FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE SLABS SUPPORTED BY DIFFERENT STIFFNESS OF BEAM

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A project report submitted in partial fulfilment of the requirements for the award of Bachelor of Engineering (Honours) Civil Engineering

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DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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APPROVAL FOR SUBMISSION

I certify that this project report entitled **"FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE SLABS SUPORTED BY DIFFERENT STIFFNESS OF BEAMS"** was prepared by **CHAI KHEM FEI** has met the required standard for submission in partial fulfilment of the requirements for the award of Bachelor of Engineering (Honours) Civil Engineering at Universiti Tunku Abdul Rahman.

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ABSTRACT

Reinforced concrete slab is one of the most important structure members that provide spacious platform for occupants to carry out activities. In the pre-computer era, the structural analysis and design of slab were limited and tend to be too conservative. Moreover, the important relationship between the slabs and the supporting beams was ignored for simplicity and due to insufficient study towards the field such as provided in clause 3.5 in design code BS8110. This study will be focusing on the effects of different beam stiffness on the slabs internal loading through modelling in Scia Engineer Software, a structural finite element software. After that, the resulting bending moment and shear force obtained from Scia Engineer of linear analysis will then be compared with the corresponding values obtained based on the bending moment coefficients and shear force coefficients provided in BS8110 (British Standard: Structural use of concrete). The results shows that slab supported by flexible beam will exercise flat slab behaviour. In the case of slab supported by stiff beam, it shows ordinary beam-slab behaviour. For stiff beam supported slab, when the long span to short span ratio is relatively low, it also shows two-way slab behaviour, as the span ratio increase to a certain extent, the slab will show one-way slab behaviour which the bending moment and shear force at long span is very minute as compared to those in short span. BS8110 only adequately estimated the internal loading (namely bending moment and shear force) for slab supported by stiff beam of small l_y/l_x ratio. The bending moment and shear force of slab supported by flexible beam are generally underestimate by BS8110 whereas for slab supported by stiff beam of large l_y/l_x ratio are overestimated by BS8110.

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LIST OF SYMBOLS / ABBREVIATIONS

a	shear span, m
A	stiffness of beam in x direction, mm^3
B	stiffness of beam in y direction, mm^3
C	stiffness of slab in x direction, mm^3
D	stiffness of slab in x direction, mm^3
E	modulus of elasticity, N/mm
I	moment of inertia, mm^4
g_k	characteristic permanent load, kN/m^2
k	stiffness of beam, N/mm
L	length of beam, m
l_x	short span of slab, m
l_y	long span of slab, m
M_0	ratio of M_1 to M_2
M_1	hogging moment obtained from Scia Engineer, kN.m/m
M_2	hogging moment calculated based on BS8110, kN.m/m
M_{sx}	bending moment at short span, kN.m/m
M_{sy}	bending moment at long span, kN.m/m
M_y	bending moment in longitudinal direction of beam, kN.m/m
M_x	bending moment in short span, kN.m/m
M_y	bending moment in long span, kN.m/m
n	total design ultimate load per unit area, kN/m^2
q_k	characteristic variable load, kN/m^2
r	total number of force and moment reaction components
t	thickness of slab, mm
V_{vx}	shear force at short span, kN/m per meter length
V_{vy}	shear force at long span, kN/m per meter length
V_z	shear force in longitudinal direction of beam kN/m
x	distance from origin, m
X	formulated independent variable
α	constant depending on the support condition
β_{sx}	short span bending moment coefficient

β_{sy}	long span bending moment coefficient
β_{vx}	short span bending moment coefficient
β_{vy}	long span bending moment coefficient

ANCOVA	analysis of covariance
BIM	building information modelling
BMD	bending moment diagram
BVP	boundary value problem
FDM	finite difference method
FEA	finite element analysis
FEM	finite element method
LCS	local coordinate system
PDE	partial differential equation
RC	reinforced concrete
SFD	shear force diagram
SPSS	statistical package for the social sciences
UDL	uniformly distributed load

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CHAPTER 1
INTRODUCTION

1.1 General Introduction

The major elements of a structure consist of slab, beam, column, and foundation. Slab is usually flat and horizontal element in a building that provides useful platform for the occupants to carry out activities. Conventionally, the load transfer follows the sequence from slab to beam (exceptional for flat slab), followed by column, and eventually to foundation. The loading on slab is first analysed before the design for thickness of concrete slab and the amount of reinforcement.

Looking into most common type of building, the reinforced concrete buildings, the slabs and beams are poured and cast as continuous members through the joints and over the support. The two key elements in connection between slab and beam consists of:

(i) The joint, which is the volume common to the slab and the supporting element.

(ii) The portion of the slab and beam adjacent to the joint (ACI-ASCE Committee 352, 2004).

This monolithic concrete structure is seamlessly integrated (or connected) to prevent leakage and ensure proper load transfer. In addition to that, due to monolithic casting, a certain width of slab act together with beam and form T-flanged or L-flange beam. Thus, the close relationship between slab and beam should be studied more in detail.

In the pre-computer era, the structural analysis and design of slab tend to be too conservative (Tan, et al., 2015). The methods of analysis are limited, even though some methods have been proposed but it is impractical to solve by hand. Moreover, the important relationship between the slabs and the supporting beams was ignored for simplicity and due to insufficient deep study towards the field such as provided in clause 3.5 in design code BS8110. Few decades ago, the invention of computer has initiated the usage of finite element method. Which this allows the complex stress relationship between the slabs and the supporting beams to be determined through simplifying the complex and continuous differential equation into finite numbers of numeric differential equations.

Since slab is the most concrete-consuming element in reinforced concrete buildings which build up more than 65 % of the building (Buidling and Construction Authority, 2012), therefore appropriate slab analysis and design by considering the effect of beam stiffness on the slab is vital key to optimize material cost, minimize wastage and to produce a safe design.

1.2 Importance of Study

British Standard Structural use of Concrete (Part 1), BS8110-1 provides an easy-to-use guideline for manual slab analysis. The latest amendment of this code was remained on 1997 which was twenty over years from now. Despite it should be replaced by Eurocode 2, EN1992, many reference books yet refer to BS8810 in manual calculation for bending moment and shear force in slab. Generally, in cast in-situ reinforced concrete (or RC for short) structure, concrete slabs and concrete beams are cast simultaneously. Due to monolithic property between RC slabs and beams, RC slabs provide lateral restraint and T-flange mechanism toward RC beams. Thus, in the opposite manner, this study seek to discover the effect of supporting beam stiffness on the bending moment and shear force in slabs which was not mentioned in BS8110.

1.3 Problem Statement

Despite partial fixity exist along the side of slab, it is neglected in the analysis as according to clause 9.3.1.2 in EN 1992-1-1. Meanwhile the relationship between slabs and supporting beams were not explained too in design code BS8110. These simplified analyses assumed that the slabs and beams are acting separately as they are not interconnected which this might not represent the actual behaviour of slab as the stiffness of supporting beam might alter the slab behaviour.

The bending moment and shear force coefficients in BS8110 had been formulated for more than twenty years (since 1997) and update and maintenance were ceased since the withdrawal of British Standard in year 2008 (Chiang, 2014). In BS8110, there are several limitations and criteria must be fulfilled in order to implement the coefficients. Moreover, the coefficients assume that the bending moment in long span is constant over all ranges of long span to short span ratio, as shown in Figure 1.1. This assumption for simplification makes manual calculation easy but in fact it does not represent the actual slab behaviour and is inappropriate for slab design. This issue might be ignored in small slab panel but will probably arise significant consequences in the case of considerably long short span, l_x. Eventually the factors mentioned above might lead to over-design or conditionally under-design of slabs as extra stiffness contributed by the underneath beams are not considered.

Type of panel and moments considered	Short span coefficients, β_{sx}								Long span coefficients, β_{sy} for all values of l_y/l_x
	Values of l_y/l_x								
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Interior panels									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at mid-span	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024

Figure 1.1: Constant Coefficient over Long Span for All Range of l_y/l_x Ratio.

The slab analysis in BS8110 only relies on long span to short span ratio of slab in determining bending moment and shear force. Literally it also means that the slab analysis by BS8110 has ignored the effect of supporting beam width and beam depth. If more parameters is taken into consideration, the more accurate result will be.

Thus, it is necessary to verify the reliability and suggest improvements on the coefficients provided in BS8110 with the current best structural analysis approach in hand, which is finite element analysis.

1.4 Aims and Objectives

The aim is to study the effect of beam stiffness on finite element analysis of slab under different conditions (ratio of long span over short span, and type of panel) with the help of Scia Engineer software. The beam stiffness refers to the width of beam, total depth of beam and length of beam. The objectives are:

(i) To study the effect of beam size on slab behaviour of slabs supported by different stiffness of beam.

(ii) To compare the results between Scia Engineer and BS8110.

(iii) To suggest a complementary empirical equation for future user of BS8110 when preforming slab analysis.

1.5 Scope and Limitation of the Study

The scope of this research is to model slab and flat slab with different conditions in Scia engineering software. The major focuses of this research are internal reactions of shear force and bending moment. The three different conditions refer to:

(i) Different supporting beam dimensions which the width range from 0mm (which means flat slab) to 900 mm; whereas the depth range from 0 mm (which means flat slab) to 900 mm.

(ii) The ratio of long span to short span, *l*y/*lx* which range from 1.0 to 2.25.

(iii) The 9 types of panel condition listed in Appendix B and Appendix C in BS8110.

There are several limitations of study in this paper:

(i) The finite element analysis software is limited to Scia engineering software. All the modelling results were made comparison with Appendix B and Appendix C in BS8110 and no comparison was made with other structural analysis software such as Midas Gen, Esteem, Tekla, Etabs or Lusas.

(ii) Only 11 selected beams sizes were studied to limit the workload on tabulation of results.

(iii) The beam-slab models are supported by a point support, which means the stiffness contribution of column (support) is not studied and was neglected.

(iv) Only slabs with *ly*/*lx* range from 1.0 to 2.25 were studied which means slab with ratio beyond 2.25 were not covered.

(v) The supporting beams at all edges are of the same dimension, which might not reflect the case in real life construction, for example the supporting beams for a piece of rectangular slab might be 150 mm x 300 mm at one side and 250 mm x 750 mm on the 3 other sides as they comprise of primary beam, secondary beam and even tertiary beam.

(vi) Linear analysis is performed, therefore no moment redistribution is allowed.

1.6 Contribution of Study

The outcome of this research served as a reference for further studies of limitations and suggest improvement to the coefficients of bending moment and shear force in BS8110 for manual slab analysis.

1.7 Outline of Report

In Chapter 1 Introduction, a brief general introduction, importance of study, problem statement, aim and objectives, scope and limitation of the study, and contribution of the study are discussed.

In Chapter 2 Literature Review, statically determinacy and kinematic determinacy are discussed. Next, the types of structural analysis approaches are discussed followed by brief review on computer analysis software for structural analysis. The statistical analysis methods involved in this study are presented. After that, the beam classification is discussed. Slab comparison is discussed as well. An overview of solid slab supported by beam and flat slab by BS8110 is made. Briefing on functions used in Scia Engineer was made. Previous researches that relates to this study was discussed.

Chapter 3 Methodology describes the workflow of this study. The procedures on structural analysis and statistical analysis are discussed in this chapter.

In Chapter 4 Results and Discussion, the results of slab supported by different beam size from Scia Engineer are displayed and compared. After that, the results from Scia Engineer is compared to results from BS8110. Next, an empirical formula for bending moment is formulated

Chapter 5 summarized the study with conclusion and recommendations for future study.

CHAPTER 2

LITERATURE REVIEW

2.1 Statically Determinacy

Static determinacy describes the force equilibrium conditions that can be used to determine the magnitude of forces and moments.

Statically indeterminant structure is defined as structure with the number of reactions or internal forces exceeding the number of equilibrium equations available for its analysis. Where total number of force and moment reaction components, *r* is three times greater than total number of member parts, *n* (for 2D member), see Equation 2.1.

$$r > 3n \tag{2.1}$$

where

r = total number of force and moment reaction components

n = total number of member parts

In this case, static equilibrium equations (which includes summation of force in x-direction is equal to zero, summation of force in y-direction is equals to zero, and summation of moment at any point is equal to zero) are no longer sufficient for determining the internal forces and reactions in the structure members. Thus, more complex and reliable method is required to determine the internal loadings.

Most structural members nowadays are partially fixed connected or even fully fix connected (as concrete beams and slabs are cast as continuous members, as the structures have continuous span instead of single simply supported span). Since fixed support incurs more restraints, therefore the extra reactions result in greater static indeterminacy.

There are two main benefits of adopting statically indeterminant structure. Firstly, it gives relatively smaller internal loadings as the internal loadings can be distributed among the redundant or extra supports. Secondly, it allows the redistribution of load that maintains the stability and prevent collapse in case of faulty design or structure overload occurs. This is crucial when sudden lateral load or unforeseen impact such as wind, earthquake or explosion strikes the structure (Hibbeler, 2017).

Adopting indeterminant structure can be a double-edged sword despite it allows smaller supporting members and has higher stability. Thus, the indeterminant structure must carefully analysed and design to prevent differential settlement. This is because a minute differential settlement will result an additional and significant stress built up in the supports. Same goes to thermal changes and fabrication errors.

Both flexibility method (or also known as force method) and stiffness method (or also known as displacement method) can be used to solve structures with high degree of static indeterminacy. Both methods will create a large amount of simultaneous equations to solve the unknowns (which include unknown force or unknown displacement) in the structures. (Megson, 1996). However, this is not a big issue as these simultaneous equations are simple and can be solved by computer easily.

2.1.1 Flexible Method (Force Method)

Force method was first developed in 1864 and was one of the first available method for analysing statically indeterminate structure (Hibbeler, 2017). The primary unknowns in flexibility method are forces. The indeterminacy of structure is first determined, the number of indeterminacies is literally the number of additional equations required for solution. The additional equations are formulated by using the principle of superposition and considering the compatibility of displacement at support. The redundant reactions are temporarily removed so that the structure becomes statically determinate and stable. The magnitude of statically redundant forces required to restore the geometry boundary conditions of the original structure are then calculated. Once these redundant forces are determined, the remaining reactive forces are determined by satisfying the equilibrium requirements. The selection of redundant forces requires engineering judgement, therefore it is not preferable for computer implementation.

2.1.2 Displacement Method (Stiffness method)

The primary unknowns in stiffness method are displacements. The first step of this method is writing force-displacement relationships for the members and then satisfying the equilibrium requirements for the structure. Once the displacements are found, the forces are obtained from compatibility and force-displacement equations. There are several methods categorized under stiffness method such as slope-deflection method, moment distribution method, direct stiffness matrix method and finite element method (Derucher, Putcha, & Kim, 2013).

The stiffness method is preferred over flexibility method due to several reasons. Firstly, the stiffness method follows the same procedure for both statically determinate and indeterminate structure, but flexibility method requires different procedure for different cases. Besides that, when using stiffness method, the unknowns (such as translations and rotation at joints) are automatically chosen once the structural model is defined, unlike analysing by force method which requires judgement of designer on which redundant forces to be temporarily removed to form a statically determinate structure. Recently, the matrix method and finite element method are widely used, as the calculation by solving matrices is easier for computer program.

2.2 Kinematic Determinacy (Degree of Freedom)

Kinematic determinacy describes the material compatibility conditions that can be used to determine the magnitude of deflection (which includes displacement and rotation). Compatibility refers to a condition where the displacement is known. Compatibility is a method used to provide extra equations when solving unknown in an indeterminate structure (Ali, 2015).

A structure is said to be kinematically indeterminate when the number of unknown displacements is greater than the available compatibility equations. In another word, kinematic indeterminacy of a structure is the unconstrained degree of freedom, which is obtained by subtracting the constrained degrees of freedom such as support points from total degrees of freedom of the nodes. In line elements (which is one-dimensional) such as beam, each node possesses three degrees of freedom (which are vertical translation, horizontal translation, and rotation, but the horizontal translation is usually neglected for beam) whereas for plate element (which is two-dimensional) such as slab, each node possesses six degrees of freedom (which include three translations in x-axis, y-axis, z-axis, and three rotations in x-y plane, y-z plane, and x-z plane).

2.3 Structural Analysis Approaches

Structural analysis is an essential part of structural design. Structural analysis is made up by various mechanics theories that obey physical laws. It allows the designer to predict the behaviour of the structures (such as support reactions, stresses and deformations) without relying on direct testing (Chang, 2013). Which this ensure the performance and soundness of the structure designed.

The structural analysis approaches can be classified into analytical method and numerical method. The selection of approaches depends on the intended use of the structural member, whether it is solving a simple elastostatic problem or detailed design of a critical member in a megastructure. The reliability increases as more and more uncertainties taken into consideration (but this will require more complex theories and longer calculation time).

Analytical method employs simple linear elastic model that leads to closed-form solution which is solvable by hand. On the other hand, numerical method makes use of numerical algorithm in solving differential equations based on mechanic theories. The tedious but more accurate numerical method can often solve by computer. Adequate understanding of analytical method and underlying theories of mechanics are important to verify the numerical results obtained from computer software. Regardless of approach, both methods are formulated based on three same fundamental relations, which are equilibrium, constitutive and compatibility (Chang, 2013).

Performing an accurate analysis requires important information such as structural loads, material properties, geometry and support conditions. Advance structural analysis may require more information for example dynamic response and nonlinear behaviour.

2.3.1 Analytical Method

Analytical methods make use of simple linear elastic model that leads to closed-form solution. The analysis is based on infinitesimal elasticity which assumed that:

(i) The material behaves elastically, and the stress is linearly proportional to the strain.

(ii) All deformations are small. These assumptions for simplicity distort the model from reality and thus reduce the reliability of the model.

Despite the limitations mentioned above, analytical solution has help in verification for numerical solutions of complex structures. Analytical method account several aspects into consideration such as strength of materials, energy method, and linear elasticity.

2.3.2 Numerical Method

Numerical method applies theories of mechanics (such as mechanics of materials and continuum mechanics) based on specific conditions and is actually a technique to approximate solution (somewhere close enough but not the 'exact' solution) for partial differential equations (or PDE for short) of the governing equation (Abdusamad A. Salih). Example of numerical method includes finite difference method (or FDM for short), finite element method (or FEM for short) (Muspratt, 1978).

2.3.2.1 Finite Difference Method

Finite difference method is a less complex approach to boundary value problem (or BVP for short). However due to its simplicity, it is difficult to be used to solve problems with irregular boundaries and to write general purpose code for Finite Different Method.

2.3.2.2 Finite Element Method

There are many engineering problems that cannot be solved analytical, which means it is tough and tedious process to obtain the exact solution for a problem, or sometimes even impossible to do so due to complex geometry, material properties and loading. Finite element analysis can be a reasonable solution to these problems. Finite element analysis uses numerical method to approximate the solution, where the answer obtained is close to but not the exact solution. This means that FEM will give an answer with error to certain degree (due to rounding error, truncation error, assumption made). The degree of error depends on several factors which include, type of numerical method adopted, assumption made, number of iterations and more. Thus, it is important that the user has to make judicious choices on selecting an appropriate method to analyse (and also to avoid divergence of result) and perform the calculation with sufficient amount of iteration to obtain a result with desire accuracy (Strang, 2013).

FEM is computational technique used to solve the BVP. Boundary value problems in physical structure can easily be imagined. Taking a simply supported beam for example, $y(x)$ is deflection function in term of x, distance from the origin. There are no deflection at both end of supports. Thus, the boundary conditions for this case is $y(0) = 0$ and $y(L) = 0$, which L is the total length of beam considered.

Some common mathematical approaches used in FEM problem are:

(i) Direct approach.

(ii) Variational approach.

(iii) Weighted residual method (which includes Galerkin method).

These approaches give a close approximation but only if the domain is small, thus this is the reason why discretization of elements is needed.

The steps of FEM are as shown below:

(i) In finite element analysis (or FEA in short), the complex and whole structure is reduced (or also known as sub-divided or discretised) into simpler elements which is described by variables at a finite number of points (which in term of set of algebraic equations).

(ii) Select an approximating shape function such as polynomial function to represent the physical behaviour of the variables (such as translation and rotation) in element.

(iii) Form element equations (which is also known as local stiffness matrix).

(iv) Assemble the element equations to form a global matrix of the whole system.

(v) Apply system constraints such as apply boundary conditions (such as known external forces, known translation and known rotation).

(vi) Solve the primary unknowns (which include support reactions, translation and rotation).

(vii) Solve the derived unknowns (internal loadings).

Nowadays, computer can work with hundreds and thousands of functions. Hence, with the help of computer, hundreds of functions, but just simple functions are needed, and their combination can lead the solution close to the correct answer. This is an important approach to make the differential equation discrete finite solvable by computer (Fish & Belytschko, 2007).

2.3.3 Slab Analysis

The Civil and Structural Engineering technical Division of The Institution of Engineer Malaysia had conducted an in-depth study of EN 1992-1-1, BS8110 and other concrete codes of practice for United States of America, New Zealand and etcetera. As BS8110 has been withdrawn since 2008 and no further maintenance, in the form of updates and amendments will be made, the committees recommend that EN 1992 should be adopted as the concrete code of practice for the local construction industry after year 2008. Up to 2012, the transition period still allows the co-existence of EN1992 and BS8110 for all states in Malaysia except for Selangor and Terengganu (Chiang, 2014).

In EN 1992, section 5.1.1 clause (6) stipulates the common idealisations of the behaviour used for analysis are:

(i) Linear elastic behaviour.

(ii) Linear elastic behaviour with limited redistribution.

(iii) Plastic behaviour.

(iv) Non-linear elastic behaviour.

There are numerous analysis methods for reinforced concrete slab design. Each method has advantage over others under different conditions which include numerical method, yield line method (which is useful for slab with complex geometry), Hillerborg strip method (which useful for slab with opening) and coefficient from design code (for example shear force and moment coefficient in BS8110).

As EN1992 section 5.1.1 clause (6) only outlined the major theories but does not specified the analysis method, the bending moment and shear force coefficient in BS8110 is still commonly adopted for manual slab analysis under the transition period. Many reference books also refer to BS8810 in manual calculation for bending moment and shear force in slab.

In EN1992, section 6.1 shows that linear strain is considered, which in another word, the design of slab is based on linear elastic theory. However, the coefficients of bending moment and shear force in BS8110 are based on inelastic analysis. Thus, there are inconsistencies between the methods of analysis and design.

In reinforced concrete slab, the limitations in elastic analysis include:

(i) Slab panels should be rectangular.
(ii) One-way slab panels must be only supported along two opposite sides (which means the other two sides remain unsupported or forced one-way slab).
(iii) Two-way slab panels must be supported along two pairs of opposite sides.
(iv) All the supports remain unyielded.
(v) Applied load must be uniformly distributed.
(vi) No large opening is allowed on slab panels.

As elastic analysis has very strict limitations, it is less favourable in slab analysis as compared to inelastic analysis such as yield line analysis.

2.3.3.1 Yield Line Method

Yield line method is a popular and reliable method especially for analysing slabs with complex shape (such as triangular or circular slabs), complex load distribution, and with different arrangement of support condition (for example 3 sides supported with 1 free edge for a rectangular slab).

Inelastic analysis of yield line theory is based on formation of plastic hinges to form a collapse mechanism. As slabs are mostly under reinforced, this property provides slabs a large rotation capacity.

A yield line is a crack on reinforced concrete slab which the reinforcing steel bars have yielded (which means plastic hinges are formed) and act as the axis of rotation for the slab segment. When a slab is loaded to failure, yield lines are formed in the area that is highly stress (Kennedy & Goodchild, 2004). Thus, yield lines are lines of maximum yielding moments of the reinforcement in slabs.

The yield line analysis comprises of two steps:

(i) Assume possible yield patterns and locate the axis of rotation.
(ii) Determine the locations of axes of rotation and collapse load for the slab.

The first step in yield line method is to identify the valid yield line pattern. Thus, some of the important concepts must be outlined. The yield line for sagging moment is denoted as positive yield line whereas for hogging moment is negative yield line. There are several rules of yield line pattern presented by (Kennedy & Goodchild, 2004) and (Mustafa & Bilal, 2015):

(i) Yield lines are straight lines as they represent the intersection of two (or more) planes.
(ii) Yield lines represents axis of rotation.
(iii) The supporting edges of slab also serve as axes of rotation.
(iv) Yield lines end either at a slab boundary or at another yield line.
(v) An axis of rotation will pass over any column
(vi) Yield lines form under concentrated load will radiating outward from the point of application.
(vii) A yield line between two segments must pass through the point of intersection of the axes of rotation.

Figures 2.1 and 2.2 show possible yield line patterns based on the assumptions above.

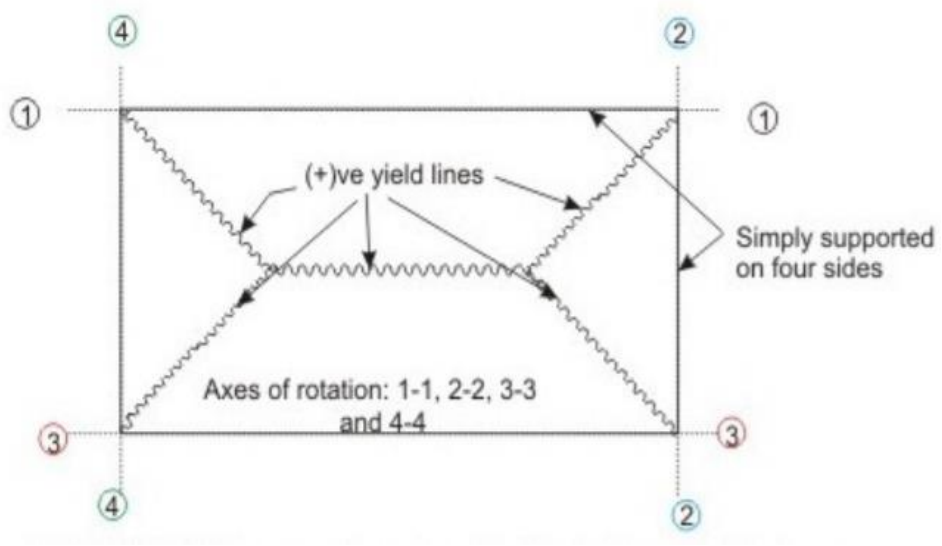

Figure 2.1: Two-way Rectangular Slab with Simply Supported Edges (Mustafa & Bilal, 2015).

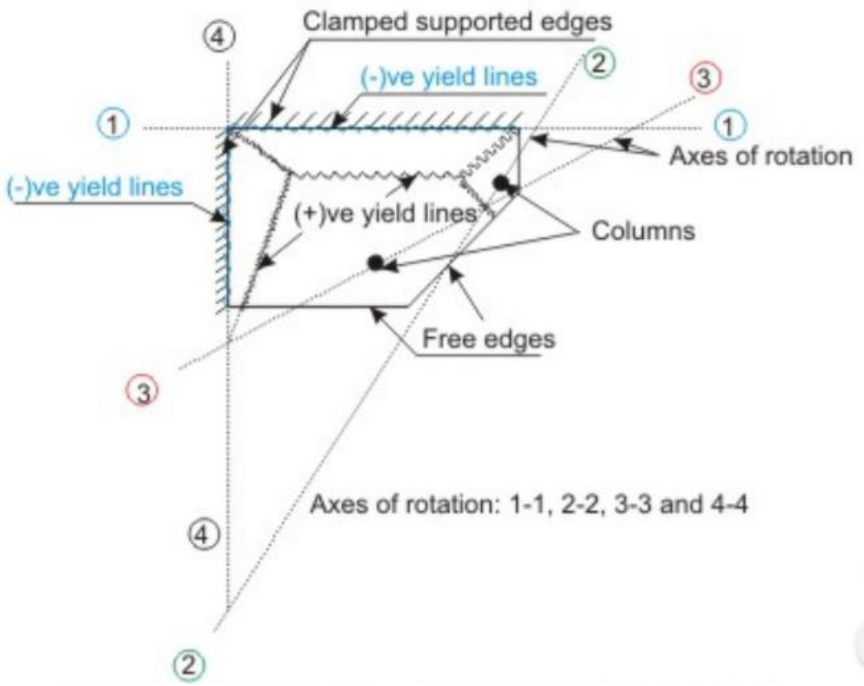

Figure 2.2: Slab with Two Supported Edges and Two Columns (Mustafa & Bilal, 2015).

After determining the yield line patterns, the method of segmental equilibrium or virtual work method is then applied. Both method forms equations to determine:

(i) Location of maximum bending moment (which is equivalent to location of yield line.

(ii) Allowable load on slab.

In order to solve these equations, the input parameters required are:

(i) Factored moment capacity of RC slabs at the yield lines.

(ii) Span length of slab.

This method of analysis is an upper bound approach where the true or actual collapse load will never be higher but only equal or lower than the load predicted (Mustafa & Bilal, 2015).

In short, yield line method search the location and magnitude of maximum moment of slab.

2.4 Computer Analysis Software

In computer analysis software for structural analysis, the analysis of complex problems essentially involves the three procedures: selection of appropriate mathematical model, execute analysis of the model, and interpretation of the results generated. For the past decade, finite element method implemented on computer has been successfully used in modelling very complicated problems in various fields which allows safer and economical design. However, finite element method is reliable only if the fundamental assumptions of the procedures implemented are well studied and thereby users can execute with computer confidently.

2.5 Statistical Analysis

In this study, two statistical tools are involved, which are covariance analysis and liner regression.

2.5.1 Analysis of covariance (ANCOVA)

Analysis of covariance (ANCOVA) is a statistical method that compare sets of data that comprised two or more variables (Gad, 2010). Covariance analysis serves the function to find the definite effect of the 'independent variables' on the 'dependent variable'.

The word 'independent variable' is interchangeable with 'covariate' and the 'dependent variable' is interchangeable with the word 'variate'. In covariance analysis, variate is correlated with the independent variable which in another word, the dependent variable is adjusted due to the effects of covariate has on it. The output of covariance analysis is independent variable with high correlation.

One of the restrictive assumptions for covariance analysis is that the relationship between the covariate and variate is assumed to be linear. Thus, the variate should linearly proportional to the covariate.

2.5.2 Linear Regression

Linear regression is a model that represent the relationship between two variables by fitting them into a linear equation ($y = mx+c$). Linear regression consists of two variables, which are independent variable (y), and dependent variable (x).

One should first determine whether or not there is a relationship between the variables of interest (independent variable and dependent variable) before trying to fit a linear model to the observed data. Which in this study, covariance analysis is performed in seeking covariate that has significant effects on the dependent variable before applying the linear regression. The output of linear regression is empirical formula.

2.5.3 Statistical Package for the Social Sciences

Statistical Package for the Social Sciences (or SPSS in short) is a statistical software design to solve business and research problems. This software was use in this study in statistical analysis as it is a free-to-use software that include covariance analysis and linear regression model.

2.6 Beam Classification

Construction industry is a multidisciplinary field which the civil engineers always cooperate with other professionals. As for most cases, structural engineers are asked to design based on the architectural drawing. There will be a case where the depth of beam will be limited by architects in order to provide sufficient head. The beam will behave like shallow beam if it's depth is reduced greatly to a level which the beam is no longer rigid enough to provide support to the above slab.

Thus, shallow beam is one of the structural elements that should be paid attention in some specific conditions when a normal depth beam is not allowed. However, no much provision was made in EN1992 and BS8110, the two commonly used code of design in Malaysia. According to ACI 318-95 (another code of design), a beam with a shear span (a) to depth (D) ratio equal or greater than 2.5 or, length to depth ratio, L/D less than 6 as a shallow beam.

Shallow beam can be analysed by simple bending theory which generally assume that the plane section remain plane after bending. In shallow beam analysis, linear stress distribution assumption is made as well. Shallow beam usually only allow in resisting longitudinal bending and shear as it is assumed as one-dimensional linear element.

Moderate beam range from $1.0 < a/D < 2.5$ or $2.0 < L/D < 6.0$

Deep beam range from $a/D < 1.0$ or $0.5 < L/D < 2.0$

This study covers the modelling of slab supported by shallow beam and moderate deep beam only.

2.7 Slab Comparison

There are numerous slab types in practice, including conventional solid slab supported by beam (also known as beam-slab system), flat slab, ribbed slab, composite slab, hollow-core slab (Designing Buildings Ltd, 2019). Each type of slabs come along with respective benefits and disadvantages in term of material cost, clear head, construction speed and flexibility in design.

The code of design, BS8110 had outlined the design of conventional solid slab and flat slab. In beam-slab system, the loading on slab is transmitted to the supporting beams at the edge, and then to column. Whereas for flat slab, the beams are absence, therefore the loading on slab is transmitted to the column directly (which caused flat slab is usually thicker than conventional solid slab in order to meet requirement for both ultimate limit state and serviceability limit state).

This subsection will discuss the concept of two-way slab and one-way slab. When a rectangular slab is supported by beams on all 4 edges, the simplified load distribution will typically be divided by yield lines and follow the path as shown in Figure 2.3. Figure 2.3 compares the load distribution and deflection four cases which:

(i) Case 1 is slab with long span (represented by l_y) to short span (represented by l_x) ratio, l_y/l_x equals to 1.

(ii) Case 2 is slab with l_y/l_x ratio in between of 1 and 2.

(iii) Case 3 is slab with l_y/l_x ratio greater than 2.

(iv) Case 2 is slab with l_y/l_x ratio greater than 2 and with simplified loading.

The comparisons are made in term of per unit length (say per meter run) located at the centre of short span and long span. The main reason is because for symmetric slab, the maximum bending is located at the middle of both spans.

(i) Case 1 (l_y/l_x = 1):

- Area load: In the case of relatively square slab, the yield lines pass through the diagonal and therefore the area load is distributed among the slab in the triangular manner. The load is distributed evenly among both long span and short span (as there are no difference between short span and long span).
- Line load: This results a M-shape line load on the slab in both ways.
- Deflection: In this Case 1, both spans carry the equal portion of loading. Thus, the deflection profiles are of the same at both spans and shows a parabolic deflection.

- Remark on load distribution by percentage: Both spans support equal portion of load.

- Short span: 50 %
- Long span: 50 %

(ii) Case 2 ($1 < l_y/l_x < 2$):

- Area load: When one of the span increases (say long span) and the short span remain (as same length as short span in Case 1), the area load is distributed as shown in Case 2.
  - Short span: Increase in long span increase the area of the slab, and the additional area load is taken by short span. The total area load distributed to the short span changed and increase from triangular to trapezoidal area load distribution as shown in Case 2.
  - Long span: The loading distributed to the long span of the slab remain unchanged which support the same triangular load at the near (edge) support part as in long span of Case 1, and merely zero loading at the middle.
- Line load:
  - Short span: The line load at near (edge) support is of M-shape and when moving towards the centre, the magnitude of line load change to uniformly distributed load (or UDL in short) gradually. The line load distribution is so called 'hourglass-shape'.
  - Long span: Based on the area load distribution, the resulting line load is shown in Figure 2.3. Which the load decrease when moving from the edge towards the middle, and merely zero loading at the middle.

- Deflection:
  - Short span: Parabolic deflection.

- Long span: Parabolic deflection at near edge and constant deflection along the middle of the span.

- Remark on load distribution by percentage: As the long span over short span ratio, l_y/l_x increase, the portion taken by short span increases accordingly whereas the portion taken by short span decreases.
  - Short span: say 33 %
  - Long span: say 67 %

(iii) Case 3 and Case 4 ($l_y/l_x > 2$):

- Area load: In Case 3, as the long span over short span ratio further increased.
  - Short span: Larger trapezoidal area load.
  - Long span: The area load taken by the long span remain as triangular load even though l_y/l_x ratio increase.
- Line load:
  - Short span: Same UDL as in Case 2
  - Long span: ‘Hourglass-shape’ but with longer ‘necking’ at middle.
- Deflection:
  - Short span: Same as Case 2.
  - Long span: Same as Case 2.
- Remark on load distribution by percentage:
  - The overall area load increase but the line load per unit width remain unchanged, as the line load is considered in per unit width manner.
  - As the ratio is increased up to certain extend, the contribution of long span in supporting the loading is diminishing and can be ignored for simplified calculation. Which this the case of one-way where the load is assumed to be

primarily distributed to only short span (one way) for simplification.

In a nutshell, for a rectangular slab supported in 4 edges, when the l_y/l_x ratio is less than 2, the load is supported in both directions, leading bending in both directions. When the l_y/l_x ratio increase, the load carry behaviour of slab shift from two-way slab to one-way slab. Which means more and more load is carried by the short span of the slab, therefore mainly causing bending in one direction.

Also, if the support in two parallel edges are absent, even for $l_y/l_x < 2$, the slab will be forced to distribute the load in one-way slab manner.

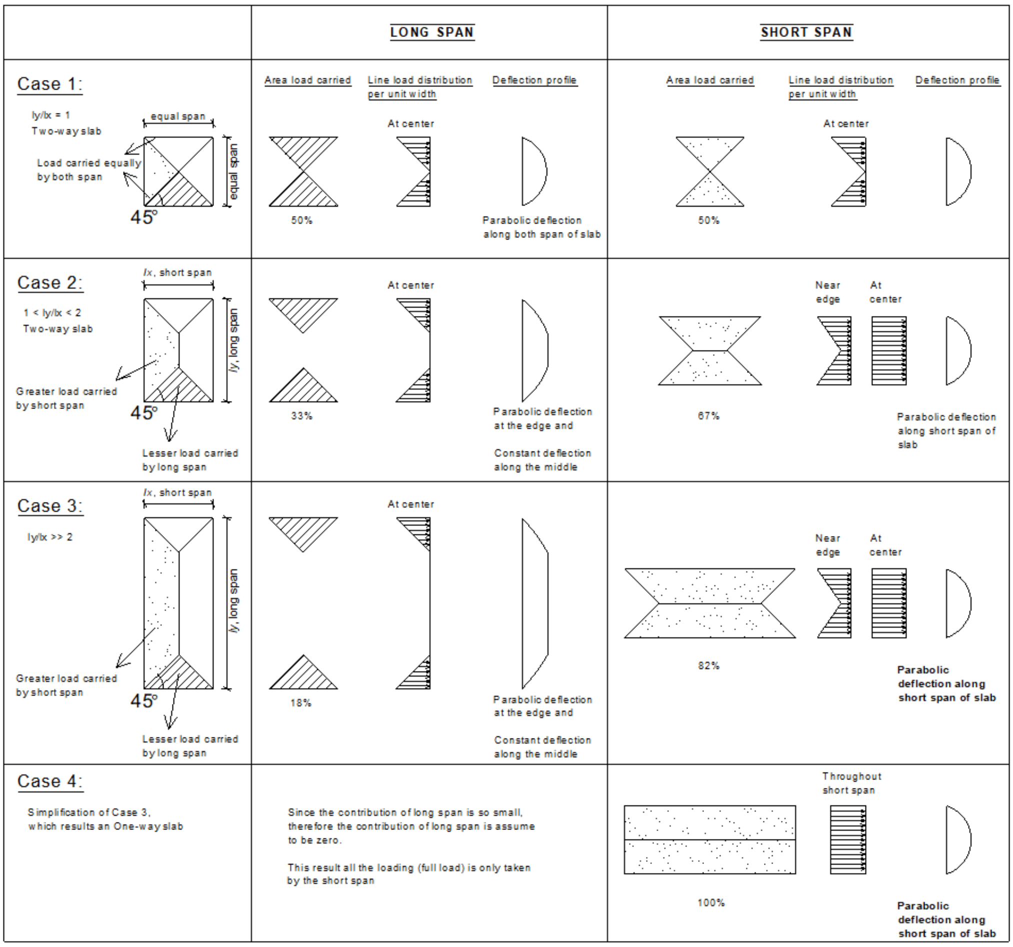

Figure 2.3: Deflection of Two-way Slab and One-way Slab.

2.8 Overview of Solid Slab Design by BS 8110

The solid slab design guidance is outlined in clause 3.5 BS8110. Elastic analysis, Yield line method and Strip method are all recommended by BS8110 (clause 3.5.2.1) in determining the bending moment and shear force. In addition to that, this code of design has established a simplified and easy-to-use formulation for determining bending moment and shear force in solid slab.

2.8.1 Restrained Slabs

According to BS8110, restrained slab is defined as slab where the corners were prevented from lifting and adequate provision was made for torsion.

Technically, a slab should be designed to resist the most unfavourable arrangements of design loads. However, a slab will meet this requirement if it is designed to withstand the moments and forces imposed by single load case of maximum design load if the following conditions are met:

(i) The characteristic variable load (q_k) does not exceed 1.25 times of characteristic permanent load (g_k) (${q_k}/{g_k} \leq 1.25$).

(ii) The characteristic variable load does not exceed 5kN/m^2 ($q_k \leq$ 5kN/m^2).

The criteria above aims to limit the live load. For two-way continuous slab at right angles that support uniformly distributed load, the Equations 2.2 and 2.3 below can be used to determine the bending moment. Equation 2.2 calculates the bending moment at short span. Equation 2.3 calculates the bending moment at long span.

$$M_{sx} = \beta_{sx} n {l_x}^2 \tag{2.2}$$

$$M_{sy} = \beta_{sy} n {l_x}^2 \tag{2.3}$$

where

M_{sx} = bending moment at short span, kN.m per meter length

M_{sy} = bending moment at long span, kN.m per meter length

β_{sx} = short span bending moment coefficient, unitless

β_{sy} = long span bending moment coefficient, unitless

n = total design ultimate load per unit area, kN/m^2

l_x = length of short span, m

*Noted that β_{sx} *and* β_{sy} are derived from a series of formula (see Appendix A) The values for β_{sx} *and* β_{sy} are attached in Appendix B.

2.8.1.1 Restrained Slabs Where the Corners are Prevented from Lifting and Adequate Provision is Made for Torsion

According to clause 3.5.3.5 in BS8110, for continuous slabs, the following two criteria (or limitations) should be met in order to apply the two equations above (Equations 2.2 and 2.3):

(i) The characteristic permanent and variable loads (g_k and q_k) on adjacent panels should not differ much from the panel considered.

(ii) The span of adjacent panels is approximately the same as the span of the slab considered in that direction.

For restrained slabs (both continuous or discontinuous), there are several rules should be complied when applying the Equations 2.2 and 2.3:

(i) Slabs are virtually divided in each direction (x and y) into one middle strip and two edge strips, where middle strip is ¾ of the width and edge strips are 1/8 of the width as shown in Figure 2.4 below. Figure 2.4 shows the division of slab into middle strip and edge strip.

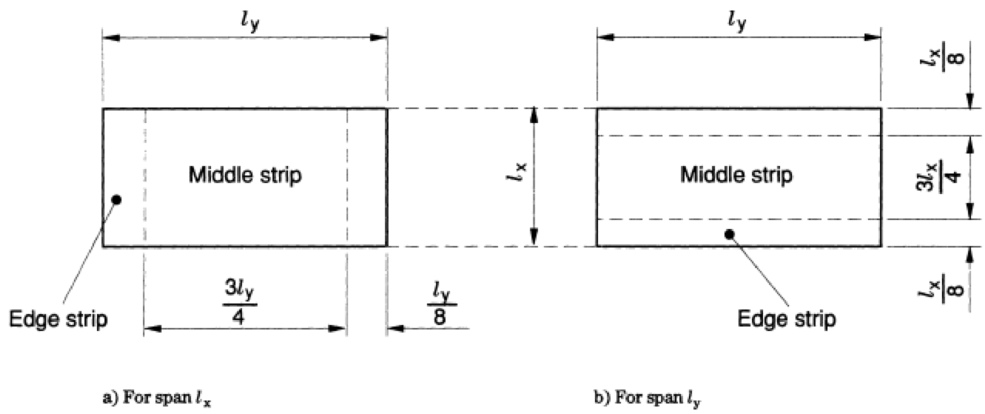

Figure 2.4: Division of Slab into Middle and Edge Strips (Sector Board for Building and Civil Engineering).

