EMPIRICAL STUDIES OF THE DIRICHLET PARAMETRIC FAMILY OF UNIVERSAL PORTFOLIOS

By

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ABSTRACT

EMPIRICAL STUDIES OF THE DIRICHLET PARAMETRIC FAMILY OF UNIVERSAL PORTFOLIOS

Khoo Ee Sin

In Chapter 1, we give a brief introduction of the universal portfolio. In Chapter 2, the Dirichlet parametric family of universal portfolios and the Helmbold-Schapire-Singer-Warmuth (HSSW) universal portfolios are introduced together with their computational algorithms. In Chapter 3 and 4, we compare the performances of the Cover-Ordentlich and the Helmbold-Schapire-Singer-Warmuth (HSSW) universal portfolios using some stock-price data selected from the Kuala Lumpur Stock Exchange. In Cover (1991) and Cover and Ordentlich (1996), the study is focussed on the parametric values α = (1, 1, …, 1) and α = (1/2, 1/2, …, 1/2) of the Dirichlet family of universal portfolios. The empirical studies of the Dirichlet family of universal portfolios is extended to a combination values of α different from (1, 1, …, 1) and (1/2, 1/2, …, 1/2). It is shown in this thesis that these general universal portfolios can outperform the Dirichlet (1, 1, …, 1) and Dirichlet (1/2, 1/2, …, 1/2) universal portfolios in both non-volatile and volatile periods of trading. In particular, we show that a universal portfolio can overcome the uncertainties of an economic downturn like the financial crisis in 2008 with regard to investment returns. Comparative study of the performances of the Dirichlet and HSSW parametric families of universal portfolios is also conducted. In

Chapter 5, we introduce a simple market model known as the DOSES model that is, Dominating Stock with Equal Subordinates model to study the asymptotic behaviour of the ratio of capitals $\frac{\hat{S}_n}{S_n^*}$ where $\hat{S}_n$ is the universal capital achieved in n trading days by the universal portfolio and S_n^* is the optimal capital achieved by the best constant rebalanced portfolio. The asymptotic behavior of $\frac{\hat{S}_n}{S_n^*}$ is a measure of the performance of the universal portfolio strategy. Cover (1991) proved the asymptotic behaviour of $\frac{\hat{S}_n}{S_n^*}$ for the uniform universal portfolio, Dirichlet (1, 1, …, 1) subject to certain regularity conditions on the maximizing sequence $\boldsymbol{b}_n^*$ achieving S_n^* and the doubling rate of capital W_n^* and the determinant of the sensitivity matrix J_n^*. For the general Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolios, a conjecture on the asymptotic behaviour of $\frac{\hat{S}_n}{S_n^*}$ due to Tan (2002a) is available. The objective of Chapter 5 is to test the validity of the conjectured behaviour of $\frac{\hat{S}_n}{S_n^*}$ by using real stock data modelled as a DOSES market. The conjectured behaviour is observed to be true based on investment in a DOSES market using the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolio.

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APPROVAL SHEET

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PERMISSION SHEET

It is hereby certified that **KHOO EE SIN** (ID No: **08UEM08105**) has completed this thesis entitled "EMPIRICAL STUDIES OF THE DIRICHLET PARAMETRIC FAMILY OF UNIVERSAL PORTFOLIOS" under the supervision of Assoc. Prof. Dr Tan Choon Peng from the Department of Mathematical and Actuarial Sciences, Faculty of Engineering and Science.

I hereby give permission to my supervisor to write and prepare a manuscript of these research findings for publishing in any form, if I did not prepare it within six (6) months time from this date, provided, that my name is included as one of the authors for this article. Arrangement of names will depend on my supervisor.

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTAR or other institutions.

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CHAPTER 1

INTRODUCTION

1.1 Portfolios in Investment

In the literature, the distributional approach is the most common approach for determining portfolio strategy by minimizing certain risk measures. This approach assumes that there is an underlying distribution of the stock price relative vectors or they follow a certain stochastic process like Brownian motion. In this thesis we do not assume any probability distribution of the stock prices. The investment portfolio obtained is 'universal' in this sense or distribution-free.

A constant rebalanced portfolio is an investment strategy that keeps the same distribution of wealth among a set of stocks from day to day. That is, the proportion of total wealth in a given stock is the same at the beginning of each day. Some define a 'constant rebalanced portfolio (CRP)' as one which preserves the proportion of investment in the various assets: that is, 'shedding' for stocks that rise, 'topping up' for those that fall. There is a certain CRP which is best in terms of investment return. However this best CRP can only be determined in hindsight.

Recently there are works on on-line investment strategies which are comparable with the best CRP determined in hindsight (Cover, 1991, Helmbold et al, 1998, Blum and Kalai, 1997, Cover and Ordentlich, 1996a, Ordentlich and Cover, 1996b, Cover, 1996c). Specifically, the daily performance of these algorithms on a market approaches that of the best CRP for that market, chosen in hindsight, as the lengths of these markets increase without bound.

There has been much work on Cover's universal portfolio, which is comparable with the best constant rebalanced portfolio determined in hindsight (Cover, 1991, Helmbold et al, 1998, Blum and Kalai, 1997, Cover and Ordentlich, 1996a, Cover, 1996c). Cover (1991) has exhibited an algorithm for portfolio selection that asymptotically outperforms the best stock in the market in accordance with theory. In this thesis, we focus on the work of Cover (1991) and Cover and Ordentlich (1996). Cover (1991) has proved that the wealth achieved by his uniform universal portfolio algorithm is optimal compared with the best constant rebalanced portfolio. Cover and Ordentlich (1996) introduced the notion of side information and generalized Cover's uniform universal portfolio to the Dirichlet parametric family of universal portfolios. Emphasis in Cover (1991) and Cover and Ordentlich (1996) are on the Dirichlet (1, 1, …, 1) and (1/2, 1/2, …, 1/2) universal portfolios. There is no study on the general Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolio until the theoretic work of Tan (2002b) which generalizes the performance bounds in Cover and Ordentlich (1996). In this thesis, we intend to perform the empirical studies of the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$

universal portfolio by using real stock data from the Malaysian Stock Exchange. Emphasis will be on two and three stock portfolios and investment returns by using the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolios. The ability of the universal portfolio to overcome the economic downturn brought by the financial crisis of 2008 has been shown in the study.

1.2 Literature Review

1.2.1 Stock Market, Portfolios and Wealth

Consider a portfolio consisting of *m* stocks and the investment model known as the uniform universal portfolio, introduced by Cover (1991). Cover and Ordentlich (1996) have introduced the Dirichlet weighted universal portfolio. For each trading day, the performance of the stocks can be denoted by the nonnegative price-relative stock vectors $\boldsymbol{x}_n = (x_{n1}, x_{n2}, ..., x_{nm})$ where

$$x_{ni} = \frac{\text{price of } i\text{th stock at end of day } n}{\text{price of } i\text{th stock at beginning of day } n}$$

Typically x_{ni} is near 1.

For example:

- If the price at end of day = price at beginning of next day, then

  Price relative = 1 $\Rightarrow$ no change in price.
- If Price = \$1.50 at beginning of day, and

  Price = \$2.00 at end of day, then

  Price relative = $\frac{2.00}{1.50} = \frac{4}{3}$ = 1.333.

This implies that the price increases by 1/3.

- If Price relative = 0.9 $\Rightarrow$ price decreases by 10%.

Definition 1.1

A stock market is a sequence of random vectors of price relatives where the stock-price-relative vector on day n is:

$$\boldsymbol{x}_n = (x_{n1}, x_{n2}, ..., x_{nm}), \quad x_{ni} \geq 0, \quad i = 1, 2, ..., m$$

where m is the number of stocks in the stock market and x_{ni} is the price relative or return of stock i on day n that can be expressed as:

$$x_{ni} = \frac{\text{price of } i\text{th stock at end of day } n}{\text{price of } i\text{th stock at beginning of day } n} \tag{1.1}$$

Definition 1.2

An investment portfolio is a probability vector, i.e. the portfolio on day n is:

$$\boldsymbol{b}_n = (b_{n1}, b_{n2}, ..., b_{nm}), \quad \sum_{i=1}^{m} b_{ni} = 1, \quad b_{ni} \geq 0$$

where b_{ni} is the proportion of the investor's capital invested in stock i on day n for i = 1, 2, …, m. The portfolio $\boldsymbol{b}_n$ is also an allocation of investment wealth across the stocks.

The portfolio and the capital on day n are listed in the following table:

Number of stock	**1**	**2**	**…**	***i***	**…**	***m***
Portfolio on day *n*	b_{n1}	b_{n2}	…	b_{ni}	…	b_{nm}
Capital at beginning of day *n*	$b_{n1}S_{n-1}$	$b_{n2}S_{n-1}$	…	$b_{ni}S_{n-1}$	…	$b_{nm}S_{n-1}$
Capital at end of day *n*	$b_{n1}S_{n-1}\, x_{n1}$	$b_{n2}S_{n-1}\, x_{n2}$	…	$b_{ni}S_{n-1}\, x_{ni}$	…	$b_{nm}S_{n-1}\, x_{nm}$

where S_{n-1} = capital of the previous day (that is at end of day $n-1$) ,

= capital at beginning of day n.

Suppose we hold y units of stock i, then

$$x_{ni} = \frac{(\text{price of } i\text{th stock at end of day } n)(y \text{ unit})}{(\text{price of the } i\text{th stock at beginning of day } n)(y \text{ unit})}$$

$$= \frac{\text{capital at end of day } n \text{ for stock } i}{b_{ni} S_{n-1}}$$

where $b_{ni}\, x_{ni}\, S_{n-1}$ = capital at end of day n for stock i.

Assuming that:

Price at beginning of day n = Price at end of day $n-1$.

Thus, the capital or the wealth relative at end of day n,

$$S_n = b_{n1}x_{n1}S_{n-1} + b_{n2}\, x_{n2}S_{n-1} + \ldots + b_{ni}x_{ni}S_{n-1} + \ldots + b_{nm}x_{nm}S_{n-1}$$

$$= (b_{n1}x_{n1} + b_{n2}\, x_{n2} + \ldots + b_{ni}x_{ni} + \ldots + b_{nm}x_{nm})S_{n-1}$$

$$= \left(\sum_{i=1}^{m} b_{ni}x_{ni}\right) S_{n-1}$$

$$= (\boldsymbol{b}_n^t \cdot \boldsymbol{x}_n) S_{n-1}$$

where $\boldsymbol{x}_n = (x_{n1}, x_{n2}, \ldots, x_{nm})$ and the portfolio is defined by the weight vector

$$\boldsymbol{b}_n = (b_{n1}, b_{n2}, \ldots, b_{nm}) \quad \text{such that} \quad b_{ni} \geq 0 \ \text{ for } \ i = 1, 2, \ldots, m$$

and $\boldsymbol{b}_n^t$ is the transpose of $\boldsymbol{b}_n$.

Thus,

$$S_n = (\boldsymbol{b}_n^t \cdot \boldsymbol{x}_n)(\boldsymbol{b}_{n-1}^t \cdot \boldsymbol{x}_{n-1})\, S_{n-2}$$

$$= (\boldsymbol{b}_n^t \cdot \boldsymbol{x}_n)(\boldsymbol{b}_{n-1}^t \cdot \boldsymbol{x}_{n-1})(\boldsymbol{b}_{n-2}^t \cdot \boldsymbol{x}_{n-2})\, S_{n-3}$$

$$\vdots$$

$$= (\boldsymbol{b}_n^t \cdot \boldsymbol{x}_n)(\boldsymbol{b}_{n-1}^t \cdot \boldsymbol{x}_{n-1}) \ldots (\boldsymbol{b}_1^t \cdot \boldsymbol{x}_1)\, S_0$$

$$= \prod_{j=1}^{n} (\boldsymbol{b}_j^t \cdot \boldsymbol{x}_j) S_0 \qquad (1.2)$$

where S_0 = initial capital on day 0.

Assume that the initial investment, S_0 = 1 unit i.e. the initial capital is normalized to one, then the investment capital at the end of the nth trading day will be

$$S_n = \prod_{j=1}^{n} (\boldsymbol{b}_j^t \cdot \boldsymbol{x}_j) \tag{1.3}$$

1.3 Constant Rebalanced Portfolio

A constant rebalanced portfolio is rebalanced back to vector $\boldsymbol{b}$ on each trading day in order that a fixed fraction of the wealth invested is held in each of the underlying investment stocks. Thus, the resulting wealth, S_n after n trading days is as follows:

$$S_n = \prod_{j=1}^{n} (\boldsymbol{b}^t \cdot \boldsymbol{x}_j) \tag{1.3}$$

We take note that at the end of a particular trading period, say the n^{th} day, the proportion of wealth invested in each stock has changed from

$$(b_{n1}, b_{n2}, \ldots, b_{nm}) \text{ to } \left(\frac{x_{n1}b_{n1}}{\boldsymbol{b}^t\boldsymbol{x}_j}, \frac{x_{n2}b_{n2}}{\boldsymbol{b}^t\boldsymbol{x}_j}, \ldots, \frac{x_{nm}b_{nm}}{\boldsymbol{b}^t\boldsymbol{x}_j}\right)$$

and therefore stock must be bought and sold to restore the proportions of wealth to $(b_{n1}, b_{n2}, \ldots, b_{nm})$ for the next trading period.

The best constant rebalanced portfolio vector $\boldsymbol{b}_n^*$ with respect to a sequence of known daily price relatives, $\boldsymbol{x}_n$ = $(x_{n1}, x_{n2}, \ldots, x_{nm})$ is the portfolio $\boldsymbol{b}_n^*$ which will result in a maximum S_n over all $\boldsymbol{b}$ in B_m, where the maximum is taken over all possible portfolio vectors in B_m, where the portfolio or probability simplex B_m in R^m is defined as:

$$B_m = \left\{ \boldsymbol{b} = (b_1,...,b_m) : \sum_{i=1}^{m} b_i = 1,\ \ 0 \le b_i \le 1,\ i = 1,...,m \right\} \qquad (1.4)$$

1.3.1 Example of Constant Rebalanced Portfolio

As an example of a useful constant rebalanced portfolio, CRP, consider the following market with just two stocks (Helmbold et al 1998, Ordentlich and Cover 1996b). The first is a risk-free, no-growth investment stock whose value never changes. The second investment is a hypothetical highly volatile stock. The price relative of one stock remains constant and the price relative of the other stock alternately halves and doubles. Investing in a single stock will not increase the wealth by more than a factor of two. However, a (1/2, 1/2) CRP will increase its wealth exponentially.

Stock 1 – Price relative stays constant
Stock 2 – Price relative doubles / halves alternatingly every two days

	Price Relative of Stock 1	Price Relative of Stock 2
Day 1	1	1/2
Day 2	1	2
Day 3	1	1/2
Day 4	1	2
⋮	⋮	⋮

If we let $\boldsymbol{b}$ = (1/2, 1/2) then the wealth of this example is:

$$
\begin{aligned}
S_1 &= 1 \\
S_2 &= S_1\left(\frac{1}{2}\cdot 1+\frac{1}{2}\cdot\frac{1}{2}\right)=\frac{3}{4}S_1 \\
S_3 &= S_2\left(\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 2\right)=\frac{3}{2}S_2 \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
S_{t+2} &= \frac{3}{2}S_{t+1}=\frac{3}{2}\cdot\frac{3}{4}S_t=\frac{9}{8}S_t
\end{aligned}
$$

The last equation shows that every two days the money grows by a factor of $\frac{9}{8}$ and over $2n$ trading days the money grows by a factor of $\left(\frac{9}{8}\right)^n$. This implies that the money is growing exponentially fast.

1.4 An Overview of the Thesis

This thesis consists of five chapters. In Chapter 1, we give the literature review and introduction of the universal portfolio. In Chapter 2, we discuss the algorithms necessary to implement the Dirichlet parametric family of universal portfolios for two and also three stocks. The Helmbold-Schapire-Singer-Warmuth (HSSW) universal portfolios are introduced together with their computational algorithms. In Chapter 3 and 4, the two-stock and three-stock universal portfolios are run on selected data sets from the Kuala Lumpur Stock Exchange and the results are discussed respectively. The empirical studies of the Dirichlet family of universal portfolios is extended to a combination values of α instead of the usual α = (1, 1, …, 1) or α = (1/2, 1/2, …, 1/2) recommended in Cover and Ordentlich (1996). The significance of the results is that higher investment returns in both non-volatile and volatile periods of trading can be achieved by changing the parametric vector α. In particular, we show that a universal portfolio can overcome the uncertainties of an economic downturn like the financial crisis at the end of 2008, which is considered a volatile period with regard to investment returns. Comparative study of the performances of the Dirichlet and HSSW parametric families of universal portfolios is also conducted. In Chapter 5, we introduce a simple market model known as the DOSES market of data sets from the local stock exchange and applying scaling on the parameters of the DOSES market to obtain a sequence of maximizing vectors of the capital converging in the simplex of portfolio vectors that satisfies the regularity conditions. The finding, by running the portfolio on real data sets provides some empirical evidence on the truth of a conjecture on the asymptotic behaviour of the ratio of capitals, $\frac{\hat{S}_n}{S_n^*}$ where $\hat{S}_n$ is the universal capital achieved in n trading days and S_n^* is the optimal capital achieved by the constant rebalanced portfolio.

CHAPTER 2

THEORY AND METHODOLOGY

2.1 Cover and Ordentlich Universal Portfolio

Cover (1991) introduced the uniform universal portfolio. In Cover and Ordentlich (1996), they generalized this to the general Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ parametric family of universal portfolios. The uniform universal portfolio is the Dirichlet (1, 1, …, 1) universal portfolio. In their study, Cover and Ordentlich focussed on the Dirichlet (1, 1, …, 1) and Dirichlet (1/2, 1/2, …, 1/2) universal portfolios. Formally, we define the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolio as follows:

Definition 2.1

Let $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n$ be a sequence of price-relative vectors in n trading days. The universal portfolio on day $k + 1$ generated by the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ distribution μ is given as follows:

$$\hat{\boldsymbol{b}}_{k+1} = \frac{\int_{B_m} \boldsymbol{b} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_m} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})} \tag{2.1}$$

where $S_k(\boldsymbol{b}) = \prod_{i=1}^{k} \boldsymbol{b}^t \boldsymbol{x}_i$ (2.2)

and $d\mu(\boldsymbol{b}) = \dfrac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} b_1^{\alpha_1 - 1} ... b_{m-1}^{\alpha_{m-1}-1} b_m^{\alpha_m - 1} db_1 db_2 ... db_m$

$$= \frac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} \prod_{j=1}^{m} b_j^{\alpha_j - 1} d\boldsymbol{b} \tag{2.3}$$

where $\alpha_j > 0$ for j = 1, 2, …, m and refers to the differential $d\boldsymbol{b}$ with respect to any m – 1 independent variables from $b_1, b_2, ..., b_{m-1}$.

Since $b_1 + b_2 + ... + b_m = 1$

$$b_m = 1 - (b_1 + b_2 + ... + b_{m-1})$$

$$= 1 - \sum_{j=1}^{m-1} b_j$$

The integration of any function $f(\boldsymbol{b})$ over B_m means integration of $f(\boldsymbol{b})$ with respect to $db_1, db_2, ..., db_{m-1}$.

Note that: The gamma function is defined as:

$$\Gamma(\alpha) = \int_0^\infty u^{\alpha-1} e^{-u} du \qquad \text{for} \qquad \alpha > 0. \tag{2.4}$$

and

$$\Gamma(x) = \frac{e^{-u}}{-1} \cdot u^{x-1} \Big|_0^\infty + \int_0^\infty e^{-u}(x-1)u^{x-2} du$$

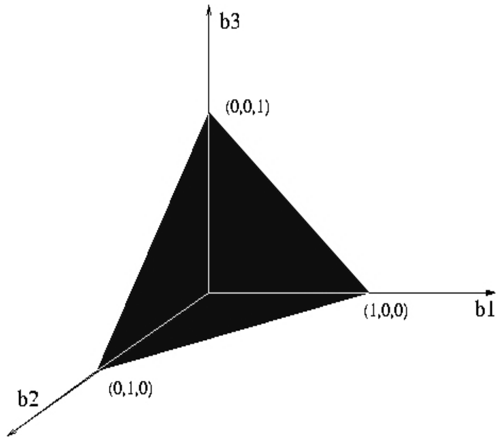
$$= (x-1)\int_0^\infty u^{(x-2)} e^{-u} du$$

$$\therefore \qquad \Gamma(x) = (x-1)\Gamma(x-1) \text{ for } \quad x > 1. \tag{2.5}$$

For the uniform universal portfolio, (2.3) yields

$$d\mu(\boldsymbol{b}) = (m-1)!\, db_1 db_2 ... db_{m-1} \qquad \text{for} \qquad \alpha_1 = \alpha_2 = ... = \alpha_m = 1$$

The following 3-dimensional graph shows a simple 3-stock uniform universal portfolio, $\boldsymbol{b}^t = (b_1, b_2, b_3)$:



$d\mu(\boldsymbol{b}) = 2db_1 db_2$ since $\alpha_1 = \alpha_2 = \alpha_3 = 1$

$$b_{k+1,1} = \frac{2\iint_{B_2} b_1 S_k(b_1,b_2)db_1 db_2}{\iint_{B_2} S_k(b_1,b_2)db_1 db_2},$$

$$b_{k+1,2} = \frac{2\iint_{B_2} b_2 S_k(b_1,b_2)db_1 db_2}{\iint_{B_2} S_k(b_1,b_2)db_1 db_2}, \tag{2.6}$$

where $S_k(b_1,b_2) = \prod_{i=1}^{k}(b_1 x_{i1} + b_2 x_{i2})$ and $\boldsymbol{x}_i = (x_{i1}, x_{i2})$

2.1.1 Method for Calculating the Universal Portfolio

The universal portfolio $\left\{\hat{\boldsymbol{b}}_j\right\}_{j=1}^{n}$ achieves a wealth of

$$\hat{S}_n = \prod_{j=1}^{n}(\hat{\boldsymbol{b}}_j^t \cdot \boldsymbol{x}_j) \tag{2.7}$$

at the end of *n* trading days where $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_n$ is the price-relative sequence. Instead of calculating each $\hat{\boldsymbol{b}}_j$ and the resulting product $\prod_{j=1}^{n}(\hat{\boldsymbol{b}}_j^t \cdot \boldsymbol{x}_j)$, it is convenient to calculate $\hat{S}_n$ directly by using the telescoping of the product as follows (see Cover and Ordentlich (1996)):

$$\hat{S}_n = \prod_{j=1}^{n} (\hat{\boldsymbol{b}}_j^t \cdot \boldsymbol{x}_j)$$

$$= \prod_{j=1}^{n} \frac{\int_{B_m} \boldsymbol{b}^t \boldsymbol{x}_j S_{j-1}(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_m} S_{j-1}(\boldsymbol{b}) d\mu(\boldsymbol{b})} \qquad \text{where} \quad S_k(\boldsymbol{b}) = \prod_{i=1}^{k} \boldsymbol{b}^t \boldsymbol{x}_i$$

$$= \prod_{j=1}^{n} \frac{\int_{B_m} S_j(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_m} S_{j-1}(\boldsymbol{b}) d\mu(\boldsymbol{b})}$$

$$= \int_{B_m} S_n d\mu(\boldsymbol{b}) \tag{2.8}$$

where $$\int_{B_m} S_0 d\mu(\boldsymbol{b}) = \int_{B_m} d\mu(\boldsymbol{b}) = 1 \tag{2.9}$$

and the initial investment capital S_0 is assumed to be 1.

When $k = 1$,

$$\hat{\boldsymbol{b}}_1 = \frac{\int_{B_m} \boldsymbol{b} S_0 d\mu(\boldsymbol{b})}{\int_{B_m} S_0 d\mu(\boldsymbol{b})}$$

$$= \frac{\int_{B_m} \boldsymbol{b} d\mu(\boldsymbol{b})}{\int_{B_m} d\mu(\boldsymbol{b})}$$

$$= \int_{B_m} \boldsymbol{b} d\mu(\boldsymbol{b}) \qquad \text{since} \quad \int_{B_m} d\mu(\boldsymbol{b}) = 1 \tag{2.10}$$

Asymptotically for large n, we have the property that

$$\lim_{n\to\infty} \sup_{x^n} \left(\ln \hat{S}_n - \ln S_n^*\right) = 0 \qquad \text{where} \qquad S_n^* = \max_b \prod_{i=1}^{n} \boldsymbol{b}^t \boldsymbol{x}_i \tag{2.11}$$

(see Cover and Ordentlich (1996)).

The algorithms are referred to as universal portfolios because they achieve optimal growth universally over price paths. Thus, a universal portfolio selection algorithm exhibits the same asymptotic growth rate in logarithmic wealth as the best rebalanced portfolio for any sequence of price relatives.

2.1.2 Modified Algorithm for Computing the Two-Stock Universal Portfolio

A recursive method is available for the computation of the universal portfolio generated by the Dirichlet (α_1, α_2) distribution where $\alpha_i > 0$ for i = 1, 2. This algorithm is developed by Cover and Ordentlich (1996) for $\alpha_1 = \alpha_2 = \frac{1}{2}$ to compute the universal portfolio recursively. Chan (2002) modified the algorithm for any $(\alpha_1, \alpha_2, ..., \alpha_m)$ where m = 2, 3, 4. We describe the modified algorithm by Chan (2002) in this chapter.

Let $\boldsymbol{x}_n = (x_{n1}, x_{n2})$ be a sequence of price-relative vectors corresponding to n trading days in a two-stock market where m = 2. The universal portfolio generated by the Dirichlet (α_1, α_2) distribution is a sequence of portfolio vectors given in (2.1), that is

$$\hat{\boldsymbol{b}}_{k+1} = \frac{\int_{B_2} \boldsymbol{b} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_2} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})} \tag{2.12}$$

for k = 1, 2, …, where $B_2 = \{(b_1, b_2) : 0 \le b_1 \le 1,\ 0 \le b_2 \le 1,\ b_1 + b_2 = 1\}$

Thus, the investment capital achieved by universal portfolio at the end of the n^{th} trading day is given by the simplified form of (2.8) as:

$$\hat{S}_n = \int_{B_2} S_n d\mu(\boldsymbol{b})$$

$$= \int_{B_2} \prod_{i=1}^{n} (b_1 x_{i1} + b_2 x_{i2}) d\mu(\boldsymbol{b})$$

$$= \int_{B_2} \sum_{j^n \in \{1,2\}^n} \prod_{i=1}^{n} b_{j_i} x_{ij_i} d\mu(\boldsymbol{b})$$

$$= \int_{B_2} \sum_{l=0}^{n} b_1^l b_2^{(n-l)} \left(\sum_{j^n \in T_n(l)} \prod_{i=1}^{n} x_{ij_i} \right) d\mu(\boldsymbol{b})$$

$$= \int_{B_2} \sum_{l=0}^{n} b_1^l b_2^{(n-l)} X_n(l) d\mu(\boldsymbol{b}) \tag{2.13}$$

where $T_n(l)$ is the set of all sequences $j^n = (j_1, j_2, ..., j_n) \in \{1, 2\}^n$

with l 1's and $n - l$ 2's.

Letting $X_n(l) = \sum_{j^n \in T_n(l)} \prod_{i=1}^{n} x_{ij_i}$ for $l = 0, 1, \ldots, n.$ (2.14)

The wealth accrued by the universal portfolio at time n can be rewriting as:

$$\hat{S}_n = \sum_{l=0}^{n} X_n(l) \int_{B_2} b_1^l b_2^{(n-l)} \, d\mu(\boldsymbol{b}) \tag{2.15}$$

$$\hat{S}_n = \sum_{l=0}^{n} X_n(l) C_n(l) \tag{2.16}$$

where $C_n(l) = \int_{B_2} b_1^l b_2^{n-l} \, d\mu(\boldsymbol{b})$ for $l = 0, 1, \ldots, n$ (2.17)

and $d\mu(\boldsymbol{b}) = \dfrac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} b_1^{\alpha_1 - 1} \, b_2^{\alpha_2 - 1} db_1$

From (2.1), (2.8) and (2.16), the universal portfolio $\hat{\boldsymbol{b}}_n$ on the $(n + 1)^{\text{th}}$ trading day can be obtained as:

$$\hat{\boldsymbol{b}}_n = \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \int_{B_2} b_1 S_{n-1}(\boldsymbol{b}) d\mu(\boldsymbol{b}) \\ \int_{B_2} b_2 S_{n-1}(\boldsymbol{b}) d\mu(\boldsymbol{b}) \end{bmatrix}$$

$$= \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l=0}^{n-1} X_{n-1}(l) \int_{B_2} b_1^{(l+1)} b_1^{(n-l-1)} d\mu(\boldsymbol{b}) \\ \sum_{l=0}^{n-1} X_{n-1}(l) \int_{B_2} b_1^{l} b_1^{(n-l)} d\mu(\boldsymbol{b}) \end{bmatrix}$$

$$= \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l=0}^{n-1} C_n(l+1) X_{n-1}(l) \\ \sum_{l=0}^{n-1} C_n(l) X_{n-1}(l) \end{bmatrix} \tag{2.18}$$

From (2.16) and (2.18), we notice that the computation of the above universal portfolio expressions depend on $C_n(l)$ and $X_n(l)$. Both of these quantities can be computed recursively.

Recursive Computations for $C_n(l)$

Two recursions for $C_n(l)$ are given as follows:

$$C_n(l) = \frac{l + \alpha_1 - 1}{n + \alpha_1 + \alpha_2 - 1} C_{n-1}(l-1) \qquad \text{for} \qquad 1 < l \le n \tag{2.19}$$

$$C_n(l) = \frac{n - l + \alpha_2 - 1}{n + \alpha_1 + \alpha_2 - 1} C_{n-1}(l) \qquad \text{for} \qquad 0 \le l \le n-1 \tag{2.20}$$

where $C_0 = \int_{B_2} d\mu(\boldsymbol{b}) = 1$.

Recursive Computations for $X_n(l)$

The recursion for $X_n(l)$ is given as follows:

$$X_n(l) = x_{n1} X_{n-1}(l-1) + x_{n2} X_{n-1}(l) \quad \text{for} \quad 1 \le l \le n-1 \tag{2.21}$$

With the endpoint conditions given as:

$$X_n(0) = x_{n2} X_{n-1}(0) \tag{2.22}$$

$$X_n(n) = x_{n1} X_{n-1}(n-1) \tag{2.23}$$

Note that:

When there are no 1's then $l = 0$, thus $X_n(0) = x_{n2}X_{n-1}(0)$

When there are n numbers of 1's and no 2's, then $l = n$, thus

$$X_n(n) = x_{n1}X_{n-1}(n-1). \tag{2.24}$$

Consider the following example:

Letting $X_n(l) = \sum_{j^n \in T_n(l)} \prod_{i=1}^{n} x_{ij_i}$ for $l = 0, 1, \ldots, n$.

where $T_n(l)$ is the set of all sequences $j^n = (j_1, j_2, ..., j_n) \in \{1, 2\}^n$

with l 1's and $n - l$ 2's.

