respectively. Subsequently, we are able to construct the paper gyrocopter with these optimum dimensions. In our case, we have found out that when the wing length is 8.92 cm, body width is 2.36 cm and shoulder length is 2.06 cm, the paper gyrocopter is performing at its best and contributing to the longest duration of flight time.

The experiment is then carried out again based on these optimum dimension and the results are as following:

Table 4.6: Flight time for before and after optimization

	Before	After	Target
1	3.30	4.3	4
2	3.40	4.1	4
3	3.50	4.3	4
4	3.30	4.2	4
5	3.40	4.1	4
6	3.40	4.1	4
7	3.50	4.3	4
8	3.60	4.2	4
9	3.30	4.2	4
10	3.40	4.1	4

According to the table, the overall trend of the flight time after optimization has improved significantly in comparison to the initial conditions. The optimized results are even better than our targeted value which is 4 seconds. The following graph will clearly displays that the values after optimization are greater than the targeted value and the values before optimization.

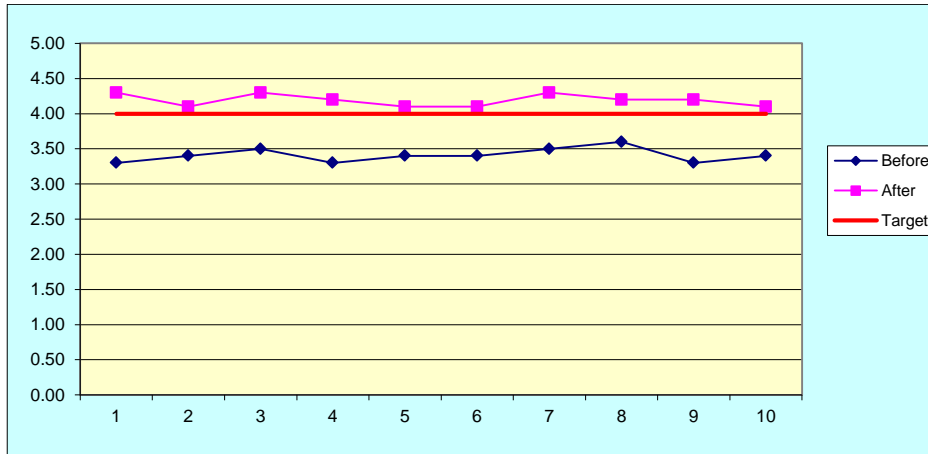


Figure 4.8: Graph of before and after optimization

4.2.4 Quality loss calculation

Table 4.7: Quality Loss Calculation for CCF

	Before	After
Loss	1.00	1.00
Spec	4.00	4.00
k	16.00	16.00
Mixture	1.38	0.93
Total	1.38	0.93
Cost		32.42

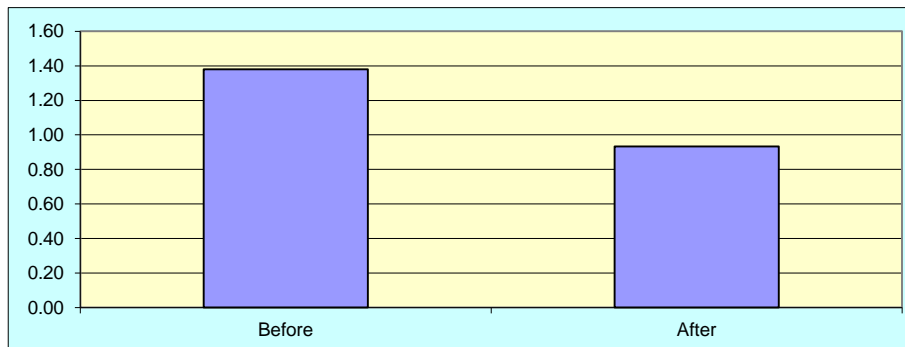


Figure 4.9: Graph which shows the reduction of cost

From the table and graph above, we could notice that the total cost of one paper gyrocopter has been reduced from RM1.38 to RM0.93. In terms of percentage, we have made an improvement of 32.42%. In other words, it also shows that face-centred composite design is not only useful in obtaining the optimum condition of a paper gyrocopter but also to save the overall production cost of a paper gyrocopter.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In conclusion, the main objective of this final year project, which is to apply central composite design on a paper gyrocopter system, has been achieved. This final year project has led us to understand the importance of design of experiment (DOE) in general and central composite design (CCD) in particular.

Basically, this experiment is to determine the optimum factor which will contribute most to the flight time of a paper gyrocopter system. After carrying out the experiment, we are then able to generate the ANOVA table and plot the response surface graph to clearly identify the significant factors involve. Later on, the optimum condition for X_1 , X_2 and X_3 of a paper gyrocopter was also determined. The optimization is able to bring a significant decrease in cost per unit of a paper gyrocopter and an increment in flight time. In other words, CCF is proven useful and effective.

Besides, we have also obtained the knowledge and skill of using Microsoft Excel through this project. Microsoft Excel is no doubt very powerful as it is equipped with numerous mathematical functions which ease our calculation.

Comment [N1]: Reword this part.

5.2 Recommendations

Although we are able to achieve the objective of this research design, there are also room for improvements to further enhance the results and reducing even more cost.

5.2.1 Design parameters

For this experiment, we have set the range for body width (X2) and shoulder length (X3) to be 2cm to 3cm. After running the experiment, the ANOVA table shown in table 4.5 indicates that the factor X2 and X3 are pooled, in other words not significant. Therefore, we may claim that the intervals for these two factors might be too close to show any significant changes and this will certainly cause the overall result of the experiment. Hence, we suggest that the intervals for all factors should not be too close with a difference of more than one.

5.2.2 Amount and types of factors

For this experiment, we only involve three factors, namely wing length, body width and shoulder length. There are certainly many more factors which will affect the flight time of the paper gyrocopter, such as the type of paper, body length and etc. However, we are uncertain as we did not involve all of the other factors. Plus, adding more factors might also help in determining the significant factors involve.

5.2.3 Location of the experimental environment

The location and environment also plays an important role when carrying out the experiment. For our case, the experiment was done in an open air condition within the university compound. Not to mention that there was wind blowing during the

course of our experiment. Therefore, we suggest that the experiment should be done in a confined area where there is no external disturbance so that the data would be more precise and variability of results could be reduced.

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