(ii) The bending moments calculated based on Equations 2.2 and 2.3 are the maximum hogging moment at the ends or maximum sagging moment at the middle. Besides that, no redistribution should be made when applying the Equations 2.2 and 2.3.

2.8.2 Loading on Supporting Beams

According to clause 3.5.3.7 in BS8110, the following equations may be used to calculate design load on beams supporting solid slabs spanning in two direction at right angles and carrying uniformly distributed load. Equation 2.4 calculates the shear force at short span. Equation 2.5 calculates the shear force at long span.

$$V_{vx} = \beta_{vx} n l_x \quad (2.4)$$

$$V_{vy} = \beta_{vy} n l_x \quad (2.5)$$

where

V_{vx} = Shear force at short span, kN/m per meter length

V_{vy} = Shear force at long span, kN/m per meter length

β_{vx} = short span bending moment coefficient, unitless

β_{vy} = long span bending moment coefficient, unitless

*The values for β_{vx} and β_{vy} are shown in Appendix C.

2.9 Overview of Flat Slab Design by BS8110

Table 2.1 shows bending moment and shear force in flat slab before distribution among middle strip and column strip. Clause 3.7.2.7 in BS8110 suggested that Table 2.1 may be referred as manual calculation for slab moments with the following provisions:

(i) Flat slab is designed to withstand single load case of maximum design load.

(ii) The slabs are continuous and made up of at least 3 panels with the similar span length in the direction being considered.

Table 2.1: Bending Moment and Shear Force in Flat Slab (Before Distribution among Middle Strip and Column Strip) (Sector Board for Building and Civil Engineering).

	End support/slab connection				At first interior support	Middle interior spans	Interior supports
	Simple		Continuous				
	At outer support	Near middle of end span	At outer support	Near middle of end span			
Moment	0	$0.086Fl$	$-0.04Fl$	$0.075Fl$	$-0.086Fl$	$0.063Fl$	$-0.063Fl$
Shear	$0.4F$	—	$0.46F$	—	$0.6F$	—	$0.5F$

NOTE F is the total design ultimate load ($1.4G_k + 1.6Q_k$);
l is the effective span.

Clause 3.7.2.10 in BS8110 stipulates that the moment obtained from Table 2.1 should be divided (or distributed) among the column strip and middle strip with the proportion in Table 2.2, which Table 2.2 shows the distribution of design moment in panel of flat slab. Figure 2.5 shows the length of column strip and middle strip for flat slab.

Table 2.2: Distribution of Design Moments in Panels of Flat Slabs (Sector Board for Building and Civil Engineering).

Design moment	Apportionment between column and middle strip expressed as percentages of the total negative or positive design moment	
	Column strip %	Middle strip %
Negative	75	25
Positive	55	45

A remark can be drawn based on Table 2.2 is that BS8110 assumed that the majority of the bending moment are taken by the column strip which is 75 % for hogging moment (or also known as negative moment) and 55 % for sagging moment (or also kwon as positive moment). Besides that, it shows that in the middle strip, the hogging moment is much lower than the sagging moment.

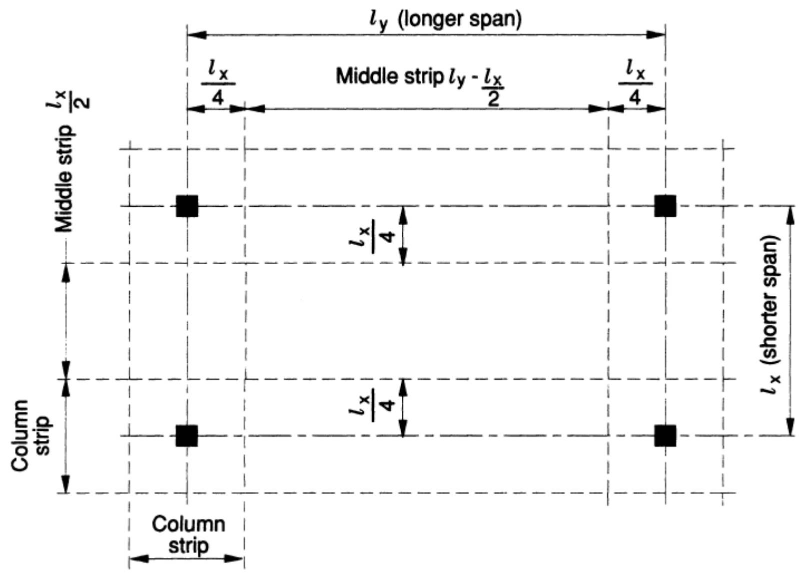

Figure 2.5: Division of Panels in Flat Slab (without Drop Panel) (Sector Board for Building and Civil Engineering).

2.10 Scia Engineer Software

Scia Engineer software is a product of Nemetschek Group. It is an open Building Information Modelling (BIM) software for analysis, code-design and optimisation of structures. Scia Engineer is a comprehensive and robust tool that helps structural engineers and designers to model, analyse and drawing of steel, concrete, timber, aluminium and composite structure (Apptech Group, 2013).

2.10.1 Plate Element in Scia Engineer

In Scia Engineer, a standard plate is a planar 2D member with an arbitrary number of edges which either straight or curved. Concrete slab is modelled as plate element in Scia Engineer. Figure 2.6 shows the input parameters for plate element in Scia Engineer. There are several input parameters when creating a plate element in Scia Engineer (Scia Engineer, 2017):

(i) Name
(ii) Type: the slabs are modelled as plate.
(iii) Material.
(iv) Concrete with the characteristic strength of 25 N/mm^2 is selected and applied for all to ensure the all the slabs are of the same Modulus of Elasticity, E.

(v) FEM model: isotropic model is checked so that the slab has identical properties in all direction.

(vi) Thickness.

(vii) Location of member system plane: mid-surface, top-surface or bottom-surface.

(viii) Top surface is selected as it can most represent the actual case.

(ix) Eccentricity.

(x) Local Coordinate System (LCS) type.

(xi) Local Coordinate System (LCS) axis.

(xii) Local Coordinate System (LCS) angle.

(xiii) Layer (for better selection. When the layer is activated, the elements in that layer will be visible; on the opposite, if the layer is inactivated, all the elements in that layer will be hide from view).

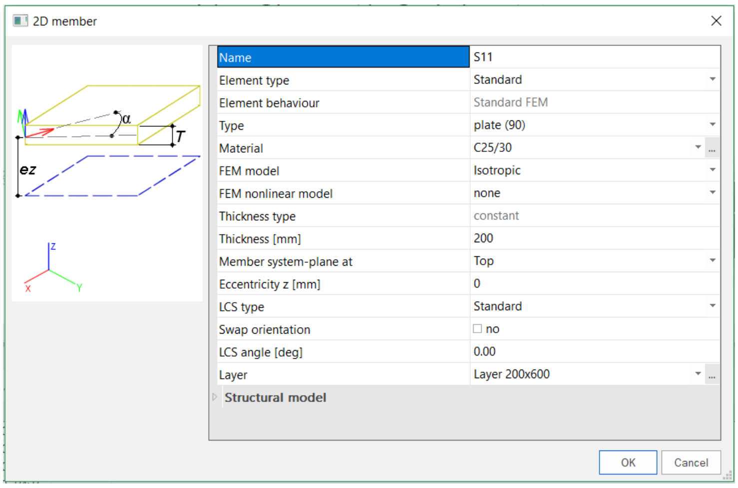

Figure 2.6: Input Parameters for Plate Element in Scia Engineer.

2.10.2 Plate Rib in Scia Engineer

Plate rib is a function in Scia Engineer that connects the internal beam to slab. Figure 2.7 shows the 3D deformation of slab without plate rib. Figure 2.8 shows the 3D deformation of slab with plate rib.

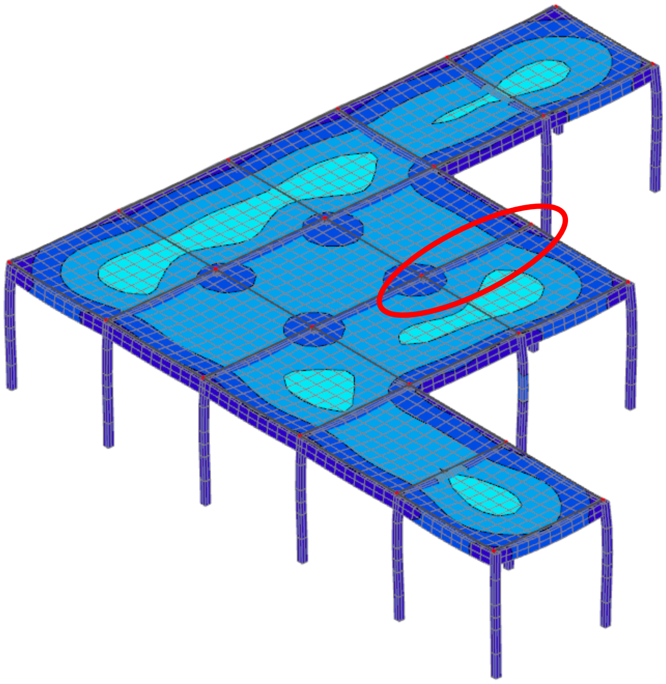

Figure 2.7: Result of 3D Deformation without Plate Rib.

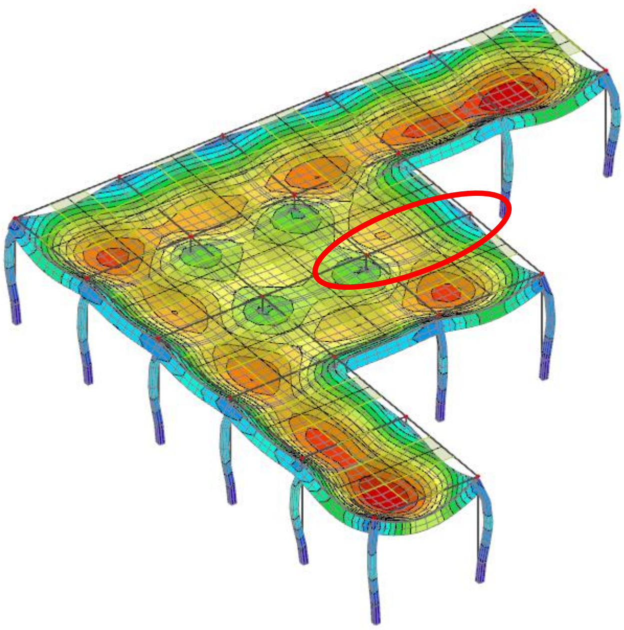

Figure 2.8: Result of 3D Deformation with Plate Rib.

Comparing the beam at same location (circled in red) but one without plate rib (as shown in Figure 2.7), and another with plate rib (as shown in Figure 2.8).

Figure 2.7 shows that the deformation of beam was of dark blue colour whereas the deformations of surrounding slabs were in light blue colour. Literally, the beam deflects more than the surrounding slabs which indicates that the beam and slab deform separately.

In Figure 2.8, the deformation of both beam and surrounding slabs were in same colour which means the beam and adjacent slabs deform with same magnitude. As a nutshell, concrete beams and slabs are usually cast as continuous member, thus modelling with plate rib as illustrated in Figure 2.8 is recommended for structural modelling in this study.

2.10.3 Mesh Size in Scia Engineer

Mesh size is also known as the finite element size. The recommended mesh size by Scia Engineer Help is 1 to 2 times of the slab thickness (Scia Engineer, Results on 2D member - What is the influence of the option 'location'?, 2019). The smaller the mesh size indicate that the elements are discretized into smaller piece for analysis. This results a smoother shear force diagram (or SFD in short) and bending moment diagram (or BMD in short) and also more precise results. However, smaller mesh size increases the computation load which lead to extra computation time. Thus, it is fair enough to perform analysis with an optimum mesh size that give results to desire accuracy and reasonable computation time.

Figure 2.9 shows the slabs with same 150 mm thickness but analysis with different mesh size. The mesh size ranges from 0.5 to 2 times of the slab thickness (150 mm) gave rather similar results which verify the recommendation by Scia Engineer Help (1-2 times of slab thickness). Based on comparison above, it is fair to say that the optimum mesh size for modelling is 150 mm (which is equal to the slab thickness).

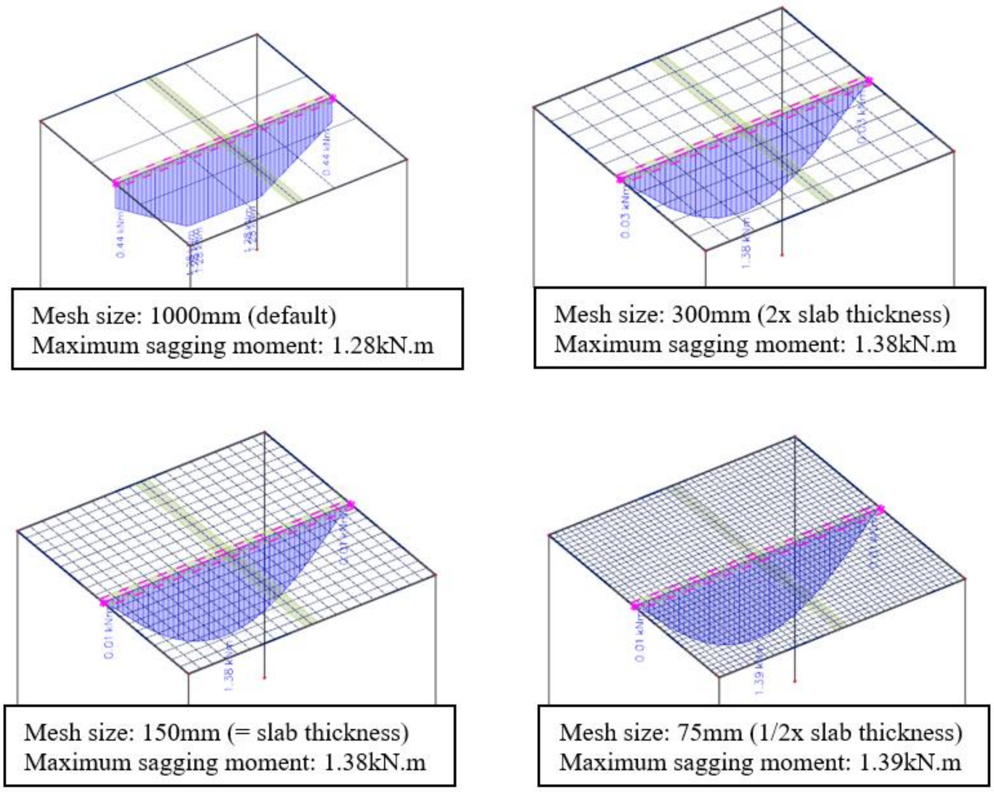

Figure 2.9: Models and Results with Different Mesh Size.

2.10.4 Integration Strip in Scia Engineer

Figure 2.10 shows result on slab without integration strip. Figure 2.11 shows results on slab with different width of integration strip. The default slab output result of Scia Engineer are in term of stress (‘kN per meter run’ and ‘kN.m per meter run’) as shown in Figure 2.10, instead of in the unit of ‘kN’ for forces and ‘kN.m’ for bending moment. A helpful function called ‘Integration Strip’ was include in newer edition of Scia Engineer, which is included from Scia Engineer 17 onwards. Integration strip is helpful in viewing the results on 2D members (for example slabs and walls). It allows user to view the results on slabs as on beam member, display results in a wall like on a column which mean the results in specific width of the element.

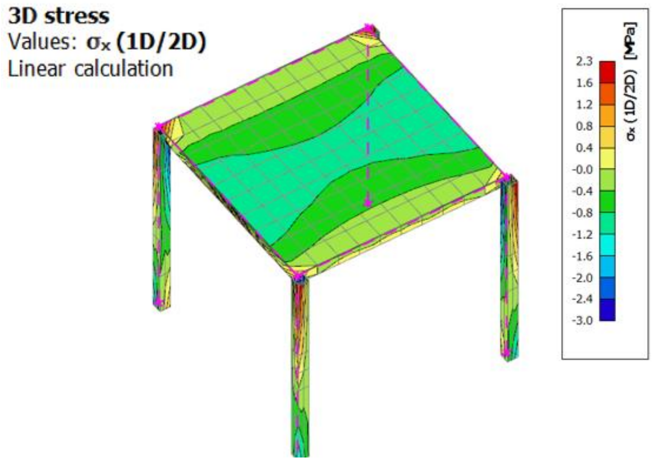

Figure 2.10: Result on Slab (without Integration Strip) in the Form of Stress.

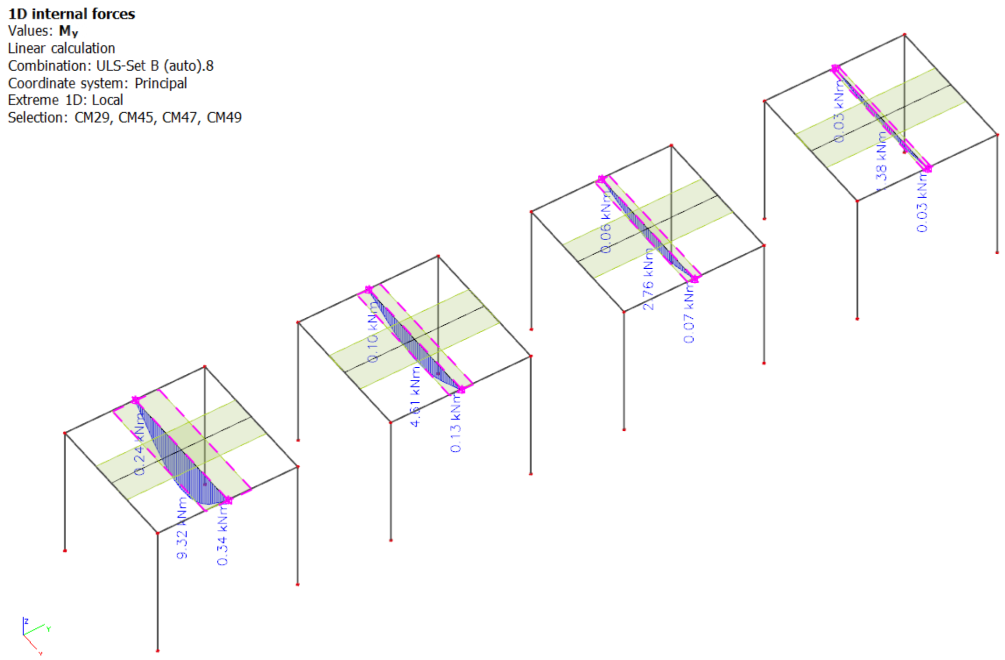

Figure 2.11: Result on Slab (with Different Width of Integration Strip).

In Figure 2.11, four integration strips of the different width are inserted on slabs of same dimensions and loading. The width of the integration strip is reduced by half from left to right, and so as the magnitude of bending moment (9.32, 4.61, 2.76 and 1.38 kN.m/m). In another word, integration strip is a powerful modelling tool in Scia Engineer which sum up the all the internal loading within the strip width that will help in data extraction for this study. Thus, it is necessary to insert an integration strip of reasonable width, else the internal loading will be under-estimated or over-estimated.

Conventionally, integration strips are input at the centre of the slab in both directions (x and y directions) as for symmetric simple supported slab, the maximum sagging moment will locate around the centre. However, for a non-symmetric continuous slab, the bending moment diagram will skew to certain side. Thus, the width and integration strip should be adjusted accordingly for different situations. In this study, the integration strip of 1-meter width is adopted for 2 reasons:

(i) Firstly, the slab is usually designed in per meter run.

(ii) Secondly, the span of slabs that will be model in this study range from 3 m to 6.75 m. Thus, it is reasonable to insert integration strip of 1m which will be able to cover 15 % (1 over 6.75) to 33 % (1 over 3) of the slab width.

2.11 Previous Research

Several researches had compared different method of slab analysis and software. Besides that, several researches had done in seeking the relationship between RC slab and beam.

2.11.1 Modelling Slab Contribution

This final year research paper seeks to find the contribution of beam stiffness to the slab in supporting the internal loading, and previously in 1992, a research was done in studying the relationship between slabs and beams in the inverse manner with the title of '*Modelling Slab Contribution in Frame Connection'*.

Shahrooz, B. M., Pantazopoulou, S. J., and Chern, S. P. (1992) had conduct a research in studying the contribution of monolithic floor slabs to the negative (or also known as hogging) flexural resistance of beams in RC frames in service.

Since early 1980s, many research studies focus on behaviour of beam-column connections with floor slabs. The researchers highlighted that even though floor slabs are generally recognized to improve the structural system as it provides infinite degree of redundancy to horizontal diaphragms, but they are typically analysed and design as a loading mechanism for transferring gravity loads to beams. Thus, contribution of slabs for structural support was assumed to be zero out of simplicity in analysis and design.

Shahrooz et al. had produced a qualitative model which establish the kinematic relations between beam deformations and slab strains. As the slab bars were connected to the beams, this model assumed that slab act as a membrane element attached to the top part of longitudinal beam and transverse beam (which established the T and L-flange beams nowadays). Shahrooz et al. had evaluated the contribution of slab in flexural (particularly hogging moment at near support), torsional and lateral bending behaviour of beams by considering the stiffness of beam, slab reinforcement bars stress, bond slip and strain. Figure 2.12 demonstrate the slab contribution to flexural resistance of beam. Figure 2.13 demonstrate the slab contribution to torsional resistance of beam.

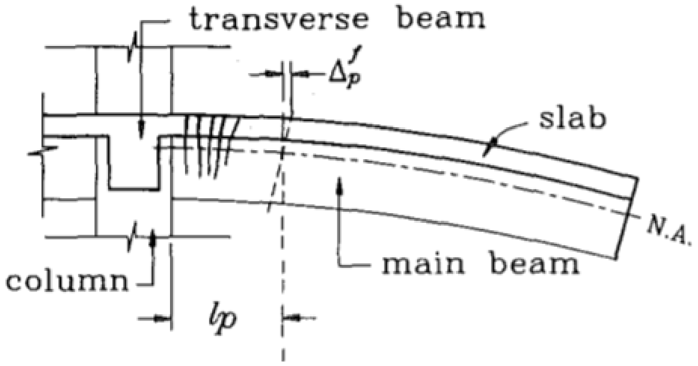

Figure 2.12: Slab Contributing to Flexural Resistance of Beam (Shahrooz, Pantazopoulou, & Chern, 1992).

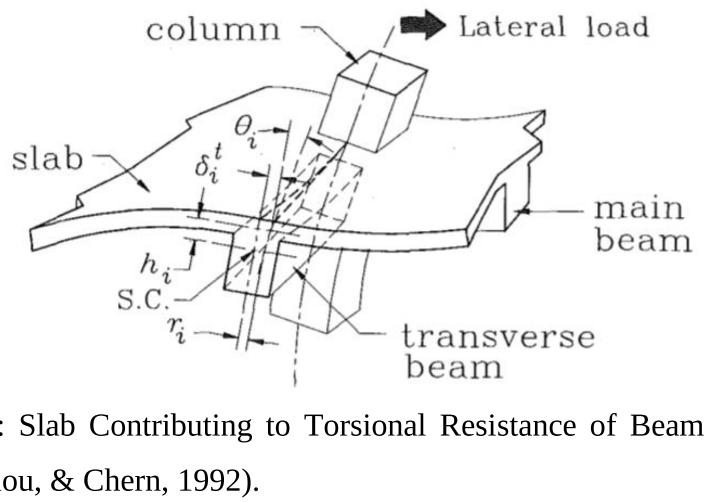

Figure 2.13: Slab Contributing to Torsional Resistance of Beam (Shahrooz, Pantazopoulou, & Chern, 1992).

2.11.2 Analysing the Slabs by Different Method

Sucharda, O. and Kubosek, J. (2013) had made a comparison between results obtained through finite difference method in Matlab and finite element analysis in Scia Engineer. Matlab and Scia Engineer are different software exercising different analysis method with respective assumptions.

A square thin slab of 5000 mm width with 180 mm thickness with 15kN/m^2 uniformly distributed load was model in both Matlab and Scia Engineer. The results compared were in term of bending moment in two directions (x and y-direction), torsional moment and deflection. Figure 2.14 shows internal forces and deflections of slab calculated using FDM. Figure 2.15 shows internal forces and deflection of slab using Scia Engineer of FEA. Table 2.3 compares result between FDM and FEA in Scia Engineer.

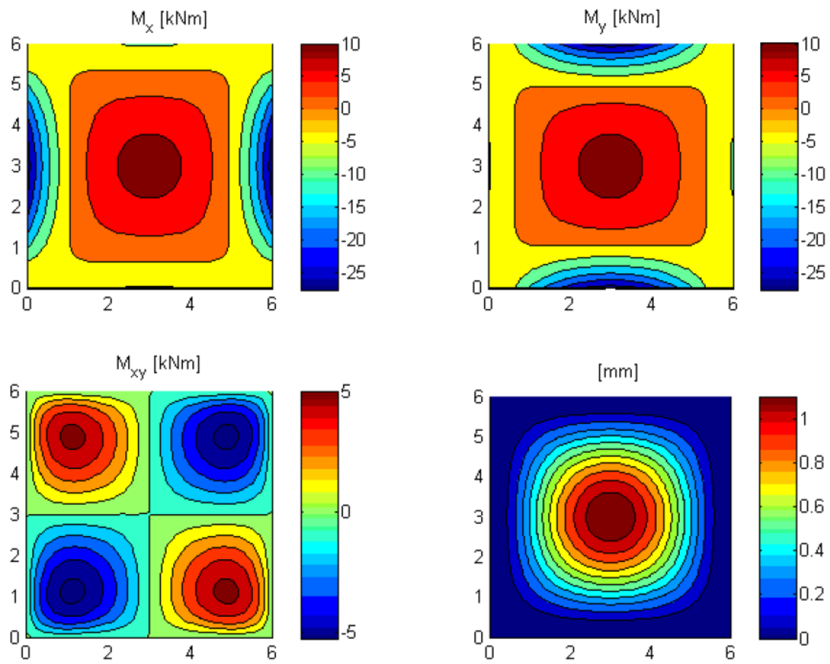

Figure 2.14: Internal Forces and Deflections Calculated using the Finite Difference Method (Sucharda & Kubosek, 2013).

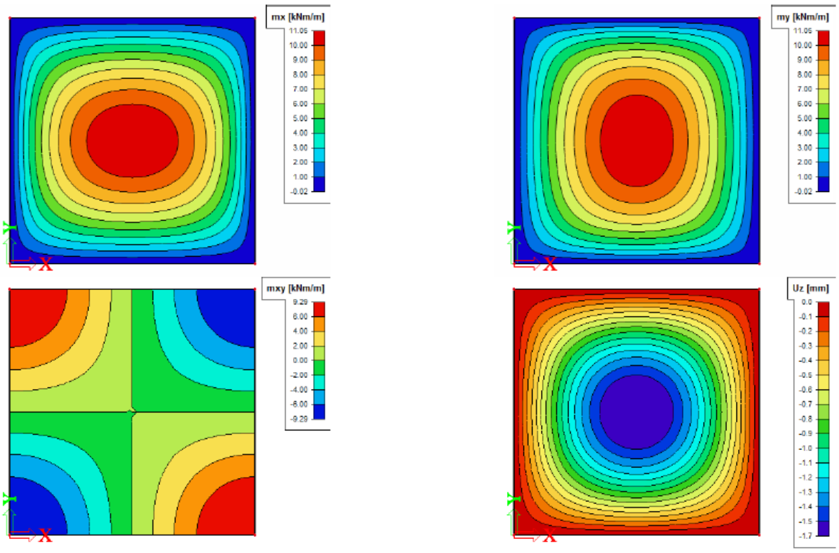

Figure 2.15: Internal Forces and Deflection Calculated in Scia Engineer (Sucharda & Kubosek, 2013).

Table 2.3: Comparison between Results from Matlab and Scia Engineer.

	FDM in Matlab	FEM in Scia Engineer	Difference
Maximum bending moment (kN.m/m)	10	11	+10 %
Maximum torsional moment (kN.m/m)	5	9.3	+86 %
Maximum deflection (mm)	1.0	1.7	+70 %

The results show that there was big difference between torsional moment and deflection. The researchers conclude that FEM gives better performance than FDM, as FDM is a relatively simple method that should only be used for rather small systems of equations.

2.11.3 Comparison of Two FEM Programs

Another research done by Cajka, R.; and Labudkova, J. in 2014 compared the experimental results of foundation slab with two FEM software, namely Scia Engineer and Mkpinter.

In this research, a foundation slab with dimension of 500 mm x 500 mm x 48 mm was casted as test sample. A centric load at 100 mm x 100 mm at the centre was applied on foundation slab until failure. The actual deformation upon failure was recorded and compared with results obtained from Mkpinter and Scia Engineer. Figure 2.16 shows centric load at test sample of foundation slab. Figure 2.17 compares the slab deformation at the middle of foundation slab. Table 2.4 compares the result between test sample, Mkpinter and Scia Engineer.

Figure 2.16: Centric Load at Test Sample (Cajka & Vaskova, 2014).

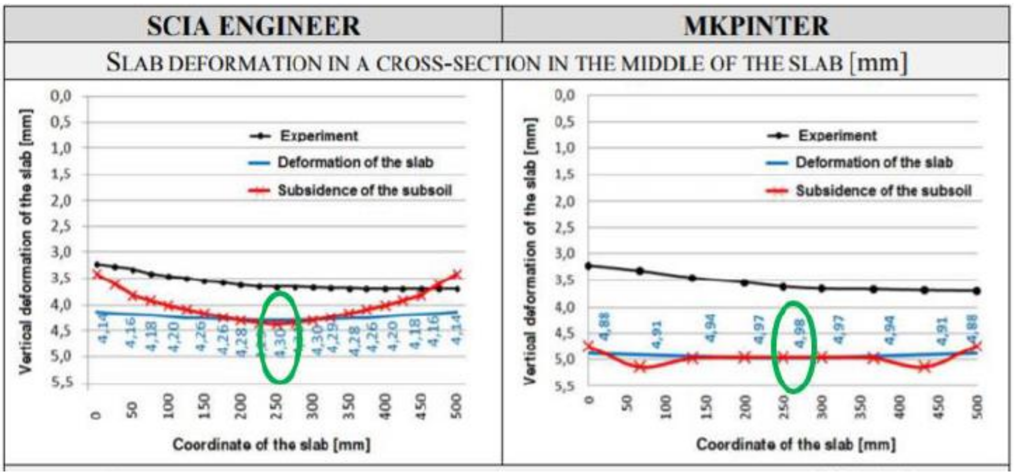

Figure 2.17: Slab Deformation at the Middle of Slab (Cajka & Vaskova, 2014).

Table 2.4: Comparison of Results between Test Sample and FEM Software (Cajka & Vaskova, 2014).

	Scia Engineer	Mkpinter	Actual
Deformation (mm)	4.30	4.98	3.60
Difference from actual	19.44 %	38.33 %	-

The results shown that both software calculated deformation that was higher than that measure during the experiment, which was over-estimating the deformation. However, the deformation obtained from Scia Engineer was closer than that obtained from Mkpinter. The remark drawn is that Scia Engineer tend to be a structural analysis software which provide more reliable results.

2.11.4 Shallow Beam Supported RC Slab

A research regarding behaviour of shallow beam supported rectangular RC slabs was conducted by 3 Indian researchers, H. Singh, M. Kumar, and N. Kwatra in the year of 2009.

‘Most of the RC design codes, ACI 318 (2008), CSA A23.3 (1994) and IS 456 (2000) limit the slabs to be supported by beams with smaller span to depth ratio when using the design code’ – H. Singh et al.. Two problems were raised by the researchers. Firstly, for many cases, the depth of beams were strictly limited by the architects to a level that are insufficient to provide rigid support to slabs. Secondly, as no clearer provision is made for the relation between slabs and supporting beams, wasteful overdesign or worse case, under-design might be done.

Hence, this research seek to suggest analytical equations for proportioning the rectangular RC slabs cast monolithically with equally spaced shallow beams. This paper also performed experimental test of two-panel and three-panel rectangular RC slabs to validate the analytical results.

The researchers conclude that when a slab is supported by shallow-flexible beams, it will result a slab will no yield line along the top face. In another word, the slab does not resist any hogging moment at the continuous edges if it is supported by shallow-flexible beams.

2.12 Summary

Finite element method is currently the most reliable structural analysis approach that allows safer and effective structural design. However, the structural modelling requires certain understanding level of underlying mechanics of theories and adequate modelling skills to ensure the modelled structures behave like actual structure without relying on direct testing.

Despite knowing that the stiffness of beam and slabs contribute structural resistance to each other, yet not much research have been conducted in studying the relationship.

In this study, the results (shear force and bending moment) obtained from Scia Engineer modelling were made comparison with results obtained by manual calculation based on shear force and bending moment coefficient provided in Appendix B and Appendix C as in BS8110.

This study seeks to determine the effect of beam stiffness (beam width and depth) on the internal loading (shear force and bending moment) of both flat slab and solid slab.

CHAPTER 3

METHODOLOGY AND WORK PLAN

3.1 Flowchart

The flowchart of methodology is shown in Figure 3.1.

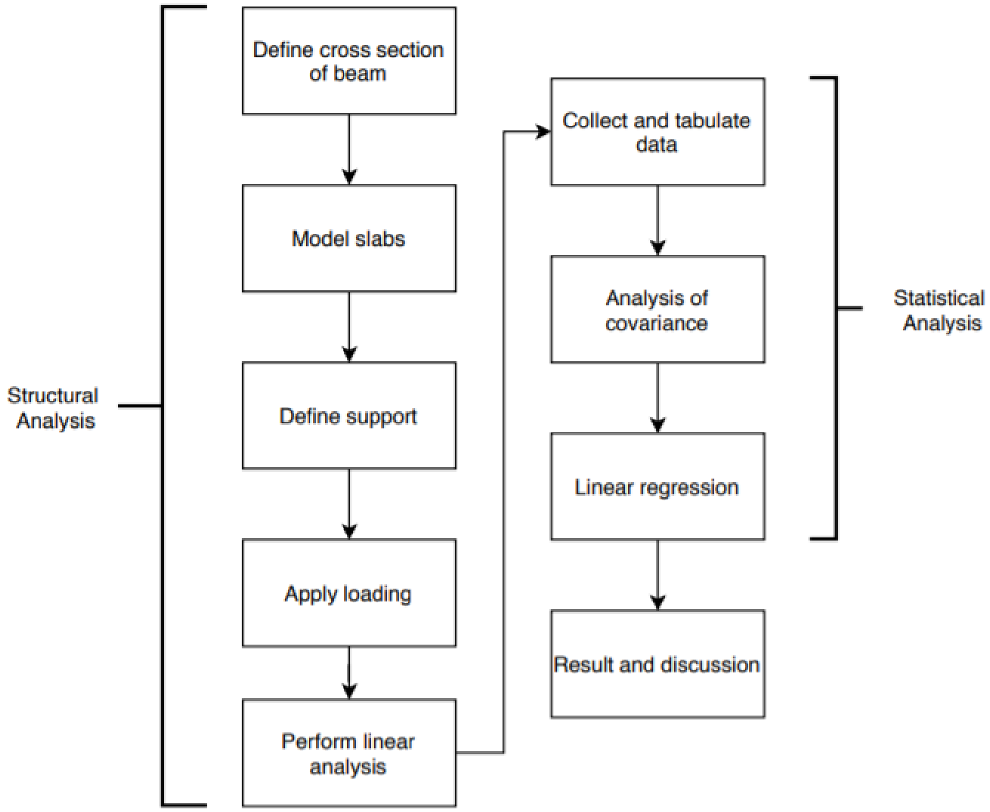

Figure 3.1: Flowchart of Methodology.

3.2 Variables in Model

In this study, 3 types of variable were considered:

(i) Supporting beam size which range from zero (which is flat slab) up to 300 mm x 900 mm.

(ii) *ly/lx* range from 1.00 to 2.25.

(iii) Type of panel as included in BS 8110.

The three variables are expanded as shown in Table 3.1.

Table 3.1: Slabs to be Modelled.

Supporting beam size (mm x mm)	*ly/lx*	Type of panel
Flat slab (not supported by beam)	1.00	(1) Four edges continuous
150 x 300	1.10	(2) One short edge discontinuous
150 x 450	1.20	(3) One long edge discontinuous
200 x 400	1.30	(4) Two adjacent edges discontinuous
200 x 600	1.40	(5) Two short edges discontinuous
250 x 500	1.50	(6) Two long edges discontinuous
250 x 750	1.75	(7) One long edge continuous
300 x 600	2.00	(8) One short edge continuous
300 x 900	2.25	(9) Four edges discontinuous
600 x 300		
900 x 300		
Total = 11	9	9

3.3 Structural Analysis Modelling

The main functions used in Scia Engineer are boxed out in Figure 3.2.

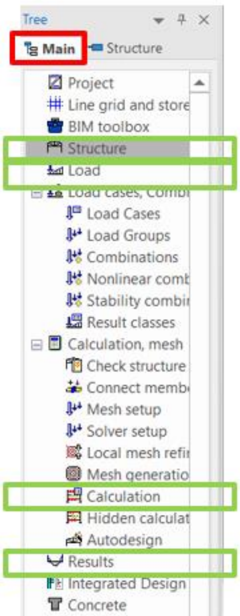

Figure 3.2: Functions to be used under ‘Main’ Tab.

The modelling steps are:

(i) Define cross section of beam which determine the structural properties such as moment of inertia.

(ii) Model the structure (which the structural members include beams, slabs, plate ribs, integration strips and supports).

(iii) Assign area load (both permanent load and variable load).

(iv) Perform linear analysis.

(v) Extract and tabulate results.

(vi) Step (i) to (v) are repeated for all the beam sizes.

3.3.1 Define Cross Section

10 cross sections of supporting beams were created in the library based on dimensions as in Table 3.1.

3.3.2 Modelling of Structure

The elements to be modelled are boxed out Figure 3.3 which includes:

(i) Beam that is 1D member that span in one direction and mainly take vertical line load.

(ii) Slab that is 2D member that span in two direction and mainly taking vertical area load.

(iii) Supports that restrict the structural members from deformation.

(iv) Integration strip that enable user to extracted results at desired location.

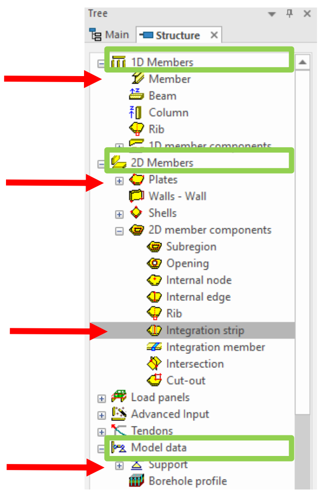

Figure 3.3: Type of Structure to be used under ‘Structure’ Tab.

3.3.2.1 Modelling of Beam and Slab

The conventional slab was set to be 150 mm thick. The thickness of flat slab was set to be 150 mm as well.

The integration strips of 1-meter width are inserted to all the middle of all slabs in both x and y-direction.

Figure 3.4 shows a slab model configuration which covers all 9 types of panel as mentioned in Table 3.1. In Figure 3.4:

(i) Grey colour strip represents plate rib.

(ii) Yellowish-green strip represents integration strip.

(iii) Blue colour joint with red cross represents pin support.

If a same panel is presented more than once, for example there are 2 panels are of two adjacent edges discontinuous (panels labelled as '4' in Figure 3.4), then the average of these 2 data is taken for tabulation of data.

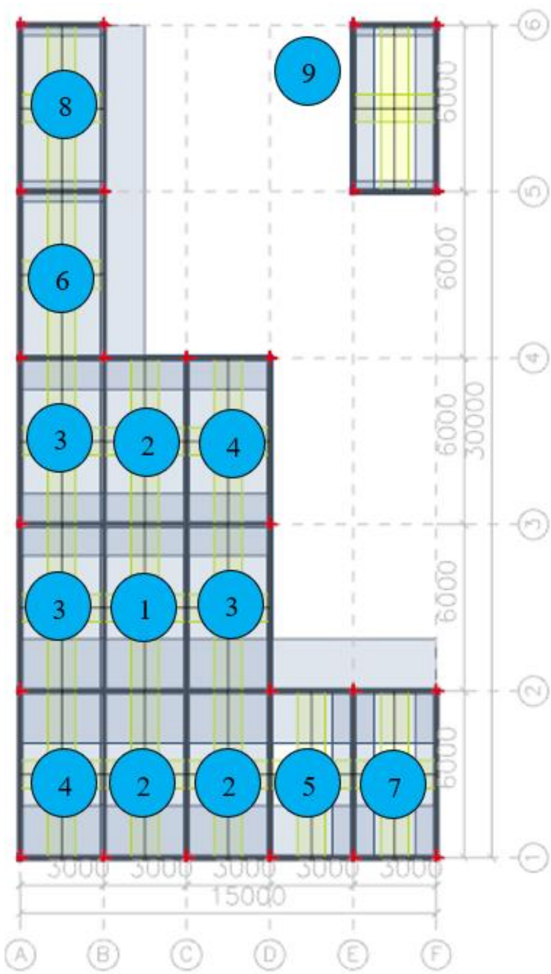

Figure 3.4: Configuration of One Model which Simulates All 9 Types of Panel.

3.3.2.2 Define Supports

The support is modelled as point support with fixed translations and free rotations in all direction (pinned support).

The translations are fixed to restrict the support from settlement whereas the rotations are allowed so that the moment taken by the structural members (beams and slabs) will the maximum (for conservative purpose).

3.3.3 Assign Loading

The load applied on the slab is in term of area load and the values are shown in Table 3.2.

Table 3.2: Load Assignment on Slabs.

Type of loading	**Surface area load (kN/m^2)**	**Reference**
g_k: Self-weight	(depends on thickness)	-
g_k: Floor finish	1.0	-
q_k: Live load	2.5	EN 1991-1-1 Table NA.2 and Table NA.3 (sub-categories of A5 and B1)

Surface area load was assigned as 2.5 kN/m^2 under sub-categories of A5 and B1. A5 represents the greatest loading in Residential area whereas B1 represents the common loading in Office area (by this, two frequently used of structure, residential and office area are covered). By adopting this value of q_k, the slab is said to be loaded under ‘general’ condition for the daily usage of most structure.

3.3.4 Performing Analysis

Linear analysis was performed with the mesh size of 150 mm, after the structural modelling was done.

3.4 Collect and Tabulate Results

The results in term of bending moment and shear force of slab is extracted from Scia Engineer and are tabulated.