When $n = 1$,

$T_1(0)$ = the set of all sequences $j_1 \in \{1, 2\}^1$ with no 1's and one 2.

= {2}

Thus, $X_1(0) = \{x_{12}\}$

$T_1(1)$ = the set of all sequences $j_1 \in \{1, 2\}^1$ with one 1 and no 2's.

= {1}

Thus, $X_1(1) = \{x_{11}\}$

When $n = 2$, the endpoints are:

$$X_2(0) = x_{22}X_1(0) = x_{22}x_{12}$$

$$X_2(2) = x_{21}X_1(1) = x_{21}x_{11}$$

For the non-end point:

$$X_2(1) = x_{21}X_1(0) + x_{22}X_1(1)$$

$$= x_{21}x_{12} + x_{22}x_{11}$$

When $n = 3$, the endpoints are:

$$X_3(0) = x_{32}X_2(0) = x_{32}x_{22}x_{12}$$

$$X_3(3) = x_{31}X_2(2) = x_{31}x_{21}x_{11}$$

For the non-end points:

$$X_3(1) = x_{31}X_2(0) + x_{32}X_2(1)$$

$$= x_{31}x_{22}x_{12} + x_{32}(x_{21}x_{12} + x_{22}x_{11})$$

$$X_3(2) = x_{31}X_2(1) + x_{32}X_2(2)$$

$$= x_{31}(x_{21}x_{12} + x_{22}x_{11}) + x_{32}x_{21}x_{11}$$

2.1.3 Recursive Computations of the Universal Capital $\hat{S}_n$ and Universal Portfolio $\hat{\boldsymbol{b}}_n$

By recursively computing and storing the quantities $X_n(l)$ and $C_n(l)$, we can compute the universal capital $\hat{S}_n$ and universal portfolio $\hat{\boldsymbol{b}}_n$. Since $\hat{S}_n$ and $\hat{\boldsymbol{b}}_n$ depend on the product of $X_n(l)$ and $C_n(l)$, we can further simply the computation of $\hat{S}_n$ and $\hat{\boldsymbol{b}}_n$ as follows:

$$Q_n(l) = X_n(l)C_n(l) \tag{2.25}$$

From (2.16), the accrued wealth $\hat{S}_n$ can be rewritten as:

$$\hat{S}_n = \sum_{l=0}^{n} Q_n(l) \tag{2.26}$$

The quantities $C_n(l+1)X_{n-1}(l)$ and $C_n(l)X_{n-1}(l)$ that appear in the expression for $\hat{\boldsymbol{b}}_n$ (3.21) can be expressed in terms of $Q_n(l)$ as:

$$C_n(l+1)X_{n-1}(l) = \frac{l+\alpha_1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l) \tag{2.27}$$

$$C_n(l)X_{n-1}(l) = \frac{n-l+\alpha_2-1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l) \tag{2.28}$$

By using (2.26), (2.27) and (2.28), the universal portfolio $\hat{\boldsymbol{b}}_n$ can be expressed as:

$$\hat{\boldsymbol{b}}_n = \frac{1}{\sum_{l=0}^{n-1} Q_{n-1}(l)} \begin{bmatrix} \sum_{l=0}^{n-1} \frac{l+\alpha_1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l) \\ \sum_{l=0}^{n-1} \frac{n-l+\alpha_2-1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l) \end{bmatrix} \tag{2.29}$$

It is reasonable to compute only the $Q_n(l)$ from a numerical standpoint since the quantities of $C_n(l)$ and $X_n(l)$ are respectively exponentially decreasing and increasing in *n*.

Recursive Formulae for $Q_n(l)$

The computation of the $Q_n(l)$ can be done recursively by combining the recursions for $X_n(l)$ and $C_n(l)$.

The recursions for $Q_n(l)$ are given as:

$$Q_n(l) = x_{n1} \frac{l+\alpha_1-1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l-1) + x_{n2} \frac{n-l+\alpha_2-1}{n+\alpha_1+\alpha_2-1} Q_{n-1}(l)$$

$$\text{for } 1 \le l \le n-1. \tag{2.30}$$

The endpoint conditions are given by:

$$Q_n(0) = x_{n2} \frac{n + \alpha_2 - 1}{n + \alpha_1 + \alpha_2 - 1} Q_{n-1}(0) \tag{2.31}$$

$$Q_n(n) = x_{n1} \frac{n + \alpha_1 - 1}{n + \alpha_1 + \alpha_2 - 1} Q_{n-1}(n-1) \tag{2.32}$$

With the initial condition $Q_0(0) = \int_{B_2} d\mu(\boldsymbol{b}) = 1$.

Summary

The two expressions $\hat{\boldsymbol{b}}_n$ and $\hat{S}_n$ on the n^{th} trading day are expressed respectively in term of $Q_n(l)$ as follows:

The universal portfolio

$$\hat{\boldsymbol{b}}_n = \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l=0}^{n-1} \frac{l + \alpha_1}{n + \alpha_1 + \alpha_2 - 1} Q_{n-1}(l) \\ \sum_{l=0}^{n-1} \frac{n - l + \alpha_2 - 1}{n + \alpha_1 + \alpha_2 - 1} Q_{n-1}(l) \end{bmatrix} \tag{2.33}$$

and the universal capital yields by $\hat{S}_n$ is given by:

$$\hat{S}_n = \sum_{l=0}^{n} Q_n(l)$$

2.1.4 Modified Algorithm for Computing the Three-Stock Universal Portfolio

We describe the modified algorithm by Chan (2002) for computing the three-stock universal portfolio generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution where $\alpha_i > 0$ for i = 1, 2, 3 and

$$d\mu(\boldsymbol{b}) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} b_1^{\alpha_1 - 1} b_2^{\alpha_2 - 1} b_3^{\alpha_3 - 1} db_1 db_2 \tag{2.34}$$

Let $\boldsymbol{x}_n = (x_{n1}, x_{n2}, ..., x_{nm})$ be a sequence of price-relative vectors corresponding to n trading days where $\boldsymbol{x}_i = (x_{i1}, x_{i2}, x_{i3})$ and use the definition of the universal portfolio generated by the probability μ measure is a sequence of portfolio vectors given in (2.1), that is

$$\hat{\boldsymbol{b}}_{k+1} = \frac{\int_{B_3} \boldsymbol{b} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_3} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}$$

for k = 1, 2, …, where $B_3 = \{(b_1, b_2, b_3) : 0 \le b_i \le 1,\ i = 1, 2, 3,\ b_1 + b_2 + b_3 = 1\}$

Thus, the investment capital achieved by universal portfolio at the end of the n^{th} trading day is given by the simplified form of (2.8) as:

$$\begin{aligned}
\hat{S}_n &= \int_{B_3} S_n d\mu(\boldsymbol{b}) \\
&= \int_{B_3} \prod_{i=1}^{n} (b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3}) d\mu(\boldsymbol{b}) \\
&= \int_{B_3} \sum_{j^n \in \{1,2,3\}^n} \prod_{i=1}^{n} b_{j_i} x_{ij_i} d\mu(\boldsymbol{b}) \\
&= \int_{B_3} \sum_{l_1=0}^{n} \sum_{l_2=0}^{n-l_1} b_1^{l_1} b_2^{l_2} b_3^{(n-l_1-l_2)} \left(\sum_{j^n \in T_n(l_1,l_2)} \prod_{i=1}^{n} x_{ij_i} \right) d\mu(\boldsymbol{b}) \\
&= \int_{B_3} \sum_{l_1=0}^{n} \sum_{l_2=0}^{n-l_1} b_1^{l_1} b_2^{l_2} b_3^{(n-l_1-l_2)} X_n(l_1,l_2) d\mu(\boldsymbol{b}) \qquad (2.35)
\end{aligned}$$

where $T_n(l_1,l_2)$ is the set of all sequences $j^n = (j_1, j_2, ..., j_n) \in \{1, 2, 3\}^n$ with l_1 1's, l_2 2's and $n - l_1 - l_2$ 3's.

Letting $X_n(l_1,l_2) = \sum_{j^n \in T_n(l_1,l_2)} \prod_{i=1}^{n} x_{ij_i}$ for l = 0, 1, …, n. (2.36)

The wealth accrued by the universal portfolio at time n can be rewriting as:

$$\hat{S}_n = \sum_{l_1=0}^{n} \sum_{l_2=0}^{n\text{-}l_1} X_n(l_1,l_2) \int_{B_3} b_1^{l_1} b_2^{l_2} b_3^{(n-l_1-l_2)}\, d\mu(\boldsymbol{b}) \tag{2.37}$$

$$\hat{S}_n = \sum_{l_1=0}^{n} \sum_{l_2=0}^{n\text{-}l_1} X_n(l_1,l_2) C_n(l_1,l_2) \tag{2.38}$$

where $C_n(l_1,l_2) = \int_{B_3} b_1^{l_1} b_2^{l_2} b_3^{(n-l_1-l_2)}\, d\mu(\boldsymbol{b})$ (2.39)

By using (2.1), (2.8) and (2.38), we obtain the universal portfolio $\hat{\boldsymbol{b}}_n$ on the n^{th} trading day as follows:

$$\hat{\boldsymbol{b}}_n = \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \int_{B_3} b_1 S_{n-1}(\boldsymbol{b}) d\mu(\boldsymbol{b}) \\ \int_{B_3} b_2 S_{n-1}(\boldsymbol{b}) d\mu(\boldsymbol{b}) \\ \int_{B_3} b_3 S_{n-1}(\boldsymbol{b}) d\mu(\boldsymbol{b}) \end{bmatrix}$$

$$= \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} X_{n-1}(l_1,l_2) \int_{B_3} b_1^{l_1+1} b_2^{l_2} b_3^{n-l_1-l_2-1} d\mu(\boldsymbol{b}) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} X_{n-1}(l_1,l_2) \int_{B_3} b_1^{l_1} b_2^{l_2+1} b_3^{n-l_1-l_2-1} d\mu(\boldsymbol{b}) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} X_{n-1}(l_1,l_2) \int_{B_3} b_1^{l_1} b_2^{l_2} b_3^{n-l_1-l_2} d\mu(\boldsymbol{b}) \end{bmatrix} \tag{2.40}$$

$$= \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} C_n(l_1+1,l_2) X_{n-1}(l_1,l_2) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} C_n(l_1,l_2+1) X_{n-1}(l_1,l_2) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} C_n(l_1,l_2) X_{n-1}(l_1,l_2) \end{bmatrix} \tag{2.41}$$

Note: The above expressions $\hat{S}_n$ and $\hat{\boldsymbol{b}}_n$ depend on $C_n(l_1,l_2)$ and $X_n(l_1,l_2)$ which can be computed recursively.

Recursive Computations for $C_n(l_1,l_2)$

Three recursions for $C_n(l_1,l_2)$ are given as follows:

(i) $$C_n(l_1,l_2)=\frac{l_1+\alpha_1-1}{n+\alpha_1+\alpha_2+\alpha_3-1}C_{n-1}(l_1-1,l_2)$$

$$\text{for } 1\le l_1\le n \quad \text{and} \quad 0\le l_2\le n-l_1 \tag{2.42}$$

(ii) $$C_n(l_1,l_2)=\frac{l_2+\alpha_2-1}{n+\alpha_1+\alpha_2+\alpha_3-1}C_{n-1}(l_1,l_2-1)$$

$$\text{for } 0\le l_1\le n-1 \quad \text{and} \quad 1\le l_2\le n-l_1 \tag{2.43}$$

(iii) $$C_n(l_1,l_2)=\frac{n-l_1-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1}C_{n-1}(l_1,l_2)$$

$$\text{for } 0\le l_1\le n-1 \quad \text{and} \quad 0\le l_2\le n-l_1-1 \tag{2.44}$$

where the initial condition, $C_0(0,0)=\int_{B_3} d\mu(\boldsymbol{b})=1$.

Recursive Computations for $X_n(l_1,l_2)$

The recursion for $X_n(l_1,l_2)$ is given as follows:

$$X_n(l_1,l_2)=x_{n1}X_{n-1}(l_1-1,l_2)+x_{n2}X_{n-1}(l_1,l_2-1)+x_{n3}X_{n-1}(l_1,l_2)$$

$$\text{for } 1\le l_1\le n-1 \quad \text{and} \quad 1\le l_2\le n-l_1-1 \tag{2.45}$$

With the endpoint conditions given as:

$$X_n(l_1,0)=x_{n1}X_{n-1}(l_1-1,0)+x_{n3}X_{n-1}(l_1,0) \qquad \text{for } 1\le l_1\le n-1 \tag{2.46}$$

$$X_n(0,l_2)=x_{n2}X_{n-1}(0,l_2-1)+x_{n3}X_{n-1}(0,l_2) \quad \text{for } 1\le l_2\le n-1 \tag{2.47}$$

$$X_n(l_1,l_2)=x_{n1}X_{n-1}(l_1-1,l_2)+x_{n2}X_{n-1}(l_1,l_2-1)$$

$$\text{for } 1\le l_1,\ 1\le l_2 \quad \text{and} \quad l_1+l_2=n \tag{2.48}$$

$$X_n(n,0)=x_{n1}X_{n-1}(n-1,0) \tag{2.49}$$

$$X_n(0,n)=x_{n2}X_{n-1}(0,n-1) \tag{2.50}$$

$$X_n(0,0)=x_{n3}X_{n-1}(0,0) \tag{2.51}$$

The $Q_n(l_1,l_2) = X_n(l_1,l_2)C_n(l_1,l_2)$ formulae:

We can compute the universal portfolio expressions $\hat{S}_n$ and $\hat{\boldsymbol{b}}_n$ by recursively computing and storing the quantities of $X_n(l_1,l_2)$ and $C_n(l_1,l_2)$.

$$Q_n(l_1,l_2) = X_n(l_1,l_2)C_n(l_1,l_2) \qquad (2.52)$$

$\hat{S}_n$ can be rewritten as follows:

$$\hat{S}_n = \sum_{l_1=0}^{n}\sum_{l_2=0}^{n-l_1} Q_n(l_1,l_2) \qquad (2.53)$$

and the computation of $Q_n(l_1,l_2)$ can be done recursively.

The quantities of $C_n(l_1+1,l_2)X_{n-1}(l_1,l_2)$, $C_n(l_1,l_2+1)X_{n-1}(l_1,l_2)$ and $C_n(l_1,l_2)X_{n-1}(l_1,l_2)$ that appear in (2.41) can be expressed in terms of the quantity $Q_n(l_1,l_2)$ as follows:

$$C_n(l_1+1,l_2)X_{n-1}(l_1,l_2) = \frac{l_1+\alpha_1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2) \qquad (2.54)$$

$$C_n(l_1,l_2+1)X_{n-1}(l_1,l_2) = \frac{l_2+\alpha_2}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2) \qquad (2.55)$$

$$C_n(l_1,l_2)X_{n-1}(l_1,l_2) = \frac{n-l_1-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2)\,. \qquad (2.56)$$

By using the results from (2.53) to (2.56), the universal portfolio $\hat{\boldsymbol{b}}_n$ from (2.41) is given by:

$$\hat{\boldsymbol{b}}_n = \frac{1}{\sum_{l_1=0}^{n-1}\sum_{l_2=0}^{n-l_1-1} Q_{n-1}(l_1,l_2)} \begin{bmatrix} \sum_{l_1=0}^{n-1}\sum_{l_2=0}^{n-l_1-1} \frac{l_1+\alpha_1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2) \\ \sum_{l_1=0}^{n-1}\sum_{l_2=0}^{n-l_1-1} \frac{l_2+\alpha_2}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2) \\ \sum_{l_1=0}^{n-1}\sum_{l_2=0}^{n-l_1-1} \frac{n-l_1-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1,l_2) \end{bmatrix} \qquad (2.57)$$

Recursion formulae for $Q_n(l_1, l_2)$

The recursion formula for $Q_n(l_1, l_2)$ is given as:

$$Q_n(l_1,l_2) = x_{n1}\frac{l_1+\alpha_1-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1-1,l_2) + x_{n2}\frac{l_2+\alpha_2-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1,l_2-1)$$
$$+ x_{n3}\frac{n-l_1-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1,l_2).$$

$$\text{for} \quad 1 \le l_1 \le n-1 \quad \text{and} \quad 1 \le l_2 \le n-l_1-1 \tag{2.58}$$

The six endpoint conditions are given as follows:

(i) $$Q_n(l_1,0) = x_{n1}\frac{l_1+\alpha_1-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1-1,0) +$$
$$x_{n3}\frac{n-l_1+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1,0) \quad \text{for} \quad 1 \le l_1 \le n-1 \tag{2.59}$$

(ii) $$Q_n(0,l_2) = x_{n2}\frac{l_2+\alpha_2-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(0,l_2-1) +$$
$$x_{n3}\frac{n-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(0,l_2) \quad \text{for} \quad 1 \le l_2 \le n-1 \tag{2.60}$$

(iii) $$Q_n(l_1,l_2) = x_{n1}\frac{l_1+\alpha_1-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1-1,l_2) +$$
$$x_{n2}\frac{l_2+\alpha_2-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(l_1,l_2-1)$$
$$\text{for } 1 \le l_1, \;\; 1 \le l_2 \quad \text{and} \quad l_1 + l_2 = n \tag{2.61}$$

(iv) $$Q_n(n,0) = x_{n1}\frac{n+\alpha_1-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(n-1,0) \tag{2.62}$$

(v) $$Q_n(0,n) = x_{n2}\frac{n+\alpha_2-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(0,n-1) \tag{2.63}$$

(vi) $$Q_n(0,0) = x_{n3}\frac{n+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1}Q_{n-1}(0,0) \tag{2.64}$$

Note that the initial condition is, $Q_0(0,0) = \int_{B_3} d\mu(\boldsymbol{b}) = 1$.

To summarize, we have written $\hat{S}_n$ and $\hat{\boldsymbol{b}}_n$ in term of $Q_{n-1}(l_1, l_2)$. Thus, the universal portfolio $\hat{\boldsymbol{b}}_n$ on the n^{th} trading day can be expressed as follows:

$$\hat{\boldsymbol{b}}_n = \frac{1}{\hat{S}_{n-1}} \begin{bmatrix} \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} \frac{l_1+\alpha_1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1, l_2) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} \frac{l_2+\alpha_2}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1, l_2) \\ \sum_{l_1=0}^{n-1} \sum_{l_2=0}^{n-l_1-1} \frac{n-l_1-l_2+\alpha_3-1}{n+\alpha_1+\alpha_2+\alpha_3-1} Q_{n-1}(l_1, l_2) \end{bmatrix} \tag{2.65}$$

The capital achieved by $\hat{\boldsymbol{b}}_n$ is given as:

$$\hat{S}_n = \sum_{l_1=0}^{n} \sum_{l_2=0}^{n-l_1} Q_n(l_1, l_2)$$

Note:

For a fixed number of *n*, there are $\sum_{l=1}^{n+1} i = \frac{1}{2}(n+1)(n+2)$ distinct quantities of $Q_n(l_1, l_2)$. The computation of the three-stock universal portfolio on the n^{th} trading day thus simplifies to recursively computing and storing the quantities of $\frac{1}{2}(n+1)(n+2)$ and $Q_n(l_1, l_2)$. Notice that the computer storage requirements for these recursive computations grow like n^2.

Concluding remarks:

We can modify and extend the above algorithm in a similar way for computing the *m*-stock universal portfolio when *m* = 4, 5, 6, … etc.

The computer storage requirements for the quantities of $Q_n(l_1, l_2, l_3, \ldots, , l_{m-1})$ grow as n^{m-1}.

2.2 Helmbold-Schapire-Singer-Warmuth Universal Portfolio (HSSW)

Helmbold et. al. (1998) introduced a universal portfolio, which we shall call the Helmbold portfolio or HSSW portfolio. They presented an on-line investment algorithm that can achieve close to the wealth of the best constant-rebalanced portfolio determined in hindsight from the actual market outcomes. The algorithm employs a multiplicative update rule derived using a framework introduced by Kivinen and Warmuth (1997) for on-line regression.

Their algorithm is very simple to implement and requires only constant storage and computing time per stock in each trading period. They tested the performance of their algorithm on real stock data from the New York Stock Exchange accumulated during a 22-year period. On this data, they showed that their algorithm clearly outperforms the best single stock as well as Cover's uniform universal portfolio.

2.2.1. The EG(η)-Update Universal Portfolio Algorithm

Let $\boldsymbol{x}_n = (x_{n1}, x_{n2}, ..., x_{nm})$ be a sequence of price-relative vectors of m stocks corresponding to n trading days. HSSW portfolio is also known as the exponentiated gradient, EG(η) update universal portfolio. This is the sequence of portfolios $\boldsymbol{b}_{n+1}$ where the i th component of the portfolio vector $\boldsymbol{b}_{n+1}$ (the proportion of the total portfolio value invested in the i th stock on the $(n + 1)$ th trading day) is given by:

$$b_{(n+1)i} = \frac{b_{ni} \exp\left(\eta \frac{x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right)}{\sum_{i=1}^{m} b_{ni} \exp\left(\eta \frac{x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n}\right)} \quad \text{for} \quad i = 1, 2, \ldots, m \tag{2.66}$$

where (i) m = number of stocks in the investment portfolio

(ii) $\boldsymbol{b}_n^t \boldsymbol{x}_n = \sum_{i=1}^{m} b_{ni} x_{ni}$

(iii) $\sum_{i=1}^{m} b_{(n+1)i} = 1$

(iv) η = learning parameter, where $\eta > 0$

The EG(η)-update universal portfolio is generated by (2.66) and the learning parameter η is chosen as follows:

$$\eta = 2r\sqrt{\frac{2 \ln m}{n}} \tag{2.67}$$

where (i) m = number of stocks in the investment portfolio

(ii) n = number of trading days in a fixed period of study

(iii) r is chosen such that

$$0 < r \leq \frac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}} \tag{2.68}$$

The universal capital or wealth, $\hat{S}_n$ achieved by the portfolio $\hat{\boldsymbol{b}}_n$ at the end of the n^{th} trading day is given as follows:

$$\hat{S}_n = \prod_{i=1}^{n} (\hat{\boldsymbol{b}}_i^t \cdot \boldsymbol{x}_i)$$

We assume an initial capital or wealth of 1 unit, the wealth S_n achieved by a constant rebalanced portfolio $\boldsymbol{b}$ is given as:

$$S_n = \prod_{i=1}^{n} (\boldsymbol{b}_i^t \cdot \boldsymbol{x}_i)$$

The best or optimum constant rebalanced portfolio $\boldsymbol{b}^*$ is the portfolio that maximizes S_n over all $\boldsymbol{b}$ in the simplex B_m given by:

$$B_m = \left\{ \boldsymbol{b} = (b_1,...,b_m) : \sum_{i=1}^{m} b_i = 1, \quad 0 \le b_i \le 1, \ i = 1,...,m \right\}$$

In this research we shall focus on $m = 2$ and $m = 3$ stocks.

For the two-stock portfolio we select, $n = 410$ and $r = \dfrac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}}$.

For the three-stock portfolio we select, $n = 305$ and $r = \dfrac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}}$.

For example:

Consider $m = 2, \quad n = 410$

$$r = \frac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}} \qquad \text{and} \qquad \eta = 2r\sqrt{\frac{2\ln 2}{410}}$$

Let $c_{ni} = b_{ni} \exp\left(\eta \dfrac{x_{ni}}{\boldsymbol{b}_n^t \boldsymbol{x}_n} \right)$, where t = transpose. We may calculate the portfolio components as a function of c_{ni}.

If the starting portfolio is (b_{11}, b_{12}), and the price-relative vector is (x_{11}, x_{12}) then $\boldsymbol{b}_1^t \boldsymbol{x}_1 = b_{11}x_{11} + b_{12}x_{12}$ and

$$c_{11} = b_{11} \exp\left(\eta \frac{x_{11}}{\boldsymbol{b}_1^t \boldsymbol{x}_1} \right),$$

$$c_{12} = b_{12} \exp\left(\eta \frac{x_{12}}{\boldsymbol{b}_1^t \boldsymbol{x}_1} \right).$$

Hence, $b_{21} = \dfrac{c_{11}}{c_{11} + c_{12}}$, $\qquad b_{22} = \dfrac{c_{12}}{c_{11} + c_{12}}$

On day 2:

$$\boldsymbol{b}_2^t \boldsymbol{x}_2 = b_{21} x_{21} + b_{22} x_{22}$$

$$c_{21} = b_{21} \exp\left(\eta \frac{x_{21}}{\boldsymbol{b}_2^t \boldsymbol{x}_2} \right),$$

$$c_{22} = b_{22} \exp\left(\eta \frac{x_{22}}{\boldsymbol{b}_2^t \boldsymbol{x}_2} \right)$$

$$b_{31} = \frac{c_{21}}{c_{21} + c_{22}} \quad , \qquad b_{32} = \frac{c_{22}}{c_{21} + c_{22}} \quad .$$

In general, for a *m*-stock market, $b_{ni} = \dfrac{c_{ni}}{\sum_{i=1}^{m} c_{ni}}$

CHAPTER 3

DIRICHLET-WEIGHTED AND HELMBOLD-SCHAPIRE-SINGER-WARMUTH UNIVERSAL PORTFOLIOS FOR TWO STOCKS

3.1 The Investment in the Stocks of Digi and LPI

In this study we have chosen two sets of stock price data from the Kuala Lumpur Stock Exchange (KLSE) namely that of Digi. Com Bhd (abbreviated as Digi) and LPI Capital Bhd (abbreviated as LPI) for investment. These stocks were chosen based on their good performance. The stock data of a total of 410 trading days from 27th October 2005 until 23rd May 2007 were collected. Digi was chosen as the first constituent stock in the two-stock universal portfolio based on its liquidity and popularity. The period of trading mentioned here is considered as the non-volatile period of stock price fluctuations. The opening and closing prices in Ringgit Malaysia (RM) for each stock were observed for each trading day and the corresponding price relatives were calculated.

Based on the stock-price data collected, we shall calculate the resulting capital after 410 days of investment using the Cover-Ordentlich universal portfolio in the next section, where we assume that the initial investment capital is 1 unit.

3.2 The Performance of Universal Portfolios in Investing in Digi and LPI during the Non-Volatile Period of Trading

In this project, we had chosen MATLAB as a tool for the computation of universal portfolios and capitals. MATLAB is a user-friendlier tool for doing numerical computations with matrices and vectors. A MATLAB m-file was written based on the modified algorithm for two-stock universal portfolio as discussed in Section 2.1.2. Different parameters of the Dirichlet (α_1, α_2) distribution have been used for generating the universal portfolio strategies.

The Dirichlet (1.0, 1.0) strategy was first introduced by Cover (1991). Subsequently, Cover and Ordentlich (1996) introduced the general class of Dirichlet (α_1, α_2) strategies. But the focus of their study is on the Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies. The aim of this project is to study the parametric class of Dirichlet (α_1, α_2) strategies, especially when the (1.0, 1.0) and (0.5, 0.5) strategies do not perform well. Appendix 1 shows the investment capitals achieved for the constituent stocks Digi and LPI by using the Cover and Ordentlich universal portfolios namely the Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (2.0, 2.0) universal portfolios. These capitals were calculated using the modified algorithm mentioned in Section 2.1.2. The period of trading from 27th October 2005 until 23rd May 2007 is considered to be the non-volatile period of stock price fluctuations.

Figure 3.1 shows the investment capital (return) after 410 trading days i.e. from 27th October 2005 until 23rd May 2007, for investment in the constituent stocks Digi and LPI against the universal portfolios generated by Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (2.0, 2.0) distributions. In this Figure, we have chosen the investment capital achieved with the universal portfolio generated by the Dirichlet (2.0, 2.0) distribution as it achieves the highest capital among all the 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI for this period of study. The other two investment capitals achieved with the universal portfolios generated by the Dirichlet (0.4, 0.3) and Dirichlet (0.5, 0.7) distributions are also chosen in Figure 3.1 as these two Dirichlet (α_1, α_2) distributions have different parametric α values.

It was observed that after 410 trading days the universal portfolios generated by these three Dirichlet distributions outperform the constituent stocks. This is also evident in the cases discussed in Cover (1991), "Universal Portfolios", *Mathematical Finance*, vol. 1. pp. 1-29. The performance of universal portfolio is exhibited in the examples in Section 8 page 21 and the concluding remarks in Section 10 page 24.

In Figure 3.1, the five different colour curves representing the investment capitals achieved after 410 trading days for the constituent stocks Digi and LPI and the universal portfolios generated by the Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (2.0, 2.0) distributions are quite closed to each other; this shows that their investment capitals achieved during this period of study do not differ much. If data for a longer trading period is made available, there is some evidence that the investment capitals achieved are apart from each other as shown in the later part of our study.