3.4.1 Conversion of Coefficients

The values provided in Appendix B and Appendix C, BS 8110 are coefficients. Thus, it is necessary to convert the coefficients into bending moment and shear force in order to make comparison from results generated. Equations 2.2 to 2.5 were used in the conversion.

3.4.2 Results Collection and Tabulation

The results were observed and collected. The results to be collected are shown in Figure 3.5 which include:

(i) *M_y* (bending moment in longitudinal direction of beam).

(ii) *V_z* (shear force in longitudinal direction of beam).

Figure 3.6 shows how the bending moment of slab is extracted by using integration strip. Table 3.3 shows the template for tabulation of bending moment which all value tabulated are with the unit of ‘kN.m per meter width’. Table 3.4 shows template for tabulation of shear force all value tabulated are with the unit of ‘kN per meter width’. The bending moment and shear force for solid slab supported by beam according to BS8110 are shown in Tables 3.5 and 3.6. The bending moment and shear force for flat slab according to BS8110 are shown in Tables 3.7 and 3.8.

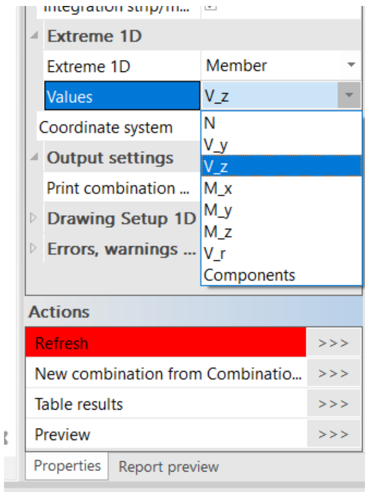

Figure 3.5: Results to be Extracted.

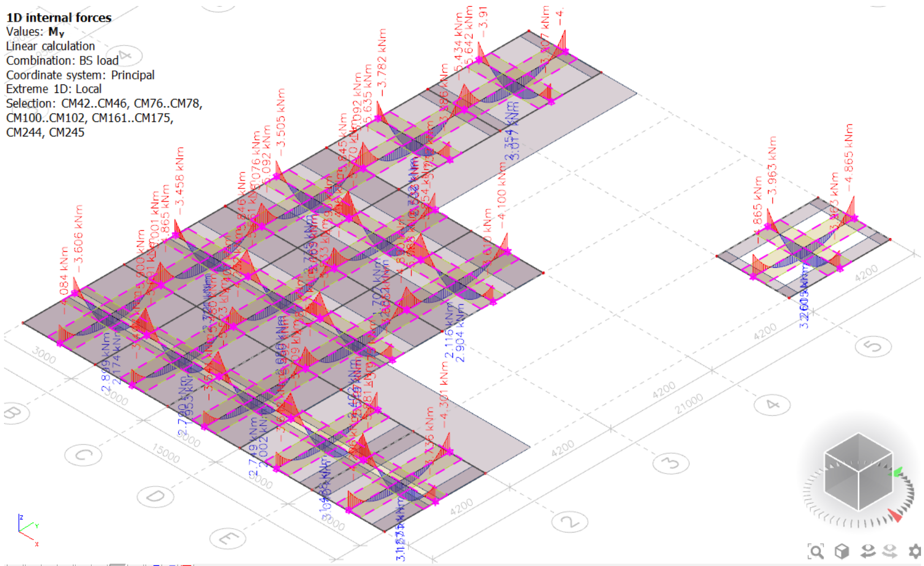

Figure 3.6: Bending Moment Results to be Extracted from Integration Strip.

Table 3.3: Sample Table for Tabulation of Bending Moment.

flat slab																			
span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'																		
	mid																		
1 short	cont'																		
dis'	mid																		
1 long	cont'																		
dis'	mid																		
2 adj	cont'																		
dis'	mid																		
2 short	cont'																		
dis'	mid																		
2 long	cont'																		
dis'	mid																		
1 long	cont'																		
cont'	mid																		
1 short	cont'																		
cont'	mid																		
4 edge	cont'																		
dis'	mid																		

Table 3.4: Sample Table for Tabulation of Shear Force.

flat slab																			
Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't																		
	-																		
1 short	con't																		
dis'	dis't																		
1 long	con't																		
dis'	dis't																		
2 adj	con't																		
dis'	dis't																		
2 short	con't																		
dis'	dis't																		
2 long	con't																		
dis'	dis't																		
1 long	con't																		
cont'	dis't																		
1 short	con't																		
cont'	dis't																		
4 edge	-																		
dis'	dis't																		

Table 3.5: Bending Moment for Solid Slab Supported by Beams as per Appendix B in BS8110.

Code: Solid slab																			
span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.07	3.07	3.07	3.07	3.07	3.07	3.07	3.07	3.07	2.97	3.55	4.03	4.41	4.79	5.08	5.66	6.04	-
	mid	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.68	3.07	3.35	3.55	3.83	4.22	4.60	-

Table 3.6: Shear Force for Solid Slab Supported by Beams as per Appendix C in BS8110.

Code: Solid slab																			
Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	11.50	12.46	13.10	13.74	14.38	15.34	15.98	-

Table 3.7: Bending Moment for Flat Slab as per Appendix D and Appendix E in BS8110.

Code: Flat slab																			
span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	4.12	4.99	5.94	6.97	8.08	9.27	12.62	16.49	20.87	4.12	3.78	3.53	3.35	3.21	3.09	2.89	2.75	2.65
	mid	5.43	6.58	7.83	9.18	10.65	12.23	16.64	21.74	27.51	5.43	4.98	4.66	4.42	4.23	4.08	3.80	3.62	3.49

Table 3.8: Shear Force for Flat Slab as per Appendix D and Appendix E in BS8110.

Code: Flat slab																			
Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	19.17	21.09	23.00	24.92	26.84	28.76	33.55	38.34	43.13	19.17	19.17	19.17	19.17	19.17	19.17	19.17	19.17	19.17

3.5 Statistical Analysis

Statistical analysis is a powerful tool that aids the user in determining the relationship between two or more sets of data. However, it can be a tedious process especially when the relationship has not been studied thorough yet and the correlation between the variables is complex. Thus, among the results above, only one set of the data from above is shortlisted in performing this statistical analysis. The 'hogging moment at long span of interior panel' was shortlisted to perform statistical analysis as it is underestimated the most by the code of design BS8110.

3.5.1 Rules for Covariance Analysis

The covariance analysis requires two input parameters, firstly the dependent variable and secondly the independent variables.

Since the title of this study is '*Finite Analysis of reinforced concrete slabs supported by different stiffness of beam*'. Thus, the dependent variable will be the internal loading in slabs (including bending moment and shear force) whereas the initial independent variables will be the stiffness of beams and slabs in both direction (long span and short span), namely:

(i) Stiffness of beam in x-direction.
(ii) Stiffness of beam in y-direction.
(iii) Stiffness of slab in x-direction.
(iv) Stiffness of slab in y-direction.

Numerous new independent variables were established based on these 4 initial independent variables. The establish of 'new independent variable' applies the 4 basic mathematical operations (addition, subtraction, multiplication and division) to form a new combination which include all the four 'initial dependent variables'. The underlying technique of this process is to obtain an independent variable that shows linear relation with the dependent variable. In another word, the formulated independent variable should increase linearly when the internal loading increase.

The formation of new independent variable was repeated until a high correlation is found (for example obtaining a correlation of 0.8 or higher), which this final independent variable will be the suggested empirical formula for calculating bending moment or shear force. The procedure in formulating the new independent variable is shown in Figure 3.7.

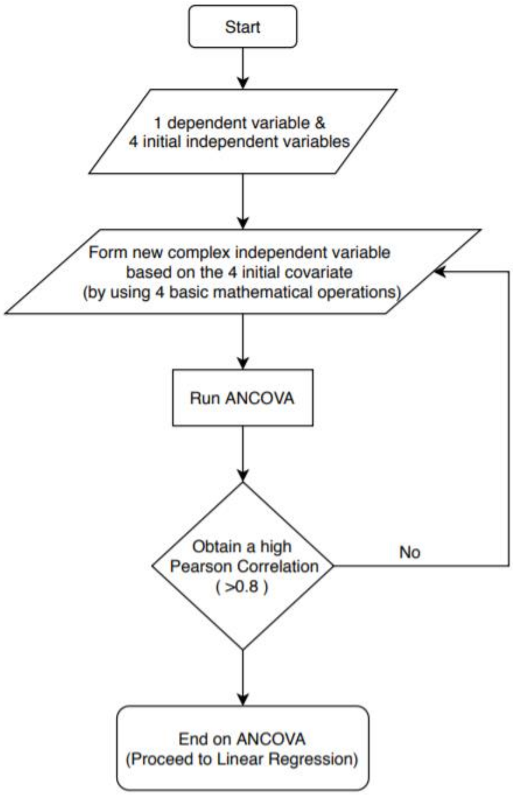

Figure 3.7: The Flow Chart of Covariance Analysis.

There are several rules need to be complied when forming new independent variable such as:

(i) It shows a linear relation with the dependent variable

(ii) The magnitude and unit of the covariates are same

For example: addition of length of beam and stiffness of beam should be avoided.

Length of beam = 6 m

Stiffness of beam = 0.0001125 m^4

6 m + 0.0001125 m^4 is prohibited and meaningless

In such case, the effect of independent variable, ‘stiffness of beam’ will be insignificant.

The output of covariance analysis is Pearson Correlation, the closer to 1.0 the better.

3.5.1.1 Modelling

The dependent variable of this analysis of covariance is named as M_0 and the computation is shown in Equation 3.1.

$$M_0 = \frac{M_1}{M_2} \tag{3.1}$$

where

M_0 = ratio of M_1 to M_2

M_1 = hogging moment obtained from Scia Engineer, kN.m/m

M_2 = hogging moment calculated based on BS8110, kN.m/m

If the value of M_0 is smaller than one, it means that BS8110 has overestimated the hogging moment on contrary, if the value of M_0 is greater than one, it means that the code of design BS8110 has underestimated the hogging moment.

The computation four initial independent variables are shown in Equations 3.2 to 3.5.

$$A = \frac{I}{l} = \frac{\frac{bh^3}{12}}{l_x} \tag{3.2}$$

where

A = stiffness of beam in x direction, mm^3

I = moment of inertia, mm^4

l = length of member, mm

b = width of beam, mm

h = depth of beam, mm

l_x = short span length, mm

$$B = \frac{I}{l} = \frac{\frac{bh^3}{12}}{l_y} \tag{3.3}$$

where

B = stiffness of beam in y direction, mm^3

I = moment of inertia, mm^4

l = length of member, mm

b = width of beam, mm

h = depth of beam, mm

l_y = long span length, mm

$$C = \frac{I}{l} = \frac{\frac{l_y t^3}{12}}{l_x} \tag{3.4}$$

where

C = stiffness of slab in x direction, mm^3

I = moment of inertia, mm^4

l = length of member, mm

l_y = long span length, mm

t = thickness of slab, mm

l_x = short span length, mm

$$D = \frac{I}{l} = \frac{\frac{l_x t^3}{12}}{l_y} \tag{3.5}$$

where

D = stiffness of slab in x direction, mm^3

I = moment of inertia, mm^4

l = length of member, mm

l_y = long span length, mm

t = thickness of slab, mm

l_x = short span length, mm

The covariance analysis between M_0 and l_y/l_x ratio is also performed to made comparison. The sample input of variables (which include M_0, M_1, M_2, l_y/l_x, A, B, C, D, and X) is shown in Figure 3.8. Which the X is the formulated independent variable. The complete input of the variables are attached in Appendix F.

Input for ANCOVA (Hogging moment at long span)										
Suporting beam size		M1	M2	M0	ly/lx	A	B	C	D	X
150*300		2.333	3.067	0.760629	1	0.000113	0.000113	0.000281	0.000281	1
		3.068	3.067	1.000261	1.1	0.000113	0.000102	0.000309	0.000256	0.751315
		3.904	3.067	1.272822	1.2	0.000113	9.38E-05	0.000338	0.000234	0.578704
		4.834	3.067	1.57603	1.3	0.000113	8.65E-05	0.000366	0.000216	0.455166
		5.853	3.067	1.908255	1.4	0.000113	8.04E-05	0.000394	0.000201	0.364431
		6.956	3.067	2.267866	1.5	0.000113	0.000075	0.000422	0.000188	0.296296
		10.050	3.067	3.276604	1.75	0.000113	6.43E-05	0.000492	0.000161	0.186589
		13.578	3.067	4.426839	2	0.000113	5.63E-05	0.000563	0.000141	0.125
		17.500	3.067	5.705529	2.25	0.000113	0.00005	0.000633	0.000125	0.087791
150*450		3.069	3.067	1.000587	1	0.00038	0.00038	0.000281	0.000281	1
		3.503	3.067	1.142084	1.1	0.00038	0.000345	0.000309	0.000256	0.751315
		3.946	3.067	1.286515	1.2	0.00038	0.000316	0.000338	0.000234	0.578704
		4.399	3.067	1.434207	1.3	0.00038	0.000292	0.000366	0.000216	0.455166
		4.863	3.067	1.585485	1.4	0.00038	0.000271	0.000394	0.000201	0.364431
		5.338	3.067	1.74035	1.5	0.00038	0.000253	0.000422	0.000188	0.296296
		6.576	3.067	2.143975	1.75	0.00038	0.000217	0.000492	0.000161	0.186589
		7.871	3.067	2.566184	2	0.00038	0.00019	0.000563	0.000141	0.125
		9.205	3.067	3.001109	2.25	0.00038	0.000169	0.000633	0.000125	0.087791

Figure 3.8: Sample Input of Covariance Analysis.

3.5.2 Linear Regression

The formulated long span to short span ratio (l_y/l_x), independent variable (X) and moment ratio (M_0) are used to performed linear regression. Two scatter graphs namely '$M_0 - X$' and graph of '$M_0 - l_y/l_x$' are plotted, and the linear equations (best fit line) are obtained.

3.6 Summary

The linear structural analysis is first performed. Thereafter, one set of 'critical data' from the former analysis is selected as the input of statistical analysis.

Among the 6 internal loadings of interior span:

(i) Hogging moment at long span.
(ii) Hogging moment at short span.
(iii) Sagging moment at long span.
(iv) Sagging moment at short span.
(v) Shear force at long span.
(vi) Shear force at short span.

The 'hogging moment at long span' was shortlisted to perform statistical analysis as it is underestimated the most by the code of design BS8110.

The objective of this statistical analysis is to seek the empirical relationship between the hogging moment (in long span) and the stiffness of beams and slabs (in x and y-direction).

CHAPTER 4

RESULT AND DISCUSSION

4.1 Introduction

There are two sets of result which include:

(i) Result and discussion of linear structural analysis.

(ii) Result and discussion of statistical analysis.

4.2 Result of Structural Analysis

All results shown in this session consist of bending moment and shear force for all conditions covered in BS8110. Figure 4.1 shows the internal loading to be extracted.

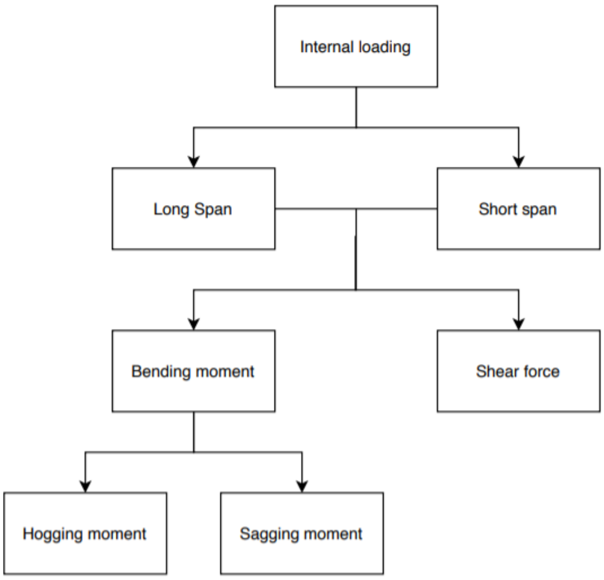

Figure 4.1: Results Extracted.

Among all results, only the internal loading of 'interior panel' (one out of 9 types of panel as stated in Table 3.1) will be discussed in this Chapter 4 for the following reasons:

(i) The interior panel has only one adjacent panel at each side in each direction (x direction and y direction), therefore the results are less skewed (see section 4.4.1 clause (ii) and Figure 4.8).

(ii) Average value is taken when there are more than one result. Since the results in interior panel at two edges do not differ much, therefore the results from interior panel are more consistent (see section 4.4.1 clause (iv) and Figure 4.11).

(iii) Moreover, the results and phenomena in other types of panel can be generally explained with the similar behaviour therefore no repetition explanation is needed.

Table 4.1 to Table 4.22 are tables of results for bending moment and shear force, the results are shown in different colour with the respective reasons for better illustration purpose:

(i) majority of the internal loading shows increase or decrease trend when the l_y/l_x ratio increase, and these results will be shown in green.

(ii) On the other hand, when the results increase to the maximum and decrease thereafter, or decrease to a minimum and increase thereafter, the ‘turning point’ (maximum or minimum) results will be shown in purple.

Hogging moment with negative sign indicates that the support undergoes settlement (which is common case in flat slab or solid slab supported by relatively flexible beam).

FLAT SLAB: BENDING MOMENT

Table 4.1: Result of Bending Moment for Flat Slab.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	2.08	3.20	4.52	6.02	7.71	9.57	14.97	21.41	28.88	2.08	1.44	0.86	0.34	-0.17	-0.61	-1.55	-2.35	-3.08
	mid	3.13	4.01	4.97	6.01	7.12	8.30	11.47	14.93	18.72	3.13	2.89	2.67	2.49	2.35	2.27	2.27	2.54	3.02
1 short	cont'	1.97	3.11	4.44	5.97	7.68	9.58	15.12	21.79	29.58	1.26	0.44	-0.29	-1.03	-1.74	-2.43	-4.03	-5.50	-6.93
dis'	mid	6.65	8.42	10.38	12.52	14.85	17.33	24.49	32.23	41.15	4.59	4.66	4.73	4.85	4.99	5.18	5.81	6.66	7.77
1 long	cont'	1.26	2.32	3.55	4.96	6.54	8.29	13.43	19.66	26.99	1.97	1.30	0.67	0.10	-0.43	-0.92	-1.99	-2.90	-3.72
dis'	mid	4.59	5.44	6.38	7.41	8.51	9.70	13.04	16.89	21.33	6.65	6.26	5.89	5.55	5.25	4.99	4.50	4.28	4.32
2 adj	cont'	1.35	2.45	3.72	5.17	6.79	8.57	13.76	19.98	27.23	1.35	0.52	-0.23	-0.97	-1.69	-2.38	-3.95	-5.33	-6.53
dis'	mid	7.38	9.08	10.94	12.99	15.21	17.56	24.15	31.65	40.08	7.38	7.18	6.98	6.80	6.64	6.52	6.37	6.45	6.81
2 short	cont'	0.09	-0.09	-0.09	-0.09	-0.09	-0.09	-0.06	-0.10	-0.02	0.93	0.05	-0.95	-1.87	-2.79	-3.69	5.89	-7.98	-9.96
dis'	mid	8.79	11.09	13.62	16.36	19.30	22.44	31.04	40.78	51.63	5.48	5.94	6.23	6.64	7.08	7.55	8.87	10.36	11.97
2 long	cont'	0.93	2.05	3.34	4.82	6.47	8.31	13.69	20.22	27.91	0.09	0.10	0.10	0.11	0.10	0.10	0.09	0.08	0.07
dis'	mid	5.48	6.19	6.95	7.75	8.60	9.49	11.98	14.78	17.97	8.79	8.30	7.87	7.45	7.07	6.72	6.03	5.54	5.19
1 long	cont'	-0.07	-0.07	-0.07	-0.07	-0.07	-0.06	-0.04	-0.02	-0.02	0.94	0.03	-1.08	-2.07	3.06	-4.02	-6.33	-8.47	-10.44
cont'	mid	10.07	12.61	15.41	18.44	21.70	25.17	34.71	45.36	57.11	8.29	8.31	8.24	8.22	8.23	8.25	8.47	9.01	9.18
1 short	cont'	0.94	2.13	3.51	5.07	6.81	8.73	14.30	20.96	28.68	-0.07	-0.10	-0.09	-0.09	-0.09	-0.09	-0.09	-0.08	-0.08
cont'	mid	8.29	9.98	11.82	13.82	16.01	18.33	24.91	32.49	41.11	10.07	9.83	9.16	8.73	8.31	7.92	7.04	6.31	5.72
4 edge	cont'	-0.03	-0.04	-0.04	-0.04	-0.05	-0.04	-0.04	-0.02	0.01	-0.03	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04	-0.04
dis'	mid	10.54	13.07	15.87	18.93	22.24	25.81	35.82	47.27	60.20	10.54	10.19	9.83	9.46	9.07	8.68	7.69	6.73	5.89

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. Initially the hogging moment at the support are smaller than the sagging moment at the mid span. As the l_y/l_x ratio increase toward 2.25, the hogging moment become greater than the sagging moment.

In short span, both hogging moment and sagging moment decrease (initially) when the l_y/l_x ratio increase. The hogging moment at the support are generally smaller than the sagging moment at the mid span; and the hogging moment decrease from 2.8 kN.m/m to -3.08 kN.m/m, a negative hogging moment indicates that the support has settled. The sagging moment decrease to the minimum of 2.27 kN.m/m (when the l_y/l_x ratio reached 1.75) and starts to increase thereafter shows that the transition of slab behaviour from flat slab to solid slab.

Comparing both spans, the bending moment at long span are generally greater than those at short span.

FLAT SLAB: SHEAR FORCE

Table 4.2: Result of Shear Force for Flat Slab.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	3.12	4.19	5.32	6.57	7.83	9.18	12.55	16.06	19.69	3.12	2.38	1.78	1.30	0.92	0.87	0.75	0.59	0.41
	-																		
1 short	con't	4.97	6.24	7.62	9.02	10.48	11.97	15.85	19.83	23.95	2.82	2.18	1.74	1.34	1.02	0.89	0.89	0.86	1.17
dis'	dis't	3.60	4.13	4.71	5.35	6.08	6.83	8.89	11.10	13.41									
1 long	con't	2.82	3.70	4.63	5.64	6.70	7.83	10.74	13.84	17.12	4.97	4.18	3.51	2.92	2.41	1.97	1.11	0.88	0.73
dis'	dis't										3.67	3.42	3.26	3.12	3.00	2.89	2.67	2.52	2.43
2 adj	con't	4.41	5.49	6.64	7.83	9.05	10.30	13.56	16.89	20.32	4.41	3.68	3.19	2.70	2.28	1.91	1.21	0.76	0.51
dis'	dis't	3.61	4.09	4.61	5.13	5.69	6.30	8.03	9.85	11.82	3.61	3.46	3.36	3.26	3.18	3.11	2.99	2.94	2.94
2 short	con't										2.80	2.37	1.97	1.66	1.38	1.22	1.43	1.58	1.61
dis'	dis't	4.47	5.19	5.99	6.83	7.72	8.66	11.08	13.59	16.18									
2 long	con't	2.80	3.54	4.31	5.13	6.00	6.92	9.32	11.90	14.62									
dis'	dis't										4.47	4.29	4.12	3.99	3.87	3.75	3.48	3.23	3.01
1 long	con't										3.70	3.22	2.60	2.16	1.79	1.47	0.92	0.74	0.74
cont'	dis't	4.55	5.19	5.91	6.68	7.49	8.35	10.57	12.91	15.32	3.71	3.64	3.53	3.46	3.42	3.38	3.37	3.43	3.57
1 short	con't	3.70	4.64	5.64	6.68	7.75	8.85	11.75	14.72	17.82									
cont'	dis't	3.71	4.16	4.68	5.17	5.64	6.15	7.58	9.21	10.95	4.58	4.52	4.25	4.11	3.98	3.86	3.57	3.31	3.08
4 edge	-										4.40	4.25	4.10	3.96	3.82	3.69	3.38	3.11	2.88
dis'	dis't	4.40	4.97	5.59	6.25	6.98	7.75	9.81	12.08	14.54									

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force decrease (initially) when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally greater than those at short span.

150mm x 300mm BEAM: BENDING MOMENT

Table 4.3: Result of Bending Moment for Solid Slab Supported by Beam Size of 150 mm x 300 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	2.33	3.07	3.90	4.83	5.85	6.96	10.05	13.58	17.50	2.33	2.10	1.87	1.64	1.42	1.22	0.76	0.33	-0.11
	mid	2.28	2.87	3.51	4.18	4.88	5.60	7.48	9.41	11.42	2.28	2.15	2.01	1.87	1.75	1.66	1.56	1.66	1.92
1 short	cont'	2.31	3.03	3.85	4.76	5.75	6.82	9.82	13.27	17.13	1.94	1.58	1.27	0.91	0.54	0.17	-0.74	-1.60	-2.41
dis'	mid	4.11	5.17	6.33	7.60	8.96	10.41	14.40	18.91	23.93	3.15	3.24	3.34	3.42	3.50	3.58	3.87	4.29	4.81
1 long	cont'	1.94	2.59	3.30	4.07	4.90	5.78	8.21	10.95	14.02	2.31	2.06	1.81	1.55	1.31	1.07	0.52	0.00	-0.49
dis'	mid	3.15	3.69	4.24	4.80	5.36	5.92	7.37	8.87	10.49	4.11	3.98	3.83	3.67	3.51	3.36	3.07	2.91	2.89
2 adj	cont'	2.01	2.69	3.44	4.24	5.09	5.98	8.42	11.13	14.12	2.01	1.63	1.29	0.89	0.48	0.07	-0.92	-1.83	-2.64
dis'	mid	4.50	5.44	6.45	7.51	8.64	9.81	13.01	16.56	20.49	4.50	4.51	4.56	4.54	4.51	4.48	4.43	4.47	4.62
2 short	cont'	0.05	0.06	0.09	0.12	0.16	0.20	0.33	0.61	1.08	1.76	1.35	0.91	0.42	-0.10	-0.63	-1.99	-3.31	-4.57
dis'	mid	5.12	6.45	7.92	9.54	11.27	13.13	18.22	23.96	30.30	3.74	4.13	4.45	4.76	5.07	5.38	6.16	7.02	7.96
2 long	cont'	1.76	2.39	3.06	3.77	4.52	5.30	7.42	9.79	12.43	0.05	0.03	-0.02	-0.05	-0.06	-0.08	-0.11	-0.13	-0.14
dis'	mid	3.74	4.13	4.48	4.79	5.05	5.29	5.75	6.09	6.37	5.12	4.97	4.88	4.74	4.61	4.50	4.31	4.24	4.26
1 long	cont'	0.02	0.03	0.05	0.07	0.09	0.12	0.23	0.36	0.53	1.70	1.27	0.72	0.15	-0.45	-1.07	-2.62	-4.09	-5.44
cont'	mid	5.76	7.10	8.56	10.12	11.80	13.57	18.41	23.80	29.77	4.86	5.11	5.30	5.45	5.56	5.66	5.88	6.20	6.66
1 short	cont'	1.70	2.35	3.03	3.74	4.48	5.24	7.27	9.47	11.89	0.02	-0.02	-0.05	-0.07	-0.09	-0.11	-0.14	-0.16	-0.17
cont'	mid	4.86	5.66	6.75	7.31	8.16	9.04	11.38	13.99	16.90	5.76	5.98	5.71	5.64	5.54	5.43	5.17	4.95	4.80
4 edge	cont'	0.06	0.07	0.08	0.10	0.12	0.15	0.22	0.33	0.47	0.06	0.02	0.01	-0.03	-0.06	-0.08	-0.11	-0.14	-0.16
dis'	mid	5.68	6.79	7.98	9.24	10.57	11.99	15.91	20.34	25.35	5.68	5.87	5.98	6.04	6.04	6.00	5.77	5.46	5.17

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span.

In short span, hogging moment and sagging moment decrease (initially) when the l_y/l_x ratio increase. The hogging moment at the support are generally smaller than the sagging moment at the mid span. The hogging moment decrease from 2.33 kN.m/m to -0.11 kN.m/m, a negative hogging indicates that the support at the edge has settled. The sagging moment decrease to the minimum of 1.56 kN.m/m (when the l_y/l_x ratio reached 1.75) and starts to increase thereafter shows that the transition of slab behaviour from flat slab to solid slab.

Comparing both spans, the bending moment at long span are generally greater than those at short span.

150mm x 300mm BEAM: SHEAR FORCE

Table 4.4: Result of Shear Force for Solid Slab Supported by Beam Size of 150 mm x 300 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	4.95	5.58	6.30	7.04	7.84	8.68	10.88	13.24	15.62	4.95	4.70	4.60	4.32	4.20	4.11	4.02	4.06	4.17
	-																		
1 short	con't	6.02	6.68	7.39	8.16	9.00	9.85	12.16	14.53	17.01	4.93	4.73	4.59	4.46	4.35	4.27	4.16	4.16	4.21
dis'	dis't	5.55	5.95	6.34	6.69	7.02	7.32	8.04	8.14	8.69									
1 long	con't	4.93	5.43	6.02	6.59	7.18	7.82	9.52	11.40	13.35	6.02	5.85	5.68	5.52	5.37	5.22	4.87	4.54	4.21
dis'	dis't										5.55	5.76	5.93	6.09	6.24	6.37	6.70	7.02	7.33
2 adj	con't	5.76	6.33	6.93	7.56	8.22	8.91	10.77	12.67	14.65	5.76	5.55	5.43	5.28	5.14	5.00	4.71	4.49	4.30
dis'	dis't	5.69	6.07	6.46	6.81	7.13	7.45	8.20	8.84	9.44	5.68	5.90	6.10	6.27	6.44	6.59	6.96	7.31	7.64
2 short	con't										5.04	4.88	4.79	4.70	4.62	4.56	4.51	4.56	4.67
dis'	dis't	6.18	6.56	6.90	7.21	7.48	7.74	8.68	9.90	11.41									
2 long	con't	5.04	5.45	5.94	6.38	6.82	7.30	8.56	9.96	11.44									
dis'	dis't										6.18	6.40	6.72	6.91	7.08	7.21	7.46	7.63	7.73
1 long	con't										5.21	5.07	4.84	4.67	4.51	4.36	4.10	3.95	3.85
cont'	dis't	6.34	6.76	7.15	7.49	7.84	8.17	8.82	9.71	10.93	5.83	6.07	6.31	6.52	6.71	6.90	7.34	7.75	8.15
1 short	con't	5.21	5.69	6.17	6.67	7.19	7.72	9.12	10.61	12.14									
cont'	dis't	5.83	6.18	6.52	6.82	7.10	7.38	8.04	8.66	9.24	6.34	6.59	6.83	7.00	7.15	7.28	7.52	7.68	7.79
4 edge	-																		
dis'	dis't	6.211	6.592	6.951	7.260	7.562	7.863	8.483	8.963	9.814	6.211	6.473	6.686	6.862	7.009	7.134	7.378	7.557	7.692

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force decrease (initially) when the l_y/l_x ratio increase. The shear decrease to the minimum of 4.02 kN/m (when the l_y/l_x ratio reached 1.75) and starts to increase thereafter. Comparing both spans, the shear force at long span are generally greater than those at short span.

150mm x 450mm BEAM: BENDING MOMENT

Table 4.5: Result of Bending Moment for Solid Slab Supported by Beam Size of 150 mm x 450 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.07	3.50	3.95	4.40	4.86	5.34	6.58	7.87	9.21	3.07	3.27	3.43	3.57	3.68	3.76	3.89	3.89	3.75
	mid	2.03	2.36	2.66	2.93	3.18	3.42	3.92	4.34	4.72	2.03	2.07	2.07	2.03	1.97	1.90	1.73	1.65	1.68
1 short	cont'	3.14	3.57	4.01	4.45	4.90	5.35	6.49	7.67	8.87	3.18	3.28	3.36	3.38	3.37	3.33	3.18	2.96	2.70
dis'	mid	2.71	3.15	3.62	4.09	4.58	5.09	6.44	7.93	9.60	2.54	2.68	2.79	2.86	2.90	2.92	2.97	3.03	3.14
1 long	cont'	3.18	3.67	4.14	4.59	5.03	5.44	6.40	7.27	8.06	3.14	3.34	3.50	3.62	3.71	3.78	3.85	3.78	3.57
dis'	mid	2.54	2.87	3.17	3.36	3.64	3.81	4.10	4.22	4.24	2.71	2.86	2.97	3.05	3.11	3.16	3.27	3.38	3.53
2 adj	cont'	3.29	3.79	4.27	4.73	5.17	5.59	6.56	7.40	8.15	3.29	3.38	3.44	3.43	3.37	3.28	2.91	2.42	1.87
dis'	mid	3.07	3.51	3.93	4.34	4.74	5.13	6.07	7.01	7.99	3.08	3.29	3.50	3.64	3.77	3.87	4.09	4.30	4.54
2 short	cont'	0.62	0.67	0.73	0.79	0.85	0.92	1.12	1.36	1.63	3.34	3.36	3.36	3.27	3.13	2.96	2.42	1.79	1.13
dis'	mid	3.04	3.55	4.10	4.70	5.34	6.03	7.95	10.15	12.59	2.98	3.24	3.48	3.67	3.83	3.98	4.34	4.74	5.21
2 long	cont'	3.34	3.88	4.40	4.88	5.32	5.72	6.58	7.26	7.79	0.62	0.61	0.59	0.57	0.54	0.52	0.44	0.35	0.26
dis'	mid	2.98	3.30	3.55	3.74	3.87	3.93	3.82	3.43	3.13	3.04	3.23	3.48	3.67	3.86	4.03	4.45	4.85	5.24
1 long	cont'	0.68	0.76	0.83	0.90	0.98	1.05	1.26	1.49	1.75	3.40	3.40	3.32	3.14	2.90	2.61	1.71	0.69	-0.37
cont'	mid	3.55	4.08	4.64	5.20	5.79	6.40	8.01	9.75	11.65	3.40	3.71	4.00	4.23	4.42	4.59	4.94	5.30	5.70
1 short	cont'	3.40	3.97	4.49	4.97	5.41	5.79	6.57	7.11	7.47	0.68	0.63	0.61	0.57	0.53	0.49	0.39	0.29	0.21
cont'	mid	3.40	3.80	4.16	4.48	4.76	5.00	5.49	5.97	6.65	3.55	3.94	4.12	4.35	4.54	4.73	5.10	5.40	5.68
4 edge	cont'	0.79	0.89	0.99	1.08	1.17	1.25	1.45	1.66	1.88	0.79	0.73	0.67	0.61	0.55	0.50	0.37	0.25	0.16
dis'	mid	3.71	4.15	4.58	4.98	5.36	5.74	6.65	7.57	8.56	3.71	4.11	4.47	4.75	5.01	5.22	5.60	5.86	6.05

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span.

In short span, the hogging moment (initially) increase whereas the sagging moment (initially) decrease when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The hogging moment increase to the maximum of 3.89 kN.m/m (when the l_y/l_x ratio reached 1.75) and starts to increase thereafter shows that the transition of slab behaviour from flat slab to solid slab. The sagging moment decrease to the minimum of 1.65 kN.m/m (when the l_y/l_x ratio reached 2.00) and starts to increase thereafter shows that the transition of slab behaviour from flat slab to solid slab.

Comparing both spans, the bending moment at long span are generally greater than those at short span.

150mm x 450mm BEAM: SHEAR FORCE

Table 4.6: Result of Shear Force for Solid Slab Supported by Beam Size of 150 mm x 450 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	8.07	8.48	8.87	9.29	9.70	10.13	11.28	12.51	13.84	8.07	8.39	8.64	8.83	8.98	9.11	9.29	9.35	9.35
	-																		
1 short	con't	8.64	9.04	9.45	9.85	10.24	10.63	11.61	12.63	13.67	8.47	8.76	8.91	9.06	9.18	9.26	9.37	9.41	9.43
dis'	dis't	7.05	7.29	7.50	7.70	7.87	8.04	8.39	8.65	8.82									
1 long	con't	8.47	8.95	9.42	9.85	10.27	10.59	11.67	12.68	13.71	8.64	9.05	9.39	9.67	9.89	10.07	10.30	10.30	10.13
dis'	dis't										7.05	7.52	7.93	8.29	8.62	8.92	9.51	10.08	10.55
2 adj	con't	8.92	9.41	9.92	10.37	10.78	11.18	12.15	13.08	14.00	8.92	9.24	9.48	9.68	9.81	9.90	9.96	9.86	9.67
dis'	dis't	7.46	7.75	8.02	8.26	8.48	8.69	9.16	9.55	9.87	7.46	7.92	8.30	8.66	8.98	9.28	9.93	10.50	11.01
2 short	con't										8.89	9.04	9.22	9.32	9.37	9.40	9.42	9.41	9.43
dis'	dis't	7.47	7.68	7.87	8.03	8.17	8.29	8.50	8.56	8.47									
2 long	con't	8.89	9.46	10.02	10.50	10.92	11.32	12.26	13.11	13.94									
dis'	dis't										7.47	7.93	8.52	8.96	9.35	9.70	10.39	10.88	11.24
1 long	con't										9.04	9.23	9.39	9.45	9.46	9.43	9.25	9.00	8.73
cont'	dis't	7.89	8.18	8.44	8.68	8.89	9.08	9.48	9.75	9.89	7.86	8.26	8.71	9.08	9.42	9.74	10.46	11.11	11.70
1 short	con't	9.04	9.65	10.22	10.70	11.12	11.50	12.35	13.09	13.79									
cont'	dis't	7.86	8.18	8.47	8.72	8.95	9.15	9.57	9.90	10.15	7.89	8.26	8.85	9.26	9.61	9.93	10.56	11.02	11.36
4 edge	-																		
dis'	dis't	8.14	8.46	8.74	8.98	9.18	9.36	9.69	9.89	9.99	8.14	8.62	9.05	9.43	9.76	10.06	10.64	11.08	11.40

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally greater than those at short span.

200mm x 400mm BEAM: BENDING MOMENT

Table 4.7: Result of Bending Moment for Solid Slab Supported by Beam Size of 200 mm x 400 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	2.91	3.36	3.83	4.31	4.81	5.33	6.67	8.08	9.54	2.91	3.06	3.17	3.27	3.34	3.40	3.49	3.48	3.36
	mid	2.08	2.44	2.78	3.10	3.40	3.68	4.30	4.83	5.31	2.08	2.09	2.05	1.99	1.90	1.81	1.60	1.48	1.49
1 short	cont'	2.98	3.44	3.90	4.38	4.86	5.36	6.63	7.94	9.28	2.97	3.02	3.05	3.03	2.98	2.91	2.69	2.43	2.16
dis'	mid	2.75	3.26	3.79	4.34	4.90	5.48	7.02	8.72	10.59	2.58	2.71	2.81	2.86	2.88	2.90	2.92	2.97	3.09
1 long	cont'	2.97	3.46	3.95	4.42	4.88	5.32	6.35	7.30	8.17	2.98	3.13	3.23	3.31	3.36	3.40	3.40	3.30	3.08
dis'	mid	2.58	2.95	3.27	3.56	3.81	4.01	4.38	4.56	4.64	2.75	2.85	2.92	2.95	2.97	2.97	2.97	3.01	3.10
2 adj	cont'	3.09	3.60	4.10	4.58	5.05	5.51	6.56	7.50	8.34	3.09	3.13	3.14	3.08	2.98	2.86	2.44	1.93	1.39
dis'	mid	3.10	3.58	4.06	4.53	4.99	5.43	6.52	7.60	8.72	3.10	3.27	3.44	3.55	3.64	3.71	3.84	3.98	4.16
2 short	cont'	0.78	0.86	0.95	1.05	1.16	1.27	1.60	1.98	2.41	3.11	3.08	3.03	2.90	2.73	2.52	1.92	1.26	0.59
dis'	mid	3.06	3.64	4.27	4.96	5.69	6.47	8.62	11.06	13.76	3.02	3.27	3.51	3.69	3.85	3.99	4.34	4.73	5.20
2 long	cont'	3.11	3.65	4.16	4.65	5.11	5.53	6.45	7.18	7.78	0.78	0.76	0.71	0.68	0.65	0.61	0.52	0.41	0.30
dis'	mid	3.02	3.35	3.61	3.82	3.95	4.02	3.93	3.56	3.25	3.06	3.20	3.38	3.51	3.64	3.76	4.06	4.38	4.71
1 long	cont'	0.83	0.94	1.05	1.16	1.28	1.40	1.74	2.12	2.55	3.17	3.11	2.98	2.77	2.50	2.18	1.25	0.23	-0.80
cont'	mid	3.56	4.16	4.78	5.43	6.09	6.79	8.61	10.57	12.70	3.40	3.69	3.95	4.14	4.30	4.43	4.72	5.02	5.37
1 short	cont'	3.17	3.72	4.26	4.75	5.20	5.61	6.46	7.08	7.53	0.83	0.76	0.72	0.67	0.62	0.57	0.44	0.33	0.22
cont'	mid	3.40	3.83	4.23	4.59	4.91	5.20	5.79	6.37	7.13	3.56	3.92	4.02	4.20	4.35	4.47	4.74	4.96	5.18
4 edge	cont'	0.95	1.09	1.23	1.36	1.49	1.63	1.96	2.33	2.73	0.95	0.87	0.79	0.72	0.65	0.58	0.42	0.28	0.17
dis'	mid	3.66	4.15	4.62	5.08	5.53	5.98	7.07	8.18	9.36	3.66	4.02	4.33	4.58	4.79	4.96	5.25	5.43	5.56

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span.

In short span, the hogging moment (initially) increase whereas the sagging moment (initially) decrease when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The hogging moment increase to the maximum of 3.49 kN.m/m (when the l_y/l_x ratio reached 1.75) and starts to decrease thereafter shows that the transition of slab behaviour from flat slab to solid slab. The sagging moment decrease to the minimum of 1.48 kN.m/m (when the l_y/l_x ratio reached 2.00) and starts to increase thereafter shows that the transition of slab behaviour from flat slab to solid slab.

Comparing both spans, the bending moment at long span are generally greater than those at short span.

200mm x 400mm BEAM: SHEAR FORCE

Table 4.8: Result of Shear Force for Solid Slab Supported by Beam Size of 200 mm x 400 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	7.81	8.24	8.67	9.12	9.58	10.05	11.29	12.65	14.09	7.81	8.07	8.27	8.43	8.55	8.64	8.80	8.87	8.90
	-																		
1 short	con't	8.39	8.81	9.26	9.70	10.13	10.56	11.64	12.76	13.89	8.11	8.34	8.47	8.59	8.68	8.75	8.86	8.92	8.95
dis'	dis't	7.06	7.32	7.56	7.78	7.98	8.17	8.56	8.86	9.06									
1 long	con't	8.11	8.58	9.06	9.52	9.95	10.38	11.45	12.53	13.66	8.39	8.74	9.02	9.25	9.43	9.57	9.73	9.69	9.50
dis'	dis't										7.06	7.49	7.87	8.22	8.53	8.82	9.44	9.98	10.46
2 adj	con't	8.55	9.04	9.57	10.03	10.48	10.91	11.94	12.96	13.97	8.55	8.82	9.01	9.17	9.28	9.35	9.38	9.28	9.10
dis'	dis't	7.41	7.72	8.01	8.28	8.53	8.76	9.28	9.72	10.08	7.41	7.83	8.19	8.53	8.84	9.13	9.77	10.34	10.85
2 short	con't										8.43	8.55	8.70	8.77	8.82	8.85	8.89	8.93	8.99
dis'	dis't	7.51	7.75	7.96	8.14	8.30	8.44	8.68	8.75	8.65									
2 long	con't	8.43	8.97	9.52	10.00	10.43	10.84	11.81	12.73	13.64									
dis'	dis't										7.51	7.92	8.47	8.88	9.24	9.56	10.20	10.67	11.00
1 long	con't										8.53	8.67	8.80	8.84	8.84	8.81	8.65	8.45	8.26
cont'	dis't	7.86	8.17	8.46	8.72	8.96	9.17	9.61	9.91	10.07	7.75	8.11	8.53	8.89	9.21	9.52	10.23	10.88	11.47
1 short	con't	8.53	9.12	9.67	10.15	10.58	10.98	11.88	12.70	13.49									
cont'	dis't	7.75	8.09	8.40	8.67	8.92	9.15	9.63	10.02	10.32	7.86	8.19	8.73	9.10	9.44	9.73	10.33	10.77	11.09
4 edge	-																		
dis'	dis't	8.02	8.37	8.67	8.94	9.18	9.38	9.78	10.03	10.19	8.02	8.45	8.84	9.19	9.50	9.78	10.34	10.76	11.09

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally greater than those at short span.

200mm x 600mm BEAM: BENDING MOMENT

Table 4.9: Result of Bending Moment for Solid Slab Supported by Beam Size of 200 mm x 600 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.53	3.80	4.02	4.20	4.35	4.46	4.66	4.74	4.72	3.53	3.98	4.37	4.72	5.02	5.29	5.80	6.14	6.32
	mid	1.99	2.17	2.29	2.37	2.39	2.39	2.31	2.40	2.66	1.99	2.16	2.27	2.32	2.33	2.31	2.15	1.95	1.77
1 short	cont'	3.66	3.93	4.15	4.33	4.48	4.60	4.77	4.81	4.70	3.83	4.19	4.51	4.76	4.96	5.13	5.41	5.58	5.67
dis'	mid	2.01	2.18	2.31	2.41	2.50	2.58	2.86	3.33	4.01	2.25	2.45	2.61	2.71	2.77	2.80	2.78	2.69	2.58
1 long	cont'	3.83	4.22	4.56	4.85	5.10	5.30	5.61	5.69	5.57	3.66	4.12	4.53	4.89	5.21	5.47	5.97	6.26	6.37
dis'	mid	2.25	2.46	2.62	2.73	2.78	2.79	2.68	2.71	2.99	2.01	2.27	2.50	2.69	2.85	3.00	3.29	3.53	3.75
2 adj	cont'	3.97	4.35	4.69	4.98	5.21	5.41	5.71	5.79	5.65	3.97	4.36	4.70	4.97	5.18	5.34	5.53	5.50	5.29
dis'	mid	2.29	2.49	2.65	2.78	2.87	2.94	3.14	3.52	4.17	2.29	2.56	2.83	3.05	3.24	3.40	3.74	4.03	4.30
2 short	cont'	1.85	2.10	2.21	2.32	2.41	2.51	2.77	3.06	3.39	4.11	4.39	4.68	4.86	4.99	5.06	5.10	5.00	4.81
dis'	mid	1.94	2.07	2.18	2.28	2.37	2.47	2.72	3.04	3.47	2.52	2.75	2.98	3.13	3.24	3.32	3.45	3.54	3.65
2 long	cont'	4.11	4.60	5.05	5.43	5.76	6.04	6.48	6.62	6.48	1.97	2.04	2.09	2.12	2.11	2.09	1.94	1.73	1.49
dis'	mid	2.52	2.77	2.96	3.08	3.15	3.16	2.96	2.81	2.98	1.94	2.24	2.58	2.88	3.17	3.45	4.11	4.72	5.28
1 long	cont'	2.14	2.34	2.52	2.69	2.86	3.01	3.38	3.76	4.14	4.26	4.56	4.84	5.00	5.09	5.11	4.92	4.47	3.83
cont'	mid	2.30	2.48	2.63	2.76	2.87	2.96	3.15	3.35	3.76	2.53	2.83	3.15	3.40	3.61	3.80	4.20	4.54	4.89
1 short	cont'	4.26	4.77	5.22	5.61	5.94	6.21	6.62	6.69	6.47	2.14	2.12	2.18	2.17	2.14	2.08	1.89	1.65	1.40
cont'	mid	2.53	2.76	2.94	3.07	3.17	3.23	3.37	3.72	4.38	2.30	2.68	3.00	3.32	3.62	3.90	4.54	5.10	5.60
4 edge	cont'	2.36	2.64	2.90	3.15	3.38	3.59	4.07	4.51	4.91	2.36	2.35	2.32	2.27	2.20	2.12	1.87	1.59	1.32
dis'	mid	2.50	2.69	2.84	2.95	3.02	3.06	3.02	2.85	2.86	2.50	2.91	3.29	3.65	3.97	4.27	4.92	5.45	5.90

In long span, both hogging moment (initially) and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The hogging moment increase to the maximum of 4.74 kN.m/m (when the l_y/l_x ratio reached 2.00) and starts to decrease thereafter. The fluctuation of sagging moment in between l_y/l_x ratio of 1.40 to 1.75 will be explained in discussion (see section 4.4.1.3 and Figure 4.10).

In short span, both hogging moment and sagging moment (initially) increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The sagging moment increase to the maximum of 2.33 kN.m/m (when the l_y/l_x ratio reached 1.40) and starts to increase thereafter.

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

200mm x 600mm BEAM: SHEAR FORCE

Table 4.10: Result of Shear Force for Solid Slab Supported by Beam Size of 200 mm x 600 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	10.45	10.80	11.09	11.33	11.53	11.73	12.22	12.75	13.33	10.45	11.13	11.67	12.09	12.41	12.64	12.95	13.01	12.97
	-																		
1 short	con't	10.77	11.12	11.40	11.65	11.88	11.92	12.59	13.09	13.60	10.91	11.52	11.94	12.27	12.52	12.69	12.92	12.96	12.82
dis'	dis't	8.75	8.97	9.13	9.27	9.38	9.48	9.71	9.93	10.14									
1 long	con't	10.91	11.44	11.88	12.28	12.62	12.94	13.62	14.23	14.79	10.77	11.53	12.17	12.69	13.12	13.45	13.98	14.21	14.23
dis'	dis't										8.75	9.29	9.75	10.14	10.46	10.74	11.25	11.61	11.90
2 adj	con't	11.19	11.70	12.14	12.53	12.89	13.21	13.94	14.59	15.18	11.19	11.86	12.37	12.80	13.13	13.38	13.73	13.82	13.74
dis'	dis't	9.18	9.52	9.81	10.06	10.28	10.48	10.92	11.31	11.66	9.18	9.68	10.06	10.41	10.71	10.97	11.48	11.89	12.25
2 short	con't										11.36	11.78	12.21	12.46	12.64	12.75	12.87	12.86	12.82
dis'	dis't	9.07	9.27	9.42	9.55	9.66	9.76	10.01	10.24	10.44									
2 long	con't	11.36	12.03	12.62	13.15	13.62	14.05	14.94	15.65	16.25									
dis'	dis't										9.07	9.60	10.26	10.74	11.17	11.53	12.24	12.72	13.05
1 long	con't										11.56	12.02	12.50	12.80	13.01	13.15	13.27	13.17	12.94
cont'	dis't	9.50	9.83	10.11	10.36	10.59	10.81	11.29	11.72	12.12	9.61	9.97	10.40	10.72	11.00	11.26	11.82	12.32	12.81
1 short	con't	11.56	12.23	12.83	13.36	13.85	14.28	15.20	15.93	16.52									
cont'	dis't	9.61	10.06	10.45	10.80	11.10	11.38	11.95	12.39	12.75	9.50	9.84	10.53	10.96	11.34	11.67	12.32	12.77	13.09
4 edge	-																		
dis'	dis't	9.87	10.31	10.70	11.05	11.36	11.64	12.22	12.67	13.03	9.87	10.34	10.76	11.14	11.47	11.77	12.36	12.79	13.11

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase (initially) when the l_y/l_x ratio increase. The shear increase to the maximum of 13.01 kN/m (when the l_y/l_x ratio reached 2.00) and starts to decrease thereafter. Comparing both spans, the shear force at long span are generally slightly smaller than those at short span.

250mm x 500mm BEAM: BENDING MOMENT

Table 4.11: Result of Bending Moment for Solid Slab Supported by Beam Size of 250 mm x 500 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.30	3.60	3.87	4.10	4.31	4.50	4.89	5.18	5.36	3.30	3.66	3.98	4.27	4.51	4.73	5.16	5.45	5.60
	mid	2.03	2.26	2.44	2.58	2.67	2.72	2.73	2.75	2.93	2.03	2.15	2.21	2.22	2.19	2.13	1.92	1.69	1.50
1 short	cont'	3.43	3.74	4.01	4.25	4.47	4.65	5.03	5.16	5.38	3.53	3.81	4.05	4.24	4.38	4.49	4.67	4.76	4.80
dis'	mid	2.11	2.37	2.59	2.79	2.98	3.15	3.60	4.18	4.93	2.31	2.48	2.62	2.70	2.74	2.75	2.70	2.57	2.50
1 long	cont'	3.53	3.93	4.29	4.61	4.88	5.12	5.52	5.71	5.68	3.43	3.81	4.14	4.42	4.67	4.88	5.26	5.45	5.50
dis'	mid	2.31	2.57	2.77	2.91	3.00	3.04	2.98	2.97	3.21	2.11	2.31	2.47	2.60	2.70	2.78	2.94	3.07	3.21
2 adj	cont'	3.69	4.09	4.45	4.78	5.06	5.29	5.72	5.92	5.91	3.69	3.98	4.24	4.42	4.56	4.64	4.69	4.55	4.27
dis'	mid	2.38	2.65	2.88	3.08	3.24	3.39	3.71	4.17	4.89	2.38	2.60	2.83	3.00	3.13	3.25	3.48	3.68	3.88
2 short	cont'	2.00	2.16	2.31	2.46	2.60	2.75	3.15	3.61	4.12	3.78	3.98	4.20	4.30	4.36	4.37	4.28	4.07	3.80
dis'	mid	2.05	2.29	2.52	2.75	2.98	3.21	3.84	4.55	5.35	2.59	2.82	3.04	3.18	3.29	3.37	3.52	3.63	3.79
2 long	cont'	3.78	4.27	4.71	5.10	5.44	5.72	6.21	6.39	6.30	2.00	2.06	2.09	2.11	2.10	2.07	1.93	1.72	1.49
dis'	mid	2.59	2.85	3.06	3.19	3.26	3.28	3.07	2.92	3.10	2.05	2.29	2.55	2.77	2.99	3.19	3.69	1.17	4.65
1 long	cont'	2.13	2.36	2.57	2.78	2.98	3.18	3.68	4.21	4.77	3.92	4.13	4.31	4.38	4.38	4.32	3.96	3.38	2.67
cont'	mid	2.42	2.69	2.94	3.18	3.40	3.61	4.11	4.60	5.23	2.61	2.89	3.17	3.38	3.55	3.70	4.01	4.28	4.59
1 short	cont'	3.92	4.42	4.87	5.27	5.61	5.90	6.36	6.50	6.35	2.13	2.10	2.15	2.13	2.10	2.04	1.85	1.61	1.36
cont'	mid	2.61	2.88	3.10	3.28	3.41	3.52	3.74	4.18	4.95	2.42	2.78	3.00	3.26	3.49	3.70	4.19	4.62	5.03
4 edge	cont'	2.34	2.64	2.92	3.19	3.46	3.71	4.31	4.89	5.47	2.34	2.32	2.29	2.23	2.16	2.07	1.83	1.55	1.28
dis'	mid	2.57	2.81	3.01	3.19	3.33	3.44	3.59	3.59	3.72	2.57	2.94	3.27	3.58	3.85	4.09	4.59	5.00	5.35

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span.

In short span, both hogging moment and sagging moment (initially) increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The sagging moment increase to the maximum of 2.22 kN.m/m (when the l_y/l_x ratio reached 1.30) and starts to decrease thereafter.

Comparing both spans, the hogging moment at long span are generally slightly smaller than those at short span. The sagging moment at long span are generally greater than those at short span.

250mm x 500mm BEAM: SHEAR FORCE

Table 4.12: Result of Shear Force for Solid Slab Supported by Beam Size of 250 mm x 500 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	9.93	10.32	10.66	10.96	11.24	11.51	12.20	12.92	13.69	9.93	10.51	10.97	11.33	11.60	11.80	12.08	12.15	12.13
	-																		
1 short	con't	10.31	10.70	11.04	11.34	11.63	11.91	12.57	13.23	13.88	10.30	10.81	11.18	11.46	11.68	11.84	12.05	12.11	12.11
dis'	dis't	8.62	8.88	9.09	9.28	9.44	9.59	9.94	10.26	10.56									
1 long	con't	10.30	10.81	11.27	11.67	12.05	12.39	13.18	13.90	14.59	10.31	10.96	11.51	11.95	12.31	12.59	13.01	13.15	13.10
dis'	dis't										8.62	9.12	9.56	9.93	10.25	10.52	11.06	11.48	11.83
2 adj	con't	10.60	11.12	11.57	11.99	12.38	12.54	13.56	14.31	15.02	10.60	11.16	11.60	11.95	12.22	12.42	12.69	12.72	12.61
dis'	dis't	9.00	9.37	9.68	9.97	10.23	10.47	11.01	11.50	11.95	9.00	9.50	9.85	10.19	10.48	10.75	11.30	11.76	12.18
2 short	con't										10.65	11.01	11.38	11.60	11.75	11.85	11.97	12.00	12.00
dis'	dis't	9.01	9.26	9.46	9.64	9.81	9.96	10.12	10.60	10.86									
2 long	con't	10.65	11.28	11.84	12.35	12.81	13.24	14.15	14.93	15.61									
dis'	dis't										9.01	9.49	10.10	10.55	10.94	11.28	11.93	12.38	12.68
1 long	con't										10.80	11.17	11.56	11.80	11.97	12.07	12.12	11.99	11.77
cont'	dis't	9.37	9.74	10.06	10.36	10.63	10.89	11.48	12.00	12.46	9.39	9.74	10.16	10.49	10.78	11.05	11.66	12.21	12.74
1 short	con't	10.80	11.43	12.00	12.52	12.99	13.42	14.36	15.13	15.79									
cont'	dis't	9.39	9.84	10.24	10.60	10.93	11.23	11.87	12.39	12.82	9.37	9.70	10.33	10.72	11.07	11.38	11.99	12.41	12.72
4 edge	-																		
dis'	dis't	9.66	10.12	10.53	10.91	11.25	11.56	12.22	12.76	13.21	9.66	10.09	10.48	10.83	11.14	11.42	11.98	12.39	12.70

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase (initially) when the l_y/l_x ratio increase. The shear increase to the maximum of 12.15 kN/m (when the l_y/l_x ratio reached 2.00) and starts to decrease thereafter. Comparing both spans, the shear force at long span are generally slightly smaller than those at short span.

250mm x 750mm BEAM: BENDING MOMENT

Table 4.13: Result of Bending Moment for Solid Slab Supported by Beam Size of 250 mm x 750 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.81	4.03	4.18	4.27	4.31	4.32	4.19	3.90	3.46	3.81	4.34	4.81	5.22	5.58	5.89	6.49	6.91	7.20
	mid	1.96	2.07	2.12	2.12	2.08	2.01	1.98	2.21	2.64	1.96	2.19	2.37	2.48	2.55	2.58	2.51	2.32	2.10
1 short	cont'	3.93	4.15	4.29	4.38	4.43	4.43	4.30	3.99	3.51	4.03	4.51	4.94	5.29	5.58	5.82	6.26	6.54	6.73
dis'	mid	1.78	1.86	1.90	1.90	1.89	1.90	2.10	2.54	3.21	2.08	2.32	2.54	2.68	2.78	2.84	2.87	2.78	2.63
1 long	cont'	4.03	4.36	4.61	4.81	4.96	5.07	5.14	5.00	4.65	3.93	4.49	5.00	5.45	5.84	6.16	6.83	7.25	7.51
dis'	mid	2.08	2.21	2.29	2.31	2.30	2.24	2.14	2.30	2.70	1.78	2.06	2.29	2.49	2.67	2.82	3.13	3.37	3.58
2 adj	cont'	4.16	4.47	4.72	4.91	5.04	5.14	5.19	5.02	4.64	4.16	4.67	5.15	5.54	5.88	6.15	6.62	6.85	6.89
dis'	mid	1.95	2.05	2.12	2.14	2.15	2.15	2.28	2.68	3.33	1.95	2.22	2.50	2.71	2.90	3.07	3.40	3.66	3.89
2 short	cont'	3.22	3.42	3.58	3.70	3.81	3.90	4.13	4.36	4.61	4.26	4.67	5.10	5.41	5.65	5.84	6.14	6.27	6.29
dis'	mid	1.58	1.62	1.60	1.56	1.49	1.40	1.19	1.14	1.27	2.22	2.47	2.74	2.91	3.04	3.13	3.24	3.25	3.22
2 long	cont'	4.26	4.66	5.02	5.30	5.53	5.72	5.98	5.97	5.72	3.22	3.45	3.68	3.80	3.88	3.92	3.84	3.60	3.28
dis'	mid	2.22	2.38	2.48	2.53	2.53	2.49	2.32	2.38	2.70	1.58	1.87	2.19	2.47	2.73	2.98	3.57	4.12	4.65
1 long	cont'	3.37	3.65	3.89	4.10	4.30	4.48	4.89	5.28	5.65	4.39	4.83	5.29	5.62	5.88	6.08	6.32	6.28	6.03
cont'	mid	1.80	1.86	1.89	1.88	1.84	1.77	1.55	1.46	1.56	2.08	2.37	2.68	2.92	3.13	3.31	3.68	3.96	4.22
1 short	cont'	4.39	4.79	5.13	5.41	5.64	5.82	6.05	5.98	5.65	3.37	3.53	3.75	3.86	3.91	3.92	3.80	3.54	3.20
cont'	mid	2.08	2.21	2.29	2.34	2.35	2.36	2.48	2.85	3.51	1.80	2.15	2.45	2.74	3.02	3.27	3.86	4.38	4.88
4 edge	cont'	3.55	3.92	4.26	4.57	4.87	5.14	5.77	6.33	6.83	3.55	3.74	3.86	3.94	3.96	3.95	3.78	3.49	3.14
dis'	mid	1.94	2.10	2.05	2.05	2.02	1.95	1.67	1.37	1.25	1.94	2.30	2.65	2.97	3.26	3.53	4.13	4.65	5.11

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The fluctuation of sagging moment in between l_y/l_x ratio of 1.20 to 1.75 will be explained in discussion (see section 4.4.1.3 and Figure 4.10).

In short span, both hogging moment and sagging moment (initially) increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The sagging moment increase to the maximum of 2.58 kN.m/m (when the l_y/l_x ratio reached 1.50) and starts to decrease thereafter.

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

250mm x 750mm BEAM: SHEAR FORCE

Table 4.14: Result of Shear Force for Solid Slab Supported by Beam Size of 250 mm x 750 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	11.45	11.79	12.02	12.20	12.33	12.43	12.67	12.91	13.19	11.45	12.27	12.93	13.43	13.81	14.08	14.43	14.51	14.47
	-																		
1 short	con't	11.65	11.98	12.22	12.39	12.54	12.67	12.95	13.23	13.52	11.75	12.49	13.10	13.54	13.87	14.10	14.39	14.45	14.41
dis'	dis't	10.40	10.66	10.85	10.97	11.07	11.14	11.30	11.44	11.59									
1 long	con't	11.75	12.22	12.59	12.91	13.17	13.41	13.92	14.37	14.77	11.65	12.54	13.28	13.87	14.35	14.73	15.33	15.63	15.74
dis'	dis't										10.40	11.03	11.53	11.95	12.18	12.39	12.71	12.81	12.82
2 adj	con't	11.93	12.38	12.76	13.07	13.35	13.60	14.15	14.65	15.11	11.93	12.73	13.41	13.94	14.36	14.69	15.27	15.38	15.41
dis'	dis't	10.71	11.09	11.39	11.65	11.86	12.06	12.48	12.85	13.19	10.71	11.28	11.73	12.07	12.32	12.51	12.70	12.93	13.04
2 short	con't										12.04	12.64	13.26	13.66	13.94	14.13	14.35	14.37	14.32
dis'	dis't	10.64	10.88	11.05	11.18	11.27	11.35	11.53	11.70	11.88									
2 long	con't	12.04	12.62	13.12	13.55	13.94	14.30	15.06	15.70	16.25									
dis'	dis't										10.64	11.23	11.92	12.39	12.77	13.08	13.60	13.89	14.06
1 long	con't										12.17	12.81	13.50	13.96	14.31	14.57	14.92	14.99	14.90
cont'	dis't	10.93	11.30	11.60	11.86	12.08	12.28	12.74	13.16	13.55	11.02	11.46	11.95	12.26	12.50	12.68	12.98	13.19	13.39
1 short	con't	12.17	12.75	13.25	13.70	14.10	14.47	15.28	15.97	16.56									
cont'	dis't	11.02	11.51	11.94	12.31	12.64	12.95	13.61	14.16	14.63	10.93	11.38	12.09	12.51	12.61	13.13	13.61	13.89	14.06
4 edge	-																		
dis'	dis't	11.20	11.68	12.11	12.49	12.84	13.16	13.86	14.45	14.96	11.20	11.77	12.23	12.62	12.93	13.18	13.62	13.88	14.05

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase (initially) when the l_y/l_x ratio increase. The shear increase to the maximum of 14.51 kN/m (when the l_y/l_x ratio reached 2.00) and starts to decrease thereafter. Comparing both spans, the shear force at long span are generally slightly smaller than those at short span.

300mm x 600mm BEAM: BENDING MOMENT

Table 4.15: Result of Bending Moment for Solid Slab Supported by Beam Size of 300 mm x 600 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.56	3.80	3.98	4.11	4.20	4.25	4.22	4.03	3.68	3.56	4.03	4.45	4.81	5.13	5.41	5.96	6.36	6.64
	mid	1.99	2.15	2.25	2.30	2.30	2.26	2.21	2.41	2.84	1.99	2.18	2.31	2.38	2.41	2.39	2.24	2.00	1.75
1 short	cont'	3.71	3.95	4.13	4.26	4.35	4.40	4.38	4.17	3.78	3.79	4.19	4.56	4.85	5.09	5.28	5.64	5.97	6.05
dis'	mid	1.84	1.99	2.09	2.15	2.21	2.26	2.52	3.04	3.80	2.15	2.35	2.54	2.66	2.73	2.76	2.73	2.61	2.45
1 long	cont'	3.79	4.12	4.40	4.62	4.79	4.92	5.04	4.91	4.57	3.71	4.20	4.64	5.03	5.37	5.66	6.22	6.58	6.77
dis'	mid	2.15	2.32	2.43	2.49	2.50	2.46	2.36	2.52	2.96	1.84	2.07	2.26	2.40	2.53	2.63	2.82	2.97	3.11
2 adj	cont'	3.94	4.27	4.54	4.76	4.93	5.05	5.17	5.05	4.70	3.94	4.38	4.77	5.09	5.36	5.57	5.90	6.01	5.96
dis'	mid	2.03	2.19	2.30	2.38	2.43	2.47	2.66	3.13	3.89	2.03	2.26	2.50	2.67	2.82	2.94	3.17	3.36	3.53
2 short	cont'	3.07	3.29	3.47	3.63	3.78	3.93	4.28	4.66	5.08	4.02	4.36	4.72	4.96	5.14	5.32	5.46	5.50	5.47
dis'	mid	1.64	1.74	1.81	1.85	1.87	1.87	1.85	1.90	2.13	2.31	2.55	2.79	2.95	3.06	3.14	3.23	3.24	3.23
2 long	cont'	4.02	4.43	4.79	5.09	5.33	5.53	5.79	5.76	5.46	3.07	3.27	3.45	3.57	3.64	3.67	3.59	3.37	3.07
dis'	mid	2.31	2.49	2.62	2.68	2.69	2.66	2.47	2.55	2.92	1.64	1.88	2.14	2.36	2.57	2.77	3.24	3.70	4.16
1 long	cont'	3.20	3.49	3.75	3.99	4.22	4.44	4.96	5.48	6.00	4.16	4.52	4.89	5.13	5.31	5.42	5.47	5.26	4.87
cont'	mid	1.89	2.02	2.11	2.17	2.20	2.21	2.16	2.17	2.40	2.17	2.44	2.72	2.93	3.10	3.25	3.52	3.74	3.97
1 short	cont'	4.16	4.58	4.93	5.23	5.48	5.67	5.91	5.85	5.49	3.20	3.32	3.52	3.60	3.64	3.64	3.52	3.27	2.96
cont'	mid	2.17	2.34	2.46	2.53	2.58	2.61	2.78	3.24	4.04	1.89	2.22	2.46	2.70	2.93	3.14	3.60	4.03	4.46
4 edge	cont'	3.38	3.76	4.11	4.44	4.75	5.06	5.77	6.44	7.08	3.38	3.53	3.63	3.68	3.69	3.67	3.50	3.22	2.88
dis'	mid	2.01	2.13	2.21	2.25	2.25	2.23	2.03	1.76	1.69	2.01	2.36	2.67	2.95	3.21	3.44	3.93	4.35	4.73

In long span, both hogging moment (initially) and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The hogging moment increase to the maximum of 4.25 kN.m/m (when the l_y/l_x ratio reached 1.50) and starts to decrease thereafter. The fluctuation of sagging moment in between l_y/l_x ratio of 1.20 to 1.75 will be explained in discussion (see section 4.4.1.3 and Figure 4.10).

In short span, both hogging moment and sagging moment (initially) increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The sagging moment increase to the maximum of 2.41 kN.m/m (when the l_y/l_x ratio reached 1.40) and starts to decrease thereafter.

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

300mm x 600mm BEAM: SHEAR FORCE

Table 4.16: Result of Shear Force for Solid Slab Supported by Beam Size of 300 mm x 600 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	11.00	11.37	11.65	11.88	12.07	12.24	12.64	13.07	13.52	11.00	11.74	12.33	12.79	13.13	13.38	13.72	13.82	13.79
	-																		
1 short	con't	11.27	11.63	11.91	12.15	12.35	12.54	12.99	13.42	13.87	11.28	11.94	12.48	12.88	13.18	13.39	13.67	13.75	13.74
dis'	dis't	10.05	10.34	10.56	10.73	10.87	10.99	11.26	11.51	11.75									
1 long	con't	11.28	11.75	12.15	12.49	12.80	13.07	13.68	14.23	14.74	11.27	12.07	12.73	13.27	13.70	14.04	14.57	14.80	14.84
dis'	dis't										10.05	10.66	11.13	11.51	11.80	12.03	12.39	12.59	12.74
2 adj	con't	11.50	11.97	12.38	12.73	13.05	13.34	14.01	14.40	15.21	11.50	12.21	12.81	13.28	13.65	13.93	14.34	14.47	14.44
dis'	dis't	10.36	10.75	11.08	11.36	11.61	11.84	12.34	12.80	13.23	10.36	10.91	11.34	11.68	11.95	12.17	12.53	12.78	12.99
2 short	con't										11.54	12.06	12.62	12.97	13.22	13.40	13.61	13.65	13.63
dis'	dis't	10.36	10.64	10.85	11.02	11.16	11.29	11.58	11.85	12.12									
2 long	con't	11.54	12.12	12.62	13.07	13.47	13.84	14.65	15.35	15.96									
dis'	dis't										10.36	10.92	11.58	12.04	12.41	12.71	13.24	13.55	13.73
1 long	con't										11.67	12.22	12.82	13.21	13.50	13.71	13.95	13.95	13.79
cont'	dis't	10.64	11.03	11.36	11.65	11.92	12.16	12.72	13.23	13.71	10.67	11.09	11.57	11.89	12.15	12.37	12.78	13.11	13.43
1 short	con't	11.67	12.25	12.76	13.22	13.64	14.03	14.90	15.65	16.32									
cont'	dis't	10.67	11.16	11.59	11.97	12.32	12.64	13.34	13.94	14.46	10.64	11.06	11.74	12.15	12.48	12.76	13.24	13.54	13.73
4 edge	-																		
dis'	dis't	10.88	11.38	11.82	12.22	12.59	12.93	13.69	14.34	14.91	10.88	11.41	11.86	12.22	12.52	12.77	13.22	13.51	13.70

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase (initially) when the l_y/l_x ratio increase. The shear increase to the maximum of 13.82 kN/m (when the l_y/l_x ratio reached 2.00) and starts to decrease thereafter. Comparing both spans, the shear force at long span are generally slightly smaller than those at short span.

300mm x 900mm BEAM: BENDING MOMENT

Table 4.17: Result of Bending Moment for Solid Slab Supported by Beam Size of 300 mm x 900 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	3.98	4.19	4.32	4.39	4.40	4.37	4.16	3.78	3.25	3.98	4.56	5.07	5.50	5.88	6.19	6.79	7.19	7.48
	mid	1.93	2.01	2.02	1.99	1.92	1.84	1.83	2.06	2.50	1.93	2.19	2.41	2.56	2.67	2.73	2.74	2.61	2.42
1 short	cont'	4.09	4.29	4.41	4.47	4.45	4.45	4.22	3.82	3.25	4.14	4.66	5.16	5.56	5.89	6.16	6.65	6.95	7.15
dis'	mid	1.71	1.76	1.75	1.72	1.68	1.67	1.83	2.24	2.85	1.98	2.24	2.50	2.68	2.81	2.90	2.98	2.92	2.79
1 long	cont'	4.14	4.41	4.61	4.75	4.84	4.88	4.84	4.61	4.20	4.09	4.69	5.23	5.71	6.12	6.47	7.15	7.61	7.91
dis'	mid	1.98	2.06	2.09	2.07	2.00	1.92	1.85	2.05	2.46	1.71	1.99	2.23	2.43	2.59	2.73	2.97	3.15	3.31
2 adj	cont'	4.24	4.50	4.69	4.81	4.89	4.92	4.84	4.56	4.10	4.24	4.80	5.27	5.78	6.16	6.47	7.04	7.38	7.55
dis'	mid	1.81	1.87	1.88	1.86	1.82	1.81	1.93	2.31	2.92	1.81	2.08	2.36	2.58	2.76	2.90	3.17	3.36	3.53
2 short	cont'	3.97	4.20	4.38	4.51	4.61	4.70	4.88	5.04	5.21	4.29	4.77	5.28	5.65	5.96	6.20	6.60	6.81	6.92
dis'	mid	1.49	1.49	1.44	1.36	1.24	1.10	0.83	0.75	0.58	2.06	2.33	2.62	2.82	2.97	3.08	3.22	3.22	3.16
2 long	cont'	4.29	4.62	4.88	5.08	5.23	5.34	5.44	5.33	5.02	3.97	4.34	4.72	4.97	5.15	5.27	5.35	5.21	4.94
dis'	mid	2.05	2.15	2.18	2.17	2.12	2.04	1.92	2.06	2.42	1.49	1.76	2.05	2.29	2.51	2.72	3.16	3.56	3.95
1 long	cont'	4.06	4.35	4.59	4.80	4.98	5.14	5.50	5.83	6.14	4.39	4.89	5.43	5.84	6.17	6.44	6.88	7.05	7.03
cont'	mid	1.63	1.65	1.62	1.55	1.45	1.33	1.02	0.87	0.89	1.88	2.18	2.48	2.72	2.92	3.08	3.38	3.59	3.76
1 short	cont'	4.39	4.71	4.96	5.15	5.29	5.38	5.44	5.26	4.86	4.06	4.38	4.75	4.99	5.15	5.26	5.30	5.14	4.86
cont'	mid	1.88	1.95	1.97	1.96	1.93	1.90	2.00	2.37	3.00	1.63	1.95	2.23	2.49	2.72	2.94	3.39	3.78	4.15
4 edge	cont'	4.16	4.53	4.85	5.14	5.41	5.67	6.25	6.78	7.27	4.16	4.52	4.81	5.02	5.16	5.25	5.27	5.09	4.80
dis'	mid	1.72	1.74	1.72	1.66	1.57	1.45	1.08	0.81	0.68	1.72	2.06	2.38	2.66	2.92	3.15	3.62	4.01	4.37

In long span, both hogging moment (initially) and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span. The hogging moment increase to the maximum of 4.40 kN.m/m (when the l_y/l_x ratio reached 1.40) and starts to decrease thereafter. The fluctuation of sagging moment in between l_y/l_x ratio of 1.20 to 1.75 will be explained in discussion (see section 4.4.1.3 and Figure 4.10).

In short span, both hogging moment and sagging moment (initially) increase when the l_y/l_x ratio increase. The hogging moment at the support are greater than the sagging moment at the mid span. The sagging moment increase to the maximum of 2.74 kN.m/m (when the l_y/l_x ratio reached 1.75) and starts to decrease thereafter.

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

300mm x 900mm BEAM: SHEAR FORCE

Table 4.18: Result of Shear Force for Solid Slab Supported by Beam Size of 300 mm x 900 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	11.96	12.29	12.50	12.65	12.75	12.82	12.96	13.10	13.26	11.96	12.84	13.54	14.08	14.46	14.74	15.09	15.16	15.12
	-																		
1 short	con't	12.09	12.41	12.62	12.76	12.87	12.95	13.12	13.28	13.45	12.12	12.92	13.65	14.13	14.50	14.75	15.06	15.12	15.07
dis'	dis't	11.46	11.76	11.95	12.07	12.16	12.22	12.33	12.42	12.52									
1 long	con't	12.12	12.52	12.83	13.06	13.25	13.41	13.77	14.09	14.39	12.09	13.02	13.77	14.37	14.84	15.20	15.76	16.03	16.14
dis'	dis't										11.46	12.21	12.78	13.20	13.49	13.68	13.86	13.83	13.72
2 adj	con't	12.24	12.64	12.93	13.17	13.37	13.54	13.92	14.27	14.61	12.24	13.09	13.84	14.41	14.84	15.17	15.67	15.89	15.96
dis'	dis't	11.64	12.02	12.30	12.52	12.70	12.86	13.20	13.50	13.79	11.64	12.32	12.89	13.29	13.56	13.74	13.92	13.91	13.85
2 short	con't										12.28	12.98	13.73	14.20	14.53	14.77	15.04	15.08	15.02
dis'	dis't	11.62	11.90	12.08	12.20	12.29	12.35	12.46	12.57	12.67									
2 long	con't	12.28	12.75	13.13	13.67	13.72	13.97	14.51	15.00	15.45									
dis'	dis't										11.62	12.79	13.05	13.53	13.90	14.17	14.56	14.71	14.76
1 long	con't										12.36	13.09	13.89	14.41	14.81	15.11	15.53	15.67	15.65
cont'	dis't	11.78	12.15	12.43	12.65	12.84	13.00	13.36	13.68	13.99	11.83	12.40	13.02	13.39	13.66	13.83	14.03	14.06	14.07
1 short	con't	12.36	12.83	13.21	13.53	13.82	14.07	14.65	15.17	15.64									
cont'	dis't	11.83	12.29	12.66	12.97	13.25	13.50	14.07	14.58	15.04	11.78	12.37	13.13	13.59	13.93	14.19	14.56	14.71	14.76
4 edge	-																		
dis'	dis't	11.94	12.39	12.77	13.09	13.38	13.65	14.25	14.79	15.28	11.94	12.65	13.20	13.64	13.96	14.02	14.56	14.70	14.75

In long span the shear force increase when the l_y/l_x ratio increase. In short span the shear force increase when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally slightly smaller than those at short span.

600mm x 300mm BEAM: BENDING MOMENT

Table 4.19: Result of Bending Moment for Solid Slab Supported by Beam Size of 600 mm x 300 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	2.43	2.88	3.35	3.83	4.33	4.83	6.11	7.37	8.60	2.43	2.50	2.55	2.60	2.64	2.69	2.83	2.95	3.03
	mid	2.20	2.65	3.08	3.48	3.85	4.18	4.89	5.43	5.87	2.20	2.15	2.04	1.89	1.73	1.55	1.13	0.79	0.56
1 short	cont'	2.62	3.08	3.56	4.05	4.54	5.04	6.27	7.46	8.61	2.55	2.52	2.46	2.38	2.29	2.19	1.95	1.76	1.59
dis'	mid	2.56	3.14	3.73	4.33	4.94	5.55	7.10	8.70	10.40	2.68	2.72	2.75	2.73	2.68	2.62	2.47	2.35	2.29
1 long	cont'	2.55	3.04	3.53	4.02	4.48	4.93	5.92	6.76	7.43	2.62	2.68	2.71	2.72	2.73	2.74	2.74	2.72	2.65
dis'	mid	2.67	3.11	3.51	3.86	4.17	4.42	4.83	4.97	4.96	2.56	2.55	2.49	2.41	2.31	2.20	1.93	1.73	1.62
2 adj	cont'	2.82	3.34	3.86	4.38	4.87	5.34	6.39	7.26	7.95	2.82	2.77	2.71	2.59	2.45	2.30	1.88	1.47	1.08
dis'	mid	2.85	3.41	3.95	4.46	4.95	5.42	6.52	7.54	8.55	2.87	2.93	3.02	3.03	3.01	2.98	2.89	2.82	2.82
2 short	cont'	1.24	1.44	1.64	1.86	2.09	2.33	2.97	3.67	4.43	2.81	2.68	2.57	2.38	2.16	1.93	1.31	0.76	0.16
dis'	mid	2.57	3.21	3.89	4.61	5.37	6.15	8.25	10.54	13.01	3.04	3.23	3.39	3.49	3.57	3.64	3.80	4.01	4.30
2 long	cont'	2.81	3.35	3.88	4.39	4.86	5.29	6.18	6.82	7.22	1.24	1.21	1.07	0.99	0.91	0.84	0.69	0.55	0.41
dis'	mid	3.04	3.44	3.77	4.02	4.21	4.31	4.25	3.90	3.81	2.57	2.58	2.60	2.58	2.56	2.55	2.56	2.66	2.85
1 long	cont'	1.30	1.55	1.81	2.08	2.36	2.65	3.42	4.25	5.12	3.04	2.92	2.74	2.48	2.19	1.86	0.96	0.06	-0.83
cont'	mid	3.05	3.68	4.33	4.99	5.67	6.36	8.14	9.96	11.86	3.12	3.32	3.50	3.61	3.68	3.73	3.83	3.93	4.10
1 short	cont'	3.04	3.63	4.19	4.72	5.22	5.67	6.60	7.25	7.65	1.30	1.09	0.99	0.85	0.73	0.63	0.44	0.31	0.21
cont'	mid	3.12	3.60	4.04	4.44	4.80	5.12	5.78	6.45	7.40	3.05	3.34	3.28	3.34	3.38	3.40	3.44	3.48	3.55
4 edge	cont'	1.53	1.86	2.21	2.56	2.92	3.29	4.24	5.24	6.32	1.53	1.29	1.05	0.85	0.67	0.51	0.25	0.11	0.03
dis'	mid	3.09	3.58	4.06	4.53	4.99	5.44	6.51	7.50	8.45	3.09	3.37	3.59	3.78	3.92	4.03	4.18	4.21	4.19

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are generally greater than the sagging moment at the mid span.

In short span, the hogging moment increase whereas the sagging moment decrease when the l_y/l_x ratio increase. The sagging moment at the support decrease and almost reaches 0 when the l_y/l_x ratio increase towards 2.25 (see 'flexible beam' in section 4.4.1.2 and Figure 4.7).

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

600mm x 300mm BEAM: SHEAR FORCE

Table 4.20: Result of Shear Force for Solid Slab Supported by Beam Size of 600 mm x 300 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	7.53	8.01	8.48	8.95	9.45	9.98	11.36	12.87	14.48	7.53	7.74	7.92	8.07	8.20	8.31	8.52	8.65	8.73
	-																		
1 short	con't	7.72	8.60	9.06	9.52	10.00	10.48	11.72	12.97	14.24	7.70	7.89	7.99	8.10	8.19	8.28	8.44	8.57	8.68
dis'	dis't	6.80	7.13	7.44	7.73	8.01	8.26	8.81	9.23	9.53									
1 long	con't	7.70	8.20	8.69	9.17	9.66	10.16	11.40	12.67	14.01	8.13	8.41	8.64	8.84	9.00	9.12	9.30	9.30	9.16
dis'	dis't										6.80	7.14	7.47	7.78	8.08	8.36	9.03	9.62	10.15
2 adj	con't	8.19	8.71	9.21	9.71	10.22	10.71	11.93	13.13	14.31	8.19	8.38	8.53	8.66	8.76	8.83	8.93	8.93	8.85
dis'	dis't	7.04	7.43	7.80	8.15	8.49	8.81	9.54	10.17	10.69	7.04	7.36	7.61	7.90	8.19	8.47	9.18	9.83	10.42
2 short	con't										7.90	7.97	8.08	8.14	8.19	8.24	8.35	8.49	8.64
dis'	dis't	7.36	7.66	7.94	8.19	8.41	8.60	8.94	9.08	9.10									
2 long	con't	7.90	8.45	8.96	9.47	9.97	10.46	11.61	12.74	13.89									
dis'	dis't										7.36	7.70	8.14	8.49	8.81	9.11	9.74	10.23	10.59
1 long	con't										8.08	8.15	8.22	8.25	8.26	8.26	8.22	8.16	8.09
cont'	dis't	7.58	7.98	8.36	8.71	9.03	9.33	9.96	10.41	10.68	7.29	7.50	7.76	8.03	8.32	8.62	9.40	10.17	10.87
1 short	con't	8.08	8.64	9.18	9.72	10.23	10.72	11.86	12.92	13.96									
cont'	dis't	7.29	7.72	8.13	8.53	8.90	9.25	10.04	10.71	11.29	7.58	7.76	8.23	8.53	8.82	9.11	9.75	10.27	10.68
4 edge	-																		
dis'	dis't	7.70	8.16	8.60	9.01	9.38	9.73	10.47	11.05	11.48	7.70	7.94	8.18	8.43	8.68	8.94	9.58	10.15	10.62

In long span, the shear force increase when the l_y/l_x ratio increase. In short span, the shear force increase when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally greater than those at short span.