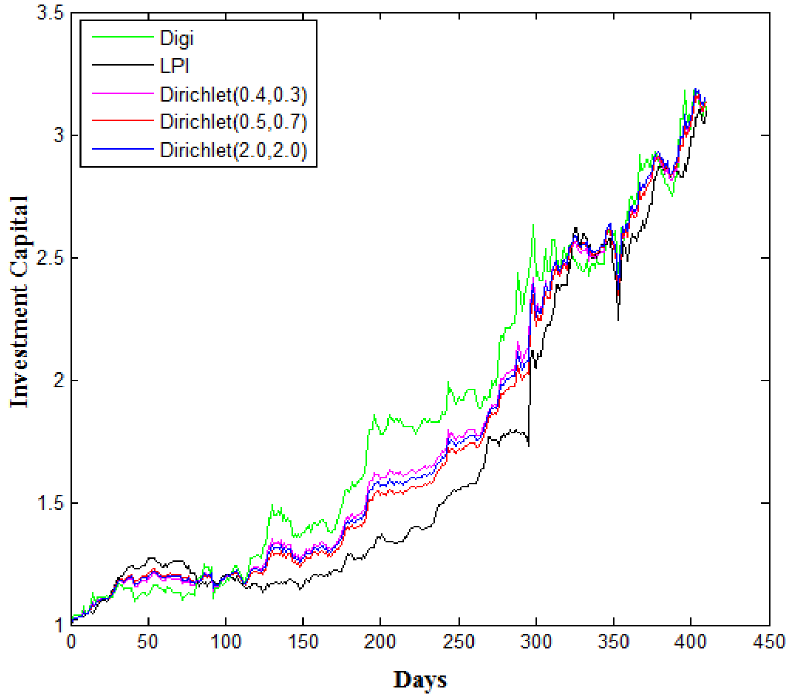


Figure 3.1: Investment capital (return) for single-stock strategy and universal portfolios generated by Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (2.0, 2.0) distributions

We observe that by varying the parameters α_1 and α_2, we can obtain Dirichlet (α_1, α_2) strategies achieving different capitals. For example, refer to Table 3.1 where the investment capitals of seventeen different pairs of (α_1, α_2) are calculated, namely for (α_1, α_2) equals to (0.4, 0.3), (0.5, 0.7), (0.2, 0.8), (0.8, 0.2), (0.8, 0.9), (0.9, 0.8), (0.1, 0.9), (0.9, 0.1), (0.5, 0.5), (1.0, 1.0), (2.0, 2.0), (1.0, 2.0), (2.0, 1.0), (1.0, 3.0), (3.0, 1.0), (1.0, 0.0) and (0.0, 1.0) respectively. Except for the detailed calculations for the Dirichlet (0.4, 0.3), (0.5, 0.7) and (2.0, 2.0) universal portfolios listed in Appendix 1, we will not show the detailed calculations for the other twelve universal portfolios listed in Table 3.1.

In Table 3.1, the investment capitals (returns) obtained after these 410 trading days with 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI are ranged from 3.09516844 units to 3.15252256 units. The Dirichlet (2.0, 2.0) strategy (as shown highlighted in grey colour) achieves the highest capital of $\hat{S}_{410}$ = 3.15252256 units. The Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of $\hat{S}_{410}$ = 3.14336022 units and $\hat{S}_{410}$ = 3.13192983 units respectively for this period of study. This shows the importance and significance of the Dirichlet (α_1, α_2) strategies other than the Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies.

Table 3.1: The investment capitals (returns) obtained after 410 trading days with 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI during the non-volatile period from 27th October 2005 until 23rd May 2007

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	Investment Capital after 410 trading days, $\hat{S}_{410}$
(0.4, 0.3)	3.12495387
(0.5, 0.7)	3.13441475
(0.2, 0.8)	3.12109718
(0.8, 0.2)	3.11810981
(0.8, 0.9)	3.14081032
(0.9, 0.8)	3.14052763
(0.1, 0.9)	3.11203093
(0.9, 0.1)	3.10801222
(0.5, 0.5)	3.13192983
(1.0, 1.0)	3.14336022
(2.0, 2.0)	3.15252256
(1.0, 2.0)	3.14414335
(2.0, 1.0)	3.14257709
(1.0, 3.0)	3.13995374
(3.0, 1.0)	3.13703658
(1.0, 0.0)	3.09516844
(0.0, 1.0)	3.10024892

Note: The investment capitals (returns) achieved after 410 trading days for single-stock strategy of the constituent stocks, Digi and LPI are listed in last two rows of the above Table 3.1, by using the universal portfolios generated by the Dirichlet (1.0, 0.0) and Dirichlet (0.0, 1.0) distributions respectively. We have chosen the different combinations of the parameters α_1 and α_2 close to 1 and 0.5. The parameters are independent, the capital returns do not show significant difference with extreme values of α, as such we have not listed the combinations with extreme values of α in the table.

3.3 The Investment in the Stocks of Digi and LPI during the Volatile Period of Trading

The recent global economic crisis of 2008 is causing problems worldwide; as such we investigate how far the stock market crash of 2008 is affecting portfolio investment. The stock price data of a total of 410 trading days from 13th September 2007 until 30th April 2009 were collected. This period of trading is considered as the volatile period. Two sets of stock price data from the Kuala Lumpur Stock Exchange (KLSE) namely that of Digi sp(v) and LPI sp(v) were collected for analysis. We use the abbreviation sp(v) for the stock prices collected during the above mentioned volatile period. Digi sp(v) was chosen as the first constituent stock in the two-stock universal portfolio. The opening and closing prices in Ringgit Malaysia (RM) for each stock were observed for each trading day and the corresponding price relatives were calculated.

Based on the stock-price data collected for this volatile period, we shall calculate the resulting capital after 410 days of investment using the Cover-Ordentlich universal portfolio in the next section, where we assume that the initial investment capital is 1 unit.

3.4 The Performance of Universal Portfolios in Investing in Digi and LPI during the Volatile Period of Trading

In this section, the parametric class of Dirichlet (α_1, α_2) distributions was again used for generating the universal portfolio strategies during the volatile period from 13th September 2007 until 30th April 2009. We use the same MATLAB m-file as mentioned in Section 3.2. In Figure 3.2, we have chosen the investment capital achieved with the universal portfolio generated by the Dirichlet (1.0, 3.0) distribution as it achieves the highest capital among all the 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi sp(v) and LPI sp(v) for this period of study. The other two investment capitals achieved with the universal portfolios generated by the Dirichlet (0.4, 0.3) and Dirichlet (0.5, 0.7) distributions are also chosen in Figure 3.2 as these two Dirichlet (α_1, α_2) distributions are used during the non-volatile period.

Figure 3.2 shows the investment capital (return) during volatile period, after 410 trading days i.e. from 13th September 2007 until 30th April 2009, for constituent stock Digi sp(v) and LPI sp(v) against the universal portfolios generated by Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (1.0, 3.0) distributions. It was observed that after 410 trading days the universal portfolios generated by Dirichlet (1.0, 3.0) distribution outperformed the constituent stocks even during this volatile period of study.

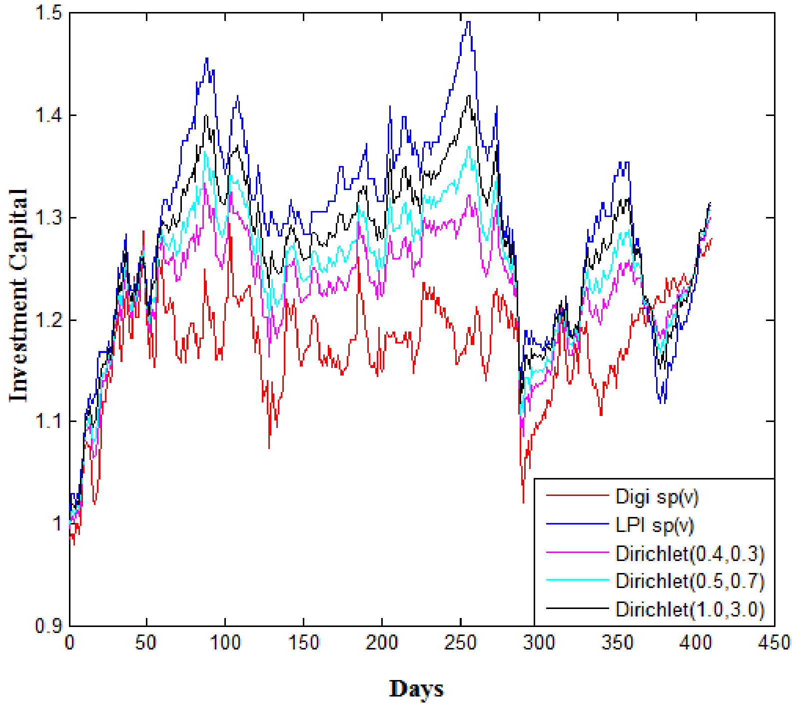


Figure 3.2: Investment capital (return) for single-stock strategy and universal portfolios generated by Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (1.0, 3.0) distributions during the volatile period

These capitals were again calculated using the modified algorithm mentioned in Section 2.1.2 with MATLAB. We observe that by varying the parameters α_1 and α_2, we can obtain Dirichlet (α_1, α_2) strategies achieving different capitals. For example, refer to Table 3.2 where the investment capitals of seventeen different pairs of (α_1, α_2) are calculated, namely for (α_1, α_2) equals to (0.4, 0.3), (0.5, 0.7), (0.2, 0.8), (0.8, 0.2), (0.8, 0.9), (0.9, 0.8), (0.1, 0.9), (0.9, 0.1), (0.5, 0.5), (1.0, 1.0), (2.0, 2.0), (1.0, 2.0), (2.0, 1.0), (1.0, 3.0), (3.0, 1.0), (1.0, 0.0) and (0.0, 1.0) respectively.

Table 3.2: The investment capitals (returns) obtained during volatile period after 410 trading days with 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi sp(v) and LPI sp(v) from 13th September 2007 until 30th April 2009

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	Investment Capital after 410 trading days, $\hat{S}_{410}$ (v)
(0.4, 0.3)	1.30180472
(0.5, 0.7)	1.30931148
(0.2, 0.8)	1.31211128
(0.8, 0.2)	1.29279134
(0.8, 0.9)	1.30950323
(0.9, 0.8)	1.30760336
(0.1, 0.9)	1.31254484
(0.9, 0.1)	1.28680500
(0.5, 0.5)	1.30602237
(1.0, 1.0)	1.30933252
(2.0, 2.0)	1.31198444
(1.0, 2.0)	1.31472550
(2.0, 1.0)	1.30393954
(1.0, 3.0)	1.31609603
(3.0, 1.0)	1.29991709
(1.0, 0.0)	1.28003542
(0.0, 1.0)	1.31217796

Note: The investment capitals achieved after 410 trading days for single-stock strategy of the constituent stocks, Digi sp(v) and LPI sp(v) are listed in last two rows of the above Table 3.1, by using the universal portfolios generated by the Dirichlet (1.0, 0.0) and Dirichlet (0.0, 1.0) distributions respectively.

In Table 3.2, the investment capitals obtained after these 410 trading days with 17 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi sp(v) and LPI sp(v) are ranging from 1.28003542 units to 1.31609603 units. The Dirichlet (1.0, 3.0) strategy (as shown highlighted in grey colour) achieves the highest capital of $\hat{S}_{410}$ (v) = 1.31609603 units. The Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of 1.30933252 units and 1.30602237 units respectively for this period of study. This again shows the importance of the Dirichlet (α_1, α_2) strategies in complementing the role of the Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies.

3.5 Constant Rebalanced Strategies

In this section, we shall use 101 constant rebalanced strategies for investment in the stocks of Digi and LPI for a total of 410 trading days for the non-volatile period from 27th October 2005 until 23rd May 2007 and 410 trading days for the volatile period from 13th September 2007 until 30th April 2009.

The initial investment capital is one unit and the resulting capitals after 410 trading days are calculated. The constant rebalanced strategies $\boldsymbol{b}$ = $(b_1, 1-b_1)$ used are for b_1 decreasing in units of 0.01 from 1.00 to 0.00. In this experiment, we used a simple programming of MATLAB to compute the resulting investment capitals for these 410 trading days. The 101 investment capitals achieved by each of the strategies after 410 trading days in each of the two periods were calculated using the algorithm discussed in Section 2.1.2. These results are listed in Table 3.3. The capitals achieved are denoted by S_{410} (CRP) and S_{410} (v) (CRP) for the non-volatile and volatile periods respectively.

Table 3.3: The investment capitals S_{410} and S_{410} (v) calculated using the constant rebalanced portfolios (CRP) for the non-volatile and volatile Digi – LPI stock data

Investment		S_{410} (CRP)	S_{410} (v) (CRP)	Investment		S_{410} (CRP)	S_{410} (v) (CRP)
Digi	LPI			Digi	LPI		
1.00	0.00	3.09516844	1.28003542	0.50	0.50	3.16632633	1.31597502
0.99	0.01	3.09792824	1.28112762	0.49	0.51	3.16633979	1.31629384
0.98	0.02	3.10063472	1.28220511	0.48	0.52	3.16629782	1.31659663
0.97	0.03	3.10328776	1.28326785	0.47	0.53	3.16620045	1.31688338
0.96	0.04	3.10588726	1.28431580	0.46	0.54	3.16604770	1.31715409
0.95	0.05	3.10843311	1.28534892	0.45	0.55	3.16583961	1.31740873
0.94	0.06	3.11092522	1.28636718	0.44	0.56	3.16557622	1.31764731
0.93	0.07	3.11336349	1.28737054	0.43	0.57	3.16525756	1.31786980
0.92	0.08	3.11574781	1.28835896	0.42	0.58	3.16488367	1.31807620
0.91	0.09	3.11807811	1.28933241	0.41	0.59	3.16445460	1.31826650
0.90	0.10	3.12035428	1.29029084	0.40	0.60	3.16397039	1.31844069
0.89	0.11	3.12257624	1.29123423	0.39	0.61	3.16343110	1.31859877
0.88	0.12	3.12474392	1.29216254	0.38	0.62	3.16283678	1.31874073
0.87	0.13	3.12685722	1.29307574	0.37	0.63	3.16218747	1.31886655
0.86	0.14	3.12891606	1.29397379	0.36	0.64	3.16148324	1.31897625
0.85	0.15	3.13092038	1.29485666	0.35	0.65	3.16072415	1.31906980
0.84	0.16	3.13287009	1.29572431	0.34	0.66	3.15991026	1.31914721
0.83	0.17	3.13476513	1.29657672	0.33	0.67	3.15904163	1.31920847
0.82	0.18	3.13660543	1.29741385	0.32	0.68	3.15811833	1.31925358
0.81	0.19	3.13839092	1.29823567	0.31	0.69	3.15714044	1.31928253
0.80	0.20	3.14012154	1.29904216	0.30	0.70	3.15610802	1.31929532
0.79	0.21	3.14179724	1.29983327	0.29	0.71	3.15502116	1.31929196
0.78	0.22	3.14341794	1.30060899	0.28	0.72	3.15387992	1.31927244
0.77	0.23	3.14498360	1.30136928	0.27	0.73	3.15268439	1.31923675
0.76	0.24	3.14649417	1.30211411	0.26	0.74	3.15143466	1.31918490
0.75	0.25	3.14794959	1.30284347	0.25	0.75	3.15013080	1.31911690
0.74	0.26	3.14934983	1.30355731	0.24	0.76	3.14877292	1.31903273
0.73	0.27	3.15069483	1.30425561	0.23	0.77	3.14736109	1.31893240
0.72	0.28	3.15198456	1.30493835	0.22	0.78	3.14589542	1.31881592
0.71	0.29	3.15321898	1.30560550	0.21	0.79	3.14437600	1.31868329
0.70	0.30	3.15439805	1.30625704	0.20	0.80	3.14280293	1.31853451
0.69	0.31	3.15552174	1.30689294	0.19	0.81	3.14117630	1.31836958
0.68	0.32	3.15659002	1.30751317	0.18	0.82	3.13949623	1.31818851
0.67	0.33	3.15760286	1.30811772	0.17	0.83	3.13776282	1.31799131
0.66	0.34	3.15856025	1.30870656	0.16	0.84	3.13597618	1.31777798
0.65	0.35	3.15946214	1.30927967	0.15	0.85	3.13413642	1.31754852
0.64	0.36	2.27514293	1.30983703	0.14	0.86	3.13224365	1.31730295
0.63	0.37	3.16109942	1.31037861	0.13	0.87	3.13029799	1.31704126
0.62	0.38	3.16183477	1.31090440	0.12	0.88	3.12829957	1.31676348
0.61	0.39	3.16251457	1.31141437	0.11	0.89	3.12624849	1.31646961
0.60	0.40	3.16313883	1.31190850	0.10	0.90	3.12414489	1.31615965
0.59	0.41	3.16370753	1.31238678	0.09	0.91	3.12198889	1.31583362
0.58	0.42	3.16422067	1.31284919	0.08	0.92	3.11978061	1.31549152
0.57	0.43	3.16467825	1.31329571	0.07	0.93	3.11752020	1.31513338
0.56	0.44	3.16508028	1.31372632	0.06	0.94	3.11520779	1.31475920
0.55	0.45	3.16542676	1.31414100	0.05	0.95	3.11284350	1.31436898
0.54	0.46	3.16571770	1.31453975	0.04	0.96	3.11042748	1.31396275
0.53	0.47	3.16595312	1.31492254	0.03	0.97	3.10795987	1.31354052
0.52	0.48	3.16613301	1.31528936	0.02	0.98	3.10544080	1.31310231
0.51	0.49	3.16625741	1.31564019	0.01	0.99	3.10287044	1.31264811
				0.00	1.00	3.10024892	1.31217796

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 410 trading days, i.e. S_{410} (CRP) based on the stock price relatives of Digi and LPI during the non-volatile period from 27th October 2005 until 23rd May 2007 is shown in Figure 3.3.

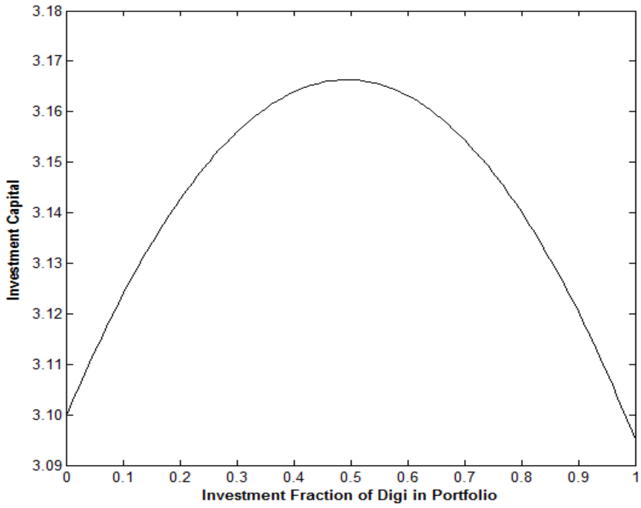


Figure 3.3: The investment capitals achieved by the constant rebalanced portfolios for the non-volatile trading period of the Digi – LPI stocks

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 410 trading days, i.e. S_{410} (v) (CRP) based on the stock price relatives of Digi sp(v) and LPI sp(v) during the volatile period from 13th September 2007 until 30th April 2009 is shown in Figure 3.4.

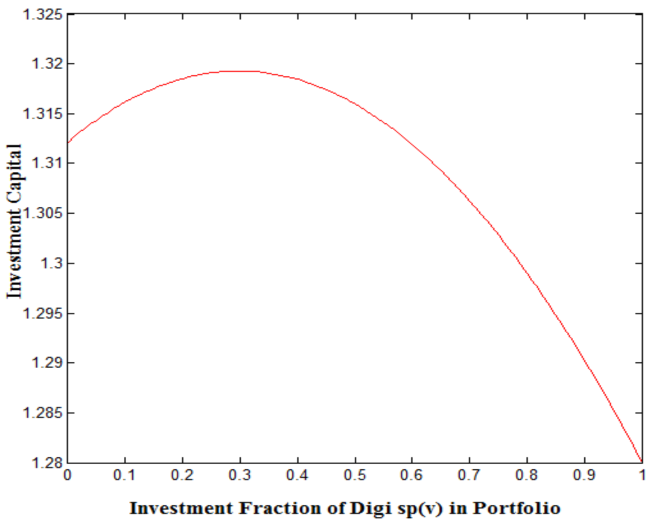


Figure 3.4: The investment capitals achieved by the constant rebalanced portfolios for the volatile trading period of the Digi sp(v) – LPI sp(v) stocks

The investment capitals computed for the non-volatile and volatile periods, i.e. S_{410} (CRP), from 27th October 2005 until 23rd May 2007 and S_{410} (v) (CRP), from 13th September 2007 until 30th April 2009, are listed in Table 3.3. It is found that the best constant rebalanced strategy for these 410 trading days during the non-volatile period (as shown highlighted in grey colour) is:

$\boldsymbol{b}^*$ = (0.49, 0.51), yielding a resulting investment capital of

S^*_{410} = 3.16633979 units.

This indicates that the investor's capital "multiplies" by a factor of 3.16633979 during this period.

On the other hand during the volatile period of 410 trading days, the best constant rebalanced strategy (as shown highlighted in grey colour) is:

$\boldsymbol{b}^*$ = (0.30, 0.70), yielding a resulting investment capital of

S^*_{410} (v) = 1.31929532 units.

This indicates that the investor's capital "multiplies" by a factor of 1.31929532 during this period.

3.6 The Ratios of the Optimal Capital to the Universal Capitals for Non-Volatile Period of Trading

The investment capitals obtained after 410 trading days with 15 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI during the non-volatile period from 27th October 2005 until 23rd May 2007 were displayed in Table 3.1. From Table 3.3 the optimal capital achieved for this period was S^*_{410} =

3.16633979 units for the CRP (0.49, 0.51). The ratios of the optimal capital to the universal capitals achieved, i.e. $\frac{S_{410}^{*}}{\hat{S}_{410}}$ are listed in Table 3.4.

Table 3.4: The ratios of the optimal capital to the universal capitals based on the Digi – LPI stock data during the non-volatile period

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	$\frac{S_{410}^{*}}{\hat{S}_{410}}$
(0.4, 0.3)	1.01324369
(0.5, 0.7)	1.01018533
(0.2, 0.8)	1.01449574
(0.8, 0.2)	1.01546770
(0.8, 0.9)	1.00812831
(0.9, 0.8)	1.00821905
(0.1, 0.9)	1.01745126
(0.9, 0.1)	1.01876684
(0.5, 0.5)	1.01098682
(1.0, 1.0)	1.00731051
(2.0, 2.0)	1.00438291
(1.0, 2.0)	1.00705961
(2.0, 1.0)	1.00756153
(1.0, 3.0)	1.00840332
(3.0, 1.0)	1.00934105
(1.0, 0.0)	1.01324369
(0.0, 1.0)	1.01018533

In Table 3.4, it is found that for the 410 trading-day period, the best Cover-Ordentlich universal portfolio is $\hat{\boldsymbol{b}}$ = (2.0, 2.0), yielding a resulting optimal universal investment capital of $\hat{S}_{410}$ = 3.15252256 units. From Table 3.3, the best constant rebalanced portfolio is $\boldsymbol{b}^*$ = (0.49, 0.51), yielding a resulting optimal investment capital of S_{410}^{*} = 3.16633979 units. Thus, the ratio of the optimal investment capital, S_{410}^{*} = 3.16633979 units to the best Cover-Ordentlich universal investment capital, $\hat{S}_{410}$ = 3.15252256 units is:

$$\frac{S_{410}^{*}}{\hat{S}_{410}} \approx 1.00438291.$$

It was proved in Cover and Ordentlich (1996) that:

$$\text{as } n \to \infty, \quad \limsup_{n\to\infty} \frac{1}{n} \ln \frac{S_n^*}{\hat{S}_n} = 0 .$$

We can show this by taking a sequence of n trading days starting from $n = 305$ increasing by 5 trading days to $n = 410$ based on the Digi – LPI stock data during the non-volatile period. The optimal investment capitals, S_n^* and corresponding universal investment capitals, $\hat{S}_n$ achieved by using the Cover and Ordentlich universal portfolios namely the Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (2.0, 2.0) are used to compute the corresponding value of $\frac{1}{n} \ln \frac{S_n^*}{\hat{S}_n}$.

For example, in Table 3.5 by using the Dirichlet (0.4, 0.3) distribution:

When n = 305, the corresponding value of $\frac{1}{305} \ln \frac{S_{305}^*}{\hat{S}_{305}}$ is -0.00000494.

When n = 310, the corresponding value of $\frac{1}{310} \ln \frac{S_{310}^*}{\hat{S}_{310}}$ is -0.00000744.

When n = 315, the corresponding value of $\frac{1}{315} \ln \frac{S_{315}^*}{\hat{S}_{315}}$ is 0.00002138.

and so on until the last trading day i.e. when n = 410.

Table 3.5: The values of S_n^*, $\hat{S}_n$ and $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ for trading days from n = 305 increasing in 5 days to n = 410 based on the Digi – LPI stock data during the non-volatile period

n	S_n^*	$\hat{S}_n$ (0.4, 0.3)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (0.5, 0.7)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (2.0, 2.0)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$
305	2.36006645	2.36362155	-0.00000494	2.32365912	0.00005097	2.35635482	0.00000516
310	2.43909498	2.44472955	-0.00000744	2.39922456	0.00005317	2.43544341	0.00000483
315	2.44930539	2.43286204	0.00002138	2.42312990	0.00003411	2.44213387	0.00000931
320	2.47733806	2.46118238	0.00002045	2.45020990	0.00003441	2.47011539	0.00000912
325	2.60063690	2.56335622	0.00004443	2.58955462	0.00001314	2.59041497	0.00001212
330	2.56432935	2.52549609	0.00004624	2.55529206	0.00001070	2.55399525	0.00001224
335	2.54820398	2.51420426	0.00004010	2.53415834	0.00001650	2.53838796	0.00001152
340	2.53716290	2.50720886	0.00003493	2.51891551	0.00002123	2.52777939	0.00001090
345	2.60838916	2.58251313	0.00002890	2.58458324	0.00002658	2.59927625	0.00001014
350	2.56824399	2.55160179	0.00001857	2.53728635	0.00003465	2.56042563	0.00000871
355	2.57043177	2.55821727	0.00001342	2.53533714	0.00003872	2.56314574	0.00000800
360	2.68271603	2.66871730	0.00001453	2.64547264	0.00003883	2.67473783	0.00000827
365	2.70466645	2.68932901	0.00001558	2.66785290	0.00003755	2.69642436	0.00000836
370	2.79426923	2.78392158	0.00001003	2.75159849	0.00004159	2.78647818	0.00000755
375	2.86835863	2.85227449	0.00001500	2.82797708	0.00003781	2.85949264	0.00000826
380	2.93490336	2.90153702	0.00003009	2.90616862	0.00002589	2.92339339	0.00001034
385	2.90341684	2.86077494	0.00003843	2.88390550	0.00001751	2.89086067	0.00001126
390	2.89681137	2.85973928	0.00003303	2.87162592	0.00002239	2.88486583	0.00001060
395	3.00184443	2.98576617	0.00001360	2.95620107	0.00003879	2.99232049	0.00000804
400	3.07101488	3.03470464	0.00002973	3.03810531	0.00002694	3.05828051	0.00001039
405	3.19543140	3.15922801	0.00002813	3.15911866	0.00002822	3.18229610	0.00001017
410	3.16633979	3.12495387	0.00003209	3.13441475	0.00002472	3.15252256	0.00001067

In Table 3.5, for trading days from n = 305 increasing in 5 days to n = 410 based on the Digi – LPI stock data during the non-volatile period, we find that all the values of $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ are approaching to 0, thus we can deduce that:

$$\text{as } n \to \infty \text{ , } \limsup_{n\to\infty} \frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n} = 0 \text{ .}$$

The finding of the above result is important as showed in Cover (1991), "Universal Portfolios", *Mathematical Finance*, vol. 1, pp. 1-29 that for an arbitrary sequence of market vectors, $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$. The universal portfolio on day k + 1 generated by the Dirichlet $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ distribution μ is given as:

$$\hat{\boldsymbol{b}}_{k+1} = \frac{\int_{B_m} \boldsymbol{b} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}{\int_{B_m} S_k(\boldsymbol{b}) d\mu(\boldsymbol{b})}$$

The wealth yields is, $\hat{S}_k(\boldsymbol{b}) = \prod_{i=1}^{k} \boldsymbol{b}^t \boldsymbol{x}_i$ such that $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n} \to 0$ for every bounded sequence $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$. This portfolio strategy has the same exponential rate of growth as S_n^*.

In T.M. Cover and E. Ordentlich (1996). "Universal portfolios with side information", *IEEE Transactions on Information Theory*, vol. 42, pp. 348-363, they showed that a sequential portfolio with side information is universal for the collection B^k of state-constant rebalanced portfolios if

$$\limsup_{n\to\infty} \frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n} = 0 \text{ .}$$

3.7 The Ratios of the Optimal Capital to the Universal Capitals for Volatile Period of Trading

The investment capitals (returns) obtained after 410 trading days, during the volatile period, with 15 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distributions based on the stock price data of Digi sp(v) and LPI sp(v) from 13th September 2007 until 30th April 2009 were calculated in Table 3.2. From Table 3.3 the optimal capital achieved for this period was S^*_{410} (v) = 1.31929532 units for the CRP (0.30, 0.70). The ratios of the optimal capital to the universal capitals achieved during the volatile period are listed.

Table 3.6: The ratios of the optimal capital to the universal capitals based on the Digi sp(v) – LPI sp(v) stock data during the volatile period from 13th September 2007 until 30th April 2009

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	$\frac{S^*_{410}\text{ (v)}}{\hat{S}_{410}\text{ (v)}}$
(0.4, 0.3)	1.01343566
(0.5, 0.7)	1.00762526
(0.2, 0.8)	1.00547517
(0.8, 0.2)	1.02050136
(0.8, 0.9)	1.00747772
(0.9, 0.8)	1.00894151
(0.1, 0.9)	1.00514305
(0.9, 0.1)	1.02524883
(0.5, 0.5)	1.01016288
(1.0, 1.0)	1.00760907
(2.0, 2.0)	1.00557238
(1.0, 2.0)	1.00347587
(2.0, 1.0)	1.01177645
(1.0, 3.0)	1.00243089
(3.0, 1.0)	1.01490728

In Table 3.6, it is found that for these 410 volatile trading days period, the best rebalanced portfolio is $\boldsymbol{b}^*$ = (0.30, 0.70), with a resulting optimal investment capital of S^*_{410} (v) = 1.31929532 units. The best universal portfolio is $\hat{\boldsymbol{b}}$ = (1.0, 3.0), with a resulting investment capital of $\hat{S}_{410}$ (v) = 1.31609603 units. Thus, the ratio of the optimal investment capital, S^*_{410} (v) = 1.31929532 units to the best universal investment capital, $\hat{S}_{410}$ (v) = 1.31609603 is:

$$\frac{S^*_{410}\text{ (v)}}{\hat{S}_{410}\text{ (v)}} \approx 1.00243089$$

It was proved in Cover and Ordentlich (1996) that:

$$\text{as } n \to \infty, \ \limsup_{n\to\infty} \frac{1}{n} \ln \frac{S^*_n}{\hat{S}_n} = 0 .$$

We can show this by taking a sequence of *n* trading days starting from *n* = 305 increasing by 5 trading days to *n* = 410 based on the Digi sp(v) – LPI sp(v) stock data during the volatile period. The optimal investment capitals, S^*_n and corresponding universal investment capitals, $\hat{S}_n$ achieved by using the Cover and Ordentlich universal portfolios namely the Dirichlet (0.4, 0.3), Dirichlet (0.5, 0.7) and Dirichlet (1.0, 3.0) are used to compute the corresponding value of $\frac{1}{n} \ln \frac{S^*_n}{\hat{S}_n}$.