900mm x 300mm BEAM: BENDING MOMENT

Table 4.21: Result of Bending Moment for Solid Slab Supported by Beam Size of 900 mm x 300 mm.

span		long span									short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	cont'	2.39	2.79	3.18	3.58	3.96	4.34	5.20	5.95	6.57	2.39	2.50	2.60	2.70	2.80	2.91	3.21	3.53	3.80
	mid	2.20	2.63	3.02	3.36	3.66	3.90	4.34	4.55	4.68	2.20	2.17	2.06	1.91	1.73	1.54	1.02	0.53	0.12
1 short	cont'	2.69	3.09	3.49	3.88	4.25	4.60	5.39	6.00	6.44	2.63	2.66	2.65	2.62	2.58	2.55	2.48	2.45	2.47
dis'	mid	2.19	2.68	3.17	3.64	4.09	4.53	5.57	6.61	7.71	2.58	2.62	2.64	2.59	2.51	2.41	2.14	1.88	1.66
1 long	cont'	2.63	3.10	3.55	3.98	4.38	4.74	5.46	5.91	6.10	2.69	2.82	2.92	3.00	3.07	3.14	3.29	3.43	3.51
dis'	mid	2.58	2.99	3.36	3.67	3.92	4.11	4.30	4.25	4.41	2.19	2.17	2.11	2.01	1.89	1.76	1.45	1.19	1.01
2 adj	cont'	3.02	3.50	3.97	4.41	4.82	5.18	5.90	6.33	6.45	3.02	3.06	3.07	3.04	2.97	2.89	2.65	2.40	2.14
dis'	mid	2.48	2.94	3.38	3.79	4.16	4.51	5.25	5.98	6.94	2.48	2.51	2.56	2.54	2.50	2.45	2.31	2.20	2.15
2 short	cont'	1.84	2.09	2.35	2.61	2.87	3.14	3.80	4.45	5.09	3.03	2.96	2.93	2.81	2.66	2.50	2.07	1.65	1.29
dis'	mid	1.94	2.45	2.97	3.51	4.05	4.59	5.99	7.44	8.95	2.87	3.02	3.13	3.17	3.19	3.19	3.18	3.19	3.29
2 long	cont'	3.03	3.57	4.08	4.56	4.99	5.36	6.05	6.39	6.38	1.84	1.87	1.77	1.72	1.65	1.59	1.41	1.23	1.02
dis'	mid	2.87	3.28	3.58	3.83	3.99	4.07	3.90	3.67	3.90	1.94	1.92	1.89	1.84	1.79	1.75	1.70	1.77	1.96
1 long	cont'	1.97	2.30	2.65	2.99	3.33	3.67	4.49	5.26	5.99	3.36	3.33	3.27	3.12	2.92	2.69	2.01	1.28	0.56
cont'	mid	2.38	2.87	3.37	3.85	4.34	4.81	5.95	7.03	8.13	2.67	2.82	2.94	3.01	3.05	3.07	3.10	3.16	3.27
1 short	cont'	3.36	3.93	4.46	4.95	5.39	5.78	6.47	6.77	6.71	1.97	1.80	1.72	1.58	1.43	1.30	1.00	0.76	0.58
cont'	mid	2.67	3.09	3.46	3.79	4.07	4.32	4.82	5.50	6.60	2.38	2.61	2.53	2.56	2.58	2.59	2.62	2.69	2.81
4 edge	cont'	2.33	2.80	3.27	3.75	4.22	4.69	5.81	6.88	7.91	2.33	2.12	1.88	1.64	1.39	1.16	0.68	0.35	0.15
dis'	mid	2.42	2.80	3.16	3.50	3.81	4.10	4.69	5.10	5.38	2.42	2.64	2.82	2.98	3.11	3.22	3.42	3.54	3.62

In long span, both hogging moment and sagging moment increase when the l_y/l_x ratio increase. The hogging moment at the support are greater than the sagging moment at the mid span.

In short span, the hogging moment increase whereas the sagging moment decrease when the l_y/l_x ratio increase. The sagging moment at the support decrease and almost reaches 0 when the l_y/l_x ratio increase towards 2.25 (see 'flexible beam' in section 4.4.1.2 and Figure 4.7).

Comparing both spans, the bending moment at long span are generally smaller than those at short span.

900mm x 300mm BEAM: SHEAR FORCE

Table 4.22: Result of Shear Force for Solid Slab Supported by Beam Size of 900 mm x 300 mm.

Span		Long span									Short span								
ly/lx ratio		1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25
Interior	con't	8.28	8.78	9.27	9.75	10.22	10.70	11.92	13.18	14.61	8.28	8.58	8.84	9.06	9.26	9.43	9.75	9.94	10.04
	-																		
1 short	con't	8.84	9.33	9.80	10.26	10.70	11.14	12.19	13.22	14.29	8.47	8.74	8.91	9.08	9.23	9.36	9.61	9.80	9.93
dis'	dis't	7.20	7.56	7.90	8.22	8.52	8.79	9.38	9.83	10.15									
1 long	con't	8.47	9.02	9.55	10.06	10.56	11.04	12.31	13.37	14.60	8.84	9.22	9.54	9.81	10.05	10.24	10.55	10.66	10.61
dis'	dis't										7.20	7.56	7.89	8.20	8.51	8.80	9.47	10.08	10.61
2 adj	con't	8.97	9.53	10.07	10.58	11.08	11.56	12.70	13.76	14.81	8.97	9.26	9.48	9.68	9.84	9.96	10.17	10.25	10.23
dis'	dis't	7.48	7.93	8.35	8.75	9.13	9.49	10.28	10.93	11.46	7.48	7.80	8.03	8.29	8.56	8.82	9.48	10.13	10.73
2 short	con't										8.72	8.85	9.02	9.12	9.21	9.29	9.45	9.60	9.76
dis'	dis't	7.78	8.11	8.41	8.68	8.91	9.12	9.48	9.64	9.57									
2 long	con't	8.72	9.33	9.90	10.45	10.97	11.47	12.64	13.75	14.88									
dis'	dis't										7.78	8.17	8.63	8.99	9.33	9.63	10.29	10.79	11.16
1 long	con't										8.94	9.09	9.26	9.36	9.43	9.47	9.52	9.52	9.47
cont'	dis't	8.02	8.46	8.87	9.25	9.60	9.91	10.55	10.97	11.17	7.83	8.01	8.23	8.44	8.66	8.91	9.57	10.29	11.00
1 short	con't	8.94	9.56	10.15	10.71	11.24	11.74	12.89	13.93	14.92									
cont'	dis't	7.83	8.35	8.86	9.33	9.78	10.21	11.16	11.96	12.62	8.02	8.21	8.69	8.98	9.26	9.53	10.15	10.69	11.14
4 edge	-																		
dis'	dis't	8.24	8.78	9.29	9.77	10.22	10.62	11.47	12.10	12.52	8.24	8.47	8.69	8.90	9.11	9.32	9.89	10.46	10.98

In long span, the shear force increase when the l_y/l_x ratio increase. The short span, the shear force increase when the l_y/l_x ratio increase. Comparing both spans, the shear force at long span are generally greater than those at short span.

4.3 Comparison between Supporting Beam Size

All the internal loading for 'interior panel' in Tables 3.5 to 3.8 and Tables 4.1 to 4.22 are summarized in Tables 4.23 to 4.25 in order to compare the results in two aspects:

(i) Compare the internal loadings as the beam size increase.

(ii) Compare the internal loadings as the l_y/l_x ratio increase.

Table 4.23 shows hogging moment of slab for interior panel. Table 4.24 shows sagging moment of slab for interior panel. Table 4.25 shows shear force of slab for interior panel. The results are shown in different colour with the respective reasons for better illustration purpose:

(i) Values calculated suggested by code of design BS8110 are shown in green.

(ii) Values obtained from Scia Engineer model are shown in blue.

(iii) Turning point of the increasing or decreasing trend when l_y/l_x ratio increase are shown in green.

(iv) Vertical colour gradient at the sides show the increasing or decreasing trend when the beam size increase.

(v) Horizontal colour gradient at the bottom show the increasing or decreasing trend when the l_y/l_x ratio increase.

In this study, the coefficients in Appendix B and Appendix C of BS8110 are converted into internal loading by using Equations 2.2 to 2.5 for comparison. The major reason of comparing 'values' instead of 'coefficients' is because comparing coefficients is straight forward and gives better visualisation especially when plotting graph (see Figures 4.18 to 4.41 in latter).

Table 4.23: Interior Panel – Bending Moment at Continuous Edge (Hogging Moment).

interior	span			long span									short span									
	ly/lx ratio			1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	
code: flat		cont'		4.12	4.99	5.94	6.97	8.08	9.27	12.62	16.49	20.87	4.12	3.78	3.53	3.35	3.21	3.09	2.89	2.75	2.65	
flat slab		cont'		2.08	3.20	4.52	6.02	7.71	9.57	14.97	21.41	28.88	2.08	1.44	0.86	0.34	-0.17	-0.61	-1.55	-2.35	-3.08	
code: solid		cont'		3.07	3.07	3.07	3.07	3.07	3.07	3.07	3.07	3.07	2.97	3.55	4.03	4.41	4.79	5.08	5.66	6.04	-	
150*300		cont'	My increase with beam size	2.33	3.07	3.90	4.83	5.85	6.96	10.05	13.58	17.50	2.33	2.10	1.87	1.64	1.42	1.22	0.76	0.33	-0.11	Mx increase with beam size
600*300		cont'		2.43	2.88	3.35	3.83	4.33	4.83	6.11	7.37	8.60	2.43	2.50	2.55	2.60	2.64	2.69	2.83	2.95	3.03	
900*300		cont'		2.39	2.79	3.18	3.58	3.96	4.34	5.20	5.95	6.57	2.39	2.50	2.60	2.70	2.80	2.91	3.21	3.53	3.80	
150*450		cont'		3.07	3.50	3.95	4.40	4.86	5.34	6.58	7.87	9.21	3.07	3.27	3.43	3.57	3.68	3.76	3.89	3.89	3.75	
200*400		cont'		2.91	3.36	3.83	4.31	4.81	5.33	6.67	8.08	9.54	2.91	3.06	3.17	3.27	3.34	3.40	3.49	3.48	3.36	
200*600		cont'		3.53	3.80	4.02	4.20	4.35	4.46	4.66	4.74	4.72	3.53	3.98	4.37	4.72	5.02	5.29	5.80	6.14	6.32	
250*500		cont'		3.30	3.60	3.87	4.10	4.31	4.50	4.89	5.18	5.36	3.30	3.66	3.98	4.27	4.51	4.73	5.16	5.45	5.60	
250*750		cont'		3.81	4.03	4.18	4.27	4.31	4.32	4.19	3.90	3.46	3.81	4.34	4.81	5.22	5.58	5.89	6.49	6.91	7.20	
300*600		cont'		3.56	3.80	3.98	4.11	4.20	4.25	4.22	4.03	3.68	3.56	4.03	4.45	4.81	5.13	5.41	5.96	6.36	6.64	
300*900		cont'		3.98	4.19	4.32	4.39	4.40	4.37	4.16	3.78	3.25	3.98	4.56	5.07	5.50	5.88	6.19	6.79	7.19	7.48	
				My increase with ratio									Mx increase with ratio									

In long span, the hogging moment at long span generally increase when the l_y/l_x ratio increase. As the beam size increase, then increment (when going across the l_y/l_x ratio)become smaller and up to a very stiff beam (such as in 200 mm x 600 mm, 300 mm x 600 mm, and 300 mm x 900 mm), the bending moment increase to a maximum and starts to decrease thereafter. This phenomenon occurs as the slabs starts to behave as a one-way beam when the l_y/l_x ratio is high, and the supporting beam is very rigid.

In short span, for the case of slab supported by very flexible beam, 150 mm x 300 mm, the hogging moment at short span decrease as l_y/l_x ratio increase. This can be explained as a flexible beam supported slab will show flat slab behaviour (refer to the first row in grey colour, where the trend as provided in code of design show decreasing trend as well, from 4.12 kN.m/m decreased to -3.08 kN.m/m) which is long span governing (high moment at long span and low moment at short span). As the beam size increases, the hogging moment at short span increase accordingly.

Table 4.24: Interior Panel – Bending Moment at Mid Span (Sagging Moment).

Interior	span			long span									short span									
Bending	ly/lx ratio			1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	
code: flat		mid		5.43	6.58	7.83	9.18	10.65	12.23	16.64	21.74	27.51	5.43	4.98	4.66	4.42	4.23	4.08	3.80	3.62	3.49	
flat slab		mid		3.13	4.01	4.97	6.01	7.12	8.30	11.47	14.93	18.72	3.13	2.89	2.67	2.49	2.35	2.27	2.27	2.54	3.02	
code: solid		mid		2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.30	2.68	3.07	3.35	3.55	3.83	4.22	4.60	-	
150*300		mid	My decrease with beam size	2.28	2.87	3.51	4.18	4.88	5.60	7.48	9.41	11.42	2.28	2.15	2.01	1.87	1.75	1.66	1.56	1.66	1.92	Mx increase with beam size
600*300		mid		2.20	2.65	3.08	3.48	3.85	4.18	4.89	5.43	5.87	2.20	2.15	2.04	1.89	1.73	1.55	1.13	0.79	0.56	
900*300		mid		2.20	2.63	3.02	3.36	3.66	3.90	4.34	4.55	4.68	2.20	2.17	2.06	1.91	1.73	1.54	1.02	0.53	0.12	
150*450		mid		2.03	2.36	2.66	2.93	3.18	3.42	3.92	4.34	4.72	2.03	2.07	2.07	2.03	1.97	1.90	1.73	1.65	1.68	
200*400		mid		2.08	2.44	2.78	3.10	3.40	3.68	4.30	4.83	5.31	2.08	2.09	2.05	1.99	1.90	1.81	1.60	1.48	1.49	
200*600		mid		1.99	2.17	2.29	2.37	2.39	2.39	2.31	2.40	2.66	1.99	2.16	2.27	2.32	2.33	2.31	2.15	1.95	1.77	
250*500		mid		2.03	2.26	2.44	2.58	2.67	2.72	2.73	2.75	2.93	2.03	2.15	2.21	2.22	2.19	2.13	1.92	1.69	1.50	
250*750		mid		1.96	2.07	2.12	2.12	2.08	2.01	1.98	2.21	2.64	1.96	2.19	2.37	2.48	2.55	2.58	2.51	2.32	2.10	
300*600		mid		1.99	2.15	2.25	2.30	2.30	2.26	2.21	2.41	2.84	1.99	2.18	2.31	2.38	2.41	2.39	2.24	2.00	1.75	
300*900		mid		1.93	2.01	2.02	1.99	1.92	1.84	1.83	2.06	2.50	1.93	2.19	2.41	2.56	2.67	2.73	2.74	2.61	2.42	
				My increase with ratio									Mx increase with ratio									

In long span, for the case of slab supported by very flexible beam, 150 mm x 300 mm, the sagging moment at long span increase tremendously as l_y/l_x ratio increase. This can be explained as it behaves as flat slab (refer to the first row in grey colour, where the trend as provided in code of design show tremendous increment trend as well, from 5.43 kN.m/m shoot up to 27.51 kN.m/m). As the beam size increase, the increment in moment is getting smaller. This shows that with increasing supporting beam size, the slab behaviour shift from two-way slab to one-way slab, which the moment generally distributed more to the short span and lesser was taken by long span. Thus, the sagging moment at long span decrease with the increase in supporting beam size.

In short span, the calculation from code of design (the two rows highlighted in grey colour) clearly show that as l_y/l_x ratio increase, the sagging moment decrease in flat slab whereas it increase in solid slab supported by beam.

The result above shows that in the case of slabs supported by relatively flexible beams (the beam size of 200 mm x 400 mm and smaller), the slabs show flat slab behaviour (moment decrease with increase in l_y/l_x ratio).

The relatively rigid beam supported slabs show increasing moment initially as l_y/l_x ratio increase. As the increment reached a turning point, the moment starts to decrease thereafter. Stiffer beam will shift the BMD upward therefore resulting a smaller sagging moment at mid span.

Table 4.25: Interior panel – Shear Force at Continuous Edge.

Interior	span			Long span									Short span									
Shear	ly/lx ratio			1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	1.00	1.10	1.20	1.30	1.40	1.50	1.75	2.00	2.25	
code: flat		cont'		19.17	21.09	23.00	24.92	26.84	28.76	33.55	38.34	43.13	19.17	19.17	19.17	19.17	19.17	19.17	19.17	19.17	19.17	
flat slab	Interior	cont'		3.12	4.19	5.32	6.57	7.83	9.18	12.55	16.06	19.69	3.12	2.38	1.78	1.30	0.92	0.87	0.75	0.59	0.41	
code: solid		cont'		10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	10.54	11.50	12.46	13.10	13.74	14.38	15.34	15.98	-	
150*300		cont'	V increase with beam size	4.95	5.58	6.30	7.04	7.84	8.68	10.88	13.24	15.62	4.95	4.70	4.60	4.32	4.20	4.11	4.02	4.06	4.17	V increase with beam size
600*300		cont'		7.53	8.01	8.48	8.95	9.45	9.98	11.36	12.87	14.48	7.53	7.74	7.92	8.07	8.20	8.31	8.52	8.65	8.73	
900*300		cont'		8.28	8.78	9.27	9.75	10.22	10.70	11.92	13.18	14.61	8.28	8.58	8.84	9.06	9.26	9.43	9.75	9.94	10.04	
150*450		cont'		8.07	8.48	8.87	9.29	9.70	10.13	11.28	12.51	13.84	8.07	8.39	8.64	8.83	8.98	9.11	9.29	9.35	9.35	
200*400		cont'		7.81	8.24	8.67	9.12	9.58	10.05	11.29	12.65	14.09	7.81	8.07	8.27	8.43	8.55	8.64	8.80	8.87	8.90	
200*600		cont'		10.45	10.80	11.09	11.33	11.53	11.73	12.22	12.75	13.33	10.45	11.13	11.67	12.09	12.41	12.64	12.95	13.01	12.97	
250*500		cont'		9.93	10.32	10.66	10.96	11.24	11.51	12.20	12.92	13.69	9.93	10.51	10.97	11.33	11.60	11.80	12.08	12.15	12.13	
250*750		cont'		11.45	11.79	12.02	12.20	12.33	12.43	12.67	12.91	13.19	11.45	12.27	12.93	13.43	13.81	14.08	14.43	14.51	14.47	
300*600		cont'		11.00	11.37	11.65	11.88	12.07	12.24	12.64	13.07	13.52	11.00	11.74	12.33	12.79	13.13	13.38	13.72	13.82	13.79	
300*900		cont'		11.96	12.29	12.50	12.65	12.75	12.82	12.96	13.10	13.26	11.96	12.84	13.54	14.08	14.46	14.74	15.09	15.16	15.12	
				Vz increase with ratio →									Vz increase with ratio →									

In long span, the shear force at long span increase when the l_y/l_x ratio increase. The shear force also generally increase when the supporting beam size increase, but at a slower rate. As it is shown in the table above, in the case of slab supported by 150 mm x 300 mm beam, the shear force increase from 4.95 kN/m to 15.62 kN/m (which is more than 3 times) but in stiffer beam, 300 mm x 900 mm, the shear force only increase from 11.96 kN/m to 13.26 kN/m (which is less than 20 % increment).

In short span, the shear force at short span generally increase when l_y/l_x ratio increase. As the supporting beam size increase, the shear force increase greatly.

4.4 Slab Behaviour

In this sub-chapter, the discussions are made based of the bending moment diagram (BMD) and shear force diagram (SFD) obtained from modelling in Scia Engineer.

4.4.1 Bending Moment

In this sub sub-chapter, the discussions are made based of the bending moment diagram (BMD) obtained from modelling in Scia Engineer. In Scia Engineer, the positive bending moment is shown in red, and blue for negative bending moment.

4.4.1.1 Comparison between Flat Slab System and Solid Slab System (Long Span Governing versus Short Span Governing)

Figure 4.2 shows bending moment of flat slab. Figures 4.3, 4.4 and 4.5 show bending moment of solid slab supported by flexible beam, moderate stiff beam, and stiff beam respectively.

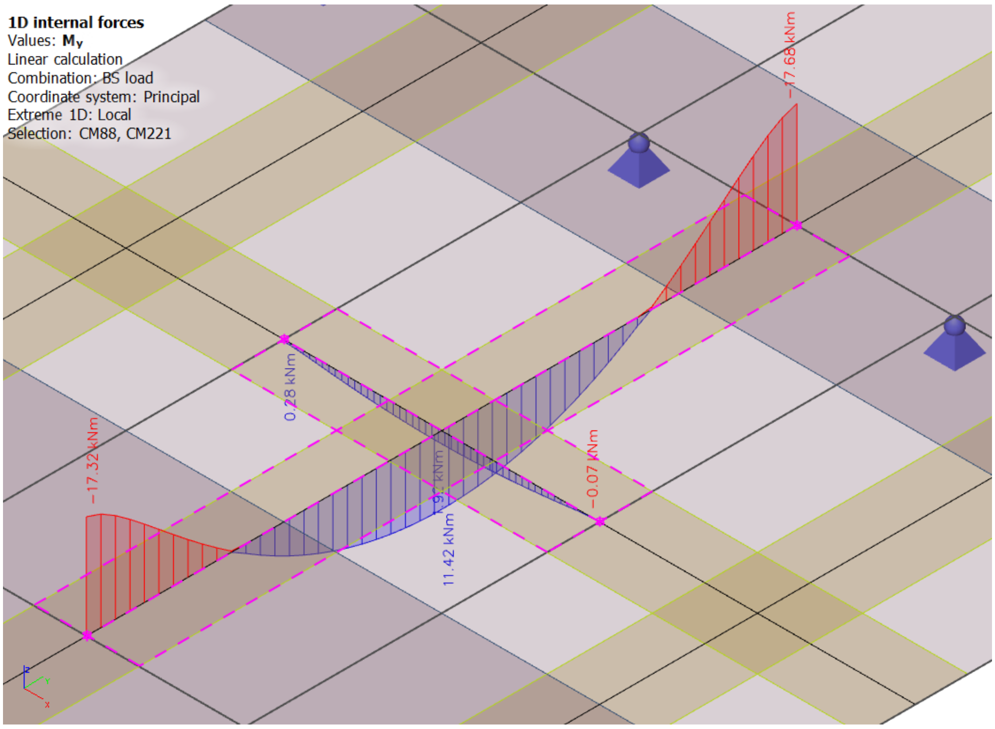

Figure 4.2: Bending Moment of Flat Slab.

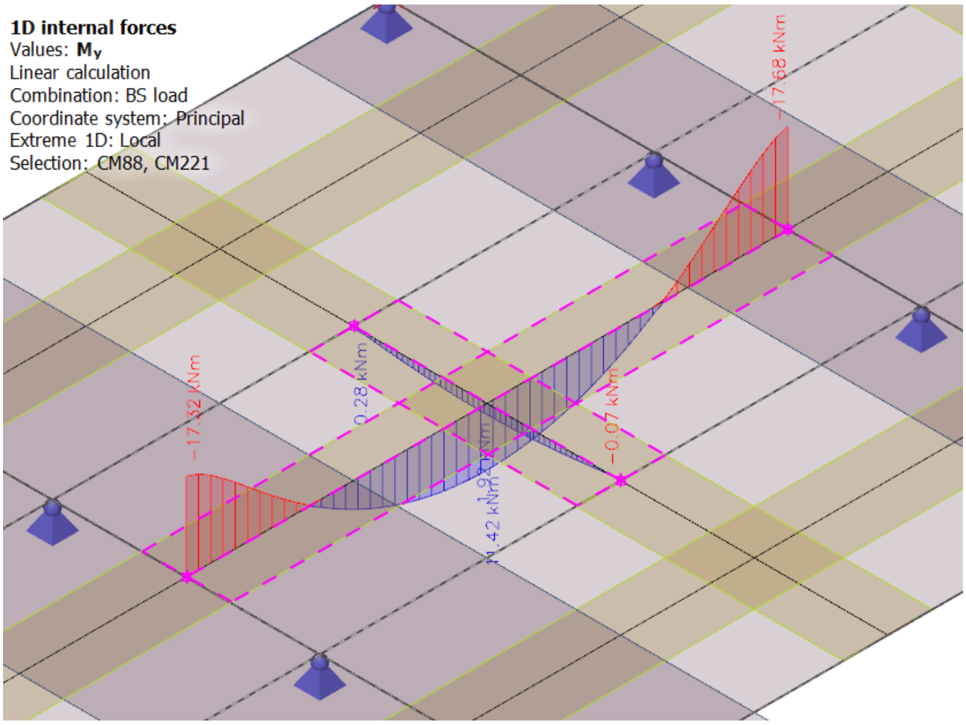

Figure 4.3: Bending Moment of Solid Slab Supported by Beam Size of 150 mm x 300 mm.

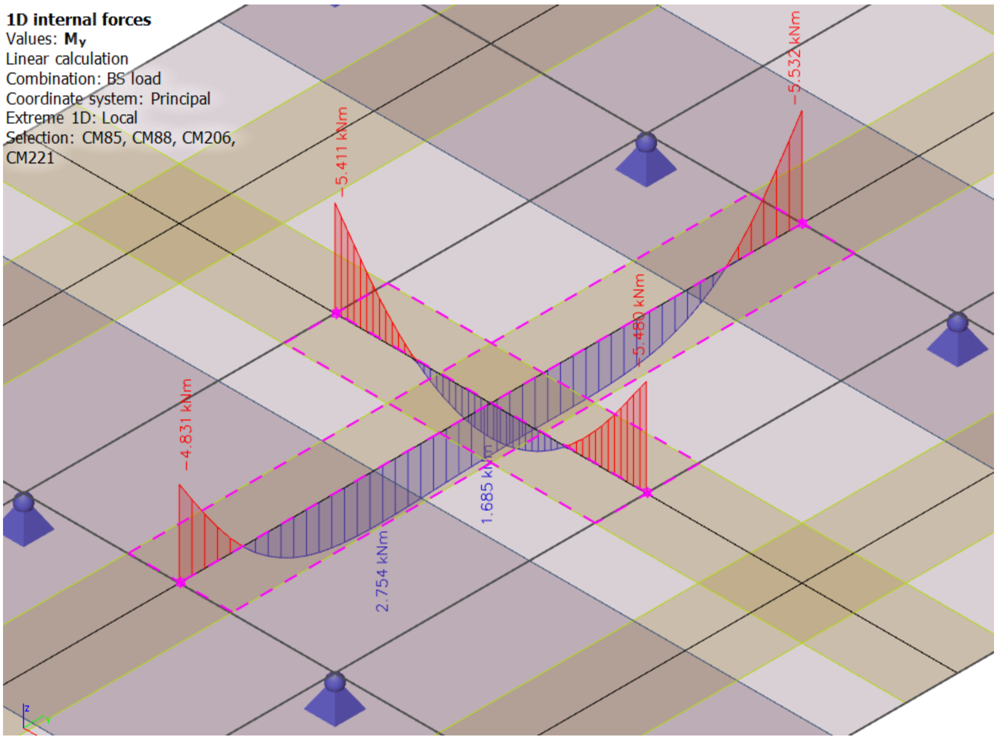

Figure 4.4: Bending Moment of Solid Slab Supported by Beam Size of 250 mm x 500 mm.

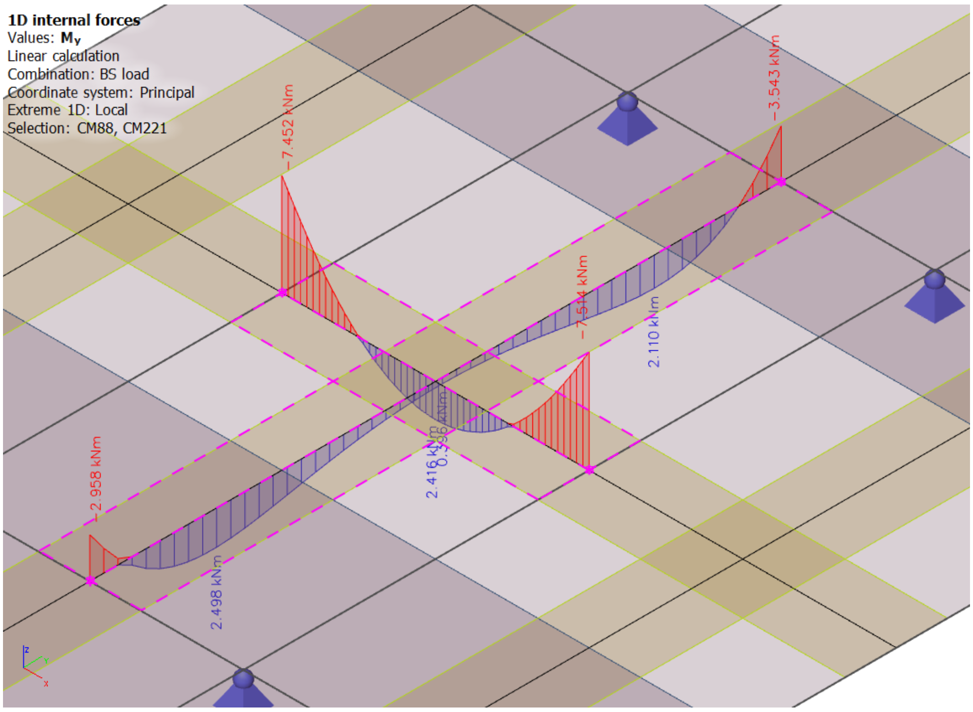

Figure 4.5: Bending Moment of Solid Slab Supported by Beam Size of 300 mm x 900 mm.

There are two span directions when discussing a rectangular slab, namely short span and long span. BS8110 clearly illustrate that in flat slab system, it is governed by long span whereas for solid slab system, it is governed by short span. The term ‘governing span’ refers to the span that takes more internal loading (which includes bending moment and shear force). Thus, in flat slab system, the long span has higher internal loading as compared to short span whereas in solid slab system, the short span has higher internal loading as compared to long span. The governing span can be explained in the following:

(i) Flat slab: As shown previously in Chapter 2.9 and Chapter 3.4.1.2, BS8110 clause 3.7.2.7, the formula for calculation of bending moment in flat slab shows that the l for flat slab calculation is dependent on the span considered, which means for short span, the length is referring to l_x ; whereas for long span, the length refers to l_y. Since the long span is definitely greater than the short span, therefore the resulting M_y (bending moment in long span) will definitely greater than M_x (bending moment in short span) as well. Moreover, the coefficients of bending moment M_y are generally greater than of M_x as provided in code of design, therefore, can say that the long span is governing in flat slab system.

(ii) Solid slab supported by beam: As shown previously in Chapters 2.8.1 and 3.4.1.1, BS8110 clause 3.5.3.4, in the formula for calculation of bending moment in solid slab, the only parameter that distinguish M_x and M_y is the coefficient, and generally the short span coefficients, β_{sx} are greater than the long span coefficient, β_{sy} according to code of design. Thus, the resulting M_x will be greater than M_y, which can say that the short span is governing in solid slab system.

Figures 4.2 to 4.5 show slab panels of l_y/l_x ratio 2.25, with increasing supporting beam size. Result shows that in flat slab (Figure 4.2), the M_y are far greater than M_x. Figure 4.3 shows that even though when there are beams supporting the slab at edges (non-flat slab) the M_y are much greater than M_x too. The explanation of this phenomenon is that the supporting beam size of 150 mm x 300 mm is too flexible (or also known as insufficient strong) to act as a support that take up the massive bending moment. Thus, the slab in Figure 4.3 still shows flat slab behaviour despite it is supported by beam at the edges.

As the supporting beam size increase to 250 mm x 500 mm (Figure 4.4), the short span moment shoots up and starts to overtake the long span moment. As the supporting beam size further increase to 300 mm x 900 mm (Figure 4-5), long span moment further decreases and shows a ‘W-shape’ bending moment diagram (which indicates that the sagging moment at the centre is almost zero). In this case, there are two greatest sagging moment at certain distance offset from the support instead of one point at the centre of long span.

As a nutshell, the results obtained from both BS8110 and Scia Engineer show that in flat slab system, it is governed by long span whereas in the solid slab system, the governing span is short span. Models in Scia Engineer further show that when the supporting beam is too flexible, the slab tend to exercise a flat slab behaviour whereas the stiffer supporting beam will lead the slab to behave as solid slab (effect of flexible beam and stiff beam on solid slab will be discussed more in section 4.4.1.2).

4.4.1.2 Comparison between Slab Supported by Flexible Beam and Stiff Beam

Another comparison was made between slab panels supported by beam size of 150 mm x 300 mm (in Figure 4.7) and 300 mm x 900 mm (in Figure 4.8) in term of stiffness. Figure 4.6 shows settlement of short span in flat slab. Figure 4.7 and Figure 4.8 show the bending moment of solid slab supported by flexible beam and stiff beam respectively. The stiffness of a beam, k is a parameter to measure 'how rigid the structure is' as shown in Equation 4.1.

$$k = \frac{\alpha EI}{L} \quad (4.1)$$

where

k = stiffness of beam (force required to cause a unit length of deflection), N/mm

α = constant depending on the support condition

E = modulus of elasticity, N/mm

I = moment of inertia, mm^4

L = length of beam, mm

Since all the beams are of same support condition and model with same material (refer to modulus of elasticity), therefore the stiffness is simplified as $k = \frac{I}{L}$. The stiffness for 150 mm x 300 mm beam will be 3.375 x 10^{-4} mm per unit of L whereas the stiffness for 300 mm x 900 mm beam is 182.25 x 10^{-4} mm per unit of L. This clearly shows that the 150 mm x 300 mm beam is much more flexible than the 300 mm x 900 mm beam as it has lower moment of inertia, I. In another word, 300 mm x 900 mm beam is much stiffer or rigid than the 150 mm x 300 mm beam. The stiffness of 150 mm x 300 mm beam will be further reduced if the length of consideration increase (when the l_y/l_x ratio increase).

In the case of slab supported by flexible beam (Figure 4.7), the beam support is insufficiently strong that causes settlement, especially in short span (as flexible beam results a flat slab behaviour which is long span governing). Figure 4.6 shows the settlement of flat slab whereas Figure 4.7 shows the weak support and settlement of solid slab.

In the case of slab supported by stiff beam, the beam support is so rigid that cause most of the bending moment are taken by the support. This will cause the BMD to shift up, which mean experiencing high hogging moment at the continuous edge and relatively low sagging moment at the mid span. In certain cases, this will significantly skew the BMD towards the support and resulting almost zero hogging moment at the centre such as case in Figure 4.8.

The other phenomena resulted from stiff beam is that it will cause notable hogging moment at discontinuous edge (see Figure 4.9). Figure 4.9 shows non-zero hogging moment at discontinuous edge of solid slab supported by stiff beam. However, the code of design assumed that the discontinuous edge act as a pin support which is free to rotate and does not take any hogging moment. Thus, a remark can be drawn is that the zero hogging moment assumption at the discontinuous edge should not be applied when a slab is supported by very rigid beam.

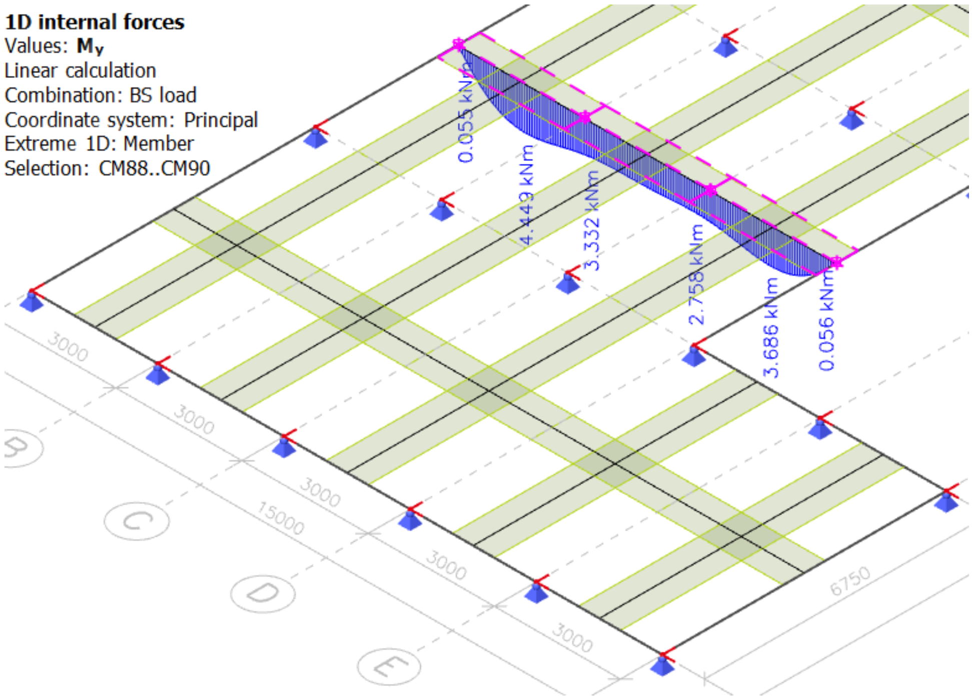

Figure 4.6: Settlement in Short Span of Flat Slab.

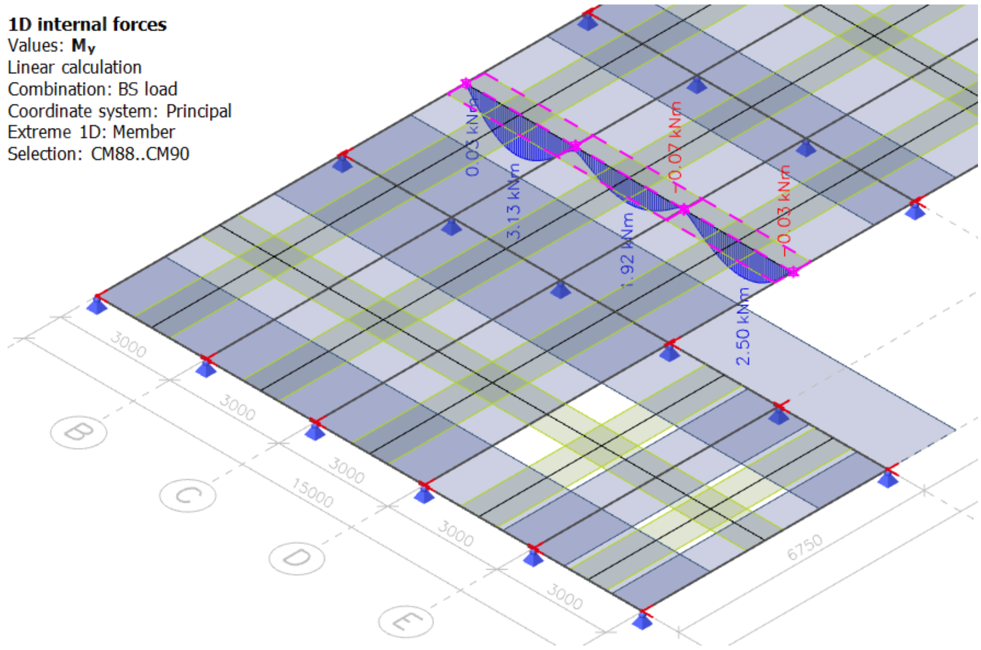

Figure 4.7: Short Span of Solid Slab Supported by Beam Size of 150 mm x 300 mm (Flexible Beam).

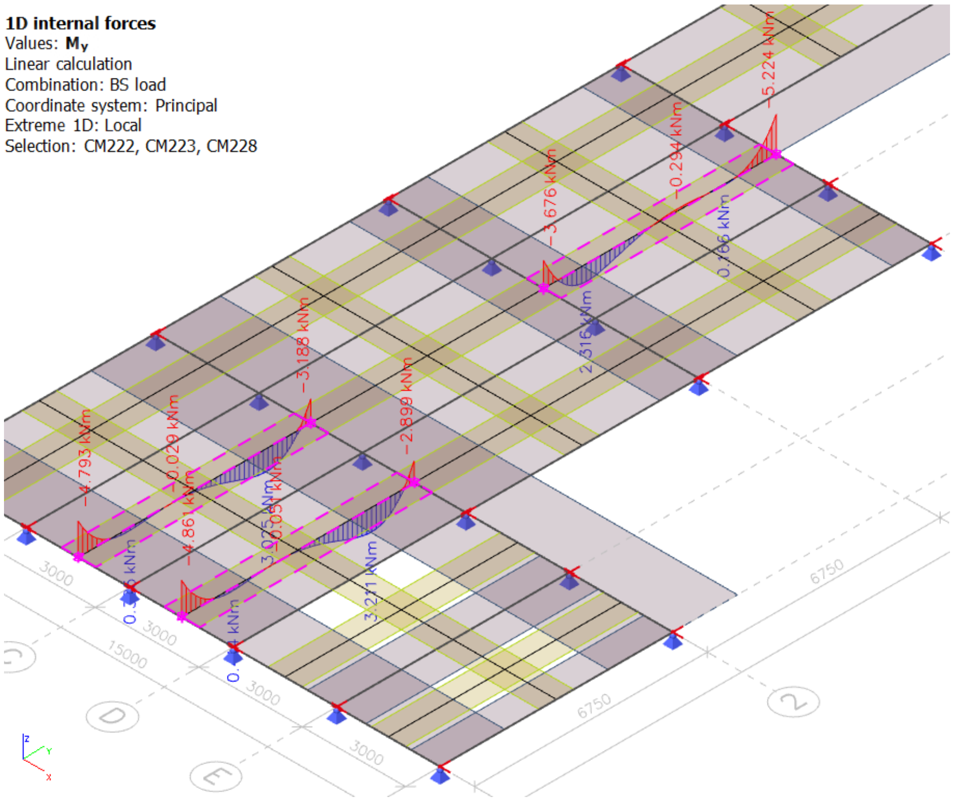

Figure 4.8: Skewed Bending Moment for Slab Panels Supported by Beam Size of 300 mm x 900 mm (Rigid Beam).

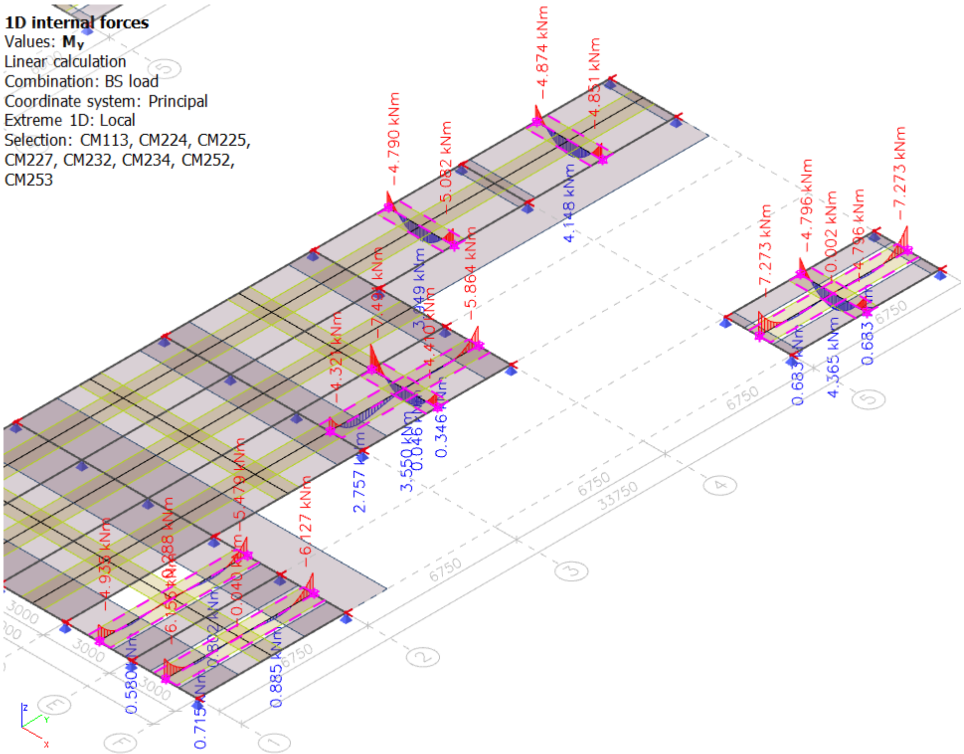

Figure 4.9: Discontinuous Edge with Notable Hogging Moment.

4.4.1.3 Comparison between One-way Slab and Two-way Slab

Figures 4.2 to 4.5 also show that the stiffer the supporting beam, the more significant the behaviour of one-way slab which result a ‘W-shape’ bending moment diagram.

The ‘W-shape’ BMD in Figure 4.10 clearly shows the one-way slab behaviour when the l_y/l_x ratio is high. In this case, the maximum value of sagging moment is taken as average of two extreme values instead of taking the sagging moment value at the mid span for conservative concern. This is also the reason of fluctuating results in Tables 4.13, 4.15 and 4.17. This fluctuating usually only happens in long span sagging moment with high l_y/l_x ratio.

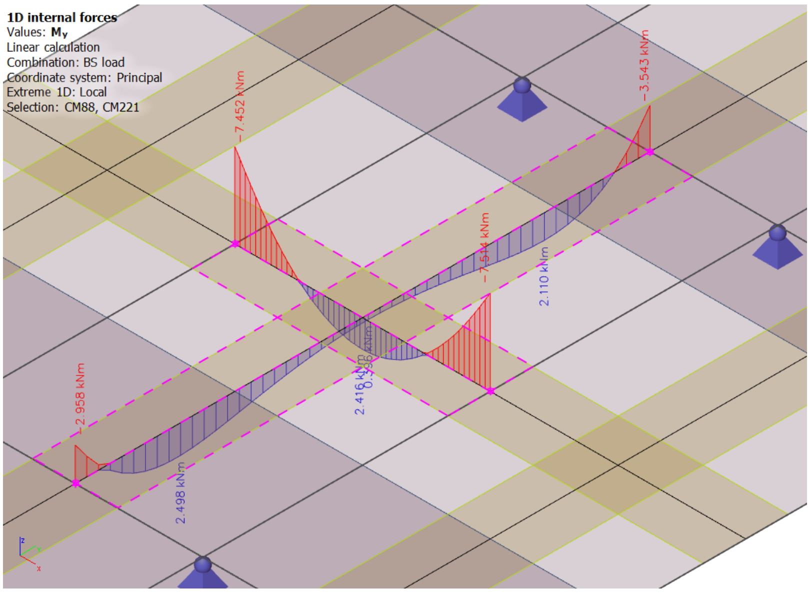

Figure 4.10: ‘W-shape’ Bending Moment when the One-way slab is Supported by Stiff Beam.

4.4.1.4 Different Results Obtained despite Same Support Condition

Figure 4.11 shows 3 pieces of slan panel with same support condition: one short edge discontinuous. The area load on all slabs are the exactly the same as well. However, the hogging moment at the continuous edges are of large differences which are 7.304, 8.633 and 9.869 kN.m/m respectively. The percentage in difference as high as = (9.869-7.304)/9.869 * 100% = 25.99 % (which is more than a quarter).

The reason behind this could be the difference in total number of continuous slab panels in the perpendicular direction of consideration. Considering long span moment, the perpendicular direction is x-direction. Looking into the x-direction, Panel A has only three continuous panels in the x-direction; whereas Panel B and C have five continuous panels in x-direction. The results turn out to be those slabs (Panel B and C) with more continuous panels (of five panels) will experience smaller bending moment as compared to slab (Panel A) with less continuous panels (of three panels). This is could be one of the reason that clause 3.7.2.7 BS8110 (which has been discussed previously in Chapter 2.9) requires at least 3 rows of panels in the direction being considered, so that the moment will not be over estimated. However, this provision is made only for flat slab and not included in solid slab. Thus, this might be the unforseen condition and limitation for solid slab in BS8110.

Looking in to the slab panel A and B, the difference between moment is significant too even though they are symmetric in the long span direction (y-direction).

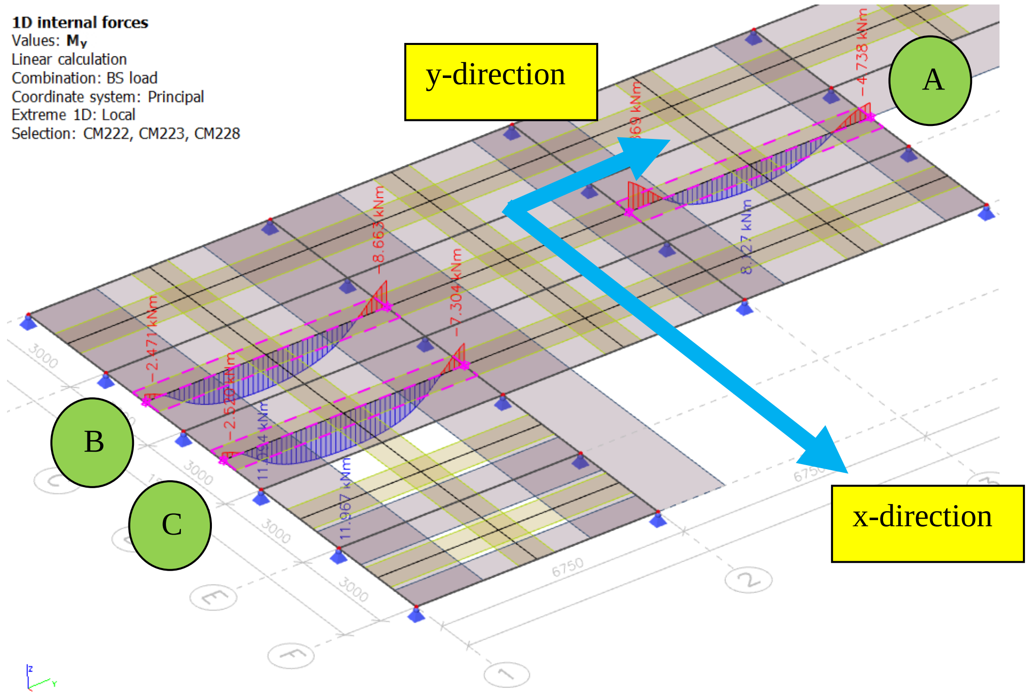

Figure 4.11: Slab Panels Supported by Beam Size of 150 mm x 300 mm.

4.4.2 Shear Force

In this sub sub-chapter, the discussions are made based on shear force diagram (SFD) obtained from modelling in Scia Engineer.

4.4.2.1 Comparison between Slab Supported by Flexible Beam and Stiff Beam

In flat slab, majority of the shear force is taken by the long span, which the shear force in short span is near zero (Figure 4.12). As the support beam size increase (such as 150 mm x 300 mm shown in Figure 4.13) small portion of the shear force in long span begin to shift to short span. Eventually, when the slab is supported by very stiff beam (such as 300 mm x 900 mm in Figure 4.14) the shear force tends to be evenly distributed among both spans (short span and long span).

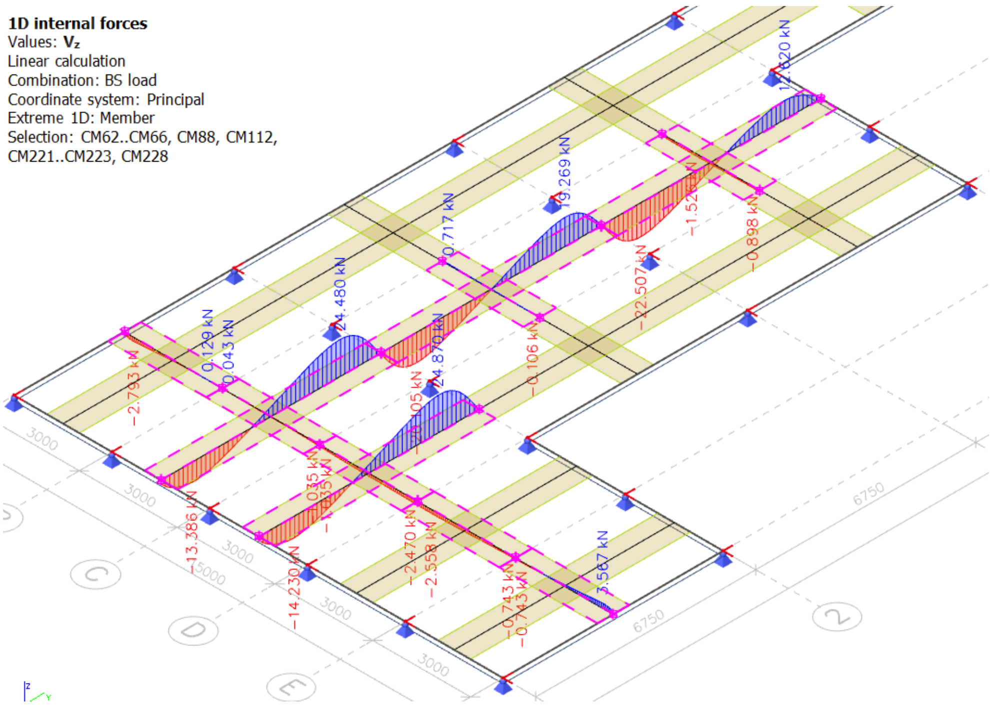

Figure 4.12: Flat Slab with Long Span Taking Majority of Shear Force.

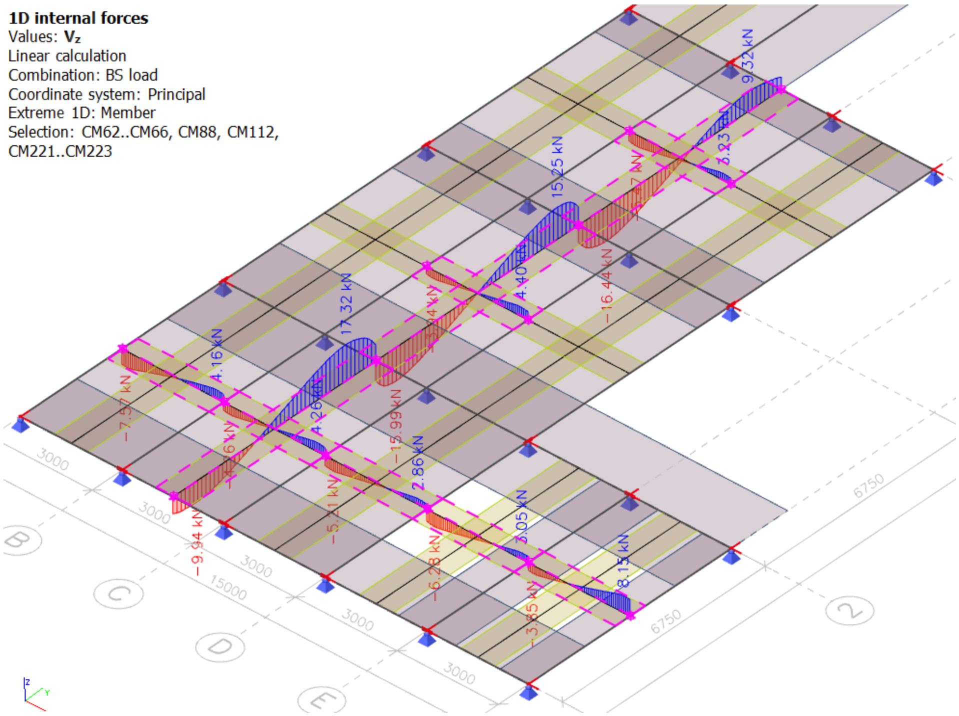

Figure 4.13: Solid Slab Supported by 150mm x 300mm Beam with Some Portion of Shear Force Distributed to Short Span.

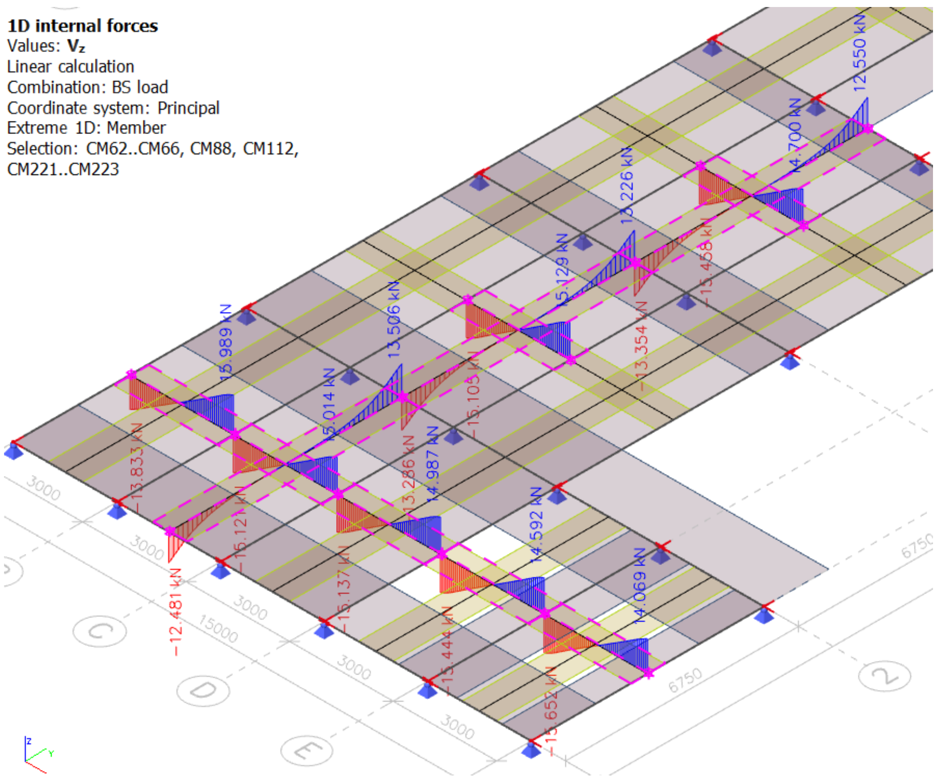

Figure 4.14: Solid Slab Supported by 300mm x 900mm Beam with Shear Force Evenly Distributed among Both Spans.

4.4.2.2 Location of Vertical Support

The location of maximum shear force also indicates the location of vertical support. Ideally in solid slab, the maximum shear force should be aligned with the edge beams as the beams are designed intentionally to support the slabs. In the case of flat slab (see Figure 4.15), the result shows that there are only two vertical supports at near outer edge and merely zero support in the interior panel. In the case of slab supported by flexible beam (see Figure 4.16), the location of vertical supports are slightly offset from the beams (tend to behave like flat slab). In the case of slab supported by rigid beam (see Figure 4.17), the location of four vertical supports are aligned with the beams.

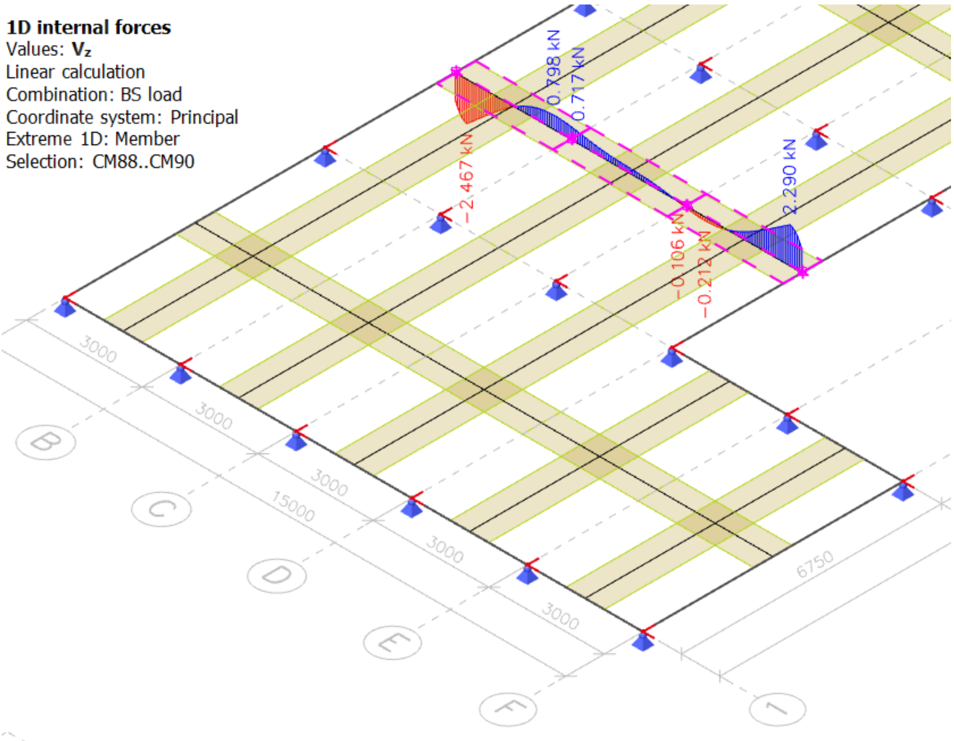

Figure 4.15: Flat Slab with Only Two Supports at the Outside Edges.

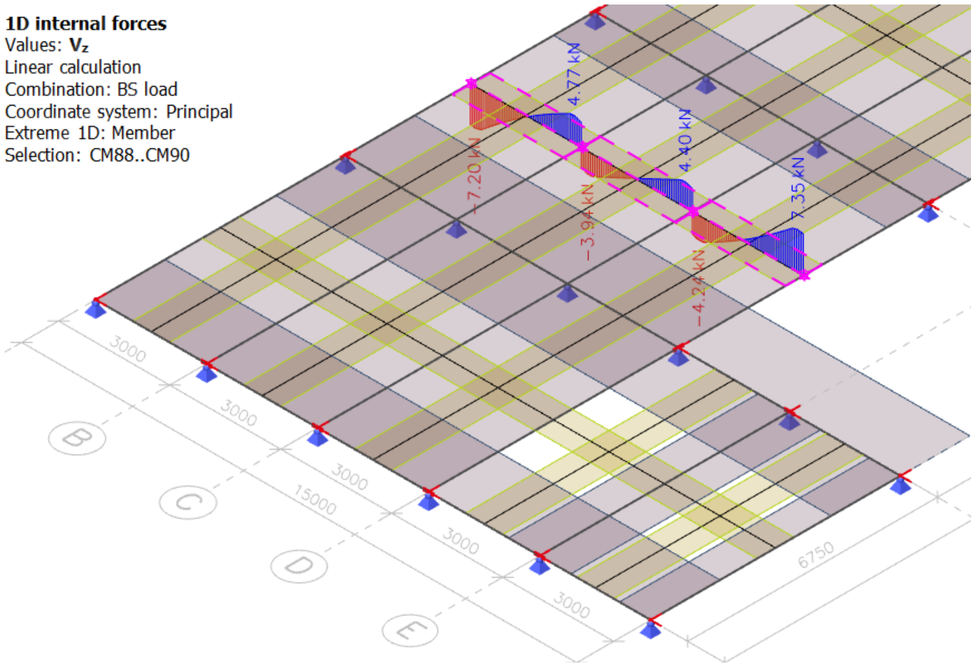

Figure 4.16: Slab Supported by Flexible Beam with Maximum Shear Slightly Offset from the Supporting Beam.

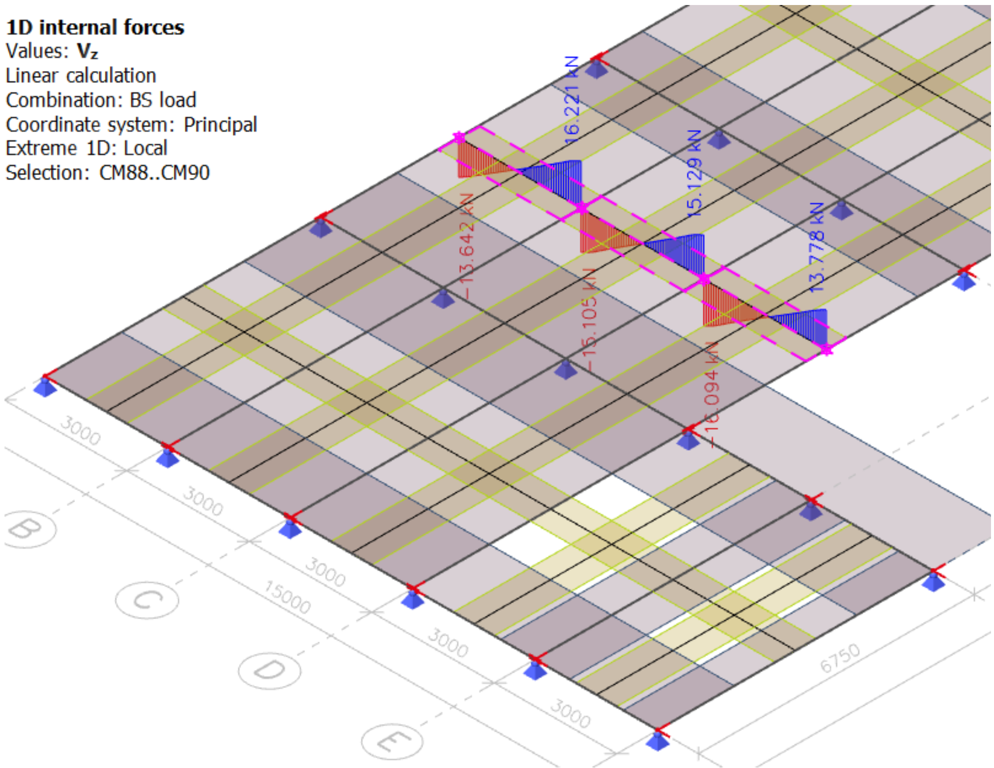

Figure 4.17: Slab Supported by Stiff Beam with Maximum Shear Aligned with the Edge of Slab.

4.5 Comparison between BS8110 and Scia Engineer

The results in Tables 4.23 to 4.25 are plotted into line graphs (as shown in Figures 4.18 to 4.41) for better visual illustration and comparison. In this sub-section, the 6 internal loading for 'interior panel' will be discussed:

(i) Hogging moment at long span.
(ii) Hogging moment at short span.
(iii) Sagging moment at long span.
(iv) Sagging moment at short span.
(v) Shear force at long span.
(vi) Shear force at short span.

Which in each internal loading, 4 combinations of beam size are further grouped and plotted into graphs for comparison:

(i) Combination 1: Slab of all eleven beam sizes
(ii) Combination 2: Slab of 4 beam sizes with the depth is two times of the width which are 150 mm x 300 mm, 200 mm x 400 mm, 250 mm x 500 mm, and 300 mm x 600 mm.
(iii) Combination 3: Slab of 4 beam sizes with the depth is three times of the width 150 mm x 450 mm, 200 mm x 600 mm, 250 mm x 750 mm, and 300 mm x 900 mm
(iv) Combination 4: Slab of 2 pairs of beams with same size but different orientation, 300mm x 600mm, 600mm x 300mm, 300mm x 900mm, and 900mm x 300mm.

Noted that the internal loading of flat slab from Tables 3.5 and 3.6 are plotted as 'code: flat'; whereas the internal loading of solid slab from Tables 3.7 and 3.8 is plotted as 'code: solid' in the graphs shown in Figures 4.18 to 4.41. The two lines, namely 'code: flat' and 'code: solid' are the control values stipulated in BS8110. The 'control value' means the values estimated according to code of design, BS8110. Thus, for those experimental values far much greater than control values, it is said to be underestimated, and vice versa for overestimated values.

4.5.1 Hogging Moment at Long Span

This sub-section compares hogging moment of slabs at long span of 4 combinations.

(i) Combination 1:

Figure 4.18 shows a line graph of with the bending moment on y-axis and l_y/l_x ratio on the x-axis. Each line represents a set of results from same supporting beam size, which all the plotted beam sizes are shown in legend on the right-hand side of the graph. The legend labels the beam size from top to bottom with the highest to lowest bending moment.

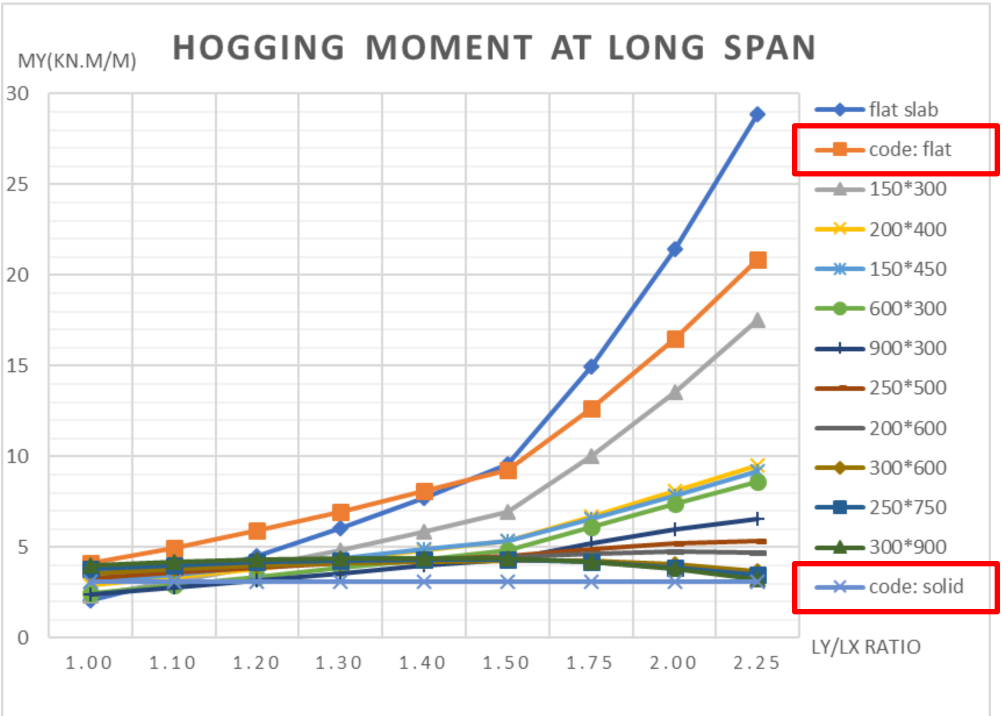

Figure 4.18: Hogging Moment at Long Span for Combination 1.

The control values for flat slab (grey line with triangular coordinate), namely 'code: flat' line shows an increasing trend when the l_y/l_x ratio increase. The control values for solid slab (light blue line with cross coordinate), namely 'code: solid' line is constant across all l_y/l_x ratio.

This graph shows that gradient of the lines changes after the l_y/l_x ratio of 1.50. This is contributed by irregular interval in the x axis: l_y/l_x ratio (the interval is 0.10 at the beginning and increased to 0.25 after the ratio of 1.50) which are provided in code of design.

The flat slab, and the solid slabs supported by relatively flexible beams, namely beam size of 150 mm x 300 mm, 200 mm x 400 mm, 150 mm x 450 mm, 600 mm x 300 mm, 900 mm x 300 mm, 250 mm x 500 mm, and 200 mm x 600 mm show flat slab behaviour, which the moment increase accordingly to the l_y/l_x ratio.

The remaining slabs supported by relatively rigid beam, namely beam size of 300 mm x 600 mm, 250 mm x 750 mm and 300 mm x 900 mm show solid slab behaviour, which the moment is relatively stagnant despite the increase of l_y/l_x ratio.

Figure 4.18a shows a graph that limits the value of y-axis to 10 kN.m/m (zoomed view) in order to show the congested part within l_y/l_x ratio of 1.00 to 1.50 in a clearer manner.

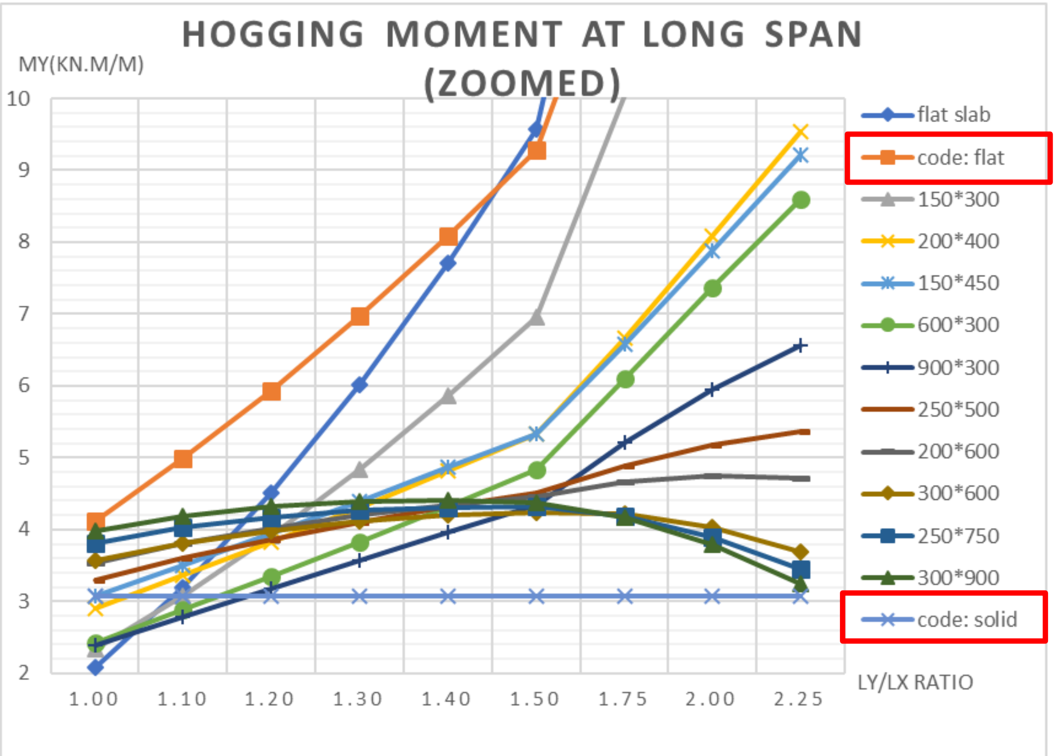

Figure 4.18a: Hogging Moment at Long Span for Combination 1 (Zoomed Version).

The 'code: flat' adequately estimates the bending moment in flat slab when the l_y/l_x ratio is less than 1.50 and underestimate the values beyond 1.50.

The ‘code: solid’ is basically a straight line of 3 kN.m/m which stay at the bottom of the graph, therefore it is underestimating most of the bending moment in solid slab.

(ii) Combination 2:

Figure 4.19 shows the result for slab supported by beam sizes which the depth is two times of the width. Slabs supported by beam size of 150 mm x 300 mm and 200 mm x 400 mm show flat slab behaviour. As the beam size increase, the slabs behaves like solid slab and only experience minute increment in moment when l_y/l_x ratio increases.

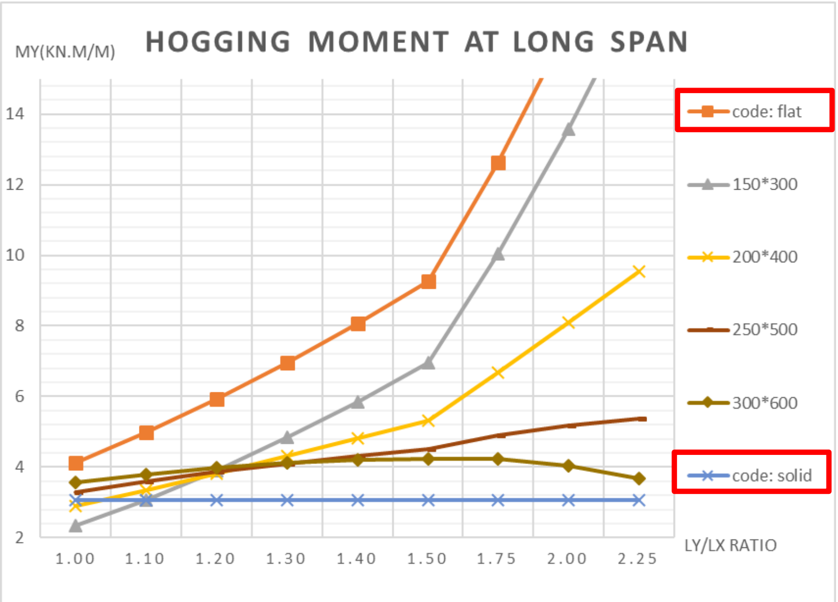

Figure 4.19: Hogging Moment at Long Span for Combination 2.

(iii) Combination 3:

Figure 4.20 shows the result for slab supported by beam sizes which the depth is three times of the width. Slab supported by beam size of 150 mm x 450 mm shows flat slab behaviour. As the size increase, the slabs behaves like solid slab and the increment in moment is small when l_y/l_x ratio increases. In the case of 250 mm x 750 mm and 300 mm x 900 mm beam, the moment even start to decrease when the l_y/l_x ratio exceed 1.50. This can be explained as the stiffer beams cause the slab to behave like one-way slab despite generally the definition of one-way slab is with l_y/l_x ratio of 2.

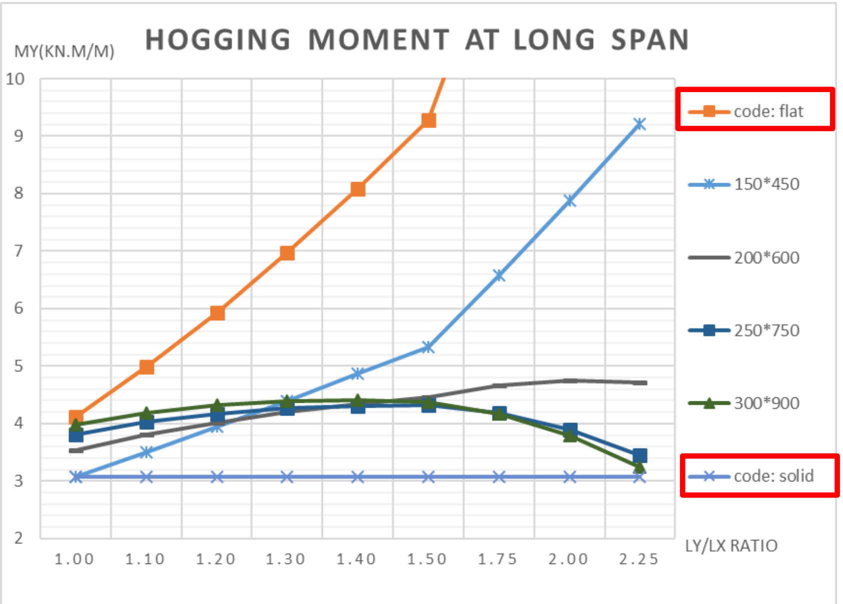

Figure 4.20: Hogging Moment at Long Span for Combination 3.

(iv) Combination 4:

Figure 4.21 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation, namely 300 mm x 600 mm compared with 600 mm x 300 mm, and 300 mm x 900 mm compared with 900 mm x 300 mm.

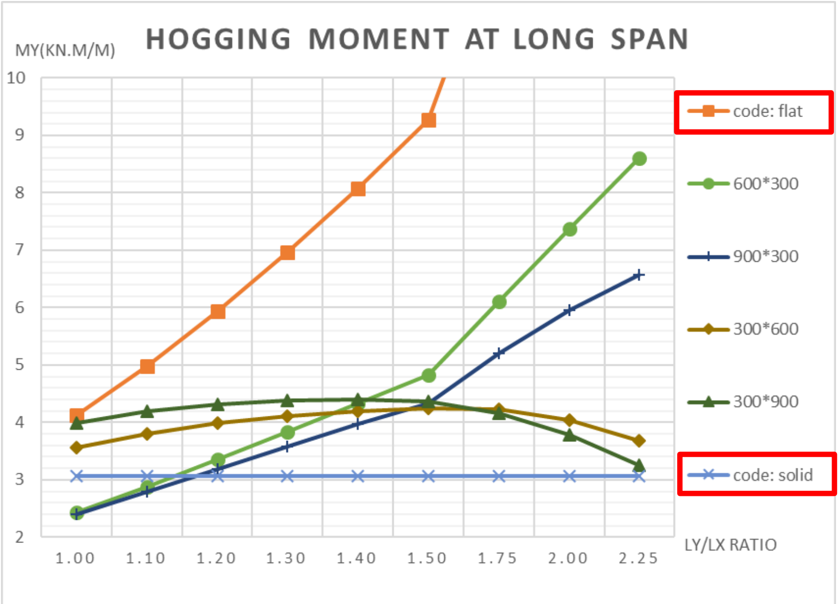

Figure 4.21: Hogging Moment at Long Span for Combination 4.

If the width of 300mm is denoted as ‘W’, then 300 mm x 600 mm and 600 mm x 300 mm are beams of ‘W x 2W’ and ‘2W x W’. Similarly, 300 mm x 900 mm and 900 mm x 300 mm are the relationship of ‘W x 3W’ and ‘3W x W’. Beam size of ‘2W x W’ and ‘3W x W’ are in fact shallow beams which there is no provision made in both code of design, BS8110 and EN1992.

Slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm (also termed as shallow beam) show flat slab behaviour. On the other hand, the slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm (also known termed as normal-depth beam) behaves like solid slab which the increment in moment is small when l_y/l_x ratio increases. In the case of slab support by normal-depth beam, the moment even start to decrease when the l_y/l_x ratio exceed 1.50. This can be explained as the stiffer beams cause the slab to behave like one-way slab despite generally the definition of one-way slab is with l_y/l_x ratio of 2. Besides that, they generally experience greater moment than their rotated pair, beam of 600 mm x 300 mm and 900 mm x 300 mm for l_y/l_x ratio less than 1.50.

4.5.2 Hogging Moment at Short Span

This sub-section compares hogging moment of slabs at short span of 4 combinations.

(i) Combination 1:

In Figure 4.22, the control values, ‘code: flat’ line shows a decreasing trend when the l_y/l_x ratio increase. The control values for solid slab, ‘code: solid’ line shows an increasing trend when the l_y/l_x ratio increase.

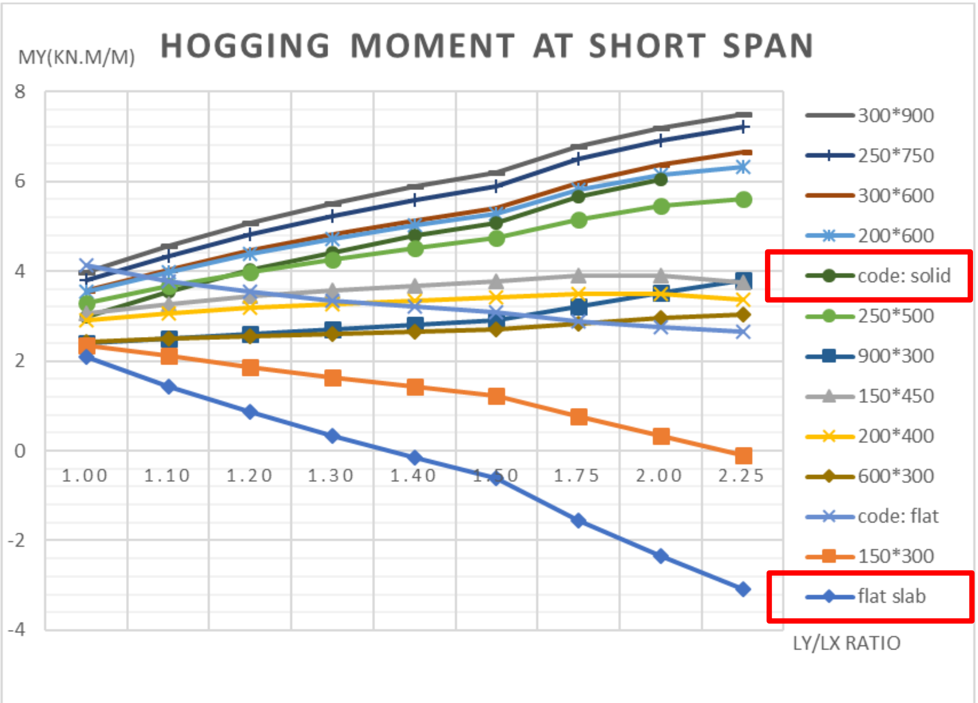

Figure 4.22: Hogging Moment at Short Span for Combination 1.

Only flat slab and slab supported by 150 mm x 300 mm show flat slab behaviour (decreasing moment with l_y/l_x ratio increase). All the remaining beams give solid slab behaviour.

The ‘code: flat’ overestimates the hogging moment in flat slab. The ‘code: solid’ underestimate the hogging moment of solid slabs supported by stiff beam (300 mm x 900 mm, 250 mm x 750 mm, 300 mm x 600 mm, and 200 mm x 600 mm), adequately estimate 250 mm x 500 mm, and overestimate those solid slabs supported by relatively flexible beam (namely 900 mm x 300 mm, 150 mm x 450 mm, 200 mm x 400 mm, 600 mm x 300 mm and 150 mm x 300 mm).

(ii) Combination 2:

Figure 4.23 shows the result for slab supported by beam sizes which the depth is two times of the width. Slab supported by beam size of 150 mm x 300 mm shows flat slab behaviour. As the beam size increase, the slabs behaves like solid slab and experience increment in moment when l_y/l_x ratio increases.

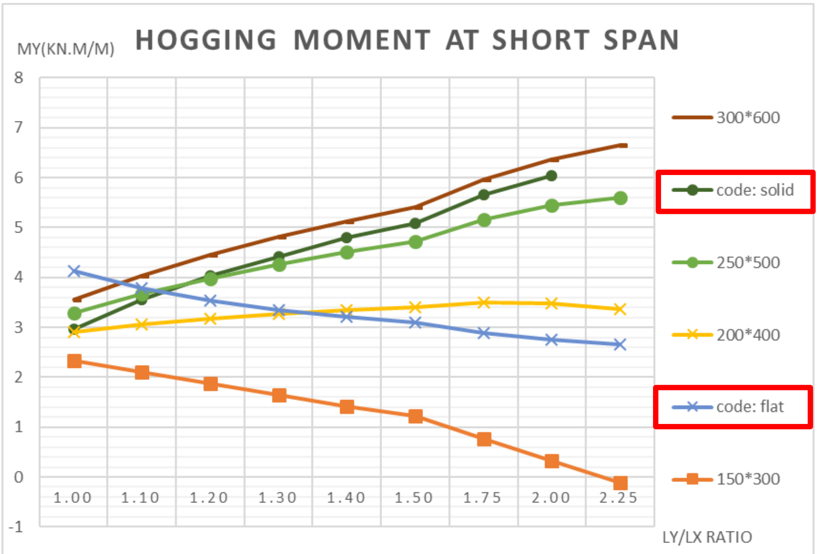

Figure 4.23: Hogging Moment at Short Span for Combination 2.

(iii) Combination 3:

Figure 4.24 shows the result for slab supported by beam sizes which the depth is three times of the width. Slab supported by all beam sizes show solid slab behaviour which the moment increase with l_y/l_x ratio. The results also shows that the hogging moment increase with beam size.