Table 3.7: The values of $S_n^*(\text{v})$, $\hat{S}_n$ and $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ for trading days from n = 305 increasing in 5 days to n = 410 based on the Digi sp(v) – LPI sp(v) stock data during the volatile period

n	$S_n^*(\text{v})$	$\hat{S}_n$ (0.4, 0.3)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (0.5, 0.7)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (1.0, 3.0)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$
305	2.36006645	2.36362155	-0.00000494	2.32365912	0.00005097	2.35635482	0.00000516
310	2.43909498	2.44472955	-0.00000744	2.39922456	0.00005317	2.43544341	0.00000483
315	2.44930539	2.43286204	0.00002138	2.42312990	0.00003411	2.44213387	0.00000931
320	2.47733806	2.46118238	0.00002045	2.45020990	0.00003441	2.47011539	0.00000912
325	2.60063690	2.56335622	0.00004443	2.58955462	0.00001314	2.59041497	0.00001212
330	2.56432935	2.52549609	0.00004624	2.55529206	0.00001070	2.55399525	0.00001224
335	2.54820398	2.51420426	0.00004010	2.53415834	0.00001650	2.53838796	0.00001152
340	2.53716290	2.50720886	0.00003493	2.51891551	0.00002123	2.52777939	0.00001090
345	2.60838916	2.58251313	0.00002890	2.58458324	0.00002658	2.59927625	0.00001014
350	2.56824399	2.55160179	0.00001857	2.53728635	0.00003465	2.56042563	0.00000871
355	2.57043177	2.55821727	0.00001342	2.53533714	0.00003872	2.56314574	0.00000800
360	2.68271603	2.66871730	0.00001453	2.64547264	0.00003883	2.67473783	0.00000827
365	2.70466645	2.68932901	0.00001558	2.66785290	0.00003755	2.69642436	0.00000836
370	2.79426923	2.78392158	0.00001003	2.75159849	0.00004159	2.78647818	0.00000755
375	2.86835863	2.85227449	0.00001500	2.82797708	0.00003781	2.85949264	0.00000826
380	2.93490336	2.90153702	0.00003009	2.90616862	0.00002589	2.92339339	0.00001034
385	2.90341684	2.86077494	0.00003843	2.88390550	0.00001751	2.89086067	0.00001126
390	2.89681137	2.85973928	0.00003303	2.87162592	0.00002239	2.88486583	0.00001060
395	3.00184443	2.98576617	0.00001360	2.95620107	0.00003879	2.99232049	0.00000804
400	3.07101488	3.03470464	0.00002973	3.03810531	0.00002694	3.05828051	0.00001039
405	3.19543140	3.15922801	0.00002813	3.15911866	0.00002822	3.18229610	0.00001017
410	3.16633979	3.12495387	0.00003209	3.13441475	0.00002472	3.15252256	0.00001067

In Table 3.7, for trading days from n = 305 increasing in 5 days to n = 410 based on the Digi sp(v) – LPI sp(v) stock data during the volatile period, we find that all the values of $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ are approaching to 0, thus not only during the non-volatile period discussed in Section 3.6 but also during the volatile period of our study in this section, we can deduce that:

$$\text{as } n \to \infty, \quad \limsup_{n\to\infty} \frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n} = 0 .$$

3.8 Investment in Longer Trading Period

3.8.1 The Investment in the Stocks of Digi and LPI for Longer Trading Period

The stock-price data of Digi and LPI were chosen as the set of two-stock portfolio for investment for a total of 900 trading days from 27th October 2005 until 30th April 2009, this included both the non-volatile period, from 27th October 2005 until 23rd May 2007 and the volatile period, from 13th September 2007 until 30th April 2009 that we have studied in separate sections earlier. The opening and closing prices for each stock were observed for each trading day and the corresponding price relatives were calculated. In this study, Digi was again chosen as the first constituent stock in the two-stock universal portfolio.

Table 3.8: The investment capitals (returns) obtained after 900 trading days with 15 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI inclusive of the non-volatile and volatile periods from 27th October 2005 until 30th April 2009

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	Investment Capital after 900 trading days, $\hat{S}_{900}$
(0.4, 0.3)	4.22404540
(0.5, 0.7)	4.40501069
(0.2, 0.8)	4.60182009
(0.8, 0.2)	3.97220769
(0.8, 0.9)	4.36233653
(0.9, 0.8)	4.30018994
(0.1, 0.9)	4.68590860
(0.9, 0.1)	3.84788976
(0.5, 0.5)	4.31289799
(1.0, 1.0)	4.33692166
(2.0, 2.0)	4.35620057
(1.0, 2.0)	4.51373770
(2.0, 1.0)	4.16010562
(1.0, 3.0)	4.59250626
(3.0, 1.0)	4.06205814

In this Table 3.8, the investment capitals (returns) obtained after these 900 trading days with 15 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Digi and LPI are ranged from 3.84788976 units to 4.68590860 units. The Dirichlet (0.1, 0.9) strategy (as shown highlighted in grey colour) achieved the highest capital of $\hat{S}_{900}$ = 4.68590860 units. The Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of $\hat{S}_{900}$ = 4.33692166 units and $\hat{S}_{900}$ = 4.31289799 units respectively for this period of study. This shows the importance and significance of the Dirichlet (α_1, α_2) strategies other than the Dirichlet (1.0, 1.0) and Dirichlet (0.5, 0.5) strategies.

Figure 3.5 shows the investment capital (return) of 900 trading days i.e. from 27th October 2005 until 30th April 2009, for investment in the constituent stocks Digi and LPI against the universal portfolios generated by Dirichlet (0.4, 0.3), Dirichlet (0.1, 0.9) and Dirichlet (1.0, 3.0) distributions.

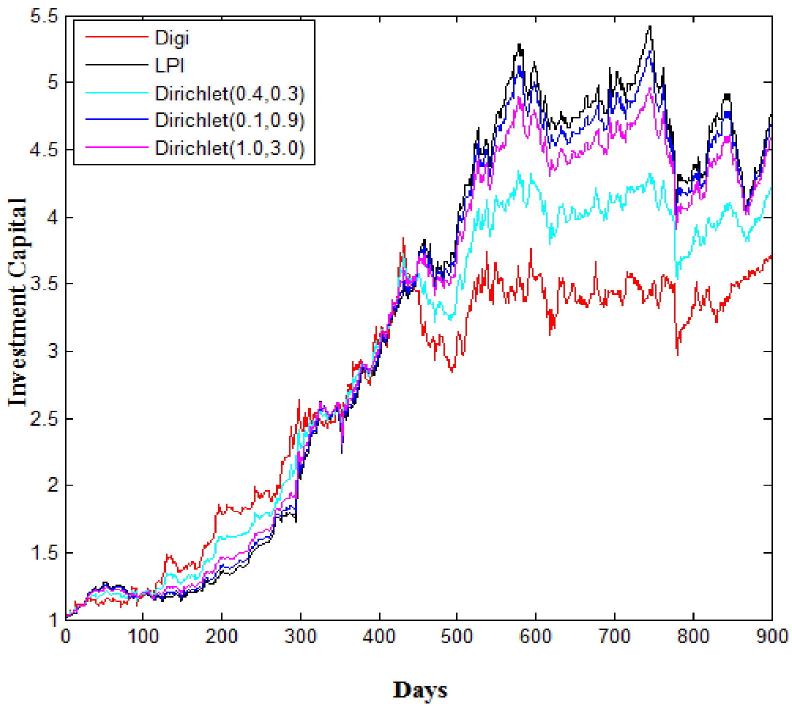


Figure 3.5: Investment capital for universal portfolios for 900 trading days from 27th October 2005 until 30th April 2009

In Figure 3.5, the blue line represents the universal portfolio generated by the Dirichlet (0.1, 0.9) distribution, yielding a resulting universal investment capital of 4.68590860 units which is the maximum capital in Table 3.8. It is observed that after these 900 trading days even inclusive of the volatile period during the economic crisis, the investment capital (return) grow at a multiple of 3.84788976 to 4.68590860. If data for a longer trading period is made available, there is some evidence that the investment capital (return) is higher for the universal portfolios.

3.8.2 The Investment in the Stocks of Resorts and Maybank for Longer Trading Period

The stock-price data of Resorts and Maybank were chosen as the second set of two-stock portfolio for investment for a total of 939 trading days from 11th July 2005 until 30th April 2009, this included both the non-volatile period, from 2nd May 2006 until 24th August 2007 and the volatile period, from 24th December 2007 until 30th April 2009. The opening and closing prices for each stock were observed for each trading day and the corresponding price relatives were calculated. In this study, Resorts was chosen as the first constituent stock in the two-stock universal portfolio.

Table 3.9: The investment capitals (returns) obtained after 939 trading days with 16 universal portfolio strategies generated by the Dirichlet (α_1, α_2) distribution based on the stock price data of Resorts and Maybank from 11th July 2005 until 30th April 2009

Cover and Ordentlich Universal Portfolio generated by Dirichlet (α_1, α_2) distribution	Investment Capital after 939 trading days $\hat{S}_{939}$
(3.0, 1.0)	1.070410588
(0.7, 0.3)	1.077691026
(0.4, 0.3)	1.116063700
(0.5, 0.5)	1.142590358
(1.0, 1.0)	1.150901996
(0.3, 0.4)	1.159500502
(0.4, 0.6)	1.172086422
(0.4, 0.8)	1.192712179
(0.3, 0.7)	1.199531989
(1.0, 2.0)	1.202337631
(0.3, 0.9)	1.214271442
(1.0, 3.0)	1.224717493
(0.2, 0.8)	1.224868113
(0.2, 1.0)	1.234196188
(0.1, 0.9)	1.248037614
(0.1, 1.0)	1.250464566

In this Table 3.9, for a longer period from 11th July 2005 until 30th April 2009, the universal portfolio generated by Dirichlet (0.1, 1.0) distribution outperforms the rest. It is observed that after a large number of trading days even inclusive of the volatile period during the economic crisis, the universal portfolios outperform the constituent stocks. This is also shown in the cases discussed by Cover (1991).

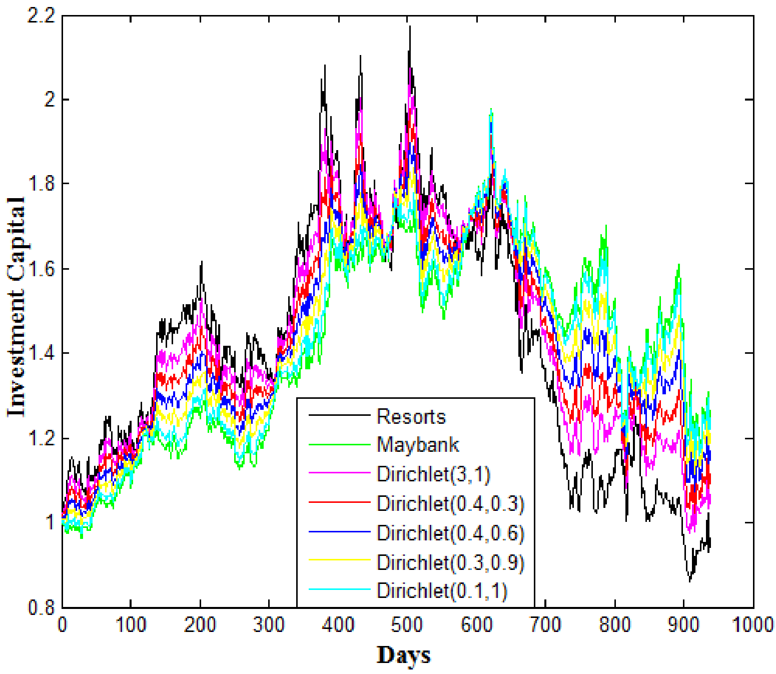


Figure 3.6: Investment capital for single-stock strategy and universal portfolios for 939 trading days from 11th July 2005 until 30th April 2009

Figure 3.6 shows that for a total of 939 trading days from 11th July 2005 until 30th April 2009, Resorts seems to outperform the rest during the first 550 trading days, but the universal portfolio generated by Dirichlet (0.1, 1.0) seems to overtake after 550 trading days. It was observed that after a large number of trading days the universal portfolios outperformed the constituent stocks, this was also shown in the cases discussed in Cover (1991). Note that during the market crashes, the constituent stock Resorts plunged from the peak of a return of 2.102034323 units after 432 trading days to the bottom of a decline of 0.859548911 units after 939 trading days, this is shown in the Figure 3.7.

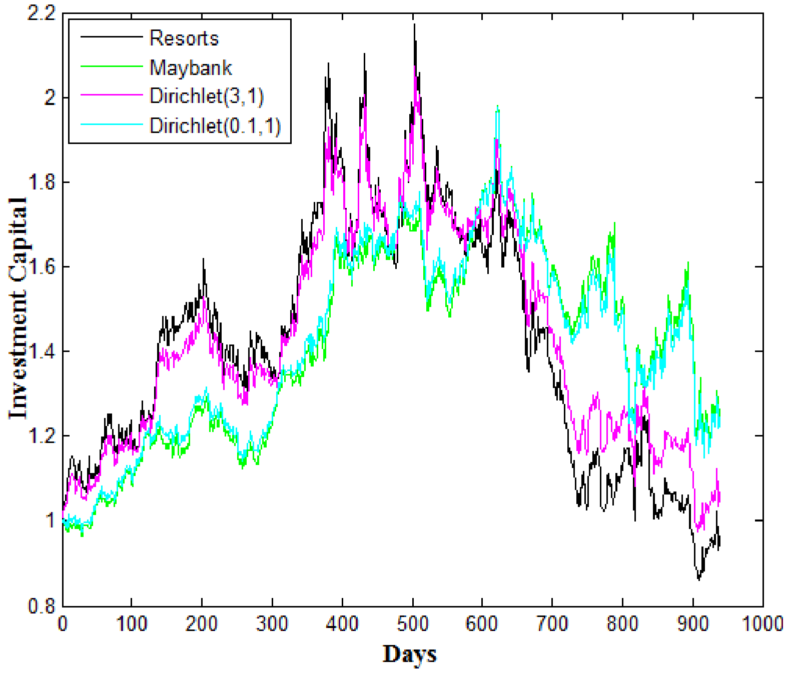


Figure 3.7: Investment capital for single-stock strategy and the universal portfolios generated by Dirichlet (3, 1) and Dirichlet (0.1, 1.0)

In Figure 3.7, the cyan line represents the universal portfolio generated by the Dirichlet (0.1, 1.0) distribution, yielding a resulting universal investment capital of 1.250464566 units which is the maximum capital in Table 3.9. The magenta line represents the universal portfolio generated by the Dirichlet (3, 1) distribution, yielding a resulting universal investment capital of 1.070410588 units which is the minimum capital in Table 3.9.

3.9 The Helmbold-Schapire-Singer-Warmuth Universal Portfolio

In this study, the investment capitals were computed by using the Helmbold-Schapire-Singer-Warmuth, (HSSW) universal portfolio algorithm with various starting portfolios. Helmbold et al. (1998) presented an on-line investment algorithm that yielded almost the same wealth as the best constant rebalanced portfolio achieved in hindsight from the actual market results. Their algorithm required the constant storage and computing time per stock in each trading day. A MATLAB m-file is written based on the modified algorithm discussed in Section 2.2.

In this section, for the two-stock portfolio of Digi and LPI, we consider n = 410 trading days and the maximum possible value of r for the corresponding price-relative set is given as follows:

$$0 < r \leq \frac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}}$$

where (i) minimum price relative, $\min_{i,n} x_{ni} = 0.90$ and

(ii) maximum price relative, $\max_{i,n} x_{ni} = 1.19$.

$$\text{Thus, } r = \frac{0.90}{1.19} \approx 0.75277504$$

The EG(η)-update universal portfolio is generated by the learning parameter η as follows:

$$\eta = 2r\sqrt{\frac{2\ln m}{n}}$$

where (i) the number of stocks in the investment portfolio, $m = 2$,

(ii) the number of trading days in a fixed period of study, $n = 410$,

(iii) r is chosen such that $r = \frac{0.90}{1.19} \approx 0.75277504$

Thus, $\eta = 2r\sqrt{\frac{2\ln m}{n}}$

$$= 2r\sqrt{\frac{2\ln 2}{410}} \approx 0.08754493$$

We assume an initial capital or wealth of 1 unit, the wealth S_n achieved by a constant rebalanced portfolio $\boldsymbol{b}$ is given as:

$$S_n = \prod_{i=1}^{n} (\boldsymbol{b}_i^t \cdot \boldsymbol{x}_i)$$

We note that the period from 27th October 2005 until 23rd May 2007 for the Digi – LPI stock data is the non-volatile period. The resulting investment capitals by using the HSSW universal portfolio are listed in Table 3.10 as S_{410} (HSSW). The resulting investment capitals by using the constant rebalanced portfolio strategies are extracted from Table 3.3 and listed in Table 3.10 as S_{410} (CRP) for comparison.

Table 3.10: The investment capitals after 410 trading days by using the HSSW universal portfolio and the constant rebalanced portfolio (CRP) strategies with various initial investment portfolios based on Digi – LPI stock data during the non-volatile period from 27th October 2005 until 23rd May 2007

Initial Investment		Investment Capital, S_{410}(HSSW)	Investment Capital, S_{410}(CRP)
Digi	LPI		
0.95	0.05	3.10732394	3.10843311
0.90	0.10	3.11823396	3.12035428
0.85	0.15	3.12789352	3.13092038
0.80	0.20	3.13629854	3.14012154
0.75	0.25	3.14344581	3.14794959
0.70	0.30	3.14933303	3.15439805
0.65	0.35	3.15395879	3.15946214
0.60	0.40	3.15732254	3.16313883
0.55	0.45	3.15942465	3.16542676
0.50	0.50	3.16026633	3.16632633
0.45	0.55	3.15984969	3.16583961
0.40	0.60	3.15817771	3.16397039
0.35	0.65	3.15525421	3.16072415
0.30	0.70	3.15108391	3.15610802
0.25	0.75	3.14567235	3.15013080
0.20	0.80	3.13902592	3.14280293
0.15	0.85	3.13115188	3.13413642
0.10	0.90	3.12205829	3.12414489
0.05	0.95	3.11175406	3.11284350

From Table 3.10, we find that the investment capitals achieved by using the HSSW universal portfolios do not differ much from that of the CRP strategies during the non-volatile period from 27th October 2005 until 23rd May 2007. This is shown in Figure 3.10 for comparison.

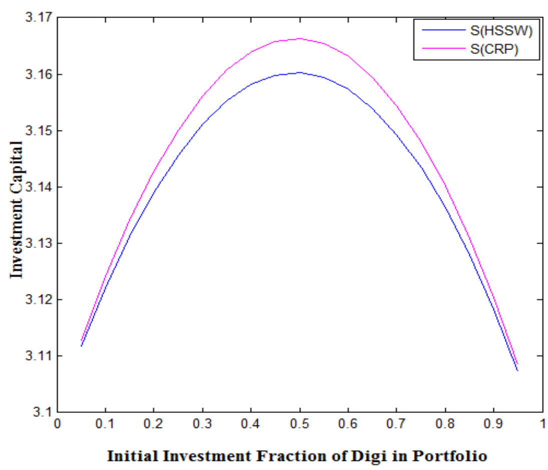


Figure 3.8: Investment capitals with initial investment fraction of Digi in portfolio using the HSSW universal portfolio and the CRP strategy during the non-volatile period

Next we consider the volatile period from 13th September 2007 until 30th April 2009, the resulting investment capitals achieved by using the HSSW universal portfolio are listed in Table 3.11 as S_{410}(v) (HSSW). The resulting investment capitals achieved by using the constant rebalanced portfolio strategies are extracted from Table 3.3 and listed in Table 3.11 as S_{410}(v) (CRP) for comparison.

Table 3.11: The investment capitals obtained during the volatile period from 13th September 2007 until 30th April 2009 after 410 trading days by using the HSSW universal portfolio and the constant rebalanced portfolio strategies with various initial investment portfolios based on Digi sp(v) – LPI sp(v) stock data

Initial Investment Portfolio		Investment Capital, S_{410}(v) (HSSW)	Investment Capital, S_{410}(v) (CRP)
Digi sp(v)	LPI sp(v)		
0.95	0.05	1.28503699	1.28534892
0.90	0.10	1.28969346	1.29029084
0.85	0.15	1.29400206	1.29485666
0.80	0.20	1.29796016	1.29904216
0.75	0.25	1.30156533	1.30284347
0.70	0.30	1.30481530	1.30625704
0.65	0.35	1.30770798	1.30927967
0.60	0.40	1.31024147	1.31190850
0.55	0.45	1.31241402	1.31414100
0.50	0.50	1.31422409	1.31597502
0.45	0.55	1.31567030	1.31740873
0.40	0.60	1.31675148	1.31844069
0.35	0.65	1.31746660	1.31906980
0.30	0.70	1.31781487	1.31929532
0.25	0.75	1.31779564	1.31911690
0.20	0.80	1.31740846	1.31853451
0.15	0.85	1.31665309	1.31754852
0.10	0.90	1.31552943	1.31615965
0.05	0.95	1.31403762	1.31436898

From Table 3.11, we find that the capital investments achieved by the HSSW universal portfolios do not differ much from that of the constant rebalanced portfolios during the volatile period from 13th September 2007 until 30th April 2009. This is shown in Figure 3.9 for comparison.

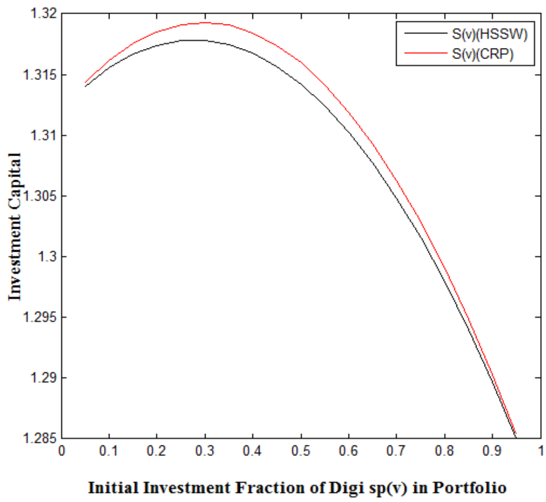


Figure 3.9: Investment capitals with initial investment fraction of Digi sp(v) in portfolio using the HSSW universal portfolio and the CRP strategy during the volatile period

3.10 Conclusion

- We have shown the importance and significance of the Dirichlet (α_1, α_2) strategies for both volatile and non-volatile periods of study in complementing the role of the Dirichlet (1, 1) and Dirichlet (0.5, 0.5) strategies introduced by Cover and Ordentlich (1996).

- It was observed that the universal portfolios generated by some Dirichlet distributions outperform the constituent stocks for both volatile and non-volatile periods of study. This is also evident in the cases discussed in Cover (1991).

- By finding the ratios of the optimal investment capitals, S_n^* and corresponding universal investment capitals, $\hat{S}_n$ during the volatile period and also non-volatile period respectively, we found that for the two-stock portfolio in investment based on empirical results:

$$\text{as } n \to \infty, \quad \limsup_{n\to\infty} \frac{1}{n} \ln \frac{S_n^*}{\hat{S}_n} = 0 .$$

(See Cover and Ordentlich (1996))

- ❖ We found that the investment capitals achieved by using the HSSW universal portfolios for the two-stock portfolio do not differ much from that of the constant rebalanced portfolio strategies during the non-volatile and volatile periods of study.

CHAPTER 4

DIRICHLET-WEIGHTED AND HELMBOLD-SCHAPIRE-SINGER-WARMUTH UNIVERSAL PORTFOLIOS FOR THREE STOCKS

4.1 The Investment in the Stocks of Daiboci, KNM and Topglov

The stock-price data of three listing companies from the Kuala Lumpur Stock Exchange (KLSE) namely that of Daibochi Plastic & Packaging Industries Berhad (abbreviated as Daiboci), KNM Group Berhad (abbreviated as KNM), Top Glove Corporation Berhad (abbreviated as Topglov) were chosen as the three-stock portfolio for investment based on their good performance. The stock data of a total of 305 trading days from 27th October 2005 until 28th December 2006 were collected. The period of trading mentioned here is considered as the non-volatile period of stock price fluctuations. Daiboci was chosen as the first constituent stock in the three-stock universal portfolio. The opening and closing prices in Ringgit Malaysia (RM) for each stock were observed for each trading day and the corresponding price relatives were calculated.

Based on the stock-price data collected for this non-volatile period from 27th October 2005 until 28th December 2006, we shall calculate the resulting capital after 305 days of investment using the Cover-Ordentlich universal portfolio in the next section, where we assume that the initial investment capital is 1 unit.

4.2 The Performance of Universal Portfolios in Investing in Daiboci, KNM and Topglov during the Non-Volatile Period of Trading

In this research, we had chosen MATLAB as the platform for the computation of universal portfolios and capitals. A MATLAB m-file was written based on the modified algorithm for three-stock universal portfolio discussed in Section 2.1.4. We used different parameters of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution for generating the universal portfolio strategies.

The Dirichlet (1.0, 1.0, 1.0) strategy was first introduced by Cover (1991). Subsequently, Cover and Ordentlich (1996) introduced the general class of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies. But the focus of their study is on the Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies. The aim of this project is to study the parametric class of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies, especially when the (1.0, 1.0, 1.0) and (0.5, 0.5, 0.5) strategies do not perform well. Appendix 2 shows the investment capitals achieved for the constituent stocks Daiboci, KNM and Topglov by using the Cover and Ordentlich universal portfolios namely the Dirichlet (0.3, 0.7, 0.9) and Dirichlet (2.0, 2.0, 1.0) universal portfolios. These capitals were calculated using the modified algorithm mentioned in Section 2.1.4. The period of trading from 27th October 2005 until 28th December 2006 is considered to be the non-volatile period of stock price fluctuations.

Figure 4.1 shows the investment capital (return) after 305 trading days i.e. from 27th October 2005 until 28th December 2006, for investment in the stocks Daiboci, KNM and Topglov by the universal portfolios generated by Dirichlet (0.3, 0.7, 0.9) , Dirichlet (0.9, 1.0, 0.8) and Dirichlet (2.0, 2.0, 1.0) distributions. It was observed that the universal portfolio generated by Dirichlet (2.0, 2.0, 1.0) outperforms the constituent stocks during this period of study.

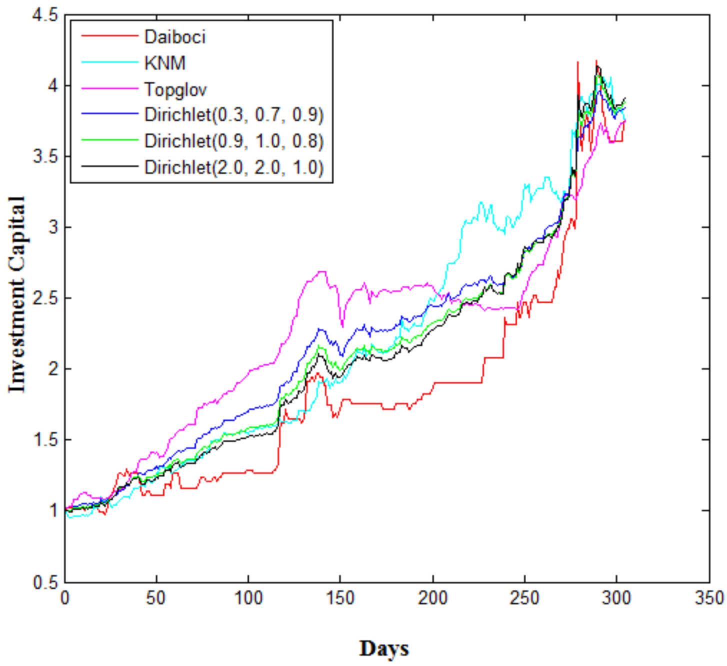


Figure 4.1: Investment capital (return) for single-stock strategy and universal portfolios during the non-volatile period

We observe that by varying the parameters α_1, α_2 and α_3, we can obtain Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies achieving different capitals. For example, refer to Table 4.1 where the investment capitals of 18 different sets of $(\alpha_1, \alpha_2, \alpha_3)$ are calculated, namely for $(\alpha_1, \alpha_2, \alpha_3)$ equals to (0.1, 0.3, 0.4), (0.2, 0.9, 0.3), (0.3, 0.4, 0.1), (0.3, 0.7, 0.9), (0.4, 0.4, 0.3), (0.5, 0.5, 0.5), (0.6, 0.6, 0.5), (0.7, 0.7, 0.7), (0.7, 0.8, 0.8), (0.8, 0.6, 0.3), (0.9, 1.0, 0.8), (1.0, 1.0, 1.0), (2.0, 1.0, 2.0), (2.0, 2.0, 2.0), (2.0, 2.0, 1.0), (1.0, 0.0, 0.0), (0.0, 1.0, 0.0) and (0.0, 0.0, 1.0) respectively. Except for the detailed calculations for the

Dirichlet (0.3, 0.7, 0.9) and Dirichlet (2.0, 2.0, 1.0) universal portfolios listed in Appendix 2, we will not show the detailed calculations for the other 16 universal portfolios listed in Table 4.1.

In Table 4.1, the investment capitals (returns) obtained after these 305 trading days with 18 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the stock price data of Daiboci, KNM and Topglov are ranged from 3.74162265 units to 3.90396078 units. The Dirichlet (2.0, 2.0, 1.0) strategy (as shown highlighted in grey colour) achieves the highest capital of $\hat{S}_{305}$ = 3.90396078 units whereas the Dirichlet (1.0, 0.0, 0.0) achieves the lowest capital of $\hat{S}_{305}$ = 3.74162265 units. The Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of $\hat{S}_{305}$ = 3.88297775 units and $\hat{S}_{305}$ = 3.85727059 units respectively for this period of study. This once again shows the importance and significance of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies other than the Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies.