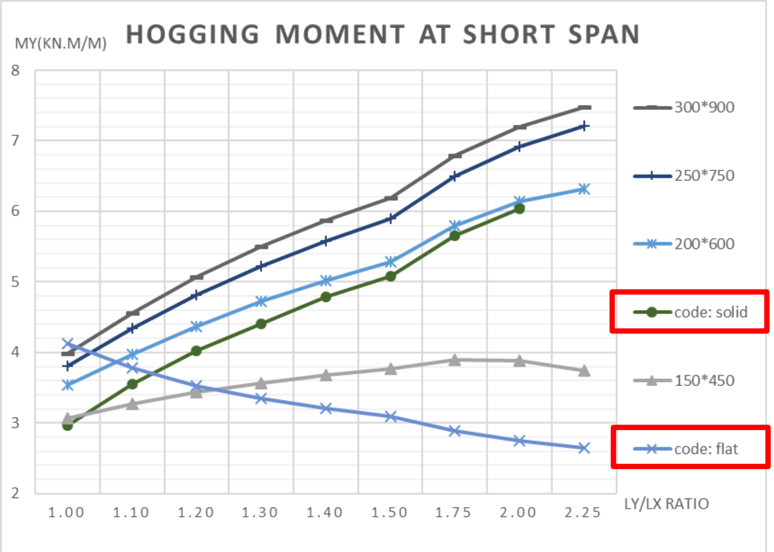

Figure 4.24: Hogging Moment at Short Span for Combination 3.

(iv) Combination 4:

Figure 4.25 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation. Slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm (also termed as shallow beam) show solid slab behaviour and the values are generally smaller than those of stiff beams (namely 300 mm x 600 mm and 300 mm x 900 mm) and even smaller than values forecasted by flat slab in code.

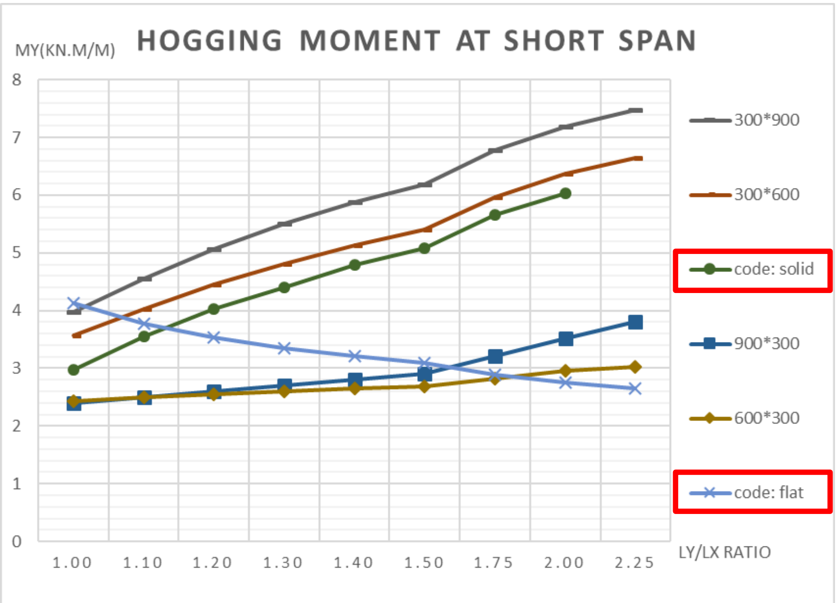

Figure 4.25: Hogging Moment at Short Span for Combination 4.

The slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm also behave like solid slab and the even exceed the values forecasted by solid slab in code, which means underestimated. They generally experience greater moment than their rotated pair, beam of 600 mm x 300 mm and 900 mm x 300 mm.

4.5.3 Sagging Moment at Long Span

This sub-section compares sagging moment of slabs at long span of 4 combinations.

(i) Combination 1:

In Figure 4.26, the control values for 'code: flat' line shows an increasing trend when the l_y/l_x ratio increase. The 'code: flat' increase with a greater rate when the l_y/l_x ratio exceed 1.50. This graph shows that the l_y/l_x ratio of 1.50 is the separation point where the gradient of many lines increases with the l_y/l_x ratio exceeding 1.50. The control values for 'code: solid' line is constant across all l_y/l_x ratio.

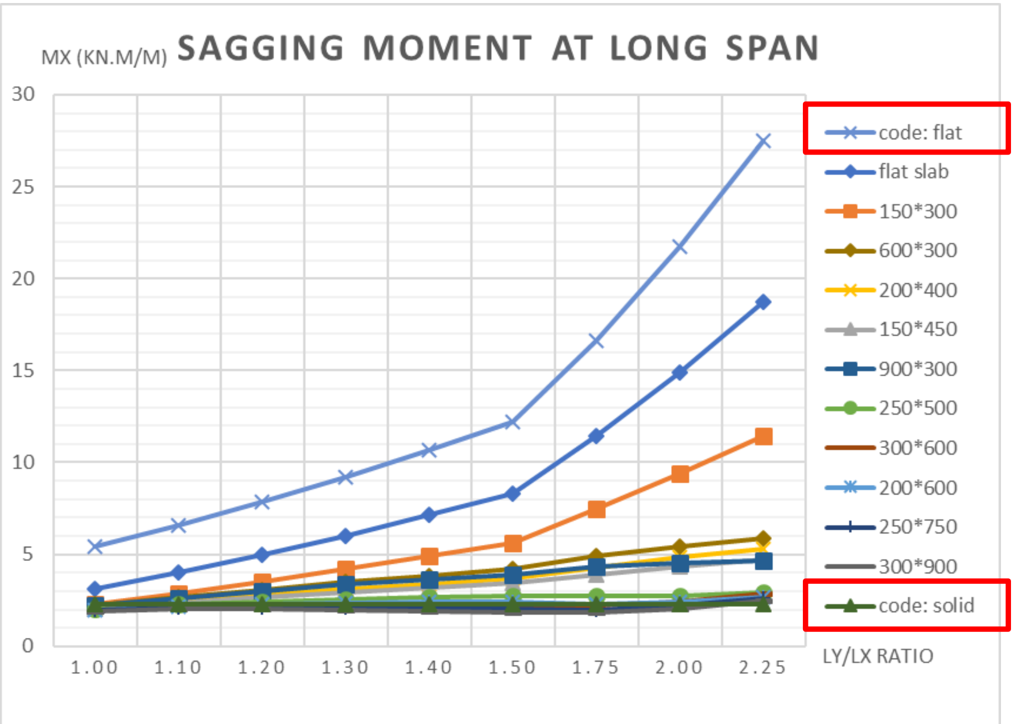

Figure 4.26: Sagging Moment at Long Span for Combination 1.

Figure 4.26a shows a graph that limits the value of y-axis to 6 kN.m/m (zoomed view) in order to show the congested part within bending moment of 1.00 to 5.00 in a clearer manner.

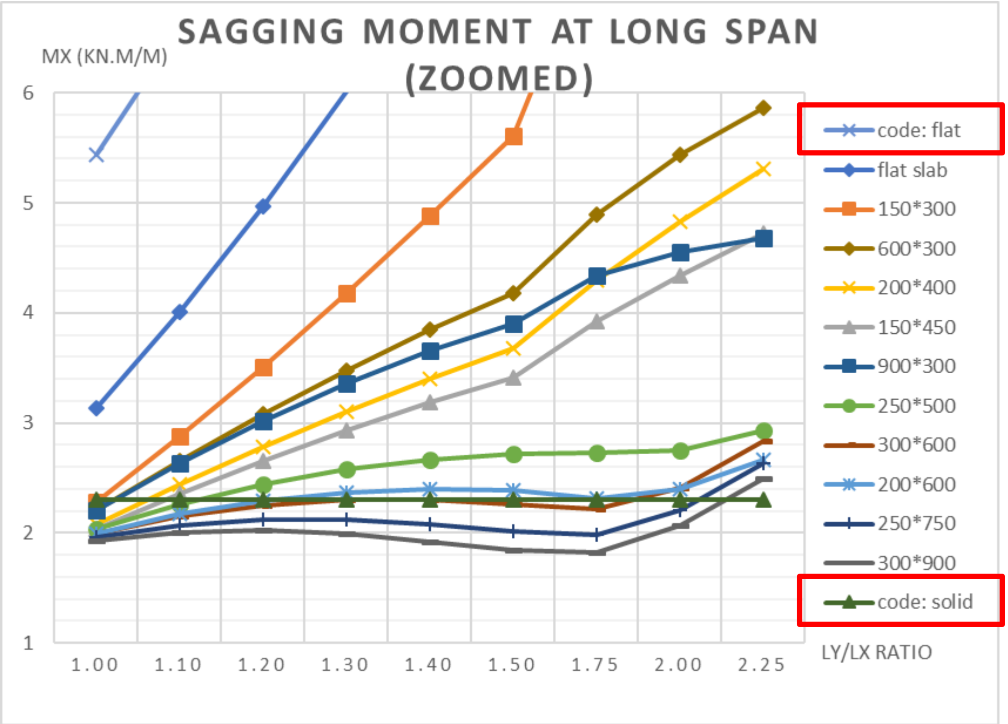

Figure 4.26a: Sagging Moment at Long Span for Combination 1 (zoomed version).

Flat slab and slabs supported by 150 mm x 300 mm, 600 mm x 300 mm, 200 mm x 400 mm, 150 mm x 450 mm, and 900 mm x 300 mm show flat slab behaviour which moment increase with l_y/l_x ratio.

All the remaining slabs (supported by 250 mm x 500 mm, 300 mm x 600 mm, 200 mm x 600 mm, 250 mm x 750 mm, and 300 mm x 900 mm) behave like solid slab which only show small increment with l_y/l_x ratio.

The ‘code: flat’ overestimates the flat slab bending moment. The ‘code: solid’ underestimate the sagging moment of solid slabs supported by relatively flexible beam (namely 150 mm x 300 mm, 600 mm x 300 mm, 200 mm x 400 mm, 150 mm x 450 mm, 900 mm x 300 mm, and 250 mm x 500 mm) and adequately estimate those solid slabs supported by stiff beam (namely 300 mm x 600 mm, 200 mm x 600 mm, 250 mm x 750 mm, and 300 mm x 900 mm).

(ii) Combination 2:

Figure 4.27 shows the result for slab supported by beam sizes which the depth is two times of the width. Slabs supported by beam size of 150 mm x 300 mm and 200 mm x 400 mm show flat slab behaviour. As the beam size increase, the slabs behaves like solid slab and only experience minute increment in moment when l_y/l_x ratio increases.

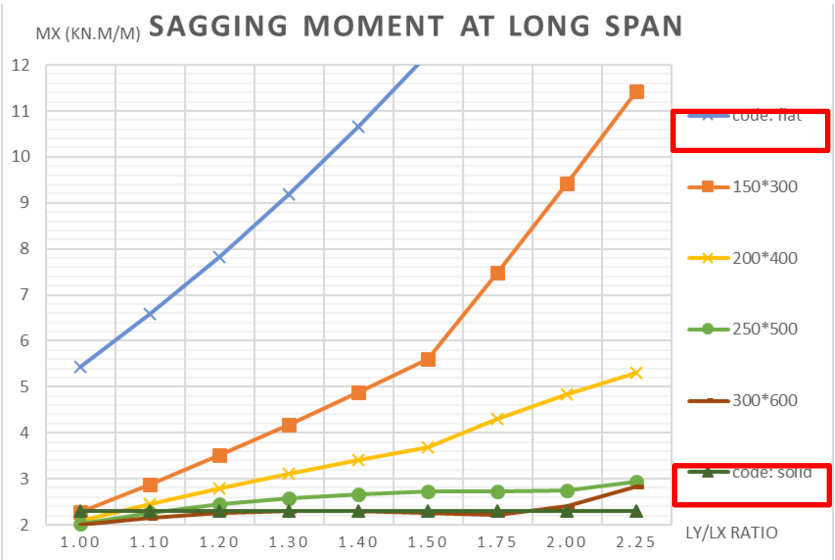

Figure 4.27: Sagging Moment at Long Span for Combination 2.

(iii) Combination 3:

Figure 4.28 shows the result for slab supported by beam sizes which the depth is three times of the width. Slab supported by beam size of 150 mm x 450 mm shows flat slab behaviour. As the size increase, the slabs behaves like solid slab and the increment in moment is small when l_y/l_x ratio increases.

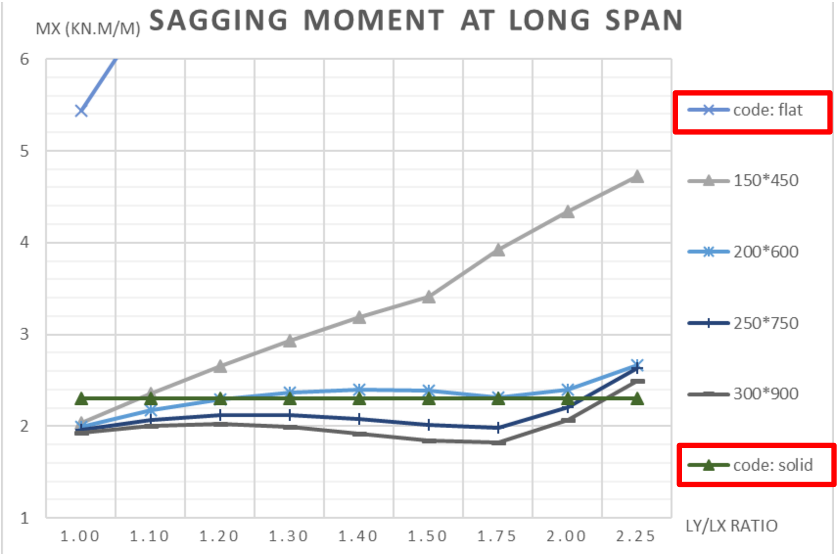

Figure 4.28: Sagging Moment at Long Span for Combination 3.

(iv) Combination 4:

Figure 4.29 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation. Slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm (also termed as shallow beam) show flat slab behaviour which bending moment increase with l_y/l_x ratio.

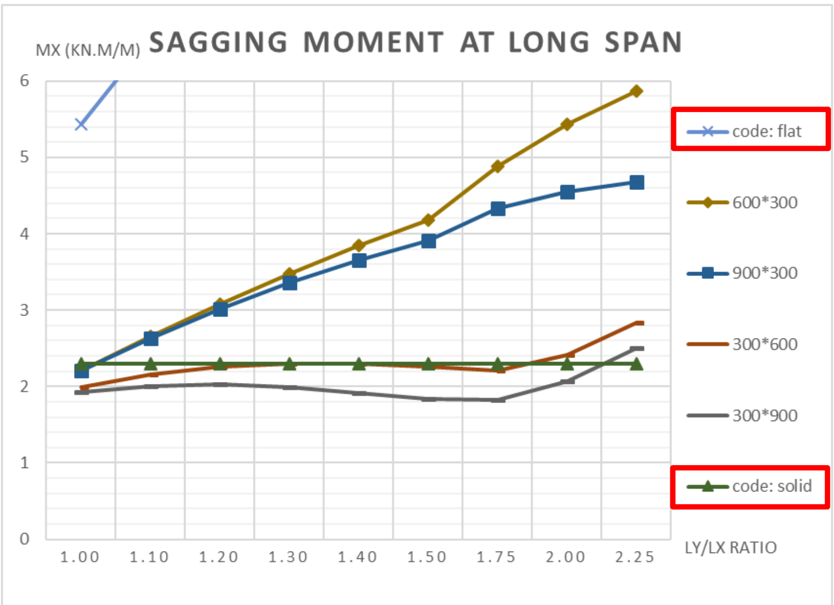

Figure 4.29: Sagging Moment at Long Span for Combination 4.

The slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm (also termed as normal depth beam) behave like solid slab and generally experience smaller sagging moment than their rotated pair, beam of 600 mm x 300 mm and 900 mm x 300 mm.

4.5.4 Sagging Moment at Short Span

This sub-section compares sagging moment of slabs at short span of 4 combinations.

(i) Combination 1:

In Figure 4.30, the control values, 'code: flat' line shows a decreasing trend when the l_y/l_x ratio increase. The control values for solid slab, 'code: solid' line shows an increasing trend when the l_y/l_x ratio increase.

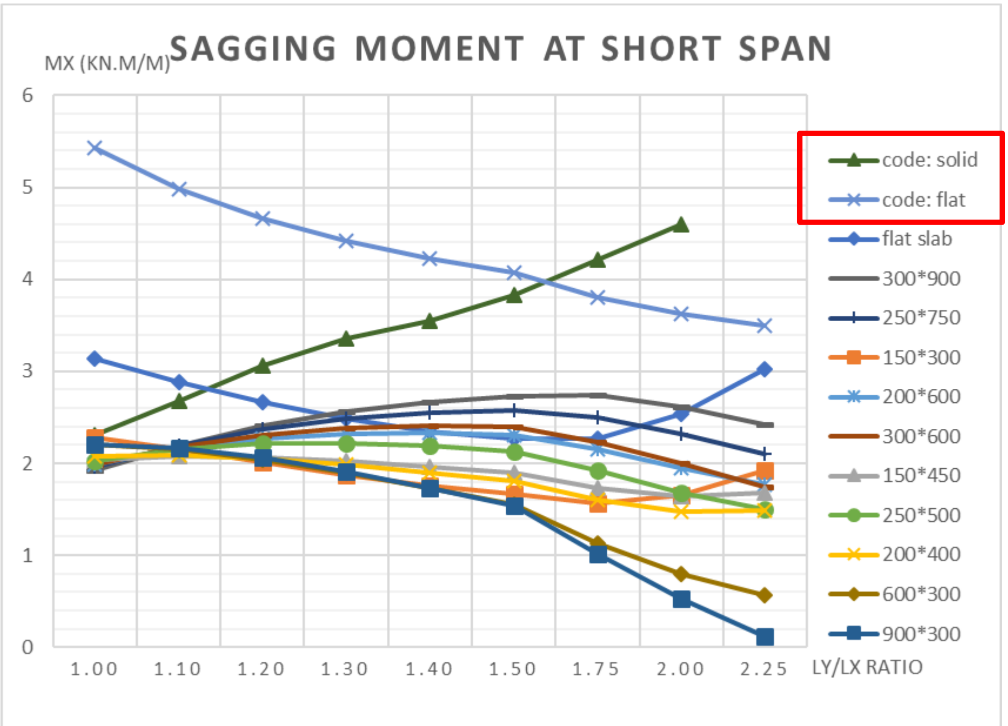

Figure 4.30: Sagging Moment at Short Span for Combination 1.

In the case of solid slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm show flat slab behaviour which bending moment decrease with l_y/l_x ratio increase.

In the case of flat slab and solid slabs supported by beam size of 150 mm x 300 mm, 150 mm x 450 mm, and 200 mm x 400 mm, sagging moment decrease initially (which shows flat slab trend) and starts to increase after the

l_y/l_x ratio of 1.75. This can be explained as when the l_y/l_x ratio increase to a certain magnitude (say l_y/l_x ratio of 1.75 in this case), the short span become flexible and eventually settled, which imposed extra sagging moment.

In solid slabs supported by stiffer beams (namely beam sizes of 300 mm x 900 mm, 250 mm x 750 mm, 300 mm x 600 mm, 200 mm x 600 mm, and 250 mm x 500 mm), the sagging moment increase initially (which shows solid slab behaviour) and starts to decrease after the ratio of 1.75. This can be explained as the stiffer beams cause the BMD to shift up (which results greater hogging moment at support and small sagging moment at mid span, which has been explained in section 4.4.1.2).

The ‘code: flat’ overestimates the sagging moment in flat slab. The ‘code: solid’ generally overestimate the sagging moment in solid slabs supported by all sizes of beam.

(ii) Combination 2:

Figure 4.31 shows the result for slab supported by beam sizes which the depth is two times of the width. Slab supported by beam size of 150 mm x 300 mm shows flat slab behaviour which the sagging moment decrease when l_y/l_x ratio increases. As the beam size increase, the slabs behaves like solid slab which the sagging moment increase when l_y/l_x ratio increases. The results also shows that the sagging moment increase with beam size.

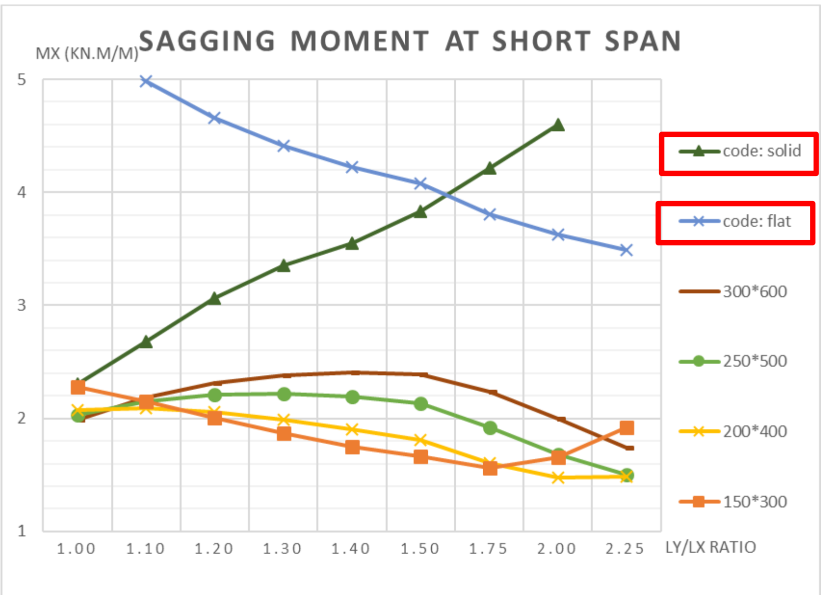

Figure 4.31: Sagging Moment at Short Span for Combination 2.

(iii) Combination 3:

Figure 4.32 shows the result for slab supported by beam sizes which the depth is three times of the width. Slab supported by beam size of 150 mm x 450 mm shows flat slab behaviour which the sagging moment decrease when l_y/l_x ratio increases. As the beam size increase, the slabs behaves like solid slab which the sagging moment increase when l_y/l_x ratio increases. The results also shows that the sagging moment increase with beam size.

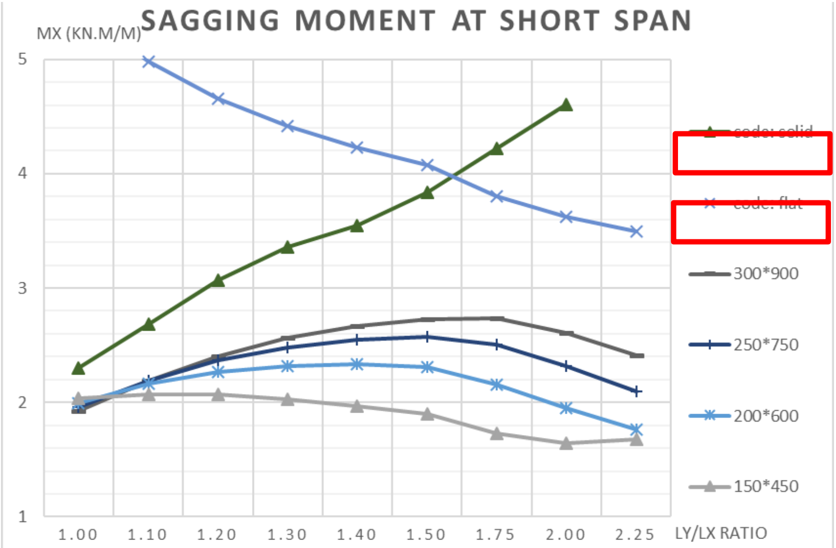

Figure 4.32: Sagging Moment at Short Span for Combination 3.

(iv) Combination 4:

Figure 4.33 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation. Slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm (also termed as shallow beam) show solid slab behaviour and the values are generally smaller than those of stiff beams (namely 300 mm x 600 mm and 300 mm x 900 mm).

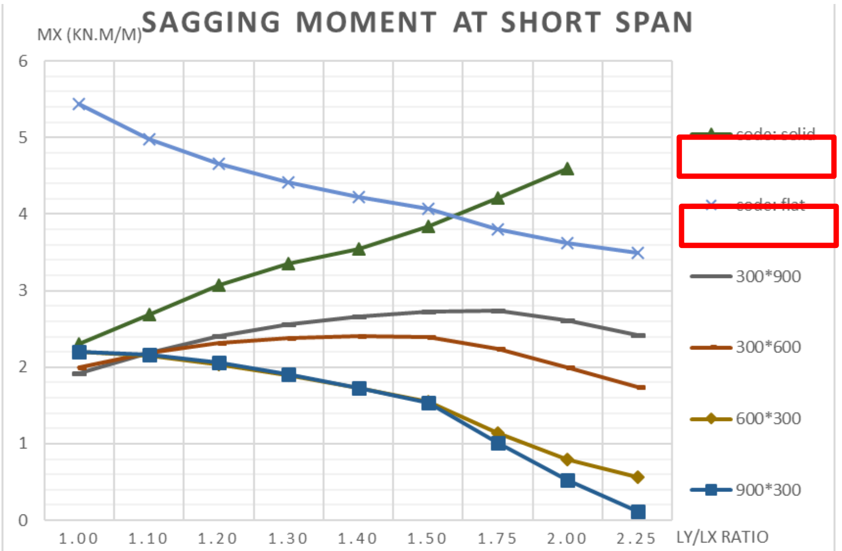

Figure 4.33: Sagging Moment at Short Span for Combination 4.

The slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm behave like solid slab and generally experience greater moment than their rotated pair, beam of 600 mm x 300mm and 900mm x 300mm.

4.5.5 Shear Force at Long Span

This sub-section compares shear force of slabs at long span of 4 combinations.

(i) Combination 1:

In Figure 4.34, the control values for ‘code: flat’ line shows an increasing trend when the l_y/l_x ratio increase. The control values for ‘code: solid’ line is constant across all l_y/l_x ratio.

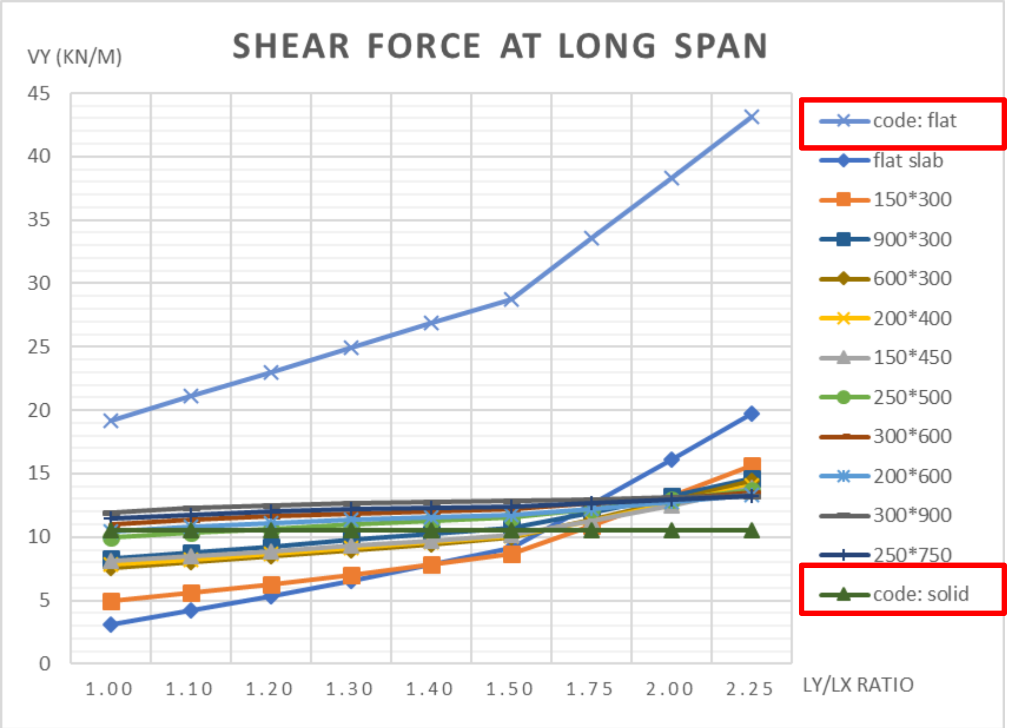

Figure 4.34: Shear Force at Long Span for Combination 1.

Figure 4.34a shows a graph that limits the value of y-axis to 20 kN/m (zoomed view) in order to show the congested part within shear force of 6 to 14 kN/m in a clearer manner. Flat slab and slabs supported by 150 mm x 300 mm, 900 mm x 300 mm, 600 mm x 300 mm, 200 mm x 400 mm, and 150 mm x 450 mm show flat slab behaviour which the shear force increase with l_y/l_x ratio.

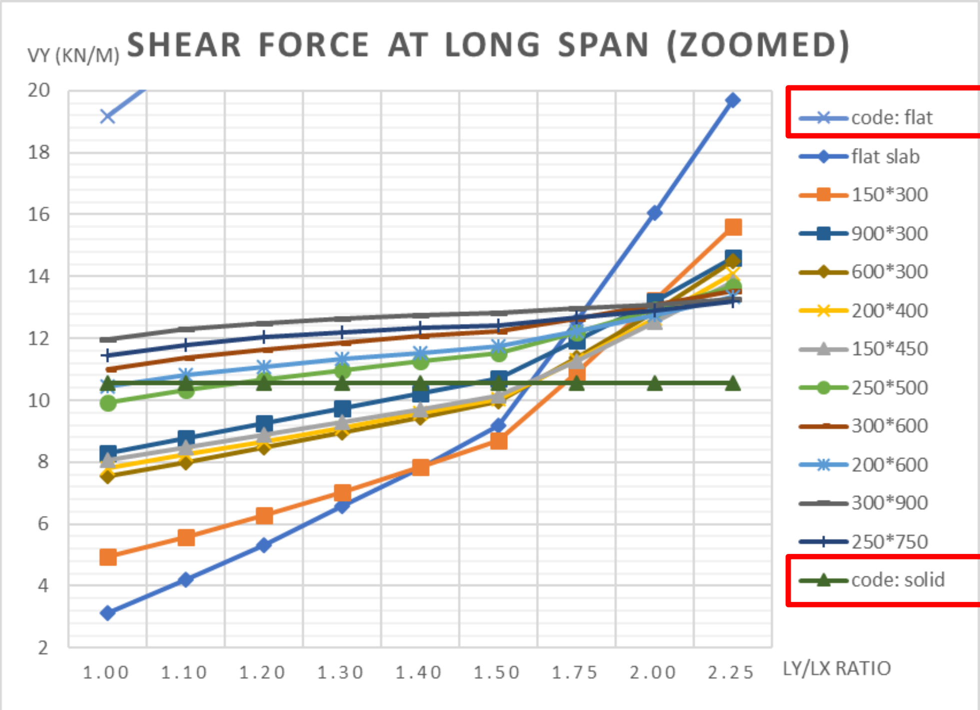

Figure 4.34a: Shear Force at Long Span for Combination 1 (Zoomed Version).

All the remaining slabs (supported by 250 mm x 500 mm, 300 mm x 600 mm, 200 mm x 600 mm, 250 mm x 750 mm, and 300 mm x 900 mm) behave like solid slab which only show small increment of shear with l_y/l_x ratio.

The 'code: flat' overestimates the flat slab shear force. The 'code: solid' underestimate the shear force of solid slabs supported by stiff beam (namely 250 mm x 500 mm, 300 mm x 600 mm, 200 mm x 600 mm, 250 mm x 750 mm, and 300 mm x 900 mm). It also underestimate the shear force of solid slabs supported by relatively flexible beam (namely 150mm x 300 mm, 900 mm x 300 mm, 600 mm x 300 mm, 200 mm x 400 mm, and 150 mm x 450 mm) of the with l_y/l_x ratio beyond 1.50.

On the other hand, 'code: solid' adequately estimate the solid slabs supported by beam size of 900 mm x 300 mm, 600 mm x 300 mm, 200 mm x 400 mm, and 150 mm x 450 mm of l_y/l_x ratio less than 1.50. It overestimates the shear force of solid slabs supported by flexible beam of 150 mm x 300 mm.

(ii) Combination 2:

Figure 4.35 shows the result for slab supported by beam sizes which the depth is two times of the width. Slabs supported by beam size of 150 mm x 300 mm and 200 mm x 400 mm show flat slab behaviour. As the beam size increase, the slabs behaves like solid slab and only experience minute increment in moment when l_y/l_x ratio increases. The results also shows that the shear force increase with beam size.

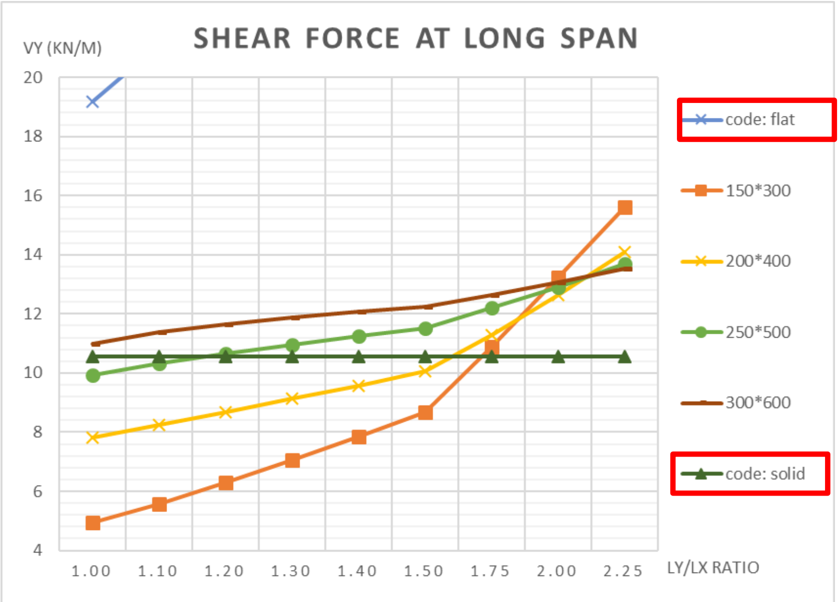

Figure 4.35: Shear Force at Long Span for Combination 2.

(iii) Combination 3:

Figure 4.36 shows the result for slab supported by beam sizes which the depth is three times of the width. Slab supported by beam size of 150 mm x 450 mm shows flat slab behaviour which the shear force increase when l_y/l_x ratio increases. As the beam size increase, the slabs behaves like solid slab which the shear force increment is small when l_y/l_x ratio increases. The results also shows that the shear force increase with beam size.

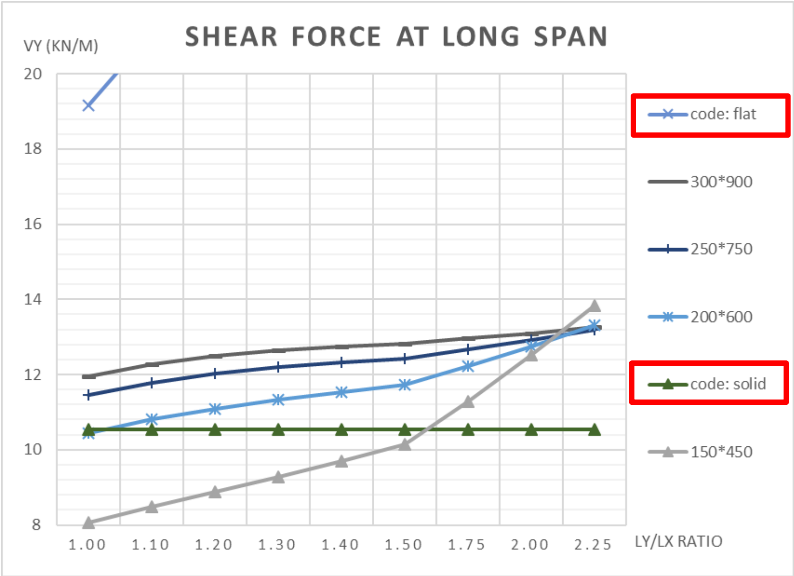

Figure 4.36: Shear Force at Long Span for Combination 3.

(iv) Combination 4:

Figure 4.37 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation. Slabs supported by beam size of 600 mm x 300 mm and 900 mm x 300 mm (also termed as shallow beam) show solid slab behaviour. The slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm behave like solid slab and generally experience greater shear force than their rotated pair, beam of 600 mm x 300 mm and 900 mm x 300 mm.

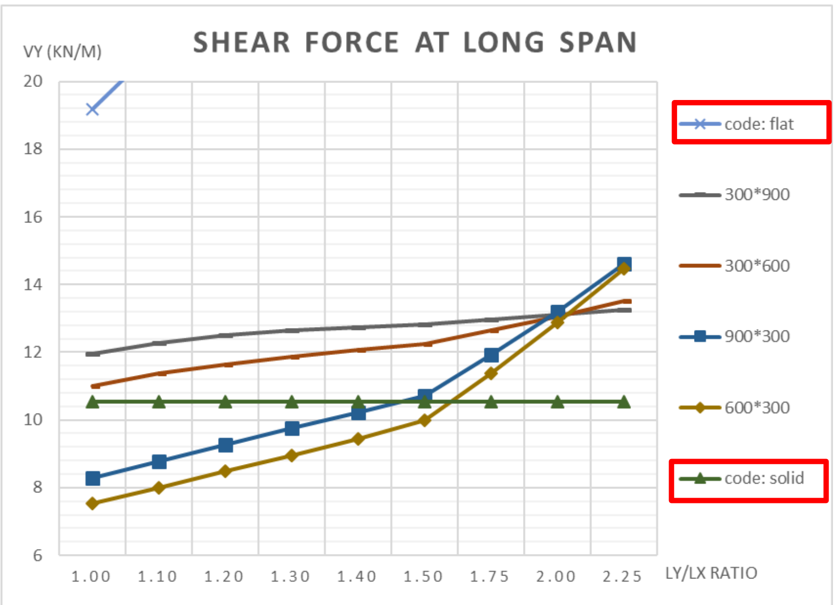

Figure 4.37: Shear Force at Long Span for Combination 4.

4.5.6 Shear Force at Short Span

This sub-section compares shear force of slabs at short span of 4 combinations.

(i) Combination 1:

In Figure 4.38, the control values for 'code: flat' line is constant across all l_y/l_x ratio. The control values for 'code: solid' line shows an increasing trend with the increased l_y/l_x ratio.

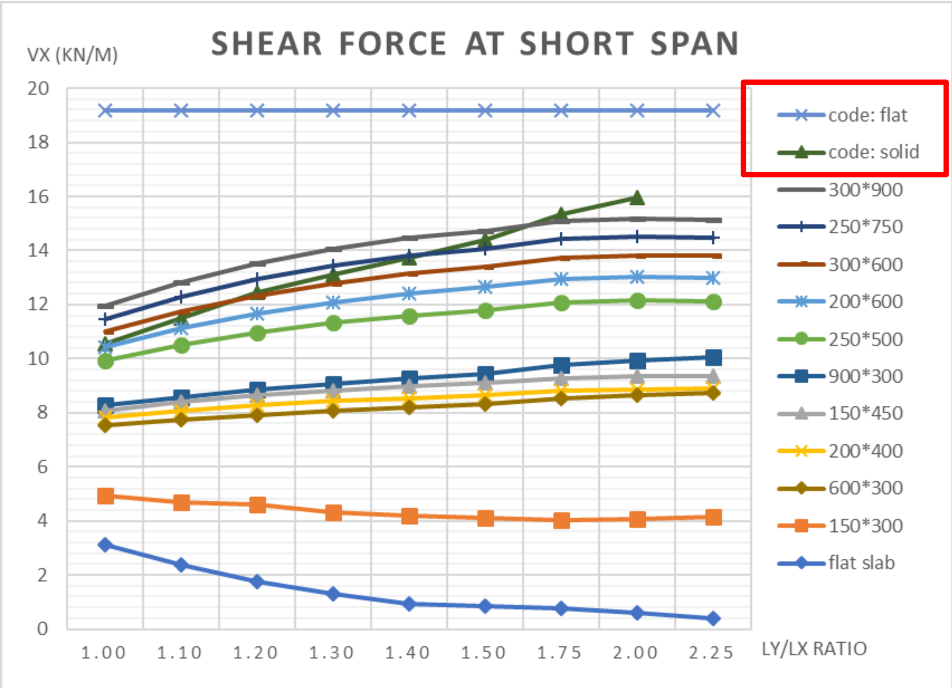

Figure 4.38: Shear Force at Short Span for Combination 1.

Flat slab shows a decreasing trend and approaches zero when the l_y/l_x ratio increase toward 2.25. The results above are short span shear force of the interior panels. In flat slab system, the short span is the less govern span, and increase in l_y/l_x ratio further reduce the short span strength which was discussed in section 4.4.1.2.

In the case of solid slabs supported by 150 mm x 300 mm beam shows a mild decreasing trend as l_y/l_x ratio increase. Other than 150 mm x 300 mm, the remaining slabs (supported by 150 mm x 450 mm, 200 mm x 400 mm, 200 mm x 600 mm, 250 mm x 500 mm, 250 mm x 750 mm, 300 mm x 600 mm, 300 mm x 900 mm, 600 mm x 300 mm, and 900 mm x 300 mm) tend to behave like solid slab which show small increment of shear with l_y/l_x ratio increment.

The ‘code: flat’ overestimates the flat slab shear force. The ‘code: solid’ adequately estimate most of the shear force of solid slabs.

(ii) Combination 2:

Figure 4.39 shows the result for slab supported by beam sizes which the depth is two times of the width. The result shows that the shear force increase with beam size.

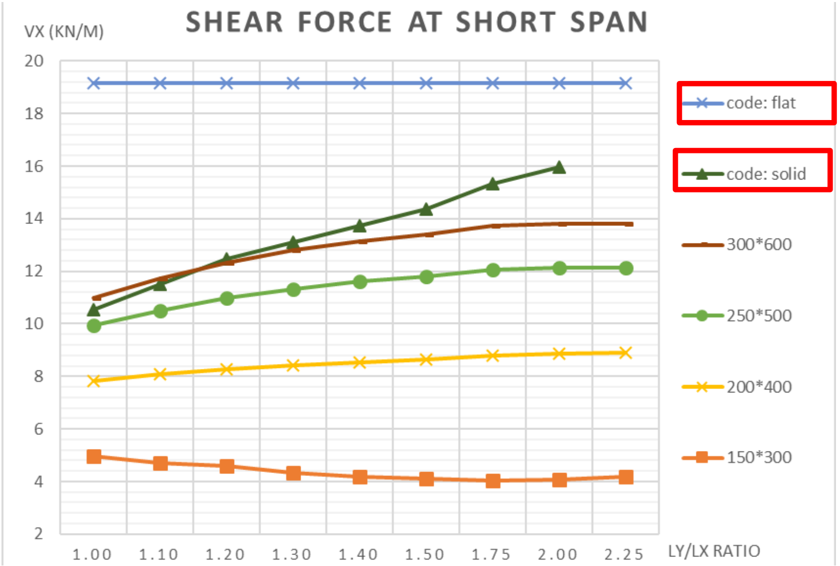

Figure 4.39: Shear Force at Short Span for Combination 2.

(iii) Combination 3:

Figure 4.40 shows the result for slab supported by beam sizes which the depth is three times of the width. The result shows that the shear force increase with beam size.