Table 4.1: The investment capitals (returns) achieved after 305 trading days, with 18 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the stock price data of Daiboci, KNM and Topglov from 27th October 2005 until 28th December 2006

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	Investment Capital after 305 trading days, $\hat{S}_{305}$
(0.1, 0.3, 0.4)	3.80932385
(0.2, 0.9, 0.3)	3.82653302
(0.3, 0.4, 0.1)	3.83286977
(0.3, 0.7, 0.9)	3.84051942
(0.4, 0.4, 0.3)	3.84652601
(0.5, 0.5, 0.5)	3.85727059
(0.6, 0.6, 0.5)	3.86426389
(0.7, 0.7, 0.7)	3.87051335
(0.7, 0.8, 0.8)	3.87123589
(0.8, 0.6, 0.3)	3.86684992
(0.9, 1.0, 0.8)	3.88042998
(1.0, 1.0, 1.0)	3.88297775
(2.0, 1.0, 2.0)	3.89555250
(2.0, 2.0, 2.0)	3.90148494
(2.0, 2.0, 1.0)	3.90396078
(1.0, 0.0, 0.0)	3.74162265
(0.0, 1.0, 0.0)	3.76411974
(0.0, 0.0, 1.0)	3.76160500

Note: The investment capitals (returns) achieved after 305 trading days for single-stock strategy of the constituent stocks, Daiboci, KNM and Topglov are listed in last three rows of the above Table 4.1, by using the universal portfolios generated by the Dirichlet (1.0, 0.0, 0.0), Dirichlet (0.0, 1.0, 0.0) and Dirichlet (0.0, 0.0, 1.0) distributions respectively.

4.3 The Investment in the Stocks of Daiboci, KNM and Topglov during the Volatile Period of Trading

The recent market crashes and finance tsunami in 2008 is causing problems worldwide; as such we investigate the impact the stock market crash of 2008 to portfolio investment. The stock price data of a total of 305 trading days from 11th February 2008 until 30th April 2009 were collected. This period of trading is considered as the volatile period. Three sets of stock price data from the Kuala Lumpur Stock Exchange (KLSE) namely that of Daiboci sp(v), KNM sp(v) and Topglov sp(v) were collected for analysis. We use the abbreviation sp(v) for the stock prices collected during the above mentioned volatile period. Daiboci sp(v) was chosen as the first constituent stock in the three-stock universal portfolio. The opening and closing prices in Ringgit Malaysia (RM) for each stock were observed for each trading day and the corresponding price relatives were calculated.

Based on the stock-price data collected for this volatile period from 11th February 2008 until 30th April 2009, we shall calculate the resulting capital after 305 days of investment using the Cover-Ordentlich universal portfolio in the next section, where we assume that the initial investment capital is 1 unit.

4.4 The Performance of Universal Portfolios in Investing in Daiboci, KNM and Topglov during the Volatile Period of Trading

In this section, the parametric class of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distributions was again used for generating the universal portfolio strategies during the volatile period from 11th February 2008 until 30th April 2009. We use the same MATLAB m-file as mentioned in Section 4.2.

Figure 4.2 shows the investment capital (return) after 305 trading days i.e. from 11th February 2008 until 30th April 2009 during the volatile period, for investment in the stocks Daiboci sp(v), KNM sp(v) and Topglov sp(v) by the universal portfolios generated by Dirichlet (0.7, 0.2, 0.1), Dirichlet (0.9, 0.1, 0.7) and Dirichlet (3.0, 0.1, 0.2) distributions. It was observed that Daiboci sp(v) outperforms the universal portfolios during this volatile period of study.

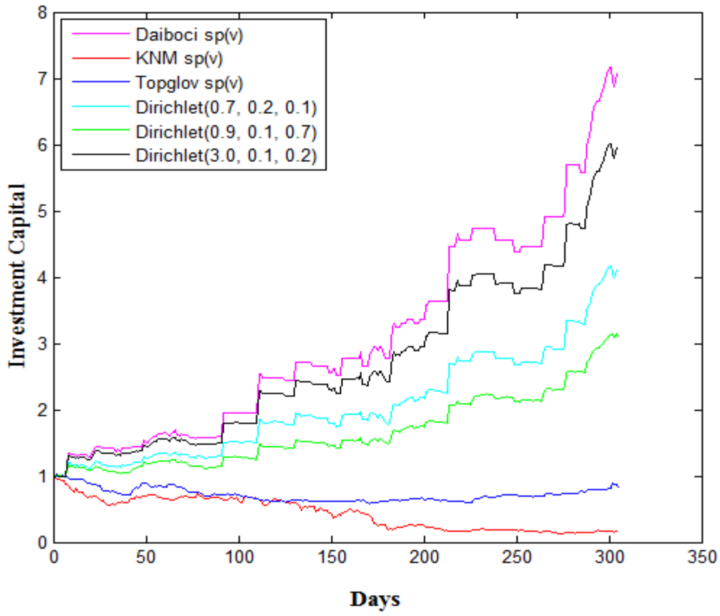


Figure 4.2: Investment capital (return) for single-stock strategy and universal portfolios during the volatile period

These capitals were again calculated using the modified algorithm mentioned in Section 2.1.4 with MATLAB. We observe that by varying the parameters α_1, α_2 and α_3, we can obtain Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies achieving different capitals. For example, refer to Table 4.2 where the investment capitals of 18 different pairs of $(\alpha_1, \alpha_2, \alpha_3)$ are calculated, namely for $(\alpha_1, \alpha_2, \alpha_3)$ equals to (0.1, 0.3, 0.4), (0.2, 0.9, 0.3), (0.3, 0.4, 0.1), (0.3, 0.7, 0.2), (0.4, 0.2, 0.3), (0.5, 0.5, 0.5), (0.6, 0.6, 0.5), (0.7, 0.7, 0.7), (0.7, 0.2, 0.1), (0.8, 0.6, 0.3), (0.9, 0.1, 0.7), (1.0, 1.0, 1.0), (2.0, 1.0, 2.0), (2.0, 2.0, 2.0), (3.0, 0.1, 0.2), (0.0, 1.0, 0.0), (0.0, 1.0, 0.0) and (0.0, 1.0, 0.0) respectively. The investment capitals (returns) obtained after these 305 trading days with 18 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the stock price data of Daiboci sp(v), KNM sp(v) and Topglov sp(v) are ranged from 0.16279082 units to 5.94093766 units. The Dirichlet (3.0, 0.1, 0.2) strategy (as shown highlighted in grey colour) achieves the highest capital of $\hat{S}_{305}$ = 5.94093766 units whereas the Dirichlet (0.0, 1.0, 0.0) achieves the lowest capital of $\hat{S}_{305}$ = 0.16279082 units. The Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of $\hat{S}_{305}$ = 1.43418118 units and $\hat{S}_{305}$ = 1.66134091 units respectively for this period of study. This once again shows the importance and significance of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies other than the Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies.

Table 4.2: The investment capitals achieved after 305 trading days, with 18 universal portfolio strategies generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the stock price data of Daiboci sp(v), KNM sp(v) and Topglov sp(v) during the volatile period, from 11th February 2008 until 30th April 2009

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	Investment Capital after 305 trading days, $\hat{S}_{305}$ (v)
(0.1, 0.3, 0.4)	0.98030904
(0.2, 0.9, 0.3)	0.69712688
(0.3, 0.4, 0.1)	1.89806601
(0.3, 0.7, 0.2)	1.12458747
(0.4, 0.2, 0.3)	2.57573156
(0.5, 0.5, 0.5)	1.66134091
(0.6, 0.6, 0.5)	1.66554092
(0.7, 0.7, 0.7)	1.54197343
(0.7, 0.2, 0.1)	4.10527155
(0.8, 0.6, 0.3)	2.29512492
(0.9, 0.1, 0.7)	3.09776713
(1.0, 1.0, 1.0)	1.43418118
(2.0, 1.0, 2.0)	1.79849356
(2.0, 2.0, 2.0)	1.28382122
(3.0, 0.1, 0.2)	5.94093766
(1.0, 0.0, 0.0)	7.07644368
(0.0, 1.0, 0.0)	0.16279082
(0.0, 0.0, 1.0)	0.81403273

Note: The investment capitals (returns) achieved after 305 trading days for single-stock strategy of the constituent stocks, Daiboci sp(v), KNM sp(v) and Topglov sp(v) are listed in last three rows of the above Table 4.2, by using the universal portfolios generated by the Dirichlet (1.0, 0.0, 0.0), Dirichlet (0.0, 1.0, 0.0) and Dirichlet (0.0, 0.0, 1.0) distributions respectively.

4.5 Constant Rebalanced Strategies

4.5.1 Constant Rebalanced Strategies for Non-Volatile Period

In this section, we shall use 101 constant rebalanced strategies for investment in the stocks of Daiboci, KNM and Topglov for a total of 305 trading days for the non-volatile period from 27th October 2005 until 28th December 2006.

The initial investment capital is one unit and the resulting capitals after 305 trading days are calculated. The constant rebalanced portfolio strategies for the investment in the stocks of Daiboci, KNM and Topglov is determined by using the cycle coordinate ascent algorithm which is known to be convergent. Table 4.3 shows a list of constant rebalanced strategies calculated in the course of applying the algorithm. We begin by fixing a coordinate in the portfolio vector $\boldsymbol{b} = (b_1, b_2, b_3)$ and then vary the other coordinates to obtain the maximum capital. For example, if we begin by fixing the b_2 coordinate at $b_2^{'}$ then maximize the capital for all portfolio vectors of $\boldsymbol{b} = (1 - b_2^{'} - b_3, b_2^{'}, b_3)$, that is maximize the capital over $0 \le b_3 \le 1 - b_2^{'}$. If the maximum is when $\boldsymbol{b}$ =

$(1-b_2^{'}-b_3^{'}, b_2^{'}, b_3^{'})$, then we fix the coordinate $b_3^{'}$ and continue to maximize the capital over ***b*** $=(b_1, 1-b_1-b_3^{'}, b_3^{'})$, until we obtain a maximum when ***b*** = $(b_1^{'}, 1-b_1^{'}-b_3^{'}, b_3^{'})$. Thus, we consider the first cycle is completed and we continue with the second cycle by maximizing the capital over ***b*** $=(b_1^{'}, b_2, 1-b_1^{'}-b_2)$, that is over $0 \le b_2 \le 1-b_1^{'}$. The process of fixing a particular coordinate b_1 at a certain point and maximizing with respect to the other free coordinate until the maximum is found, is repeated in a cyclic manner until we obtain a fixed point ***b***** = $(b_1^{*}, b_2^{*}, b_3^{*})$. At the fixed point ***b*****, by maximizing with respect to the other free coordinate until it ends up at the same point ***b*****. In this case, the algorithm is said to converge at ***b*****.

In this experiment, we begin the algorithm by fixing the third constituent stock Topglov，that is the b_3 coordinate in the portfolio vector ***b*** = (b_1, b_2, b_3) to be 0. We then maximize the capital over the constant rebalanced portfolio strategies of ***b*** = $(1-b, b, 0)$， where b increasing in units of 0.01 from 0.00 to 1.00. From all possible constant portfolio strategies ***b***, we search for the best constant rebalanced strategy and we find that the best constant rebalanced strategy is ***b*** = (0.48, 0.52, 0.00), yielding investment capital of S_{305} = 3.93131073 units. This is listed in Table 4.3 (as shown highlighted in grey colour) and is shown in Figure 4.3.

Table 4.3: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction of KNM in portfolio and the investment fraction of Topglov is fixed at zero

Investment Fraction			Investment Capital, S_{305} (CRP)	Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov		Daiboci	KNM	Topglov	
1.00	0.00	0.00	3.74162265	0.49	0.51	0.00	3.93122577
0.99	0.01	0.00	3.74861758	0.48	0.52	0.00	3.93131073
0.98	0.02	0.00	3.75549212	0.47	0.53	0.00	3.93124993
0.97	0.03	0.00	3.76224549	0.46	0.54	0.00	3.93104320
0.96	0.04	0.00	3.76887691	0.45	0.55	0.00	3.93069040
0.95	0.05	0.00	3.77538562	0.44	0.56	0.00	3.93019141
0.94	0.06	0.00	3.78177086	0.43	0.57	0.00	3.92954610
0.93	0.07	0.00	3.78803189	0.42	0.58	0.00	3.92875437
0.92	0.08	0.00	3.79416798	0.41	0.59	0.00	3.92781613
0.91	0.09	0.00	3.80017839	0.40	0.60	0.00	3.92673133
0.90	0.10	0.00	3.80606242	0.39	0.61	0.00	3.92549988
0.89	0.11	0.00	3.81181936	0.38	0.62	0.00	3.92412176
0.88	0.12	0.00	3.81744852	0.37	0.63	0.00	3.92259693
0.87	0.13	0.00	3.82294922	0.36	0.64	0.00	3.92092538
0.86	0.14	0.00	3.82832079	0.35	0.65	0.00	3.91910711
0.85	0.15	0.00	3.83356256	0.34	0.66	0.00	3.91714213
0.84	0.16	0.00	3.83867389	0.33	0.67	0.00	3.91503048
0.83	0.17	0.00	3.84365413	0.32	0.68	0.00	3.91277219
0.82	0.18	0.00	3.84850267	0.31	0.69	0.00	3.91036733
0.81	0.19	0.00	3.85321889	0.30	0.70	0.00	3.90781597
0.80	0.20	0.00	3.85780218	0.29	0.71	0.00	3.90511819
0.79	0.21	0.00	3.86225195	0.28	0.72	0.00	3.90227411
0.78	0.22	0.00	3.86656763	0.27	0.73	0.00	3.89928384
0.77	0.23	0.00	3.87074863	0.26	0.74	0.00	3.89614751
0.76	0.24	0.00	3.87479442	0.25	0.75	0.00	3.89286526
0.75	0.25	0.00	3.87870443	0.24	0.76	0.00	3.88943727
0.74	0.26	0.00	3.88247815	0.23	0.77	0.00	3.88586370
0.73	0.27	0.00	3.88611504	0.22	0.78	0.00	3.88214476
0.72	0.28	0.00	3.88961461	0.21	0.79	0.00	3.87828064
0.71	0.29	0.00	3.89297635	0.20	0.80	0.00	3.87427156
0.70	0.30	0.00	3.89619979	0.19	0.81	0.00	3.87011777
0.69	0.31	0.00	3.89928445	0.18	0.82	0.00	3.86581951
0.68	0.32	0.00	3.90222987	0.17	0.83	0.00	3.86137704
0.67	0.33	0.00	3.90503562	0.16	0.84	0.00	3.85679066
0.66	0.34	0.00	3.90770125	0.15	0.85	0.00	3.85206064
0.65	0.35	0.00	3.91022636	0.14	0.86	0.00	3.84718731
0.64	0.36	0.00	3.91261053	0.13	0.87	0.00	3.84217098
0.63	0.37	0.00	3.91485338	0.12	0.88	0.00	3.83701199
0.62	0.38	0.00	3.91695451	0.11	0.89	0.00	3.83171070
0.61	0.39	0.00	3.91891358	0.10	0.90	0.00	3.82626747
0.60	0.40	0.00	3.92073021	0.09	0.91	0.00	3.82068269
0.59	0.41	0.00	3.92240408	0.08	0.92	0.00	3.81495674
0.58	0.42	0.00	3.92393486	0.07	0.93	0.00	3.80909005
0.57	0.43	0.00	3.92532224	0.06	0.94	0.00	3.80308303
0.56	0.44	0.00	3.92656591	0.05	0.95	0.00	3.79693613
0.55	0.45	0.00	3.92766560	0.04	0.96	0.00	3.79064980
0.54	0.46	0.00	3.92862103	0.03	0.97	0.00	3.78422451
0.53	0.47	0.00	3.92943194	0.02	0.98	0.00	3.77766074
0.52	0.48	0.00	3.93009809	0.01	0.99	0.00	3.77095898
0.51	0.49	0.00	3.93061926	0.00	1.00	0.00	3.76411974
0.50	0.50	0.00	3.93099522				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of KNM in portfolio and the investment fraction of Topglov is fixed at zero, during the non-volatile period from 27th October 2005 until 28th December 2006 is shown in Figure 4.3.

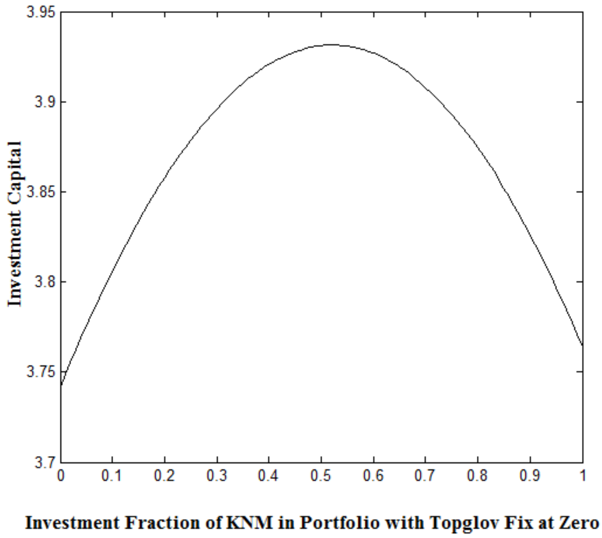


Figure 4.3: Investment capitals achieved by the CRP strategy with investment fraction of KNM in portfolio and investment fraction of Topglov fixed at zero

Now we fix the proportion in the second stock of KNM at 0.52 and search for the portfolio $\boldsymbol{b} = (1 - b - 0.52, 0.52, b)$ that maximizes the capital. We consider b increasing in units of 0.01 from 0.00 to 0.48 and obtain the maximum capital of $S_{305} = 3.93265677$ units for the best constant rebalanced strategy $\boldsymbol{b} = (0.43, 0.52, 0.05)$. These results are listed in Table 4.4 and showed in Figure 4.4.

Table 4.4: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction of Topglov in portfolio and the investment fraction of KNM fixed at 0.52

Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov	
0.48	0.52	0.00	3.93131073
0.47	0.52	0.01	3.93182457
0.46	0.52	0.02	3.93221630
0.45	0.52	0.03	3.93248570
0.44	0.52	0.04	3.93263259
0.43	0.52	0.05	3.93265677
0.42	0.52	0.06	3.93255807
0.41	0.52	0.07	3.93233631
0.40	0.52	0.08	3.93199134
0.39	0.52	0.09	3.93152301
0.38	0.52	0.10	3.93093118
0.37	0.52	0.11	3.93021572
0.36	0.52	0.12	3.92937651
0.35	0.52	0.13	3.92841344
0.34	0.52	0.14	3.92732642
0.33	0.52	0.15	3.92611535
0.32	0.52	0.16	3.92478015
0.31	0.52	0.17	3.92332075
0.30	0.52	0.18	3.92173711
0.29	0.52	0.19	3.92002916
0.28	0.52	0.20	3.91819686
0.27	0.52	0.21	3.91624020
0.26	0.52	0.22	3.91415915
0.25	0.52	0.23	3.91195370
0.24	0.52	0.24	3.90962386
0.23	0.52	0.25	3.90716964
0.22	0.52	0.26	3.90459106
0.21	0.52	0.27	3.90188816
0.20	0.52	0.28	3.89906097
0.19	0.52	0.29	3.89610956
0.18	0.52	0.30	3.89303398
0.17	0.52	0.31	3.88983432
0.16	0.52	0.32	3.88651065
0.15	0.52	0.33	3.88306308
0.14	0.52	0.34	3.87949170
0.13	0.52	0.35	3.87579664
0.12	0.52	0.36	3.87197802
0.11	0.52	0.37	3.86803598
0.10	0.52	0.38	3.86397066
0.09	0.52	0.39	3.85978223
0.08	0.52	0.40	3.85547085
0.07	0.52	0.41	3.85103670
0.06	0.52	0.42	3.84647998
0.05	0.52	0.43	3.84180087
0.04	0.52	0.44	3.83699959
0.03	0.52	0.45	3.83207637
0.02	0.52	0.46	3.82703143
0.01	0.52	0.47	3.82186501
0.00	0.52	0.48	3.81657736

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of Topglov in portfolio and the investment fraction of KNM fixed at 0.52 during the non-volatile period from 27th October 2005 until 28th December 2006 is shown in Figure 4.4.

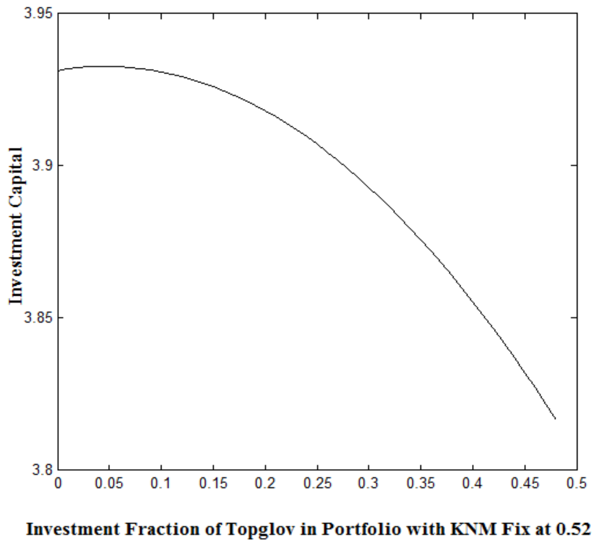


Figure 4.4: Investment capitals achieved by the CRP strategy with investment fraction of Topglov in portfolio and investment fraction of KNM fixed at 0.52

Next we fix the proportion of KNM in the second coordinate at 0.00 and search for the portfolio $\boldsymbol{b} = (1-b_3, 0.00, b_3)$ that maximizes the capital where b_3 increasing in units of 0.01 from 0.00 to 1.00. This is found when $\boldsymbol{b}$ = (0.48, 0.00, 0.52) and S_{305} = 3.90025885 units as listed in Table 4.5 (as shown highlighted in grey colour) and showed in Figure 4.5.

Table 4.5: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction Topglov in portfolio and the investment fraction KNM fixed at zero

Investment Fraction			Investment Capital, S_{305} (CRP)	Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov		Daiboci	KNM	Topglov	
1.00	0.00	0.00	3.74162265	0.49	0.00	0.51	3.90016397
0.99	0.00	0.01	3.74746009	0.48	0.00	0.52	3.90025885
0.98	0.00	0.02	3.75319649	0.47	0.00	0.53	3.90023274
0.97	0.00	0.03	3.75883127	0.46	0.00	0.54	3.90008549
0.96	0.00	0.04	3.76436385	0.45	0.00	0.55	3.89981693
0.95	0.00	0.05	3.76979365	0.44	0.00	0.56	3.89942693
0.94	0.00	0.06	3.77512011	0.43	0.00	0.57	3.89891535
0.93	0.00	0.07	3.78034269	0.42	0.00	0.58	3.89828205
0.92	0.00	0.08	3.78546082	0.41	0.00	0.59	3.89752694
0.91	0.00	0.09	3.79047397	0.40	0.00	0.60	3.89664990
0.90	0.00	0.10	3.79538160	0.39	0.00	0.61	3.89565083
0.89	0.00	0.11	3.80018317	0.38	0.00	0.62	3.89452965
0.88	0.00	0.12	3.80487818	0.37	0.00	0.63	3.89328629
0.87	0.00	0.13	3.80946610	0.36	0.00	0.64	3.89192067
0.86	0.00	0.14	3.81394643	0.35	0.00	0.65	3.89043274
0.85	0.00	0.15	3.81831867	0.34	0.00	0.66	3.88882245
0.84	0.00	0.16	3.82258232	0.33	0.00	0.67	3.88708978
0.83	0.00	0.17	3.82673691	0.32	0.00	0.68	3.88523468
0.82	0.00	0.18	3.83078194	0.31	0.00	0.69	3.88325715
0.81	0.00	0.19	3.83471696	0.30	0.00	0.70	3.88115717
0.80	0.00	0.20	3.83854151	0.29	0.00	0.71	3.87893476
0.79	0.00	0.21	3.84225512	0.28	0.00	0.72	3.87658992
0.78	0.00	0.22	3.84585735	0.27	0.00	0.73	3.87412268
0.77	0.00	0.23	3.84934776	0.26	0.00	0.74	3.87153308
0.76	0.00	0.24	3.85272593	0.25	0.00	0.75	3.86882115
0.75	0.00	0.25	3.85599143	0.24	0.00	0.76	3.86598695
0.74	0.00	0.26	3.85914385	0.23	0.00	0.77	3.86303055
0.73	0.00	0.27	3.86218277	0.22	0.00	0.78	3.85995202
0.72	0.00	0.28	3.86510781	0.21	0.00	0.79	3.85675144
0.71	0.00	0.29	3.86791857	0.20	0.00	0.80	3.85342891
0.70	0.00	0.30	3.87061467	0.19	0.00	0.81	3.84998454
0.69	0.00	0.31	3.87319574	0.18	0.00	0.82	3.84641843
0.68	0.00	0.32	3.87566142	0.17	0.00	0.83	3.84273073
0.67	0.00	0.33	3.87801135	0.16	0.00	0.84	3.83892155
0.66	0.00	0.34	3.88024518	0.15	0.00	0.85	3.83499104
0.65	0.00	0.35	3.88236257	0.14	0.00	0.86	3.83093938
0.64	0.00	0.36	3.88436320	0.13	0.00	0.87	3.82676670
0.63	0.00	0.37	3.88624674	0.12	0.00	0.88	3.82247321
0.62	0.00	0.38	3.88801289	0.11	0.00	0.89	3.81805907
0.61	0.00	0.39	3.88966133	0.10	0.00	0.90	3.81352450
0.60	0.00	0.40	3.89119178	0.09	0.00	0.91	3.80886969
0.59	0.00	0.41	3.89260395	0.08	0.00	0.92	3.80409486
0.58	0.00	0.42	3.89389756	0.07	0.00	0.93	3.79920024
0.57	0.00	0.43	3.89507234	0.06	0.00	0.94	3.79418607
0.56	0.00	0.44	3.89612805	0.05	0.00	0.95	3.78905259
0.55	0.00	0.45	3.89706442	0.04	0.00	0.96	3.78380007
0.54	0.00	0.46	3.89788122	0.03	0.00	0.97	3.77842876
0.53	0.00	0.47	3.89857823	0.02	0.00	0.98	3.77293896
0.52	0.00	0.48	3.89915521	0.01	0.00	0.99	3.76733094
0.51	0.00	0.49	3.89961196	0.00	0.00	1.00	3.76160500
0.50	0.00	0.50	3.89994828				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of Topglov in portfolio and the investment fraction of KNM fixed at zero during the non-volatile period from 27th October 2005 until 28th December 2006 are shown in Figure 4.5.

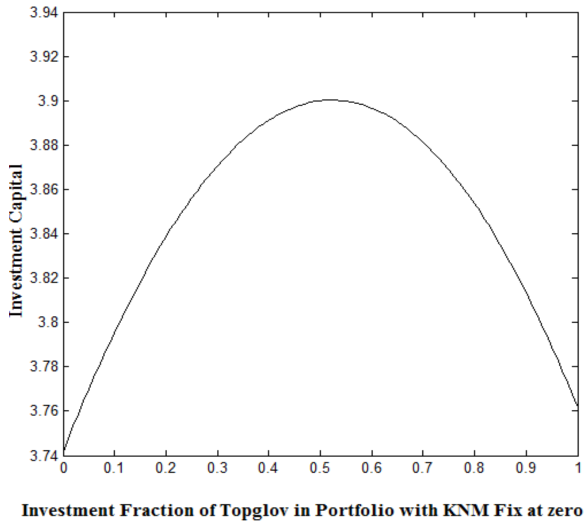


Figure 4.5: Investment capitals achieved by the CRP strategy with investment fraction of Topglov in portfolio and the investment fraction KNM fixed at zero

Next we fix the proportion in the third stock of Topglov at 0.52 and search for the portfolio $\boldsymbol{b} = (1 - b - 0.52, b, 0.52)$ that maximizes the capital. We consider b increasing in units of 0.01 from 0.00 to 0.48 and obtain the maximum capital of $S_{305} = 3.91122398$ units for the best constant rebalanced strategy $\boldsymbol{b} = (0.36, 0.12, 0.52)$. These results are listed in Table 4.6 and showed in Figure 4.6.

Table 4.6: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction of KNM in portfolio and the investment fraction of Topglov fixed at 0.52

Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov	
0.48	0.00	0.52	3.90025885
0.47	0.01	0.52	3.90197332
0.46	0.02	0.52	3.90354322
0.45	0.03	0.52	3.90496821
0.44	0.04	0.52	3.90624797
0.43	0.05	0.52	3.90738220
0.42	0.06	0.52	3.90837059
0.41	0.07	0.52	3.90921288
0.40	0.08	0.52	3.90990880
0.39	0.09	0.52	3.91045810
0.38	0.10	0.52	3.91086054
0.37	0.11	0.52	3.91111590
0.36	0.12	0.52	3.91122398
0.35	0.13	0.52	3.91118458
0.34	0.14	0.52	3.91099752
0.33	0.15	0.52	3.91066265
0.32	0.16	0.52	3.91017981
0.31	0.17	0.52	3.90954886
0.30	0.18	0.52	3.90876970
0.29	0.19	0.52	3.90784221
0.28	0.20	0.52	3.90676630
0.27	0.21	0.52	3.90554190
0.26	0.22	0.52	3.90416895
0.25	0.23	0.52	3.90264740
0.24	0.24	0.52	3.90097723
0.23	0.25	0.52	3.89915841
0.22	0.26	0.52	3.89719096
0.21	0.27	0.52	3.89507487
0.20	0.28	0.52	3.89281018
0.19	0.29	0.52	3.89039694
0.18	0.30	0.52	3.88783520
0.17	0.31	0.52	3.88512505
0.16	0.32	0.52	3.88226656
0.15	0.33	0.52	3.87925985
0.14	0.34	0.52	3.87610504
0.13	0.35	0.52	3.87280225
0.12	0.36	0.52	3.86935165
0.11	0.37	0.52	3.86575339
0.10	0.38	0.52	3.86200766
0.09	0.39	0.52	3.85811465
0.08	0.40	0.52	3.85407458
0.07	0.41	0.52	3.84988766
0.06	0.42	0.52	3.84555415
0.05	0.43	0.52	3.84107430
0.04	0.44	0.52	3.83644837
0.03	0.45	0.52	3.83167666
0.02	0.46	0.52	3.82675947
0.01	0.47	0.52	3.82169711
0.00	0.48	0.52	3.81648991

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of

Topglov in portfolio and the investment fraction of KNM fixed at 0.52 during the non-volatile period from 27th October 2005 until 28th December 2006 is shown in Figure 4.6.

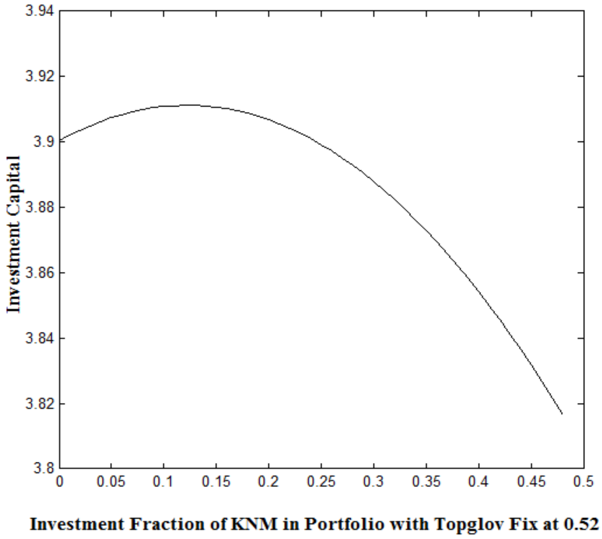


Figure 4.6: Investment capitals achieved by the CRP strategy with investment fraction of KNM in portfolio and investment fraction of Topglov fixed at 0.52

Lastly we fix the proportion of Daiboci in the first coordinate at 0.00 and search for the portfolio $\boldsymbol{b} = (0.00, b_2, 1-b_2)$ that maximizes the capital where b_2 increasing in units of 0.01 from 0.00 to 1.00. This is found when b = (0.00, 0.51, 0.49) and the maximum capital of S_{305} = 3.81662045 units as listed in Table 4.7 (as shown highlighted in grey colour) and showed in Figure 4.7.

Table 4.7: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction of KNM in portfolio and the investment fraction of Daiboci fixed at zero

Investment Fraction			Investment Capital, S_{305} (CRP)	Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov		Daiboci	KNM	Topglov	
0.00	0.00	1.00	3.76160500	0.00	0.51	0.49	3.81662045
0.00	0.01	0.99	3.76375127	0.00	0.52	0.48	3.81657736
0.00	0.02	0.98	3.76585556	0.00	0.53	0.47	3.81649100
0.00	0.03	0.97	3.76791781	0.00	0.54	0.46	3.81636136
0.00	0.04	0.96	3.76993795	0.00	0.55	0.45	3.81618846
0.00	0.05	0.95	3.77191592	0.00	0.56	0.44	3.81597232
0.00	0.06	0.94	3.77385168	0.00	0.57	0.43	3.81571295
0.00	0.07	0.93	3.77574516	0.00	0.58	0.42	3.81541037
0.00	0.08	0.92	3.77759632	0.00	0.59	0.41	3.81506460
0.00	0.09	0.91	3.77940509	0.00	0.60	0.40	3.81467566
0.00	0.10	0.90	3.78117143	0.00	0.61	0.39	3.81424357
0.00	0.11	0.89	3.78289529	0.00	0.62	0.38	3.81376835
0.00	0.12	0.88	3.78457662	0.00	0.63	0.37	3.81325002
0.00	0.13	0.87	3.78621538	0.00	0.64	0.36	3.81268862
0.00	0.14	0.86	3.78781151	0.00	0.65	0.35	3.81208417
0.00	0.15	0.85	3.78936498	0.00	0.66	0.34	3.81143670
0.00	0.16	0.84	3.79087574	0.00	0.67	0.33	3.81074623
0.00	0.17	0.83	3.79234375	0.00	0.68	0.32	3.81001280
0.00	0.18	0.82	3.79376898	0.00	0.69	0.31	3.80923645
0.00	0.19	0.81	3.79515137	0.00	0.70	0.30	3.80841720
0.00	0.20	0.80	3.79649090	0.00	0.71	0.29	3.80755509
0.00	0.21	0.79	3.79778754	0.00	0.72	0.28	3.80665017
0.00	0.22	0.78	3.79904123	0.00	0.73	0.27	3.80570246
0.00	0.23	0.77	3.80025197	0.00	0.74	0.26	3.80471200
0.00	0.24	0.76	3.80141970	0.00	0.75	0.25	3.80367884
0.00	0.25	0.75	3.80254441	0.00	0.76	0.24	3.80260303
0.00	0.26	0.74	3.80362606	0.00	0.77	0.23	3.80148459
0.00	0.27	0.73	3.80466463	0.00	0.78	0.22	3.80032359
0.00	0.28	0.72	3.80566009	0.00	0.79	0.21	3.79912006
0.00	0.29	0.71	3.80661242	0.00	0.80	0.20	3.79787406
0.00	0.30	0.70	3.80752159	0.00	0.81	0.19	3.79658563
0.00	0.31	0.69	3.80838759	0.00	0.82	0.18	3.79525482
0.00	0.32	0.68	3.80921039	0.00	0.83	0.17	3.79388169
0.00	0.33	0.67	3.80998998	0.00	0.84	0.16	3.79246628
0.00	0.34	0.66	3.81072634	0.00	0.85	0.15	3.79100866
0.00	0.35	0.65	3.81141945	0.00	0.86	0.14	3.78950888
0.00	0.36	0.64	3.81206929	0.00	0.87	0.13	3.78796700
0.00	0.37	0.63	3.81267587	0.00	0.88	0.12	3.78638307
0.00	0.38	0.62	3.81323916	0.00	0.89	0.11	3.78475716
0.00	0.39	0.61	3.81375915	0.00	0.90	0.10	3.78308933
0.00	0.40	0.60	3.81423584	0.00	0.91	0.09	3.78137964
0.00	0.41	0.59	3.81466922	0.00	0.92	0.08	3.77962815
0.00	0.42	0.58	3.81505929	0.00	0.93	0.07	3.77783493
0.00	0.43	0.57	3.81540603	0.00	0.94	0.06	3.77600005
0.00	0.44	0.56	3.81570945	0.00	0.95	0.05	3.77412358
0.00	0.45	0.55	3.81596955	0.00	0.96	0.04	3.77220558
0.00	0.46	0.54	3.81618633	0.00	0.97	0.03	3.77024612
0.00	0.47	0.53	3.81635978	0.00	0.98	0.02	3.76824528
0.00	0.48	0.52	3.81648991	0.00	0.99	0.01	3.76620313
0.00	0.49	0.51	3.81657673	0.00	1.00	0.00	3.76411974
0.00	0.50	0.50	3.81662024				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of KNM in portfolio and the investment fraction of Daiboci fixed at zero during the non-volatile period from 27th October 2005 until 28th December 2006 is shown in Figure 4.7.

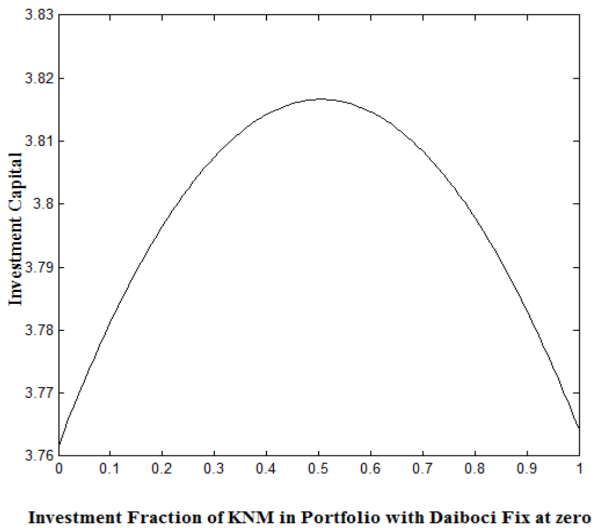


Figure 4.7: Investment capitals achieved by the CRP strategy with investment fraction of KNM in portfolio and investment fraction of Daiboci fixed at zero

Lastly, we fix the proportion in the second stock of KNM at 0.51 and search for the portfolio $\boldsymbol{b}$ = (b, 0.51, 1 – b – 0.51) that maximizes the capital. We consider b increasing in units of 0.01 from 0.00 to 0.49 and obtain the maximum capital of S_{305} = 3.93314554 units for the best constant rebalanced strategy $\boldsymbol{b}$ = (0.43, 0.51, 0.06). These results are listed in Table 4.8 and showed in Figure 4.8.

Table 4.8: Investment capitals S_{305} calculated using the constant rebalanced portfolios (CRP) for the non-volatile period with the investment fraction of Daiboci in portfolio and the investment fraction of KNM fixed at 0.51

Investment Fraction			Investment Capital, S_{305} (CRP)
Daiboci	KNM	Topglov	
0.00	0.51	0.49	3.81662045
0.01	0.51	0.48	3.82191815
0.02	0.51	0.47	3.82709465
0.03	0.51	0.46	3.83214970
0.04	0.51	0.45	3.83708306
0.05	0.51	0.44	3.84189450
0.06	0.51	0.43	3.84658379
0.07	0.51	0.42	3.85115073
0.08	0.51	0.41	3.85559511
0.09	0.51	0.40	3.85991674
0.10	0.51	0.39	3.86411545
0.11	0.51	0.38	3.86819106
0.12	0.51	0.37	3.87214341
0.13	0.51	0.36	3.87597236
0.14	0.51	0.35	3.87967777
0.15	0.51	0.34	3.88325951
0.16	0.51	0.33	3.88671746
0.17	0.51	0.32	3.89005152
0.18	0.51	0.31	3.89326159
0.19	0.51	0.30	3.89634758
0.20	0.51	0.29	3.89930941
0.21	0.51	0.28	3.90214703
0.22	0.51	0.27	3.90486038
0.23	0.51	0.26	3.90744941
0.24	0.51	0.25	3.90991408
0.25	0.51	0.24	3.91225439
0.26	0.51	0.23	3.91447030
0.27	0.51	0.22	3.91656182
0.28	0.51	0.21	3.91852895
0.29	0.51	0.20	3.92037172
0.30	0.51	0.19	3.92209014
0.31	0.51	0.18	3.92368426
0.32	0.51	0.17	3.92515412
0.33	0.51	0.16	3.92649979
0.34	0.51	0.15	3.92772133
0.35	0.51	0.14	3.92881881
0.36	0.51	0.13	3.92979234
0.37	0.51	0.12	3.93064199
0.38	0.51	0.11	3.93136789
0.39	0.51	0.10	3.93197016
0.40	0.51	0.09	3.93244891
0.41	0.51	0.08	3.93280429
0.42	0.51	0.07	3.93303645
0.43	0.51	0.06	3.93314554
0.44	0.51	0.05	3.93313173
0.45	0.51	0.04	3.93299520
0.46	0.51	0.03	3.93273614
0.47	0.51	0.02	3.93235474
0.48	0.51	0.01	3.93185121
0.49	0.51	0.00	3.93122577

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci, KNM and Topglov with the investment fraction of Daiboci in portfolio and the investment fraction of KNM fixed at 0.51 during the non-volatile period from 27th October 2005 until 28th December 2006 is shown in Figure 4.8.

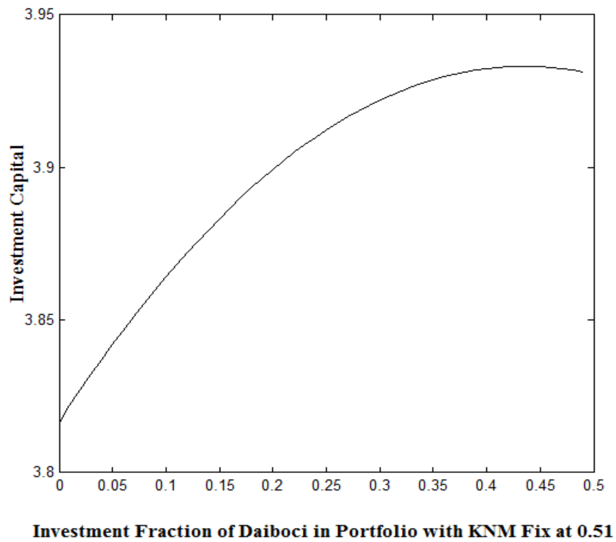


Figure 4.8: Investment capitals achieved by the CRP strategy with investment fraction of Daiboci in portfolio and investment fraction of KNM fixed at 0.51

By comparing the results obtained using the CRP from Table 4.3 to Table 4.8, based on the stock price data of 305 trading days of Daiboci, KNM and Topglov during the non-volatile period from 27th October 2005 until 28th December 2006, we find that the optimal investment capital achieved was S_{305}^{*} = 3.93314554 units with the best constant rebalanced portfolio, $\boldsymbol{b}^{*}$ = (0.43, 0.51, 0.06) (as shown highlighted in grey colour) in Table 4.8.

4.5.2 Constant Rebalanced Strategies for Volatile Period

In this section, we shall again use the 101 constant rebalanced strategies for investment in the stocks of Daiboci sp(v), KNM sp(v) and Topglov sp(v) for a total of 305 trading days for the volatile period from 11th February 2008 until 30th April 2009.

The initial investment capital is one unit and the resulting capitals after 305 trading days are calculated. In this experiment, we used the same programming code of MATLAB as for the non-volatile period to compute the resulting investment capitals for these 305 trading days. The constant rebalanced portfolio strategies for the investment in the stocks of Daiboci sp(v), KNM sp(v) and Topglov sp(v) is determined by using the cycle coordinate ascent algorithm which is known to be convergent. Table 4.9 shows a list of constant rebalanced strategies calculated in the course of applying the algorithm.

In this experiment, we begin the algorithm by fixing the third constituent stock Topglov sp(v), that is the b_3 coordinate in the portfolio vector $\boldsymbol{b}$ = (b_1, b_2, b_3) to be 0. We find that the best constant rebalanced strategy is $\boldsymbol{b}$ = (1.00, 0.00, 0.00), yielding investment capital of S_{305} (v) = 7.07644368 units. This is listed in Table 4.9 (as shown highlighted in grey colour) and is shown in Figure 4.9.

Table 4.9: The investment capitals S_{305} (v) calculated using the constant rebalanced portfolios (CRP) for the volatile period from 11th February 2008 until 30th April 2009 with the investment fraction of KNM sp(v) in portfolio and the investment fraction of Topglov sp(v) fixed at zero

Investment Fraction			Investment Capital, S_{305} (v) (CRP)	Investment Fraction			Investment Capital, S_{305} (v) (CRP)
Daiboci sp(v)	KNM sp(v)	Topglov sp(v)		Daiboci sp(v)	KNM sp(v)	Topglov sp(v)	
1.00	0.00	0.00	7.07644368	0.49	0.51	0.00	1.17356036
0.99	0.01	0.00	6.84735775	0.48	0.52	0.00	1.13008359
0.98	0.02	0.00	6.62509645	0.47	0.53	0.00	1.08810689
0.97	0.03	0.00	6.40947603	0.46	0.54	0.00	1.04758261
0.96	0.04	0.00	6.20031708	0.45	0.55	0.00	1.00846445
0.95	0.05	0.00	5.99744443	0.44	0.56	0.00	0.97070748
0.94	0.06	0.00	5.80068710	0.43	0.57	0.00	0.93426804
0.93	0.07	0.00	5.60987818	0.42	0.58	0.00	0.89910375
0.92	0.08	0.00	5.42485477	0.41	0.59	0.00	0.86517347
0.91	0.09	0.00	5.24545791	0.40	0.60	0.00	0.83243726
0.90	0.10	0.00	5.07153247	0.39	0.61	0.00	0.80085636
0.89	0.11	0.00	4.90292710	0.38	0.62	0.00	0.77039314
0.88	0.12	0.00	4.73949414	0.37	0.63	0.00	0.74101108
0.87	0.13	0.00	4.58108956	0.36	0.64	0.00	0.71267477
0.86	0.14	0.00	4.42757286	0.35	0.65	0.00	0.68534982
0.85	0.15	0.00	4.27880701	0.34	0.66	0.00	0.65900290
0.84	0.16	0.00	4.13465839	0.33	0.67	0.00	0.63360166
0.83	0.17	0.00	3.99499670	0.32	0.68	0.00	0.60911473
0.82	0.18	0.00	3.85969491	0.31	0.69	0.00	0.58551167
0.81	0.19	0.00	3.72862917	0.30	0.70	0.00	0.56276300
0.80	0.20	0.00	3.60167874	0.29	0.71	0.00	0.54084011
0.79	0.21	0.00	3.47872596	0.28	0.72	0.00	0.51971526
0.78	0.22	0.00	3.35965613	0.27	0.73	0.00	0.49936157
0.77	0.23	0.00	3.24435752	0.26	0.74	0.00	0.47975299
0.76	0.24	0.00	3.13272121	0.25	0.75	0.00	0.46086426
0.75	0.25	0.00	3.02464111	0.24	0.76	0.00	0.44267091
0.74	0.26	0.00	2.92001387	0.23	0.77	0.00	0.42514923
0.73	0.27	0.00	2.81873879	0.22	0.78	0.00	0.40827624
0.72	0.28	0.00	2.72071783	0.21	0.79	0.00	0.39202969
0.71	0.29	0.00	2.62585548	0.20	0.80	0.00	0.37638803
0.70	0.30	0.00	2.53405873	0.19	0.81	0.00	0.36133037
0.69	0.31	0.00	2.44523704	0.18	0.82	0.00	0.34683649
0.68	0.32	0.00	2.35930225	0.17	0.83	0.00	0.33288682
0.67	0.33	0.00	2.27616852	0.16	0.84	0.00	0.31946239
0.66	0.34	0.00	2.19575233	0.15	0.85	0.00	0.30654486
0.65	0.35	0.00	2.11797237	0.14	0.86	0.00	0.29411647
0.64	0.36	0.00	2.04274950	0.13	0.87	0.00	0.28216002
0.63	0.37	0.00	1.97000673	0.12	0.88	0.00	0.27065886
0.62	0.38	0.00	1.89966914	0.11	0.89	0.00	0.25959690
0.61	0.39	0.00	1.83166385	0.10	0.90	0.00	0.24895856
0.60	0.40	0.00	1.76591994	0.09	0.91	0.00	0.23872877
0.59	0.41	0.00	1.70236847	0.08	0.92	0.00	0.22889294
0.58	0.42	0.00	1.64094234	0.07	0.93	0.00	0.21943698
0.57	0.43	0.00	1.58157634	0.06	0.94	0.00	0.21034724
0.56	0.44	0.00	1.52420703	0.05	0.95	0.00	0.20161054
0.55	0.45	0.00	1.46877276	0.04	0.96	0.00	0.19321413
0.54	0.46	0.00	1.41521356	0.03	0.97	0.00	0.18514567
0.53	0.47	0.00	1.36347116	0.02	0.98	0.00	0.17739324
0.52	0.48	0.00	1.31348892	0.01	0.99	0.00	0.16994534
0.51	0.49	0.00	1.26521178	0.00	1.00	0.00	0.16279082
0.50	0.50	0.00	1.21858625				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (v) (CRP) based on the stock price relatives of Daiboci sp(v), KNM sp(v) and Topglov sp(v) with the investment fraction of KNM sp(v) in portfolio and the investment fraction of Topglov sp(v) fixed at zero during the volatile period from 11th February 2008 until 30th April 2009 is shown in Figure 4.9.

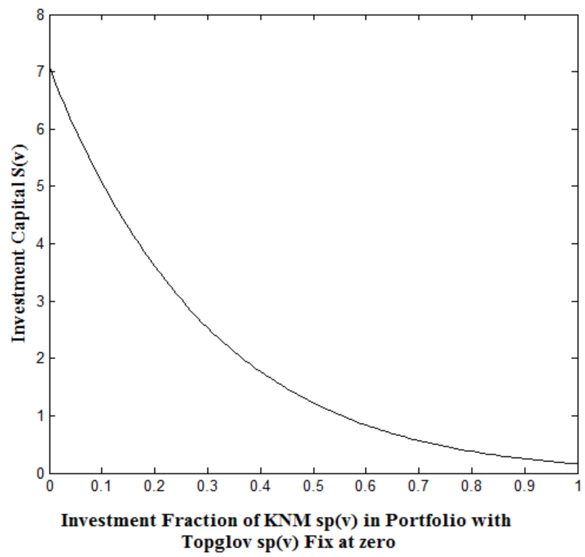


Figure 4.9: Investment capitals achieved by the CRP strategy with investment fraction of KNM sp(v) in portfolio and investment fraction of Topglov sp(v) fixed at zero

Now we fix the proportion of Daiboci sp(v) in the first coordinate at 0.00 and search for the portfolio $\boldsymbol{b}$ = $(0.00, b_2, 1-b_2)$ that maximizes the capital where b_2 increasing in units of 0.01 from 0.00 to 1.00. This is found when b = (0.00, 0.00, 1.00) and the maximum capital of S_{305} (v) = 0.81403273 units as listed in Table 4.10 (as shown highlighted in grey colour) and showed in Figure 4.10.

Table 4.10: The investment capitals S_{305} (v) calculated using the constant rebalanced portfolios (CRP) for the volatile period from 11th February 2008 until 30th April 2009 with the investment fraction of KNM sp(v) in portfolio and the investment fraction of Daiboci sp(v) fixed at zero

Investment Fraction			Investment Capital, S_{305} (v) (CRP)	Investment Fraction			Investment Capital, S_{305} (v) (CRP)
Daiboci sp(v)	KNM sp(v)	Topglov sp(v)		Daiboci sp(v)	KNM sp(v)	Topglov sp(v)	
0.00	0.00	1.00	0.81403273	0.00	0.51	0.49	0.38978100
0.00	0.01	0.99	0.80366002	0.00	0.52	0.48	0.38353451
0.00	0.02	0.98	0.79336917	0.00	0.53	0.47	0.37736262
0.00	0.03	0.97	0.78316039	0.00	0.54	0.46	0.37126491
0.00	0.04	0.96	0.77303386	0.00	0.55	0.45	0.36524097
0.00	0.05	0.95	0.76298972	0.00	0.56	0.44	0.35929038
0.00	0.06	0.94	0.75302815	0.00	0.57	0.43	0.35341271
0.00	0.07	0.93	0.74314925	0.00	0.58	0.42	0.34760752
0.00	0.08	0.92	0.73335316	0.00	0.59	0.41	0.34187437
0.00	0.09	0.91	0.72363997	0.00	0.60	0.40	0.33621280
0.00	0.10	0.90	0.71400976	0.00	0.61	0.39	0.33062238
0.00	0.11	0.89	0.70446260	0.00	0.62	0.38	0.32510264
0.00	0.12	0.88	0.69499855	0.00	0.63	0.37	0.31965311
0.00	0.13	0.87	0.68561765	0.00	0.64	0.36	0.31427333
0.00	0.14	0.86	0.67631992	0.00	0.65	0.35	0.30896282
0.00	0.15	0.85	0.66710537	0.00	0.66	0.34	0.30372110
0.00	0.16	0.84	0.65797399	0.00	0.67	0.33	0.29854770
0.00	0.17	0.83	0.64892577	0.00	0.68	0.32	0.29344211
0.00	0.18	0.82	0.63996067	0.00	0.69	0.31	0.28840386
0.00	0.19	0.81	0.63107864	0.00	0.70	0.30	0.28343244
0.00	0.20	0.80	0.62227963	0.00	0.71	0.29	0.27852735
0.00	0.21	0.79	0.61356356	0.00	0.72	0.28	0.27368809
0.00	0.22	0.78	0.60493034	0.00	0.73	0.27	0.26891416
0.00	0.23	0.77	0.59637988	0.00	0.74	0.26	0.26420505
0.00	0.24	0.76	0.58791205	0.00	0.75	0.25	0.25956023
0.00	0.25	0.75	0.57952673	0.00	0.76	0.24	0.25497920
0.00	0.26	0.74	0.57122379	0.00	0.77	0.23	0.25046143
0.00	0.27	0.73	0.56300307	0.00	0.78	0.22	0.24600641
0.00	0.28	0.72	0.55486440	0.00	0.79	0.21	0.24161360
0.00	0.29	0.71	0.54680762	0.00	0.80	0.20	0.23728249
0.00	0.30	0.70	0.53883253	0.00	0.81	0.19	0.23301255
0.00	0.31	0.69	0.53093894	0.00	0.82	0.18	0.22880323
0.00	0.32	0.68	0.52312664	0.00	0.83	0.17	0.22465402
0.00	0.33	0.67	0.51539540	0.00	0.84	0.16	0.22056437
0.00	0.34	0.66	0.50774499	0.00	0.85	0.15	0.21653376
0.00	0.35	0.65	0.50017517	0.00	0.86	0.14	0.21256164
0.00	0.36	0.64	0.49268568	0.00	0.87	0.13	0.20864747
0.00	0.37	0.63	0.48527627	0.00	0.88	0.12	0.20479072
0.00	0.38	0.62	0.47794665	0.00	0.89	0.11	0.20099084
0.00	0.39	0.61	0.47069654	0.00	0.90	0.10	0.19724730
0.00	0.40	0.60	0.46352566	0.00	0.91	0.09	0.19355955
0.00	0.41	0.59	0.45643368	0.00	0.92	0.08	0.18992705
0.00	0.42	0.58	0.44942031	0.00	0.93	0.07	0.18634926
0.00	0.43	0.57	0.44248522	0.00	0.94	0.06	0.18282562
0.00	0.44	0.56	0.43562807	0.00	0.95	0.05	0.17935561
0.00	0.45	0.55	0.42884854	0.00	0.96	0.04	0.17593867
0.00	0.46	0.54	0.42214626	0.00	0.97	0.03	0.17257427
0.00	0.47	0.53	0.41552089	0.00	0.98	0.02	0.16926186
0.00	0.48	0.52	0.40897205	0.00	0.99	0.01	0.16600089
0.00	0.49	0.51	0.40249938	0.00	1.00	0.00	0.16279082
0.00	0.50	0.50	0.39610249				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (v) (CRP) based on the stock price relatives of Daiboci sp(v), KNM sp(v) and Topglov sp(v) with the investment fraction of KNM sp(v) in portfolio and the fraction of Daiboci sp(v) fixed at zero during the volatile period from 11th February 2008 until 30th April 2009 is shown in Figure 4.10.

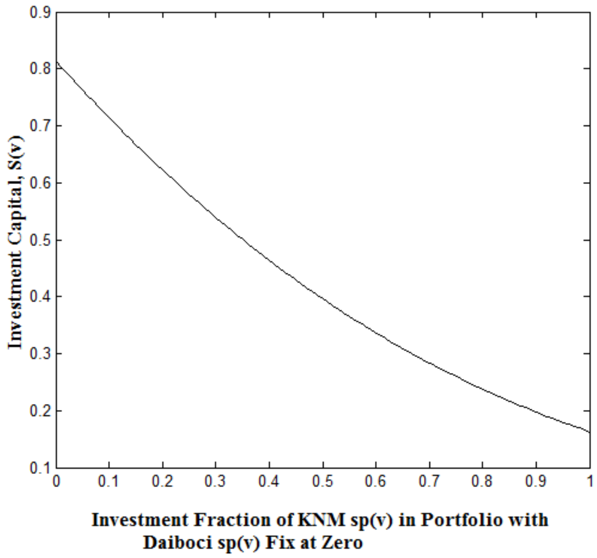


Figure 4.10: Investment capitals achieved by the CRP strategy with investment fraction of KNM sp(v) in portfolio and investment fraction of Daiboci sp(v) fixed at zero

Lastly, we fix the proportion of KNM sp(v) in the second coordinate at 0.00 and search for the portfolio $\boldsymbol{b} = (1-b_3, 0.00, b_3)$ that maximizes the capital where b_3 increasing in units of 0.01 from 0.00 to 1.00. This is found when $\boldsymbol{b}$ = (1.00, 0.00, 0.00) and S_{305} = 0.586682081 units as listed in Table 4.11 (as shown highlighted in grey colour) and showed in Figure 4.11. By comparing Table 4.9, Table 4.10 and Table 4.11, this shows that the first stock, Daiboci sp(v) with $\boldsymbol{b}$ = (1.00, 0.00, 0.00) and S_{305} (v) = 7.07644368 units outperformed during the volatile period from 11th February 2008 until 30th April 2009 indicating that the investor will have his capital multiplied by a factor of 7.07644368.

Table 4.11: The investment capitals S_{305} (v) calculated using the constant rebalanced portfolios (CRP) for the volatile period from 11th February 2008 until 30th April 2009 with the investment fraction of Topglov sp(v) in portfolio and the investment fraction of KNM sp(v) fixed at zero

Investment Fraction			Investment Capital, S_{305} (v) (CRP)	Investment Fraction			Investment Capital, S_{305} (v) (CRP)
Daiboci sp(v)	KNM sp(v)	Topglov sp(v)		Daiboci sp(v)	KNM sp(v)	Topglov sp(v)	
1.00	0.00	0.00	7.07644368	0.49	0.00	0.51	2.48976265
0.99	0.00	0.01	6.94030892	0.48	0.00	0.52	2.43647735
0.98	0.00	0.02	6.80651740	0.47	0.00	0.53	2.38422122
0.97	0.00	0.03	6.67503402	0.46	0.00	0.54	2.33297665
0.96	0.00	0.04	6.54582412	0.45	0.00	0.55	2.28272632
0.95	0.00	0.05	6.41885347	0.44	0.00	0.56	2.23345314
0.94	0.00	0.06	6.29408824	0.43	0.00	0.57	2.18514029
0.93	0.00	0.07	6.17149507	0.42	0.00	0.58	2.13777116
0.92	0.00	0.08	6.05104096	0.41	0.00	0.59	2.09132944
0.91	0.00	0.09	5.93269338	0.40	0.00	0.60	2.04579902
0.90	0.00	0.10	5.81642017	0.39	0.00	0.61	2.00116405
0.89	0.00	0.11	5.70218958	0.38	0.00	0.62	1.95740890
0.88	0.00	0.12	5.58997030	0.37	0.00	0.63	1.91451820
0.87	0.00	0.13	5.47973136	0.36	0.00	0.64	1.87247679
0.86	0.00	0.14	5.37144225	0.35	0.00	0.65	1.83126975
0.85	0.00	0.15	5.26507279	0.34	0.00	0.66	1.79088237
0.84	0.00	0.16	5.16059323	0.33	0.00	0.67	1.75130017
0.83	0.00	0.17	5.05797420	0.32	0.00	0.68	1.71250891
0.82	0.00	0.18	4.95718668	0.31	0.00	0.69	1.67449453
0.81	0.00	0.19	4.85820207	0.30	0.00	0.70	1.63724322
0.80	0.00	0.20	4.76099211	0.29	0.00	0.71	1.60074136
0.79	0.00	0.21	4.66552893	0.28	0.00	0.72	1.56497554
0.78	0.00	0.22	4.57178501	0.27	0.00	0.73	1.52993258
0.77	0.00	0.23	4.47973320	0.26	0.00	0.74	1.49559947
0.76	0.00	0.24	4.38934672	0.25	0.00	0.75	1.46196344
0.75	0.00	0.25	4.30059913	0.24	0.00	0.76	1.42901187
0.74	0.00	0.26	4.21346435	0.23	0.00	0.77	1.39673239
0.73	0.00	0.27	4.12791665	0.22	0.00	0.78	1.36511278
0.72	0.00	0.28	4.04393062	0.21	0.00	0.79	1.33414104
0.71	0.00	0.29	3.96148124	0.20	0.00	0.80	1.30380535
0.70	0.00	0.30	3.88054378	0.19	0.00	0.81	1.27409408
0.69	0.00	0.31	3.80109388	0.18	0.00	0.82	1.24499577
0.68	0.00	0.32	3.72310749	0.17	0.00	0.83	1.21649916
0.67	0.00	0.33	3.64656088	0.16	0.00	0.84	1.18859316
0.66	0.00	0.34	3.57143068	0.15	0.00	0.85	1.16126685
0.65	0.00	0.35	3.49769381	0.14	0.00	0.86	1.13450951
0.64	0.00	0.36	3.42532751	0.13	0.00	0.87	1.10831058
0.63	0.00	0.37	3.35430935	0.12	0.00	0.88	1.08265964
0.62	0.00	0.38	3.28461720	0.11	0.00	0.89	1.05754649
0.61	0.00	0.39	3.21622923	0.10	0.00	0.90	1.03296107
0.60	0.00	0.40	3.14912393	0.09	0.00	0.91	1.00889347
0.59	0.00	0.41	3.08328009	0.08	0.00	0.92	0.98533396
0.58	0.00	0.42	3.01867677	0.07	0.00	0.93	0.96227297
0.57	0.00	0.43	2.95529337	0.06	0.00	0.94	0.93970109
0.56	0.00	0.44	2.89310955	0.05	0.00	0.95	0.91760904
0.55	0.00	0.45	2.83210525	0.04	0.00	0.96	0.89598771
0.54	0.00	0.46	2.77226073	0.03	0.00	0.97	0.87482816
0.53	0.00	0.47	2.71355650	0.02	0.00	0.98	0.85412156
0.52	0.00	0.48	2.65597337	0.01	0.00	0.99	0.83385926
0.51	0.00	0.49	2.59949241	0.00	0.00	1.00	0.81403273
0.50	0.00	0.50	2.54409496				

The investment capitals achieved by the constant rebalanced portfolio strategy for a total of 305 trading days, i.e. S_{305} (CRP) based on the stock price relatives of Daiboci sp(v), KNM sp(v) and Topglov sp(v) with the investment fraction of Topglov sp(v) in portfolio and the fraction of KNM sp(v) fixed at zero during the volatile period from 11th February 2008 until 30th April 2009 is shown in Figure 4.11.