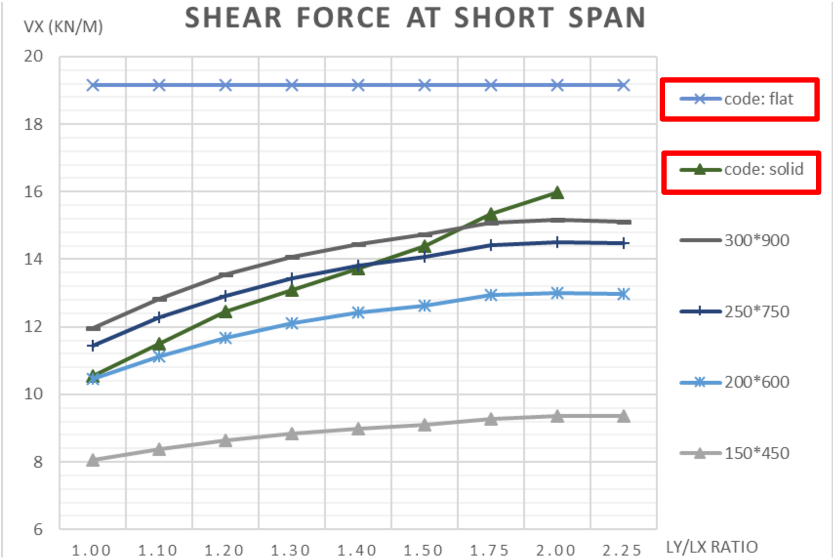

Figure 4.40: Shear Force at Short Span for Combination 3.

(iv) Combination 4:

Figure 4.41 shows the result for slab supported by 2 pairs of same beam sizes but with different orientation. The slabs supported by beam size of 300 mm x 600 mm and 300 mm x 900 mm behave like solid slab experience far greater shear force than their rotated pair, beam of 600 mm x 300 mm and 900 mm x 300 mm.

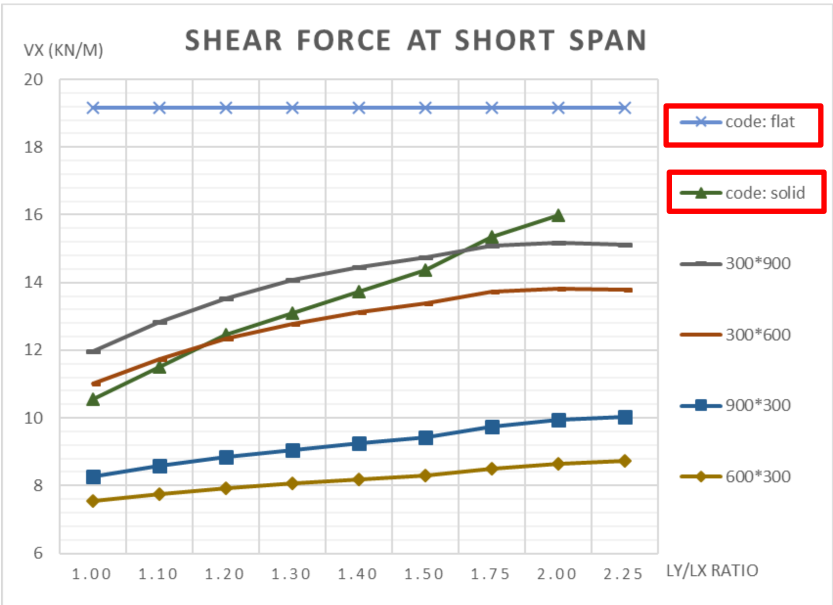

Figure 4.41: Shear Force at Short Span for Combination 4.

4.6 Result and Discussion on Statistical Analysis

The discussion above has clearly demonstrates how the supporting beam stiffness affects the bending moment and shear force in the slabs. However, the code of design BS8110 shows that the internal loading in slab is only related to l_y/l_x ratio.

The words, 'flexible supporting beam' and 'rigid supporting beam' were mentioned frequently in previous discussion. However, the definitions of 'flexible supporting beam' and 'rigid supporting beam' is vague and border between these two terms are not clearly defined.

Thus, a covariance analysis was performed with the intention to seek an empirical formula that can explain the bending moment in term of not only l_y/l_x ratio, but also including the stiffness of supporting beams and slabs itself.

4.6.1 Covariance Analysis

The l_y/l_x ratio is included in covariance analysis to represent the suggestion by BS8110, which the moment is only dependent on the l_y/l_x ratio. The formulated independent variable, X was formulated based on stiffness of beams and slabs. The result of covariance analysis is shown in Figure 4.42.

Correlations

		Mo	LYLXratio	A	B	C	D	X
Mo	Pearson Correlation	1	.579**	-.232*	-.243*	.579**	-.568**	.949**
	Sig. (2-tailed)		.000	.028	.021	.000	.000	.000
	Sum of Squares and Cross-products	47.374	15.052	-.026	-.020	.004	-.002	2830847.401
	Covariance	.532	.169	.000	.000	.000	.000	31807.274
	N	90	90	90	90	90	90	90

Figure 4.42: Result of Covariance Analysis.

The result above shows that the independent variable, l_y/l_x ratio shows a 0.579 correlation with M_0, whereas as A, B, C, D and X shows correlation of -0.232, -0.243, 0.579, -0.568 and 0.949 respectively. Where A, B, C, D are stiffness of beam and slab as mentioned in Equations 3.2 to 3.5. The formulation of independent variable, X is shown in Equation 4.2.

The Pearson correlation ranges from -1.0 to 1.0. The closer it is to 1.0, the greater the correlation between the independent variable with M_0. Negative correlation indicates that the independent variable decrease as the M_0 increase.

The formulated empirical equation, *X* gives a high Pearson correlation of 0.949. The formulation of *X* is shown in Equation 4.2:

$$X = \frac{C^2}{D^2\sqrt{AB}} \tag{4.2}$$

where

X = formulated independent variable, mm^3

A = stiffness of beam in x-direction (as shown in Equation 3.2), mm^3

B = stiffness of beam in y-direction (as shown in Equation 3.3), mm^3

C = stiffness of slab in x-direction (as shown in Equation 3.4), mm^3

D = stiffness of slab in y-direction (as shown in Equation 3.5), mm^3

Thus, expanding of *X* will give Equation 4.3:

$$X = \left(\frac{C}{D}\right)^2 \frac{1}{\sqrt{AB}} = \left(\frac{\frac{\frac{l_y t^3}{12}}{l_x}}{\frac{\frac{l_x t^3}{12}}{l_y}}\right)^2 \frac{1}{\sqrt{\frac{\frac{bh^3}{12}}{l_x} * \frac{\frac{bh^3}{12}}{l_y}}} = \frac{12\,(l_y)^{4.5}}{bh^3 (l_x)^{3.5}} \tag{4.3}$$

where

X = formulated independent variable, mm^3

b = width of beam, mm

h = depth of beam, mm

t = thickness of slab, mm

l_y = long span length, mm

l_x = short span length, mm

4.6.2 Linear Regression

Figure 4.43 shows the linear regression of M_0 - l_y/l_x ratio which yield R^2 of 0.336, and the equation of the linear regression is shown in Equation 4.4. Figure 4.44 shows the linear regression of M_0 - X which yield R^2 of 0.901, and the equation of the linear regression is shown in Equation 4.5.

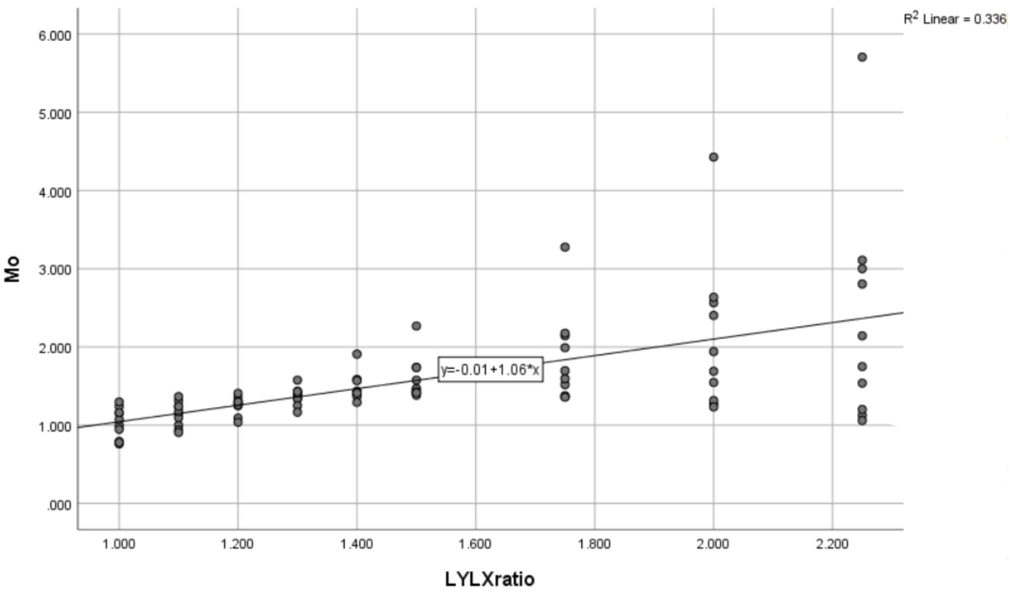

Figure 4.43: Linear Regression of M_0 - l_y/l_x Ratio.

$$M_0 = 0.01 + 1.06(ly/lx) \qquad (4.4)$$

where

M_0 = moment ratio (as shown in Equation 3.1), unitless

l_y = long span length, mm

l_x = short span length, mm

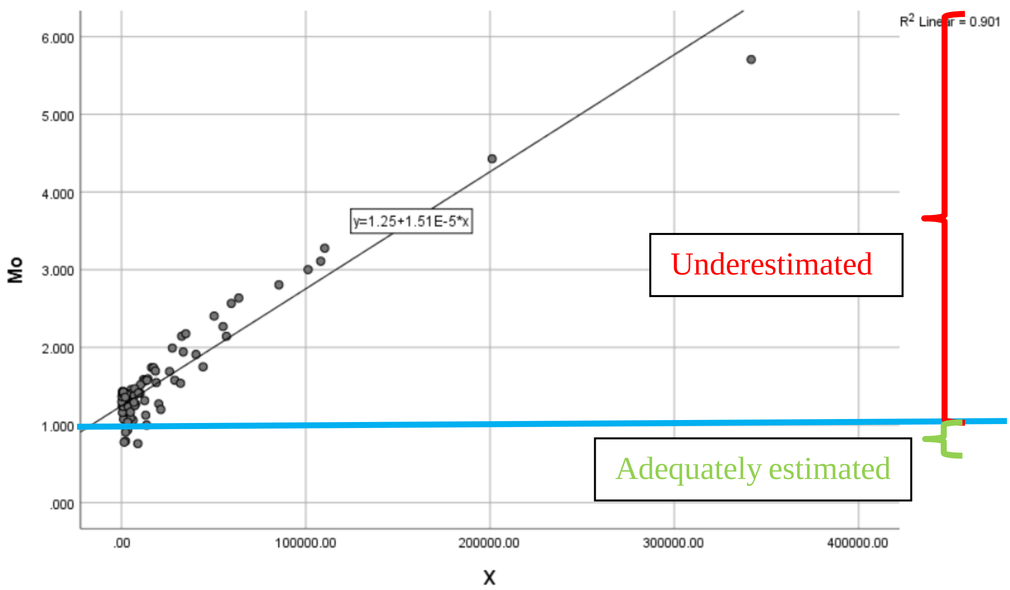

Figure 4.44: Linear Regression of $M_0 – X$.

$$M_0 = 1.25 + 0.0000151(X) \qquad (4.5)$$

where

M_0 = moment ratio (as shown in Equation 3.1), unitless

X = formulated independent variable, mm^3

As the result in Figure 4.43 shows a low R^2 of 0.336, therefore the data is more scatter around from the best fit line whereas in Figure 4.44, the higher R^2 of 0.901 results lower deviation from the best fit line.

The results in Section 4.6.1, Figures 4.43 and 4.44 are summarized in Table 4.26.

Table 4.26: Summary and Comparison between Result.

	M_0 - l_y/l_x (from BS8110)	M_0 - X (from Scia Engineer)
Pearson Correlation	0.579	0.949
R^2	0.336	0.901
Linear regression equation	$M_0 = 0.01 + 1.06(ly/lx)$	$M_0 = 1.25 + 0.0000151(X)$

As the result in M_0 - l_y/l_x ratio graph gives a correlation of 0.579, it is fair enough to say that relying solely on the l_y/l_x ratio (as suggested in BS8110) can help the user to predict the bending moment in general. However, the R^2 of 0.336 indicates that the deviation of data is high and significant. The result in Figure 4.43 also shows that the greater the l_y/l_x ratio, the greater the variation.

Substituting X (Equation 4.3) into the M_0 (Equation 4.5) will give the empirical formula, Equation 4.6:

$$M_0 = 0.0000151 \frac{12\,(l_y)^{4.5}}{bh^3 (l_x)^{3.5}} + 1.25 \tag{4.6}$$

where

M_0 = moment ratio (as shown in Equation 3.1), unitless

b = width of beam, mm

h = depth of beam, mm

l_y = long span length, mm

l_x = short span length, mm

Comparing this M_0 equation with general form of linear equation, $y = mx + c$, the gradient of this equation is 0.0000151 and the y-intercept is 1.25. The empirical equation shows that the M_0 is independent of the thickness of slab, t. The Equation 4.6 shows that the M_0 is dependent on the width of beam, b, length of long span (l_y), length of short span (l_x),and depth of beam (h).

Further substituting Equation 4.6 into Equation 3.1 will yield Equation 4.7.

$$M_1 = M_2 \left[0.0000151 \frac{12\,(l_y)^{4.5}}{bh^3 (l_x)^{3.5}} + 1.25 \right] \tag{4.7}$$

where

M_1 = empirical formula for finding bending moment, kN.m/m

M_2 = hogging moment calculated based on BS8110, kN.m/m

b = width of beam, mm

h = depth of beam, mm

l_y = long span length, mm

l_x = short span length, mm

Thus, by substituting in all the variables on the right hand side of the equation, M_1, the hogging moment obtained from SCIA Engineer (or say the actual bending moment) can be calculated which is also the empirical formula for finding bending moment. Since in general, the smaller hogging moment the better, therefore the designer can either reduce the l_y/l_x ratio or increase the beam depth.

Recalling Equation 3.1, the value of M_0 smaller than one means that BS8110 has either overestimated or adequately estimated the actual hogging moment, on contrary the value of M_0 greater than one means that the code of design BS8110 has underestimated the actual hogging moment. Thus, the value of M_0 should not exceed 1.0 for a safe analysis. If M_0 is limited to be equal or less than one and applying this ($M_0 \leq 1$)into Equation 4.6 will give inequality Equation 4.8.

$$0.0000151 \frac{12\,(l_y)^{4.5}}{bh^3 (l_x)^{3.5}} \leq -0.25 \qquad (4.8)$$

where

b = width of beam, mm

h = depth of beam, mm

l_y = long span length, mm

l_x = short span length, mm

Since the y-intercept of Equation 4.6 is at 1.25 and additional to the variables in inequality Equation 4.8 will never be negative, these two reasons show that the Equation 4.6 will never give a M_0 of less than one. In another words, for any value of l_y, l_x, width of beam and depth of beam, the hogging moment at the long span will be at least 25 % more than the value obtained based on BS8110.

Obviously, there are flaws in this proposed empirical equation in calculating hogging moment at long span. As a mitigation, the empirical equation can be improved by:

(i) Increase the number of samples.

(ii) Modelling the slabs with more parameters, such as different slab thickness, different supporting beam size at different edges.

(iii) As the value obtained from Scia Engineer are based on certain theories and assumptions, experimental casting and testing should be carried out to verify the value obtained from Scia Engineer.

4.7 Summary

Section 4.5 shows that the value by BS8110 only adequately estimated the internal loading for slab supported by stiff beam of small l_y/l_x ratio, whereas for slab supported by flexible beam are generally underestimate by BS8110. The internal loading for slab supported by stiff beam of large l_y/l_x ratio are overestimated by BS8110. The summary of comparison with BS8110 and slab behaviour according to supporting beam stiffness and l_y/l_x ratio are shown in Table 4.27.

Figures 4.45 to 4.48 show the summary in diagram form which in the diagrams, green curves represent internal loading for slab supported by flexible beam, yellow curves represent internal loading for slab supported by stiff beam, and red curves represent internal loading as calculated based on BS8110.

Table 4.27: Slab behaviour summary.

Supporting beam of slab	Small l_y/l_x ratio	Large l_y/l_x ratio
Flexible beam	- Underestimated by BS8110 - Flat slab behaviour	- Underestimated by BS8110 - Significant flat slab behaviour
Stiff beam	- Adequately estimated by BS8110 - Beam-slab behaviour (Two-way slab)	- Overestimated by BS8110 - Beam-slab behaviour (One-way slab)

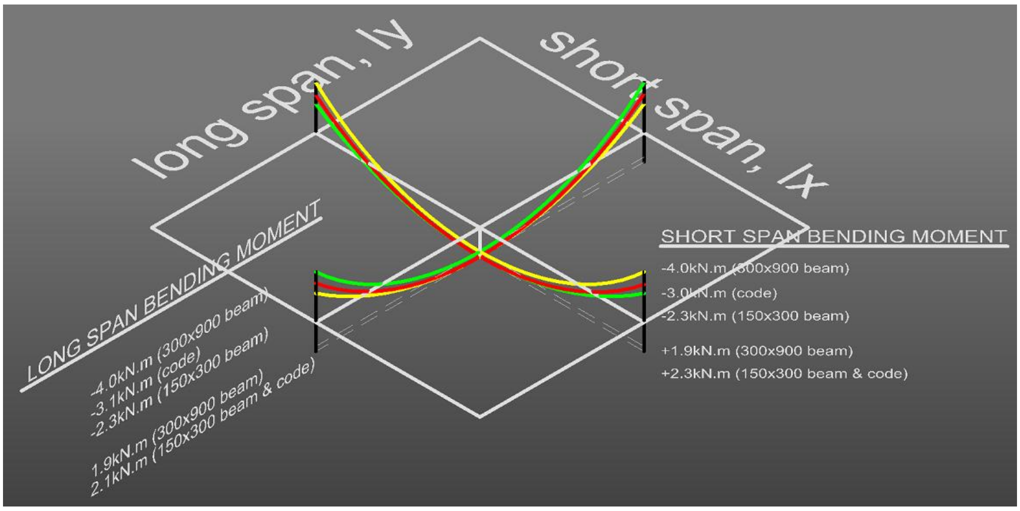

Figure 4.45: Bending Moment of Slab for l_y/l_x Ratio Equals to 1.

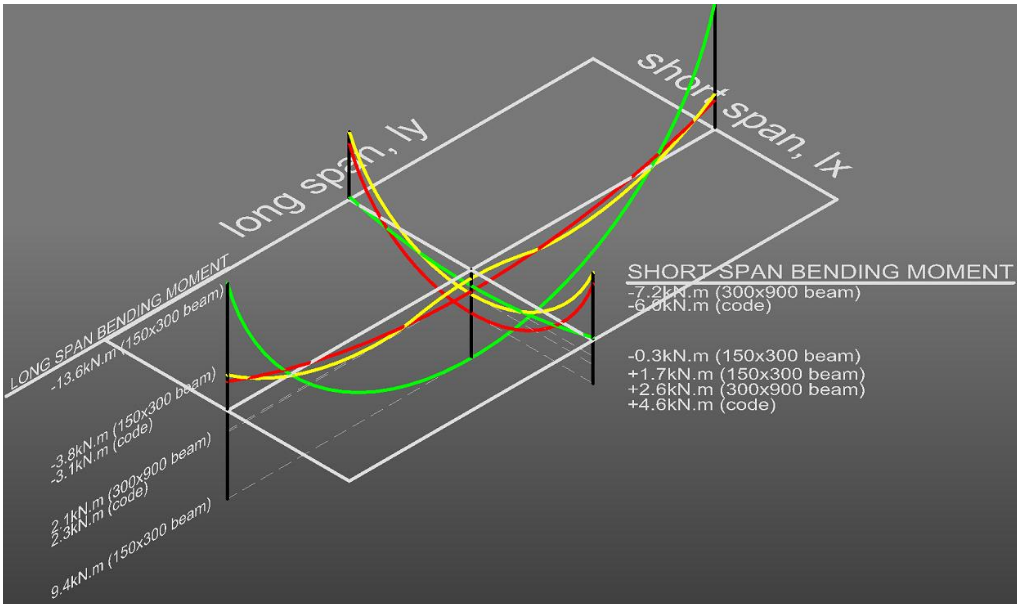

Figure 4.46: Bending Moment of Slab with l_y/l_x Ratio Equals to 2.

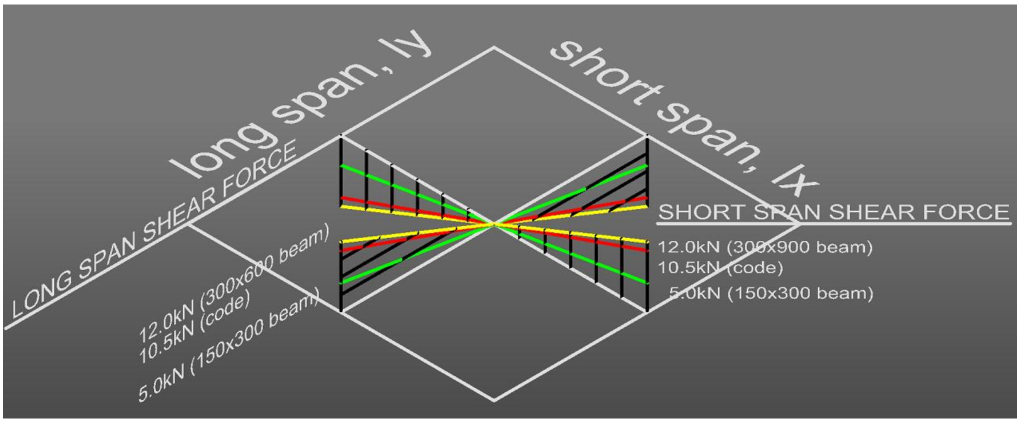

Figure 4.47: Shear Force of Slab with l_y/l_x Ratio Equals to 1.

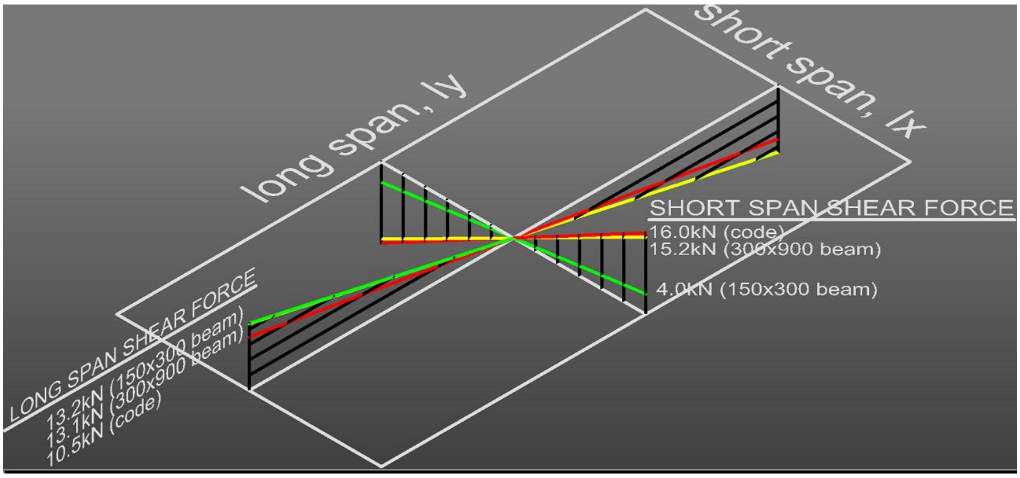

Figure 4.48: Shear Force of Slab with l_y/l_x Ratio Equals to 2.

Figure 4.44 in section 4.6.2 shows how heavily the hogging moment in long span are underestimated. Hence, a summary can be marked is that the suggested calculation by code of design BS8110 mostly underestimates the internal loading in slab which this sounds unfavourable for structural engineer.

Despite that, not much slab failure cases were reported. The main reasons were:

(i) The hogging moment obtained from Scia Engineer were those at the centre of beam. However, BS8110 stipulates that the design hogging moment for beam should be taken as the moment at the column face and not at the centre of the column. Thus, applying the same provision to slab design, the hogging moment at the edge will be reduced significantly.

(ii) The values obtained from Scia Engineer was by linear analysis (which no redistribution is considered) but BS8110 allows certain degree of moment redistribution.

(iii) The slab is always overdesign. Among results in Table 4.23 to Table 4.25, the highest moment was the hogging moment at long span of 2.25 l_y/l_x ratio supported by 150 mm x 300 mm, with the value of 17.5 kN/m (almost 6 times of value by BS8110, 3.07 kN/m). Assume that the thickness of slab is 150 mm, f_{ck} is 25 N/mm^2 and f_{yk} is 500 N/mm^2, the $A_{s,req}$ is 285 mm^2/m, and providing 10mm bar with 250 mm spacing will give reinforcement area of 314 mm^2/m which is 10 % greater than 285 mm^2/m.

(iv) The case in Case (iii) is the case of slab supported by very flexible beam with very high l_y/l_x ratio, which usually this beam size will not pass the deflection check during design. Thus, over-stiff beams will be eliminated under serviceability limit state.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusions

After going through study in Chapter 4, slabs supported by different beam size are modelling and study, hence several conclusions can be made.

The first objective of study the effect of beam size on slab behaviour of slabs supported by different stiffness of beam was achieved. In the case of slab supported by flexible beam, it shows flat slab behaviour which the bending moment and shear force are greater in long span. In the case of slab supported by stiff beam, it shows ordinary beam-slab behaviour which the bending moment and shear force are greater in short span. For stiff beam supported slab, when the long span to short span ratio is relatively low, it shows two-way slab behaviour, as the span ratio increase to a certain extent, the slab will show one-way slab behaviour which the bending moment and shear force at long span is very minute as compared to those in short span.

The second objective of compare the results between Scia Engineer and BS8110 was completed. BS8110 only adequately estimated the internal loading (namely bending moment and shear force) for slab supported by stiff beam of small l_y/l_x ratio. The bending moment and shear force of slab supported by flexible beam are generally underestimate by BS8110 whereas for slab supported by stiff beam of large l_y/l_x ratio are overestimated by BS8110.

The third objective of suggesting a complementary empirical equation for user of BS8110 when preforming slab analysis was fulfilled. The formulated empirical formula for calculating hogging moment at long span is $M_1 = M_2\left[0.0000151\frac{12\,(l_y)^{4.5}}{bh^3(l_x)^{3.5}} + 1.25\right]$ as shown in Equation 4.7.

5.2 Recommendations

The recommendation for user of BS8110 is that in the case of over-flexible or over-stiff beams supporting slabs, the table of coefficient suggested by BS8110 should be used with adequate engineering judgement or any other appropriate slab analysis should be adopted. As flexible beam will tend to behave as flat slab which is long span governing. The placement of main reinforcement bar at mid span (bottom-bottom bar) should be take note.

The definition and border between flexible supporting beam and rigid supporting beam is rather vague. In this study, only one set of data (hogging moment in long span for interior panel) was shortlisted in performing statistical analysis. It is recommended to carry out modelling with different parameters, especially those were stated in the limitation of this study, such as model with different slab thickness. The more the variety of modelling and the bigger the sample size, the more accurate the statistical analysis. More detailed and comprehensive research should be done especially in formulating the empirical equation. Once the more accurate empirical equation for different cases are found, it can be an extra provision or guideline for future user of BS8110 to get a more accurate value. The more accurate analysis can reduce the waste in overdesign and also improve the safety of the occupants.

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APPENDICES

APPENDIX A: Derivation of Bending Moment Coefficient, β Provided by BS8110 (page 36).

3.5.3.4 *Restrained slabs*

In slabs where the corners are prevented from lifting, and provision for torsion is made, the maximum design moments per unit width are given by equations 14 and 15:

$$m_{sx} = \beta_{sx} n l_x^2 \quad \text{equation 14}$$

$$m_{sy} = \beta_{sy} n l_x^2 \quad \text{equation 15}$$

Where these equations are used, the conditions and rules of 3.5.3.5 should be applied.

NOTE Values of β_{sx} and β_{sy} are given in Table 3.14.

Equations 14 and 15 and the coefficients in Table 3.14 may be derived from the following equations:

$$\beta y = (24 + 2N_d + 1.5N_d^2)/1000 \quad \text{equation 16}$$

$$\gamma = \frac{2}{9}\left[3 - \sqrt{(18)}\,\frac{l_x}{l_y}\left\{\sqrt{(\beta_y + \beta_1)} + \sqrt{(\beta_y + \beta_2)}\right\}\right] \quad \text{equation 17}$$

$$\sqrt{\gamma} = \sqrt{(\beta_x + \beta_3)} + \sqrt{(\beta_x + \beta_4)} \quad \text{equation 18}$$

NOTE β_1 and β_2 take values of $4/3\beta_y$ for continuous edges or zero for discontinuous edges. β_3 and β_4 take values of $4/3\beta_x$ for continuous edges or zero for discontinuous edges.

APPENDIX B: Table of Bending Moment Coefficient for Uniformly Loaded Rectangular Panels Supported on Four Sides with Provision for Torsion at Corners (solid slab) Provided by BS8110 (page 38).

Table 3.14 — Bending moment coefficients for rectangular panels supported on four sides with provision for torsion at corners

Type of panel and moments considered	Short span coefficients, β_{sx}								Long span coefficients, β_{sy} for all values of l_y/l_x
	Values of l_y/l_x								
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Interior panels									
Negative moment at continuous edge	0.031	0.037	0.042	0.046	0.050	0.053	0.059	0.063	0.032
Positive moment at mid-span	0.024	0.028	0.032	0.035	0.037	0.040	0.044	0.048	0.024
One short edge discontinuous									
Negative moment at continuous edge	0.039	0.044	0.048	0.052	0.055	0.058	0.063	0.067	0.037
Positive moment at mid-span	0.029	0.033	0.036	0.039	0.041	0.043	0.047	0.050	0.028
One long edge discontinuous									
Negative moment at continuous edge	0.039	0.049	0.056	0.062	0.068	0.073	0.082	0.089	0.037
Positive moment at mid-span	0.030	0.036	0.042	0.047	0.051	0.055	0.062	0.067	0.028
Two adjacent edges discontinuous									
Negative moment at continuous edge	0.047	0.056	0.063	0.069	0.074	0.078	0.087	0.093	0.045
Positive moment at mid-span	0.036	0.042	0.047	0.051	0.055	0.059	0.065	0.070	0.034
Two short edges discontinuous									
Negative moment at continuous edge	0.046	0.050	0.054	0.057	0.060	0.062	0.067	0.070	—
Positive moment at mid-span	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
Two long edges discontinuous									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.045
Positive moment at mid-span	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
Three edges discontinuous (one long edge continuous)									
Negative moment at continuous edge	0.057	0.065	0.071	0.076	0.081	0.084	0.092	0.098	—
Positive moment at mid-span	0.043	0.048	0.053	0.057	0.060	0.063	0.069	0.074	0.044
Three edges discontinuous (one short edge continuous)									
Negative moment at continuous edge	—	—	—	—	—	—	—	—	0.058
Positive moment at mid-span	0.042	0.054	0.063	0.071	0.078	0.084	0.096	0.105	0.044
Four edges discontinuous									
Positive moment at mid-span	0.055	0.065	0.074	0.081	0.087	0.092	0.103	0.111	0.056

APPENDIX C: Table of Shear Force Coefficient for Uniformly Loaded Rectangular Panels Supported on Four Sides with Provision for Torsion at Corners (Solid Slab Supported by Beams) Provided by BS8110 (page 40).

Table 3.15 — Shear force coefficient for uniformly loaded rectangular panels supported on four sides with provision for torsion at corners

Type of panel and location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	—	—	—	—	—	—	—	—	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	—
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.33	0.34	0.35	0.38	0.40	0.26
Two short edges discontinuous									
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	—
Discontinuous edge	—	—	—	—	—	—	—	—	0.26
Two long edges discontinuous									
Continuous edge	—	—	—	—	—	—	—	—	0.40
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	—
Three edges discontinuous (one long edge discontinuous)									
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	—
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Three edges discontinuous (one short edge discontinuous)									
Continuous edge	—	—	—	—	—	—	—	—	0.45
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Four edges discontinuous									
Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

APPENDIX D: Bending Moment and Shear Force for Flat Slab Provided by BS8110 (pg35).

Table 3.12 — Ultimate bending moment and shear forces in one-way spanning slabs

	End support/slab connection				At first interior support	Middle interior spans	Interior supports
	Simple		Continuous				
	At outer support	Near middle of end span	At outer support	Near middle of end span			
Moment	0	0.086*Fl*	– 0.04*Fl*	0.075*Fl*	– 0.086*Fl*	0.063*Fl*	– 0.063*Fl*
Shear	0.4*F*	—	0.46*F*	—	0.6*F*	—	0.5*F*

NOTE *F* is the total design ultimate load ($1.4G_k + 1.6Q_k$);
l is the effective span.

APPENDIX E: Distribution of Design Moments in Panels of Flat Slab Provided by BS8110 (page50).

Table 3.18 — Distribution of design moments in panels of flat slabs

Design moment	Apportionment between column and middle strip expressed as percentages of the total negative or positive design moment	
	Column strip %	Middle strip %
Negative	75	25
Positive	55	45

NOTE For the case where the width of the column strip is taken as equal to that of the drop, and the middle strip is thereby increased in width, the design moments to be resisted by the middle strip should be increased in proportion to its increased width. The design moments to be resisted by the column strip may be decreased by an amount such that the total positive and the total negative design moments resisted by the column strip and middle strip together are unchanged.

APPENDIX F: Input Parameters of Covariance Analysis (Hogging Moment at Long Span of Interior Span).

Suporting beam size	M1	M2	M0	ly/lx	A	B	C	D	X
150*300	2.333	3.067	0.760629	1	0.000113	0.000113	0.000281	0.000281	1
	3.068	3.067	1.000261	1.1	0.000113	0.000102	0.000309	0.000256	0.751315
	3.904	3.067	1.272822	1.2	0.000113	9.38E-05	0.000338	0.000234	0.578704
	4.834	3.067	1.57603	1.3	0.000113	8.65E-05	0.000366	0.000216	0.455166
	5.853	3.067	1.908255	1.4	0.000113	8.04E-05	0.000394	0.000201	0.364431
	6.956	3.067	2.267866	1.5	0.000113	0.000075	0.000422	0.000188	0.296296
	10.050	3.067	3.276604	1.75	0.000113	6.43E-05	0.000492	0.000161	0.186589
	13.578	3.067	4.426839	2	0.000113	5.63E-05	0.000563	0.000141	0.125
	17.500	3.067	5.705529	2.25	0.000113	0.00005	0.000633	0.000125	0.087791
150*450	3.069	3.067	1.000587	1	0.00038	0.00038	0.000281	0.000281	1
	3.503	3.067	1.142084	1.1	0.00038	0.000345	0.000309	0.000256	0.751315
	3.946	3.067	1.286515	1.2	0.00038	0.000316	0.000338	0.000234	0.578704
	4.399	3.067	1.434207	1.3	0.00038	0.000292	0.000366	0.000216	0.455166
	4.863	3.067	1.585485	1.4	0.00038	0.000271	0.000394	0.000201	0.364431
	5.338	3.067	1.74035	1.5	0.00038	0.000253	0.000422	0.000188	0.296296
	6.576	3.067	2.143975	1.75	0.00038	0.000217	0.000492	0.000161	0.186589
	7.871	3.067	2.566184	2	0.00038	0.00019	0.000563	0.000141	0.125
	9.205	3.067	3.001109	2.25	0.00038	0.000169	0.000633	0.000125	0.087791
200*400	2.906	3.067	0.947444	1	0.000356	0.000356	0.000281	0.000281	1
	3.358	3.067	1.09481	1.1	0.000356	0.000323	0.000309	0.000256	0.751315
	3.827	3.067	1.247718	1.2	0.000356	0.000296	0.000338	0.000234	0.578704
	4.312	3.067	1.405842	1.3	0.000356	0.000274	0.000366	0.000216	0.455166
	4.813	3.067	1.569184	1.4	0.000356	0.000254	0.000394	0.000201	0.364431
	5.328	3.067	1.737089	1.5	0.000356	0.000237	0.000422	0.000188	0.296296
	6.673	3.067	2.1756	1.75	0.000356	0.000203	0.000492	0.000161	0.186589
	8.083	3.067	2.635303	2	0.000356	0.000178	0.000563	0.000141	0.125
	9.536	3.067	3.109025	2.25	0.000356	0.000158	0.000633	0.000125	0.087791
200*600	3.534	3.067	1.152191	1	0.0012	0.0012	0.000281	0.000281	1
	3.803	3.067	1.239893	1.1	0.0012	0.001091	0.000309	0.000256	0.751315
	4.023	3.067	1.31162	1.2	0.0012	0.001	0.000338	0.000234	0.578704
	4.202	3.067	1.369979	1.3	0.0012	0.000923	0.000366	0.000216	0.455166
	4.347	3.067	1.417254	1.4	0.0012	0.000857	0.000394	0.000201	0.364431
	4.464	3.067	1.455399	1.5	0.0012	0.0008	0.000422	0.000188	0.296296
	4.660	3.067	1.519301	1.75	0.0012	0.000686	0.000492	0.000161	0.186589
	4.743	3.067	1.546362	2	0.0012	0.0006	0.000563	0.000141	0.125
	4.717	3.067	1.537885	2.25	0.0012	0.000533	0.000633	0.000125	0.087791
250*500	3.296	3.067	1.074596	1	0.000868	0.000868	0.000281	0.000281	1
	3.598	3.067	1.173057	1.1	0.000868	0.000789	0.000309	0.000256	0.751315
	3.865	3.067	1.260107	1.2	0.000868	0.000723	0.000338	0.000234	0.578704
	4.103	3.067	1.337539	1.3	0.000868	0.000668	0.000366	0.000216	0.455166
	4.314	3.067	1.406495	1.4	0.000868	0.00062	0.000394	0.000201	0.364431
	4.503	3.067	1.468114	1.5	0.000868	0.000579	0.000422	0.000188	0.296296
	4.895	3.067	1.595755	1.75	0.000868	0.000496	0.000492	0.000161	0.186589
	5.182	3.067	1.689326	2	0.000868	0.000434	0.000563	0.000141	0.125
	5.364	3.067	1.748826	2.25	0.000868	0.000386	0.000633	0.000125	0.087791

250*750		3.805	3.067	1.240545	1	0.00293	0.00293	0.000281	0.000281	1
		4.026	3.067	1.312435	1.1	0.00293	0.002663	0.000309	0.000256	0.751315
		4.177	3.067	1.361665	1.2	0.00293	0.002441	0.000338	0.000234	0.578704
		4.269	3.067	1.391823	1.3	0.00293	0.002254	0.000366	0.000216	0.455166
		4.313	3.067	1.406005	1.4	0.00293	0.002093	0.000394	0.000201	0.364431
		4.317	3.067	1.40731	1.5	0.00293	0.001953	0.000422	0.000188	0.296296
		4.188	3.067	1.365415	1.75	0.00293	0.001674	0.000492	0.000161	0.186589
		3.895	3.067	1.269888	2	0.00293	0.001465	0.000563	0.000141	0.125
		3.455	3.067	1.126435	2.25	0.00293	0.001302	0.000633	0.000125	0.087791
300*600		3.563	3.067	1.161646	1	0.0018	0.0018	0.000281	0.000281	1
		3.803	3.067	1.239893	1.1	0.0018	0.001636	0.000309	0.000256	0.751315
		3.983	3.067	1.298579	1.2	0.0018	0.0015	0.000338	0.000234	0.578704
		4.112	3.067	1.340636	1.3	0.0018	0.001385	0.000366	0.000216	0.455166
		4.197	3.067	1.368349	1.4	0.0018	0.001286	0.000394	0.000201	0.364431
		4.245	3.067	1.383998	1.5	0.0018	0.0012	0.000422	0.000188	0.296296
		4.224	3.067	1.377152	1.75	0.0018	0.001029	0.000492	0.000161	0.186589
		4.033	3.067	1.31488	2	0.0018	0.0009	0.000563	0.000141	0.125
		3.683	3.067	1.200769	2.25	0.0018	0.0008	0.000633	0.000125	0.087791
300*900		3.984	3.067	1.298905	1	0.006075	0.006075	0.000281	0.000281	1
		4.190	3.067	1.366067	1.1	0.006075	0.005523	0.000309	0.000256	0.751315
		4.319	3.067	1.408125	1.2	0.006075	0.005063	0.000338	0.000234	0.578704
		4.385	3.067	1.429643	1.3	0.006075	0.004673	0.000366	0.000216	0.455166
		4.400	3.067	1.434533	1.4	0.006075	0.004339	0.000394	0.000201	0.364431
		4.373	3.067	1.42573	1.5	0.006075	0.00405	0.000422	0.000188	0.296296
		4.164	3.067	1.35759	1.75	0.006075	0.003471	0.000492	0.000161	0.186589
		3.784	3.067	1.233698	2	0.006075	0.003038	0.000563	0.000141	0.125
		3.251	3.067	1.059924	2.25	0.006075	0.0027	0.000633	0.000125	0.087791
600*300		2.432	3.067	0.792906	1	0.00045	0.00045	0.000281	0.000281	1
		2.880	3.067	0.938804	1.1	0.00045	0.000409	0.000309	0.000256	0.751315
		3.349	3.067	1.091875	1.2	0.00045	0.000375	0.000338	0.000234	0.578704
		3.835	3.067	1.250163	1.3	0.00045	0.000346	0.000366	0.000216	0.455166
		4.331	3.067	1.411874	1.4	0.00045	0.000321	0.000394	0.000201	0.364431
		4.834	3.067	1.57603	1.5	0.00045	0.0003	0.000422	0.000188	0.296296
		6.106	3.067	1.990578	1.75	0.00045	0.000257	0.000492	0.000161	0.186589
		7.371	3.067	2.403006	2	0.00045	0.000225	0.000563	0.000141	0.125
		8.603	3.067	2.804838	2.25	0.00045	0.0002	0.000633	0.000125	0.087791
900*300		2.391	3.067	0.779538	1	0.000675	0.000675	0.000281	0.000281	1
		2.785	3.067	0.907994	1.1	0.000675	0.000614	0.000309	0.000256	0.751315
		3.183	3.067	1.037591	1.2	0.000675	0.000563	0.000338	0.000234	0.578704
		3.577	3.067	1.166047	1.3	0.000675	0.000519	0.000366	0.000216	0.455166
		3.963	3.067	1.292058	1.4	0.000675	0.000482	0.000394	0.000201	0.364431
		4.337	3.067	1.413993	1.5	0.000675	0.00045	0.000422	0.000188	0.296296
		5.204	3.067	1.696661	1.75	0.000675	0.000386	0.000492	0.000161	0.186589
		5.955	3.067	1.941347	2	0.000675	0.000338	0.000563	0.000141	0.125
		6.571	3.067	2.142182	2.25	0.000675	0.0003	0.000633	0.000125	0.087791