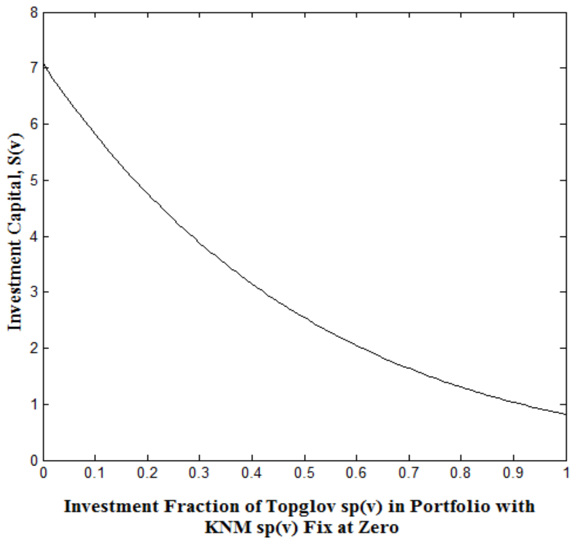


Figure 4.11: Investment capitals achieved by the CRP strategy with investment fraction of Topglov sp(v) in portfolio and investment fraction of KNM sp(v) fixed at zero

4.6 The Ratios of the Optimal Capital to the Universal Capitals

The investment capitals obtained after 305 trading days with 15 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the stock price data of Daiboci, KNM and Topglov from 27th October 2005 until 28th December 2006 were displayed in Table 4.1. The optimal investment capital achieved was S_{305}^{*} = 3.93314554 units with the best constant rebalanced portfolio, $\boldsymbol{b}^*$ = (0.43, 0.51, 0.06) (as shown highlighted in grey colour) in Table 4.8. The ratios of the optimal investment capital to the universal capitals achieved, i.e. $\frac{S^*_{305}}{\hat{S}_{305}}$ are listed in Table 4.12.

Table 4.12: The ratios of the optimal capital to the universal capitals, $\frac{S^*_{305}}{\hat{S}_{305}}$ based on the stock price relatives of Daiboci, KNM and Topglov from 27th October 2005 until 28th December 2006

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	Investment Capital after 305 trading days, $\hat{S}_{305}$	$\frac{S^*_{305}}{\hat{S}_{305}}$
(0.1, 0.3, 0.4)	3.80932385	1.03250490
(0.2, 0.9, 0.3)	3.82653302	1.02786139
(0.3, 0.4, 0.1)	3.83286977	1.02616206
(0.3, 0.7, 0.9)	3.84051942	1.02411812
(0.4, 0.4, 0.3)	3.84652601	1.02251890
(0.5, 0.5, 0.5)	3.85727059	1.01967063
(0.6, 0.6, 0.5)	3.86426389	1.01782530
(0.7, 0.7, 0.7)	3.87051335	1.01618188
(0.7, 0.8, 0.8)	3.87123589	1.01599222
(0.8, 0.6, 0.3)	3.86684992	1.01714461
(0.9, 1.0, 0.8)	3.88042998	1.01358498
(1.0, 1.0, 1.0)	3.88297775	1.01291993
(2.0, 1.0, 2.0)	3.89555250	1.00965025
(2.0, 2.0, 2.0)	3.90148494	1.00811501
(2.0, 2.0, 1.0)	3.90396078	1.00747568

In Table 4.12, it is found that for the 305 non-volatile trading-day period, the best Cover-Ordentlich universal portfolio is $\hat{\boldsymbol{b}}$ = (2.0, 2.0, 1.0), yielding a resulting optimal universal investment capital of $\hat{S}_{305}$ = 3.90396078 units. In Table 4.8, it is found that for the 305 trading-day period, the best rebalanced portfolio is $\boldsymbol{b}^*$ = (0.43, 0.51, 0.06), yielding a resulting optimal investment capital, S^*_{305} = 3.93314554 units. Thus, the ratio of the optimal investment capital, S^*_{305} = 3.93314554 units to the best Cover-Ordentlich universal investment capital, $\hat{S}_{305} = 3.90396078$ units is:

$$\frac{S^*_{305}}{\hat{S}_{305}} \approx 1.00747568.$$

It was proved in Cover and Ordentlich (1996) that:

$$\text{as } n \to \infty, \ \limsup \frac{1}{n} \ln \frac{S^*_n}{\hat{S}_n} = 0.$$

We can show this by taking a sequence of n trading days starting from $n = 210$ increasing by 5 trading days to $n = 305$ based on the Daiboci, KNM and Topglov stock data during the non-volatile period of study. The optimal investment capitals, S^*_n and corresponding universal investment capitals, $\hat{S}_n$ achieved by using the Cover and Ordentlich universal portfolios namely the Dirichlet (0.3, 0.7, 0.9), Dirichlet (0.9, 1.0, 0.8), and Dirichlet (2.0, 2.0, 1.0) distributions are used to compute the corresponding values of $\frac{1}{n} \ln \frac{S^*_n}{\hat{S}_n}$.

For example, in Table 4.13 by using the Dirichlet (2.0, 2.0, 1.0) distribution:

When $n = 210$, the corresponding value of $\frac{1}{210} \ln \frac{S^*_{210}}{\hat{S}_{210}}$ is 0.00002133.

When $n = 215$, the corresponding value of $\frac{1}{215} \ln \frac{S^*_{215}}{\hat{S}_{215}}$ is 0.00003192.

When $n = 220$, the corresponding value of $\frac{1}{220} \ln \frac{S^*_{220}}{\hat{S}_{220}}$ is 0.00006771.

and so on until the last trading day i.e. when $n = 305$.

Table 4.13: The values of S_n^*, $\hat{S}_n$ and $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ for trading days from n = 210 increasing in 5 days to n = 305 based on the Daiboci, KNM and Topglov stock data during the non-volatile period

n	S_n^*	$\hat{S}_n$ (0.3, 0.7, 0.9)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (0.9, 1.0, 0.8)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$	$\hat{S}_n$ (2.0, 2.0, 1.0)	$\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$
210	2.37887125	2.49420352	-0.00022544	2.39752616	-0.00003720	2.36824132	0.00002133
215	2.41966359	2.53253693	-0.00021206	2.43219647	-0.00002403	2.40311418	0.00003192
220	2.50201281	2.58525571	-0.00014877	2.48808174	0.00002538	2.46502080	0.00006771
225	2.54629984	2.62034004	-0.00012739	2.52196334	0.00004268	2.50054718	0.00008059
230	2.61623637	2.62498037	-0.00001451	2.56725942	0.00008216	2.56205423	0.00009099
235	2.57641681	2.58812161	-0.00001929	2.53373072	0.00007109	2.52827028	0.00008027
240	2.73927869	2.66465081	0.00011509	2.65550247	0.00012942	2.67072474	0.00010560
245	2.73859447	2.65789076	0.00012209	2.65132198	0.00013219	2.66794577	0.00010668
250	2.94334341	2.83280361	0.00015312	2.83420463	0.00015114	2.85738309	0.00011856
255	2.94822835	2.88679596	0.00008258	2.87383518	0.00010022	2.88635804	0.00008317
260	2.94740956	2.92091311	0.00003473	2.88739143	0.00007913	2.89141345	0.00007377
265	2.94876986	3.00614460	-0.00007272	2.93743878	0.00001453	2.92222310	0.00003413
270	3.06152740	3.13899477	-0.00009255	3.08280320	-0.00002565	3.06452543	-0.00000363
275	3.34160463	3.31402584	0.00003014	3.30560326	0.00003939	3.31203436	0.00003232
280	3.90365477	3.62683445	0.00026269	3.72666722	0.00016571	3.79143594	0.00010417
285	3.94102953	3.73893317	0.00018471	3.81251450	0.00011633	3.86050797	0.00007243
290	4.21466897	3.94534731	0.00022770	4.06625121	0.00012362	4.13128659	0.00006890
295	4.00977150	3.85624859	0.00013234	3.90491271	0.00008983	3.94143839	0.00005827
300	3.89993130	3.80219459	0.00008460	3.83228344	0.00005833	3.85621661	0.00003757
305	3.93314554	3.84051942	0.00007814	3.88042998	0.00004424	3.90396078	0.00002442

In Table 4.13, for trading days from n = 210 increasing in 5 days to n = 305 based on the Daiboci, KNM and Topglov stock data during the non-volatile period from 27th October 2005 until 28th December 2006, we find that all the values of $\frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n}$ are approaching 0, thus not only for two-stock data discussed in Section 3.6 during the non-volatile period and in Section 3.7 during the volatile period but also for three-stock study in this section, we can deduce that:

$$\text{as } n \to \infty, \quad \limsup \frac{1}{n}\ln\frac{S_n^*}{\hat{S}_n} = 0.$$

4.7 Helmbold-Schapire-Singer-Warmuth Universal Portfolio

In this study, the investment capitals were computed by using the Helmbold-Schapire-Singer-Warmuth, (abbreviated as HSSW or Helmbold portfolio) universal portfolio algorithm with various starting portfolios. Helmbold et al. (1998) presented an on-line investment algorithm that yielded almost the same wealth as the best constant rebalanced portfolio achieved in hindsight from the actual market results. Their algorithm required the constant storage and computing time per stock in each trading day. A MATLAB m-file is written based on the modified algorithm discussed in Section 2.2. In terms of memory requirements for the MATLAB m-file, the HSSW universal portfolio algorithm has advantage over the Cover-Ordentlich universal portfolio algorithm as the memory required is very much less.

In this section, for the three-stock portfolio of Daiboci, KNM and Topglov, we consider n = 305 trading days and the maximum possible value of r for the corresponding price-relative set is given as follows:

$$0 < r \le \frac{\min_{i,n} x_{ni}}{\max_{i,n} x_{ni}}$$

where (i) minimum price relative, $\min_{i,n} x_{ni}$ = 0.87 and

(ii) maximum price relative, $\max_{i,n} x_{ni}$ = 1.33.

$$\text{Thus, } r = \frac{0.87}{1.33} \approx 0.65413534$$

The EG(η)-update-based portfolio is generated by the learning parameter η as follows:

$$\eta = 2r\sqrt{\frac{2\ln m}{n}}$$

where (i) the number of stocks in the investment portfolio, m = 3,

(ii) the number of trading days in a fixed period of study, n = 305,

(iii) r is chosen such that $r = \frac{0.87}{1.33} \approx 0.65413534$

Thus, $\eta = 2r\sqrt{\frac{2\ln m}{n}}$

$$= 2r\sqrt{\frac{2\ln 3}{305}} \approx 0.11104140$$

We assume an initial capital or wealth of 1 unit, the wealth S_n achieved by a constant rebalanced portfolio $\boldsymbol{b}$ is given as:

$$S_n = \prod_{i=1}^{n} (\boldsymbol{b}_i^t \cdot \boldsymbol{x}_i)$$

We note that the period from 27th October 2005 until 28th December 2006 for the Daiboci, KNM and Topglov stock data is the non-volatile period.

The resulting investment capitals by using the HSSW universal portfolio are listed in Table 4.14 as S_{305} (HSSW). The resulting investment capitals by using the constant rebalanced portfolio (CRP) strategies are extracted from Table 4.5, Table 4.6 and listed in Table 4.14 as S_{305} (CRP) for comparison.

Table 4.14: The investment capitals after 305 trading days by using the HSSW universal portfolio and the CRP strategies with various initial investment portfolios based on Daiboci, KNM and Topglov stock data during the non-volatile period from 27th October 2005 until 28th December 2006

Investment Fraction			Investment Capital, S_{305} (CRP)	Investment Capital, S_{305} (HSSW)
Daiboci	KNM	Topglov		
1.00	0.00	0.00	3.74162265	3.74162265
0.95	0.00	0.05	3.76979365	3.76523840
0.90	0.00	0.10	3.79538160	3.78660574
0.85	0.00	0.15	3.81831867	3.80569415
0.80	0.00	0.20	3.83854151	3.82247497
0.75	0.00	0.25	3.85599143	3.83692144
0.70	0.00	0.30	3.87061467	3.84900875
0.65	0.00	0.35	3.88236257	3.85871406
0.60	0.00	0.40	3.89119178	3.86601657
0.55	0.00	0.45	3.89706442	3.87089755
0.50	0.00	0.50	3.89994828	3.87334037
0.48	0.00	0.52	3.90025885	3.87363150
0.43	0.05	0.52	3.90738220	3.88169609
0.38	0.10	0.52	3.91086054	3.88655072
0.33	0.15	0.52	3.91066265	3.88816979
0.28	0.20	0.52	3.90676630	3.88653124
0.23	0.25	0.52	3.89915841	3.88161665
0.18	0.30	0.52	3.88783520	3.87341123
0.13	0.35	0.52	3.87280225	3.86190390
0.08	0.40	0.52	3.85407458	3.84708728
0.03	0.45	0.52	3.83167666	3.82895782
0.00	0.48	0.52	3.81648991	3.81648991

From Table 4.14, we find that the investment capitals achieved by using the HSSW universal portfolios do not differ much from that of the CRP strategies during the non-volatile period from 27th October 2005 until 28th December 2006. This is shown in Figure 4.12 for comparison.

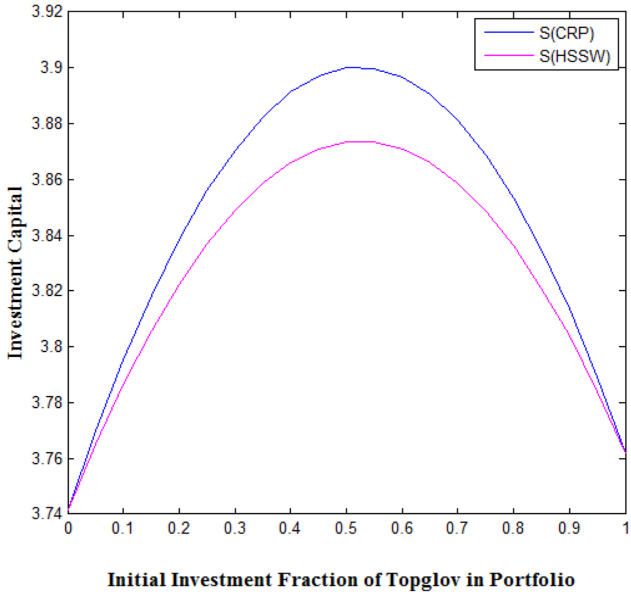


Figure 4.12: Investment capitals with initial investment fraction of Topglov in portfolio using the HSSW universal portfolio and the CRP strategy during the non-volatile period

4.8 Conclusion

- We have shown the importance and significance of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies for both volatile and non-volatile periods of study in complementing the role of the Dirichlet (1, 1, 1) and Dirichlet (0.5, 0.5, 0.5) strategies introduced by Cover and Ordentlich (1996).

- By finding the ratios of the optimal investment capitals, S_n^* and corresponding universal investment capitals, $\hat{S}_n$ during the non-volatile period, we found that for the three-stock portfolio in investment based on empirical evidence:

$$\text{as } n \to \infty, \ \limsup_{n\to\infty} \frac{1}{n} \ln \frac{S_n^*}{\hat{S}_n} = 0 .$$

- We found that the investment capitals achieved for the three-stock portfolio by using the HSSW universal portfolios do not differ much from that of the constant rebalanced portfolio strategies during the non-volatile period of study.

- The class of HSSW universal portfolios does not outperform the general parametric class of the Dirichlet-weighted universal portfolios studied by Cover and Ordentlich (1996) and Tan (2002b).

CHAPTER 5

THE DOMINATING STOCK WITH EQUAL SUBORDINATES (DOSES) MARKET AND THE ASYMPTOTIC BEHAVIOUR OF THE RATIO OF THE UNIVERSAL TO THE BEST CAPITALS

5.1 Introduction and Objective of the Study

A measure of the performance of the Dirichlet ($\boldsymbol{\alpha}$) universal portfolio is the asymptotic behaviour of the ratio of capitals $\frac{\hat{S}_n(\boldsymbol{\alpha})}{S_n^*}$ for large n, where $\hat{S}_n(\boldsymbol{\alpha})$ is the universal capital achieved by the Dirichlet ($\boldsymbol{\alpha}$) universal portfolio and S_n^* is best achievable capital by a constant rebalanced portfolio. Cover (1991) obtained the behaviour of $\frac{\hat{S}_n(\mathbf{1})}{S_n^*}$ for large n when $\boldsymbol{\alpha} = \mathbf{1} = (1, 1, \ldots, 1)$ is the uniform universal portfolio subject to certain regularity conditions. Tan (2002a) conjectured the asymptotic behaviour of $\frac{\hat{S}_n(\boldsymbol{\alpha})}{S_n^*}$ for the general $\boldsymbol{\alpha} > \mathbf{0}$ (see Equation 5.25 in this chapter) based on the m-dimensional Laplace approximation of a certain integral, but no proof of its validity is available. A mathematical model of the stock market where the determinant of the sensitivity matrix J_n^* can be calculated is needed to study this behaviour. Thus, the Kelly market with compensation introduced by Tan and Chan (2002) is used as the mathematical model. In this thesis, we use the name DOSES market for the Kelly market with compensation, with emphasis on investment rather than gambling.

In the DOSES model of the stock prices, it is possible to obtain explicit forms of the best capital S_n^* and the determinant of the sensitivity matrix J_n^*. In this chapter, we model a real 3-stock market as an approximate DOSES market. We discretize the price-relatives to become 3 price-relative vectors (a_1, c_1, c_1), (c_2, a_2, c_2) and (c_3, c_3, a_3). Suppose these vectors occur with frequencies n_1, n_2, and n_3 respectively in n trading days, where $n_1 + n_2 + n_3 = n$. If the DOSES market is ergodic, we expect the relative frequencies $\frac{n_1}{n}$, $\frac{n_2}{n}$ and $\frac{n_3}{n}$ to converge to fixed probabilities p_1, p_1 and p_3 respectively. We first check the approximation of the real market by a DOSES market. The empirical results in this chapter show that the approximated DOSES markets are reasonably ergodic where there exist probabilities p_i such that $\left|\frac{n_i}{n} - p_i\right|$ are small for large *n*. This first approximation by a DOSES market entails a problem in the sequence of maximizing vectors $\{\boldsymbol{b}_n^*\}$ for the optimal investment capitals, S_n^* where $\boldsymbol{b}_n^*$ lie on the boundary of the simplex of portfolio vectors, i.e. there is a zero component in $\boldsymbol{b}_n^*$. This does not satisfy the regularity condition for a positive, convergent sequence of $\{\boldsymbol{b}_n^*\}$ to verify the asymptotic behaviour of $\frac{\hat{S}_n(\boldsymbol{\alpha})}{S_n^*}$. To avoid this problem, we scale the price relatives a_i and c_i for i = 1, 2, 3 by some appropriate constants, which will be shown in the example in Section 5.2.1, to obtain new price relatives $\tilde{a}_i$ and $\tilde{c}_i$ for $i = 1, 2, 3$ for a second DOSES market consisting of the price relatives $(\tilde{a}_1, \tilde{c}_1, \tilde{c}_1)$, $(\tilde{c}_2, \tilde{a}_2, \tilde{c}_2)$ and $(\tilde{c}_3, \tilde{c}_3, \tilde{a}_3)$. The ergodic structure of the first DOSES market is preserved in the second DOSES market, i.e. the relative frequencies $\frac{n_i}{n}$ are the same and converge to p_i for $i = 1, 2, 3$. For example, the frequencies of (a_1, c_1, c_1) and $(\tilde{a}_1, \tilde{c}_1, \tilde{c}_1)$ are the same and the relative frequency $\frac{n_1}{n}$ converges to p_1. For the second DOSES market, there is a positive, maximizing sequence $\{\tilde{\boldsymbol{b}}_n^*\}$ of $\tilde{S}_n^*$ where $\tilde{\boldsymbol{b}}_n^*$ converges to an interior point in the simplex of portfolio vectors. Once this regularity condition for $\{\tilde{\boldsymbol{b}}_n^*\}$ is satisfied, we can proceed to investigate the asymptotic behaviour of $\frac{\hat{S}_n(\boldsymbol{\alpha})}{\tilde{S}_n^*}$ given by (5.25).

In this chapter, we have a total of 4 stock data sets, where 1 data set consist of 330 trading days and 3 data sets consist of 500 trading days. First, the price relatives of each stock data set are converted to a DOSES market by a discretization procedure. After that the price relatives of the first DOSES market are scaled by appropriate constants to become a second DOSES market where the ergodic structure of the first market is preserved. For simplicity, we call the price relatives of the second DOSES market the transformed price relatives of the original price relatives.

5.2 Dominating Stock with Equal Subordinates (DOSES)

We begin by transforming the price relatives for each trading day of stock price data from the Kuala Lumpur Stock Exchange (KLSE) to a new set of data which is named as **DOSES Market.** In any trading day, exactly one dominating stock pays off and the rest of the stocks pay a uniform amount of money known as compensation. This stock that pays off is called the "dominating" stock that is the one with the highest price relative. We elaborate the DOSES market in the following example:

Let the DOSES 4-stock market be described by the following 4 – price-relative vectors $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, $\boldsymbol{x}_3$ and $\boldsymbol{x}_4$:

$\boldsymbol{x}_1 = (a_1, c_1, c_1, c_1)$, where the first stock relative, a_1 is dominating;

$\boldsymbol{x}_2 = (c_2, a_2, c_2, c_2)$, where the second stock relative, a_2 is dominating;

$\boldsymbol{x}_3 = (c_3, c_3, a_3, c_3)$, where the third stock relative, a_3 is dominating;

$\boldsymbol{x}_4 = (c_4, c_4, c_4, a_4)$, where the last stock relative, a_4 is dominating,

where $a_i > c_i$ for $i = 1, 2, 3, 4$.

In general, the price relatives of m stocks can be described by m – price-relative vectors $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, ..., \boldsymbol{x}_m$ as follows:

$$\boldsymbol{x}_i = (c_i, \ldots, c_i, a_i, c_i, \ldots, c_i) \text{ for } i = 1, 2, \ldots, m, \text{ where } a_i > c_i.$$

In investment terminology, there is only one performance-dominating stock, let this be stock i at the end of a particular investment period with a return of a_i on that stock and a uniform return of c_i on all other stocks j for all $j \neq i$. This was called the Kelly market with compensation or KMC by Tan

(2002a). In the investment situation, it is more appropriate to call a KMC a DOSES market. The two names are synonymous. Thus, the KMC can be described by a $m \times m$ matrix of price relatives where the i^{th} row or vector, $\boldsymbol{x}_i$ is of the form: $\boldsymbol{x}_i = (c_i, \ldots, c_i, a_i, c_i, \ldots, c_i)$,

where all the components of $\boldsymbol{x}_i$ are the same as c_i except a_i, which is the ith component of the row for $i = 1, 2, \ldots, m$ and $0 \le c_i < a_i$. Typically a_i is near 1. This price relative vector is an m-dimensional vector consisting of the price relatives of m stocks. Each row of the matrix corresponds to the stock returns of some trading day which in turn corresponds to a game in the gaming situation.

For the price-relative vector, $\boldsymbol{x}_i$, the ith stock is known as the "hot" stock and the rest of the stocks j, where $j \ne i$, are non-hot stocks. If the price-relative vector $\boldsymbol{x}_i = (c_i, \ldots, c_i, a_i, c_i, \ldots, c_i)$ appears on the nth trading day, the new value of the capital of the investor on stock i (which is the hot stock) is a_i times the previous value, where else the capital placed on stocks j, where $j \ne i$, has a reduced value c_i times the previous value. This m-stock DOSES market is described by the $m \times m$ matrix of price-relative vectors as follows:

$$\begin{pmatrix} a_1 & c_1 & c_1 & \cdots & c_1 \\ c_2 & a_2 & c_2 & \cdots & c_2 \\ c_3 & c_3 & a_3 & \cdots & c_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_m & c_m & c_m & \cdots & a_m \end{pmatrix} \tag{5.1}$$

In the gaming situation, each unit of bet not on the jth outcome is uniformly compensated by a return c_i where $0 \le c_i < 1$. The value of the uniform compensation c_i satisfies $0 \le c_i < 1$ for $i = 1, 2, \ldots, m$ and $c_i = 0$ for all i correspond to the usual Kelly market. It is required that $0 \le c_i < a_i$ for all $i = 1, 2, \ldots, m$ and it is not necessary that $c_i < 1$ for any i in the gaming situation. In investment we do not require $c_i < 1$.

We will assume that the investor has an initial capital of 1 unit in order to standardize the units of calculation. For any capital exceeds 1 unit indicating that a profit has been made. We describe the other interpretation of the DOSES market that is useful to approximating the real stock market. Each day, there is a dominating stock in price and we shall convert other non-dominating stock prices to a uniform one by some discretization process like averaging. The aim is to study the asymptotic behaviour of the ratio of wealth $\frac{\hat{S}_n}{S_n^*}$ for the large number of trading days n. As a theoretical model, certain properties of the DOSES market are known, for example, the sensitivity matrix and its determinant which are derived in Tan and Chan (2002). We transform the real stock market into a DOSES market and run the Cover-Ordentlich algorithm on the DOSES market to study the behaviour of $\frac{\hat{S}_n}{S_n^*}$.

5.2.1 Discretize the Stock Prices to Approximate the Real Market as a DOSES Market

The price relatives of the real stock market for each trading day are discretized to obtain the price relatives of the DOSES market by averaging.

Each day, there is a stock with a dominating price, we shall convert other non-dominating stock prices to a uniform one by some discretization process like averaging as shown in the following example:

Example:

Assume there are n = 3 trading days and we have three price-relative vectors $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$. Approximate the three price-relative vectors $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ in a 4-stock market to become a DOSES price-relative vector where the first stock price is dominating.

Suppose $\boldsymbol{x}_1$ = (1.2, 0.9, 1.03, 1.05),

$\boldsymbol{x}_2$ = (1.3, 0.9, 1.02, 0.89) and

$\boldsymbol{x}_3$ = (1.4, 0.9, 1.04, 1.01).

To obtain the vector (a_1, c_1, c_1, c_1) we do the following.

Note that the first stock of each of the three price-relative vectors, $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ is dominating. By averaging each of the dominating first stock price of $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3$, and denoting the result as a_1 where,

$$a_1 = \frac{1.2 + 1.3 + 1.4}{3} = 1.3$$

By averaging the remaining non-dominating stock prices of $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3$, and denoting the result as c_1 where,

$$c_1 = \frac{0.9 + 1.03 + 1.05 + 0.9 + 1.02 + 0.89 + 0.9 + 1.04 + 1.01}{9}$$

$$\approx 0.97$$

The price-relative vector $\boldsymbol{x}_1$, becomes:

$$(a_1, c_1, c_1, c_1) = (1.3, 0.97, 0.97, 0.97)$$

Similarly, we can approximate the values of the price-relative vectors:

$$\boldsymbol{x}_2 \approx (c_2, a_2, c_2, c_2) \text{ and } \boldsymbol{x}_3 \approx (c_3, c_3, a_3, c_3).$$

In general, in a DOSES market the following price-relatives: $(a_1, c_1, \ldots, c_1)$ appears n_1 times, $(c_2, a_2, c_2, \ldots, c_2)$ appears n_2 times, …, $(c_m, \ldots, c_m, a_m)$ appears n_m times in n trading days, where $n_1 + n_2 + \ldots + n_m = n$, $a_i > 0$ and $c_i > 0$ for i = 1, 2, 3, …, m.

Assume the initial investment capital is one unit and the resulting capital after each trading day is in the corresponding number of units. This model of DOSES market with the above price-relatives vectors is used for our study in this chapter.

5.2.2 Method for Calculating the Optimal Capital of DOSES Market

From Section 2.1, for a given sequence of stock-price-relative vectors in n trading days, $\boldsymbol{x}^n = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$, the investment capital at the end of the nth trading day achieved by a constant rebalanced portfolio $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)$ is given as follows:

$$S_n(\boldsymbol{x}^n, \boldsymbol{b}) = \prod_{i=1}^{n} \boldsymbol{b}^t \boldsymbol{x}_i \tag{5.2}$$

The doubling rate of capital achieved by a constant rebalanced portfolio is given by:

$$W_n(\boldsymbol{x}^n, \boldsymbol{b}) = \frac{1}{n} \ln S_n(\boldsymbol{x}^n, \boldsymbol{b}) \tag{5.3}$$

The optimal doubling rate of capital achieved by the best constant rebalanced portfolio is given by:

$$W_n^*(\boldsymbol{x}^n, \boldsymbol{b}_n^*) = \max_b \frac{1}{n} \ln S_n(\boldsymbol{x}^n, \boldsymbol{b})$$

$$= \frac{1}{n} \ln S_n^*(\boldsymbol{x}^n, \boldsymbol{b}_n^*) \qquad (5.4)$$

where $S_n^*(\boldsymbol{x}^n, \boldsymbol{b}_n^*) = \max_b S_n(\boldsymbol{x}^n, \boldsymbol{b})$ and $\boldsymbol{b}_n^*$ maximizes $S_n(\boldsymbol{x}^n, \boldsymbol{b})$ over all portfolio vectors $\boldsymbol{b}$, i.e. $\boldsymbol{b}_n^*$ maximizes $W_n(\boldsymbol{x}^n, \boldsymbol{b})$ over all portfolio vectors $\boldsymbol{b}$.

Now we shall consider the DOSES market with the price-relative listed as follows:

$(a_1, c_1, \ldots, c_1)$ appears n_1 times, $(c_2, a_2, c_2, \ldots, c_2)$ appears n_2 times, …, $(c_m, \ldots, c_m, a_m)$ appears n_m times, in n trading days, where $n_1 + n_2 + \ldots + n_m = n$, $a_i > 0$ and $c_i > 0$ for $i = 1, 2, 3, \ldots, m$.

Assume the initial investment capital is one unit and the resulting capital after each trading day is in the corresponding number of units. The investment capital achieved at the end of the *n*th trading day by a constant rebalanced portfolio $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)$ is given from (2.2) as:

$$S_n(\boldsymbol{x}^n, \boldsymbol{b}) = \prod_{i=1}^{n} \boldsymbol{b}^t \boldsymbol{x}_i$$

$$= [a_1 b_1 + (1-b_1)c_1]^{n_1} [a_2 b_2 + (1-b_2)c_2]^{n_2} \ldots [a_m b_m + (1-b_m)c_m]^{n_m}$$

$$= [(a_1 - c_1)b_1 + c_1]^{n_1} [(a_2 - c_2)b_2 + c_2]^{n_2} \ldots [(a_m - c_m)b_m + c_m]^{n_m}$$

$$= \prod_{i=1}^{m} [(a_i - c_i)b_i + c_i]^{n_i} \qquad (5.5)$$

Consider a three-stock DOSES market where m = 3, the investment capital achieved at the end of the nth trading day by a constant rebalanced portfolio $\boldsymbol{b} = (b_1, b_2, 1 - b_1 - b_2)$ where $b_1 + b_2 + b_3 = 1$, is given from (5.5) as:

$$S_n(\boldsymbol{x}^n, \boldsymbol{b}) = [(a_1 - c_1)b_1 + c_1]^{n_1} \, [(a_2 - c_2)b_2 + c_2]^{n_2} \, [(a_3 - c_3)b_3 + c_3]^{n_3} \qquad (5.6)$$

It is noted that S_n is maximum over the set of portfolio vectors $\boldsymbol{b}$ if and only if $W_n(\boldsymbol{x}^n, \boldsymbol{b}) = \frac{1}{n} \ln S_n(\boldsymbol{x}^n, \boldsymbol{b})$ is maximum.

Thus by substituting (5.6),

$$W_n(\boldsymbol{x}^n, \boldsymbol{b}) = \frac{1}{n} \ln S_n(\boldsymbol{x}^n, \boldsymbol{b})$$

$$= \frac{1}{n} \ln\left\{[(a_1 - c_1)b_1 + c_1]^{n_1} \, [(a_1 - c_1)b_1 + c_1]^{n_2} \, [(a_1 - c_1)b_1 + c_1]^{n_3}\right\}$$

$$= \frac{1}{n}\{n_1 \ln[(a_1 - c_1)b_1 + c_1] + n_2 \ln[(a_2 - c_2)b_2 + c_2] + n_3 \ln[(a_3 - c_3)b_3 + c_3]\}$$

$$= \frac{n_1}{n} \ln[(a_1 - c_1)b_1 + c_1] + \frac{n_2}{n} \ln[(a_2 - c_2)b_2 + c_2] + \frac{n_3}{n} \ln[(a_3 - c_3)b_3 + c_3]$$

By partial differentiation:

$$\frac{\partial W_n}{\partial b_1} = \frac{1}{n}\left[\frac{n_1(a_1 - c_1)}{(a_1 - c_1)b_1 + c_1} + \frac{n_3(c_3 - a_3)}{b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3}\right] \qquad (5.7)$$

$$\frac{\partial W_n}{\partial b_2} = \frac{1}{n}\left[\frac{n_2(a_2 - c_2)}{(a_2 - c_2)b_2 + c_2} + \frac{n_3(c_3 - a_3)}{b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3}\right] \qquad (5.8)$$

If $\boldsymbol{b}_n^*$ maximizes W_n, then $\frac{\partial W_n}{\partial b_1} = 0$ and $\frac{\partial W_n}{\partial b_2} = 0$.

From (5.7):

$$\frac{\partial W_n}{\partial b_1} = 0 \quad \Rightarrow \frac{n_1(a_1 - c_1)}{(a_1 - c_1)b_1 + c_1} + \frac{n_3(c_3 - a_3)}{b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3} = 0 \qquad (5.9)$$

$$n_3(c_3 - a_3)\left[(a_1 - c_1)b_1 + c_1\right] + n_1(a_1 - c_1)\left[b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3\right] = 0$$

$$b_1(n_1 + n_3)(a_1 - c_1)(c_3 - a_3) + b_2 n_1(a_1 - c_1)(c_3 - a_3) + c_1 n_3(c_3 - a_3) + a_3 n_1(a_1 - c_1) = 0 \qquad (5.10)$$

From (5.8):

$$\frac{\partial W_n}{\partial b_2} = 0 \quad \Rightarrow \frac{n_2(a_2 - c_2)}{(a_2 - c_2)b_2 + c_2} + \frac{n_3(c_3 - a_3)}{b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3} = 0 \qquad (5.11)$$

$$n_3(c_3 - a_3)\left[(a_2 - c_2)b_2 + c_2\right] + n_2(a_2 - c_2)\left[b_1(c_3 - a_3) + b_2(c_3 - a_3) + a_3\right] = 0$$

$$b_2(n_2 + n_3)(a_2 - c_2)(c_3 - a_3) + b_1 n_2(a_2 - c_2)(c_3 - a_3) + c_2 n_3(c_3 - a_3) + a_3 n_2(a_2 - c_2) = 0 \qquad (5.12)$$

From (5.9) and (5.11):

$$\frac{n_1(a_1 - c_1)}{(a_1 - c_1)b_1 + c_1} = \frac{n_2(a_2 - c_2)}{(a_2 - c_2)b_2 + c_2}$$

$$\Rightarrow n_1(a_1 - c_1)\left[(a_2 - c_2)b_2 + c_2\right] = n_2(a_2 - c_2)\left[(a_1 - c_1)b_1 + c_1\right]$$

$$\Rightarrow b_2 n_1(a_1 - c_1)(a_2 - c_2) = n_2(a_2 - c_2)\left[(a_1 - c_1)b_1 + c_1\right] - c_2 n_1(a_1 - c_1)$$

$$\Rightarrow b_2 n_1(a_1 - c_1) = b_1 n_2(a_1 - c_1) + c_1 n_2 - \frac{c_2 n_1(a_1 - c_1)}{a_2 - c_2} \qquad (5.13)$$

Substituting (5.13) into (5.10):

$$b_1(n_1 + n_3)(a_1 - c_1)(c_3 - a_3) + \left[b_1 n_2(a_1 - c_1) + c_1 n_2 - \frac{c_2 n_1(a_1 - c_1)}{a_2 - c_2}\right](c_3 - a_3) + c_1 n_3(c_3 - a_3) + a_3 n_1(a_1 - c_1) = 0$$

$$\Rightarrow b_1(n_1 + n_2 + n_3)(a_1 - c_1)(c_3 - a_3) + c_1(n_2 + n_3)(c_3 - a_3) + a_3 n_1(a_1 - c_1) - \frac{c_2 n_1(a_1 - c_1)(c_3 - a_3)}{(a_2 - c_2)} = 0$$

$$\Rightarrow b_1(n_1 + n_2 + n_3) = \frac{a_3 n_1}{a_3 - c_3} + \frac{c_2 n_1}{a_2 - c_2} - \frac{c_1(n_2 + n_3)}{a_1 - c_1}$$

$$\Rightarrow b_1^* n = \left(\frac{a_3}{a_3 - c_3} + \frac{c_2}{a_2 - c_2}\right)n_1 - \frac{c_1}{a_1 - c_1}(n_2 + n_3) \qquad \because n = n_1 + n_2 + n_3$$

$$\Rightarrow b_1^* = \frac{1}{n}\left[\left(\frac{a_3}{a_3 - c_3} + \frac{c_2}{a_2 - c_2}\right)n_1 - \frac{c_1}{a_1 - c_1}(n_2 + n_3)\right] \tag{5.14}$$

From (5.13):

$$\Rightarrow b_1 n_2 (a_1 - c_1) = b_2 n_1 (a_1 - c_1) + \frac{c_2 n_1 (a_1 - c_1)}{a_2 - c_2} - c_1 n_2$$

$$b_1 n_2 (a_1 - c_1)(a_2 - c_2) = n_1 (a_1 - c_1)[b_2 (a_2 - c_2) + c_2] - c_1 n_2 (a_2 - c_2)$$

$$\Rightarrow b_1 n_2 (a_2 - c_2) = b_2 n_1 (a_2 - c_2) + c_2 n_1 - \frac{c_1 n_2 (a_2 - c_2)}{a_1 - c_1} \tag{5.15}$$

Substituting (5.15) into (5.12):

$$b_2 (n_2 + n_3)(a_2 - c_2)(c_3 - a_3) + \left[b_2 n_1 (a_2 - c_2) + c_2 n_1 - \frac{c_1 n_2 (a_2 - c_2)}{a_1 - c_1}\right](c_3 - a_3)$$
$$+ c_2 n_3 (c_3 - a_3) + a_3 n_2 (a_2 - c_2) = 0$$

$$\Rightarrow b_2 (n_1 + n_2 + n_3)(a_2 - c_2)(c_3 - a_3) + c_2 (n_1 + n_3)(c_3 - a_3) + a_3 n_2 (a_2 - c_2)$$
$$- \frac{c_1 n_2 (a_2 - c_2)(c_3 - a_3)}{(a_1 - c_1)} = 0$$

$$\Rightarrow b_2 (n_1 + n_2 + n_3) = \frac{c_1 n_2}{a_1 - c_1} - \frac{c_2 (n_1 + n_3)}{a_2 - c_2} + \frac{a_3 n_2}{a_3 - c_3}$$

$$\Rightarrow b_2^* n = \left(\frac{a_3}{a_3 - c_3} + \frac{c_1}{a_1 - c_1}\right)n_2 - \frac{c_2}{a_2 - c_2}(n_1 + n_3) \quad \because n = n_1 + n_2 + n_3$$

$$\Rightarrow b_2^* = \frac{1}{n}\left[\left(\frac{a_3}{a_3 - c_3} + \frac{c_1}{a_1 - c_1}\right)n_2 - \frac{c_2}{a_2 - c_2}(n_1 + n_3)\right] \tag{5.16}$$

Let $\gamma_i = \frac{c_i}{a_i - c_i}$ where $i = 1, 2, 3$ then from (5.14)

$$b_1^* = \frac{1}{n}\left[(1+\gamma_3+\gamma_2)n_1 - \gamma_1(n_2+n_3)\right]$$

$$b_1^* = \frac{1}{n}\left[(1+\gamma_1+\gamma_2+\gamma_2)n_1 - \gamma_1(n_1+n_2+n_3)\right]$$

$$\Rightarrow b_1^* = \frac{n_1}{n}\left(1+\sum_{j=1}^{m}\gamma_j\right) - \gamma_1$$

Similarly, from (5.16)

$$b_2^* = \frac{1}{n}\left[(1+\gamma_3+\gamma_1)n_2 - \gamma_2(n_1+n_3)\right]$$

$$b_2^* = \frac{1}{n}\left[(1+\gamma_1+\gamma_2+\gamma_3)n_2 - \gamma_2(n_1+n_2+n_3)\right]$$

$$\Rightarrow b_2^* = \frac{n_2}{n}\left(1+\sum_{j=1}^{m}\gamma_j\right) - \gamma_2$$

In general, b_i^* can be written as:

$$b_i^* = \frac{n_i}{n}\left(1+\sum_{j=1}^{m}\gamma_j\right) - \gamma_i > 0 \tag{5.17}$$

This formula is due to Tan and Chan (2008).

This time-varying log-optimal portfolio which depends on the time *n* can be expressed as:

$$\boldsymbol{b}_{\boldsymbol{n}}^* = (b_1^*, b_2^*, 1-b_1^*-b_2^*) \tag{5.18}$$

where implicitly we assume that b_1^* and b_2^* depend on *n*.

From (5.4) for three-stock market, the optimal capital, S_n^* can be written as:

$$S_n^* = \left[(a_1-c_1)b_1^*+c_1\right]^{n_1}\left[(a_2-c_2)b_2^*+c_2\right]^{n_2}\left[(a_3-c_3)b_3^*+c_3\right]^{n_3}$$

$$= \prod_{i=1}^{3}\left[(a_i-c_i)b_i^*c_i\right]^{n_i} \tag{5.19}$$

5.2.3 The Determinant of the Sensitivity Matrix for the DOSES Market

The determinant of a certain sensitivity matrix is used for measuring the volatility of the stock-price-relative sequences $\boldsymbol{x^n}$. The functional form of the determinant of the sensitivity matrix for the Kelly market with compensation was studied by Tan and Chan (2002). The results they obtained is stated as follows:

Consider a Kelly market with compensation where the price-relatives $(a_1, c_1, ..., c_1)$ appears n_1 times, $(c_2, a_2, c_2, ..., c_2)$ appears n_2 times, …, $(c_m, ..., c_m, a_m)$ appears n_m times in a period of n trading days, where $n_1 + n_2 + ... + n_m = n$, $a_i > 0$ and $c_i > 0$ for i = 1, 2, 3, …, m.

For large n, the limit of the determinant of the sensitivity matrix for this stationary and ergodic market is given by:

$$\left|J^*\right| = \lim_{n\to\infty}\left|J_n^*\right| = \frac{1}{\left[1+\sum_{j=1}^{m}\gamma_j\right]^{2(m-1)}\left[\prod_{j=1}^{m}p_j\right]} \tag{5.20}$$

where $p_j = \dfrac{n_j}{n}$ is the probability of the price-relative vector

$(c_j, \ldots, c_j, a_j, c_j, \ldots, c_j)$ and $\gamma_j = \dfrac{c_j}{a_j - c_j} > 0$ for j = 1, 2, …, m.

For our computation in this chapter, we study the case of a three-stock market where m = 3, where the real stock market is transformed to a DOSES market with the following price-relatives:

(a_1, c_1, c_1), (c_2, a_2, c_2) and (c_3, c_3, a_3) which appear n_1, n_2, and n_3 times respectively in n trading days where $n = n_1 + n_2 + n_3$, the constants $a_i > 0$ and $c_i > 0$ for $i = 1, 2, 3$.

The determinant of the sensitivity matrix,

$$\left|J_n^*\right| = \frac{1}{\left[1+\sum_{j=1}^{3}\gamma_j\right]^{2(3-1)}\left[\prod_{j=1}^{3}p_j\right]}$$

$$= \frac{1}{p_1 p_2 p_3 (1+\gamma_1+\gamma_2+\gamma_3)^4} \qquad (5.21)$$

for large n, in a stationary and ergodic market, where p_1, p_2 and p_3 are the probabilities of (a_1, c_1, c_1), (c_2, a_2, c_2) and (c_3, c_3, a_3) respectively, $\gamma_j = \frac{c_j}{a_j - c_j}$ for $j = 1, 2, 3$. As n_j and $n \to \infty$, $\frac{n_j}{n}$ approaches p_j and we use $\frac{n_j}{n}$ to estimate p_j for $j = 1, 2, 3$.

5.3 The Asymptotic Behaviour of the Ratio of the Universal Capital to the Optimal Capital

Cover (1991) showed that for the uniform Dirichlet (1, 1, …, 1) universal portfolio, the asymptotic behavior of the ratio of the investment capital achieved by the universal portfolio to the investment capital achieved by the best constant rebalanced portfolio depends on the determinant of some sensitivity matrix. Tan and Chan (2002) studied the functional form of the determinant of the sensitivity matrix for the Kelly market with compensation.

The sensitivity matrix with respect to the stock-price-relative sequence $\boldsymbol{x}^n$ is defined as:

$$J_n(\boldsymbol{b}) = -\left[\frac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b})}{\partial b_i \partial b_j}\right] \tag{5.22}$$

where the matrix $J_n(\boldsymbol{b})$ is $(m-1)\times(m-1)$, $W_n(\boldsymbol{x}^n, \boldsymbol{b})$ is given from (5.2) as:

$$W_n(\boldsymbol{x}^n, \boldsymbol{b}) = \frac{1}{n}\ln S_n(\boldsymbol{x}^n, \boldsymbol{b})$$

and the partial derivatives $\frac{\partial^2 W_n}{\partial b_i \partial b_j}$ are taken with respect to the $(m-1)$ independent variables $b_1, b_2, ..., b_m$.

The $(m-1)\times(m-1)$ sensitivity matrix, J_n^* which is the negative of the Hessian of $W_n(\boldsymbol{b}_n^*)$ is defined as:

$$J_n^* = -\left[\frac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b}_n^*)}{\partial b_i \partial b_j}\right] \tag{5.23}$$

where $\boldsymbol{b}_n^*$ maximizes $W_n(\boldsymbol{x}^n, \boldsymbol{b})$ over all portfolio vectors $\boldsymbol{b}$.

Note:

1. The sensitivity matrix, J_n^* is nonnegative definite.
2. If $\boldsymbol{b}_n^*$ achieves the maximum of $S_n(\boldsymbol{b})$ in the interior of the simplex B_m, then all the components of time-varying log-optimal portfolio, $\boldsymbol{b}_n^*$ are positive.
3. Note that the maximum of $S_n(\boldsymbol{b})$ may be achieved at $\boldsymbol{b}_n^*$ on the boundary of the simplex B_m.

Cover (1991) showed that for the universal portfolio generated by the uniform Dirichlet (1, 1, …, 1) distribution, the ratio of the universal capital, $\hat{S}_n$ to the optimal capital, S_n^* has the following asymptotic behavior:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{(m-1)!}{\left|J^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}} \tag{5.24}$$

where $\left|J_n^*\right|$ denotes the determinant of the sensitivity matrix, J_n^*.

We try to study the asymptotic behaviour of $\frac{\hat{S}_n}{S_n^*}$ for the general Dirichlet $(\alpha_1, \alpha_2, \ldots\ldots, \alpha_m)$ universal portfolio in a DOSES market, i.e.

$$\frac{\hat{S}_n(\hat{\boldsymbol{b}}_n)}{S_n^*(\boldsymbol{b}_n^*)} \sim \frac{g(\boldsymbol{b}_n^*)}{\left|J_n^*(\boldsymbol{b}_n^*)\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}} \tag{5.25}$$

Let $\boldsymbol{x}^{\boldsymbol{n}} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n)$ be a sequence of stock-price-relative vectors in n trading days. Suppose the stock-price-relative vector of the form $(c_i, \ldots, c_i, a_i, c_i, \ldots, c_i)$ appears n_i times where $n = \sum_{j=1}^{m} n_j$ trading days for i = 1, 2, …, m, in a DOSES market. The Dirichlet universal portfolio is generated by a parametric positive vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots\ldots, \alpha_m)$ where $\alpha_i > 0$ for i = 1, 2, …, m, where the Dirichlet probability measure $\mu(\boldsymbol{b})$ defined on the simplex B_m is given as follows:

$$d\mu(\boldsymbol{b}) = \frac{\Gamma(\alpha_1 + \ldots + \alpha_m)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_m)} b_1^{\alpha_1 - 1} \ldots b_m^{\alpha_m - 1} db_1 \ldots db_{m-1} \tag{5.26}$$

Cover and Ordentlich (1996) have shown that the universal wealth $\hat{S}_n(\hat{\boldsymbol{b}}_n)$, achieved by the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolio at the end of the nth trading day can be computed by using the following formula:

$$\hat{S}_n(\hat{\boldsymbol{b}}_n) = \int_{B_m} \left(\prod_{j=1}^{n} \boldsymbol{b}^t \boldsymbol{x}_j \right) d\mu(\boldsymbol{b}) \tag{5.27}$$

The universal portfolio corresponding to the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ distribution used is:

$$\hat{\boldsymbol{b}}_n = \frac{\int_{B_m} \boldsymbol{b} \left(\prod_{i=1}^{n-1} \boldsymbol{b}^t \boldsymbol{x}_i \right) g(\boldsymbol{b}) d(\boldsymbol{b})}{\int_{B_m} \left(\prod_{i=1}^{n-1} \boldsymbol{b}^t \boldsymbol{x}_i \right) g(\boldsymbol{b}) d(\boldsymbol{b})} \tag{5.28}$$

The simplex of portfolio vectors $\boldsymbol{b}$ is denoted by B_m, where

$$B_m = \left\{ (b_1, b_2, ..., b_m) : 0 \le b_i \le 1 \text{ for } i = 1,2,...,m \text{ and } \sum_{i=1}^{m} b_i = 1 \right\}$$

If $\boldsymbol{x}_j = (c_i, ..., c_i, a_i, c_i, ..., c_i)$ for some i and this stock-price-relative vector appears n_i times where $n = \sum_{j=1}^{m} n_j$ trading days, then

$$\begin{aligned} \boldsymbol{b}^t \boldsymbol{x}_j &= b_i a_i + c_i (1 - b_i) \\ &= c_i + b_i (a_i - c_i) \end{aligned} \tag{5.29}$$

From (5.27) and (5.29),

$$\hat{S}_n(\hat{\boldsymbol{b}}_n) = \int_{B_m} \left(\prod_{i=1}^{m} [c_i + b_i (a_i - c_i)]^{n_i} \right) d\mu(\boldsymbol{b})$$

$$= \int_{B_m} \prod_{i=1}^{m} c_i^{n_i} \left(1 + \frac{b_i}{\gamma_i} \right)^{n_i} d\mu(\boldsymbol{b}) \qquad \text{where } \gamma_i = \frac{c_i}{a_i - c_i}$$

$$= \frac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} \int_{B_m} \left(\prod_{i=1}^{m} c_i^{n_i} \left[\frac{b_i}{\gamma_i} + 1 \right]^{n_i} \right) b_1^{\alpha_1 - 1} ... b_m^{\alpha_m - 1} db_1 ... db_{m-1} \qquad \text{from (5.26)}$$

$$= \frac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} \left(\prod_{i=1}^{m} c_i^{n_i} \right) \int_{B_m} \prod_{i=1}^{m} \left[\sum_{j_i=0}^{n_i} \binom{n_i}{j_i} \left(\frac{b_i}{\gamma_i} \right)^{j_i} \right] b_1^{\alpha_1 - 1} ... b_m^{\alpha_m - 1} db_1 ... db_{m-1}$$

$$= \frac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} \left(\prod_{i=1}^{m} c_i^{n_i} \right) \int_{B_m} \sum_{j_1=0}^{n_1} ... \sum_{j_m=0}^{n_m} \binom{n_1}{j_1} ... \binom{n_m}{j_m} \frac{b_1^{\alpha_1 + j_1 - 1} ... b_m^{\alpha_m + j_m - 1}}{\gamma_1^{j_1} ... \gamma_m^{j_m}} db_1 ... db_{m-1}$$

$$= \frac{\Gamma(\alpha_1 + ... + \alpha_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)} \left(\prod_{i=1}^{m} c_i^{n_i} \right) \sum_{j_1=0}^{n_1} ... \sum_{j_m=0}^{n_m} \binom{n_1}{j_1} ... \binom{n_m}{j_m} \gamma_1^{-j_1} ... \gamma_m^{-j_m} \int_{B_m} b_1^{\alpha_1 + j_1 - 1} ... b_m^{\alpha_m + j_m - 1} db_1 ... db_{m-1} \tag{5.30}$$

By using the property of the Dirichlet distribution

$$\int_{B_m} b_1^{\alpha_1 + j_1 - 1} .. b_m^{\alpha_m + j_m - 1} db_1 ... db_{m-1} = \frac{\Gamma(\alpha_1 + j_1)...\Gamma(\alpha_m + j_m)}{\Gamma(\alpha_1 + ... + \alpha_m + j_1 + ... + j_m)} \tag{5.31}$$

and letting

$$T_\alpha(\mathbf{j}) = \frac{\Gamma(\alpha_1 + ... + \alpha_m)\Gamma(\alpha_1 + j_1)...\Gamma(\alpha_m + j_m)}{\Gamma(\alpha_1)...\Gamma(\alpha_m)\Gamma(\alpha_1 + ... + \alpha_m + j_1 + ... + j_m)}, \tag{5.32}$$

the expression for $\hat{S}_n(\hat{\boldsymbol{b}}_n)$ becomes:

$$\hat{S}_n(\hat{\boldsymbol{b}}_n) = \left(\prod_{i=1}^{m} c_i^{n_i} \right) \sum_{j_1=0}^{n_1} ... \sum_{j_m=0}^{n_m} \binom{n_1}{j_1} ... \binom{n_m}{j_m} \gamma_1^{-j_1} ... \gamma_m^{-j_m} T_\alpha(\mathbf{j})$$

$$= T_\alpha(\mathbf{j}) \left(\prod_{i=1}^{m} c_i^{n_i} \right) \sum_{j_1=0}^{n_1} ... \sum_{j_m=0}^{n_m} \frac{n_1! ... n_m!}{j_1! ... j_m! (n_1 - j_1)! ... (n_m - j_m)!} \gamma_1^{-j_1} ... \gamma_m^{-j_m} \tag{5.33}$$

If the time-varying log-optimal portfolio, $\boldsymbol{b}_n^* = \left(b_1^*, b_2^*, ..., b_m^*\right)$ is achieved in the interior of the simplex B_m (refer to Tan and Chan (2008)), then

$$b_i^* = \frac{n_i}{n} \left(1 + \sum_{j=1}^{m} \gamma_j \right) - \gamma_i > 0 \qquad \text{for } i = 1, 2, \ldots, m \qquad \text{and}$$

$$\lim_{\substack{n_i \to \infty \\ n \to \infty}} b_i^* = p_i \left(1 + \sum_{j=1}^{m} \gamma_j \right) - \gamma_i \tag{5.34}$$

where $\gamma_i = \dfrac{c_i}{a_i - c_i}$ and p_i is the probability of the price-relative vector

$(c_i,...,c_i,a_i,c_i,...,c_i)$ for $i = 1, 2, \ldots, m$, where the DOSES market is stationary and ergodic. Hence (Tan and Chan (2008)),

(i) $\lim_{n\to\infty} b_i^* = p_i\left(1+\sum_{j=1}^{m}\gamma_j\right)-\gamma_i$ almost surely for $i = 1, 2, \ldots, m$, where n is the number of trading days,

(ii) for large $n_1,...,n_m,n$, we have

$$S_n^*(\boldsymbol{b}_n^*) = \prod_{i=1}^{m}\left[(a_i-c_i)b_i^*+c_i\right]^{n_i}$$

$$\sim \prod_{i=1}^{m}\left\{\left[p_i\left(1+\sum_{j=1}^{m}\gamma_j\right)-\gamma_i\right](a_i-c_i)+c_i\right\}^{n_i} \quad \text{where } \gamma_i = \frac{c_i}{a_i-c_i}$$

$$=\prod_{i=1}^{m}\left[p_i\left(1+\sum_{j=1}^{m}\gamma_j\right)(a_i-c_i)\right]^{n_i}$$

$$=\left(1+\sum_{j=1}^{m}\gamma_j\right)^{n}\prod_{i=1}^{m}\left(\frac{c_i p_i}{\gamma_i}\right)^{n_i}$$

$$\approx \frac{\left(1+\sum_{j=1}^{m}\gamma_j\right)^{n}}{n^n}\prod_{i=1}^{m}\left(\frac{n_i c_i}{\gamma_i}\right)^{n_i} \quad \text{where } \frac{n_i}{n}\approx p_i \tag{5.35}$$

Subject to certain regularity conditions on the sequence $\{\boldsymbol{b}_n^*\}$ and $\{|J_n^*|\}$, the ratio of the universal capital $\hat{S}_n(\hat{\boldsymbol{b}}_n)$ to the optimal capital $S_n^*(\boldsymbol{b}_n^*)$ is conjectured to have the following asymptotic behavior:

$$\frac{\hat{S}_n(\hat{\boldsymbol{b}}_n)}{S_n^*(\boldsymbol{b}_n^*)} \sim \frac{g(\boldsymbol{b}_n^*)}{\left|J_n^*(\boldsymbol{b}_n^*)\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}$$

The above is the Laplace approximation conjecture which is due to Tan (2002a), where:

$\hat{S}_n(\hat{\boldsymbol{b}}_n)$ is the capital of the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ universal portfolio, $\hat{\boldsymbol{b}}_n$;

$S_n^*(\boldsymbol{b}_n^*)$ is the capital of the best constant rebalanced portfolio $\boldsymbol{b}_n^*$;

$\left|J_n^*(\boldsymbol{b}_n^*)\right|$ denotes the determinant of the $(m-1)\times(m-1)$ sensitivity matrix, which is the negative of the Hessian of $W_n(\boldsymbol{b}_n^*)$.

The regularity conditions are that the sequence $\{\boldsymbol{b}_n^*\}$ must be positive and has a limit in the interior of the simplex B_m, $W_n(\boldsymbol{b}_n^*)$ and $\left|J_n^*(\boldsymbol{b}_n^*)\right|$ both have limits as $n \to \infty$. For a DOSES market,

$$\begin{aligned}\left|J^*(\boldsymbol{b}_n^*)\right| &= \lim_{n\to\infty}\left|J_n^*(\boldsymbol{b}_n^*)\right| \\ &= \frac{1}{\left(1+\sum_{j=1}^{m}\gamma_j\right)^{2m-2}\left(\prod_{j=1}^{m}p_j\right)} \\ &= \left[\left(1+\sum_{j=1}^{m}\gamma_j\right)^{2m-2}\left(\prod_{j=1}^{m}p_j\right)\right]^{-1} \end{aligned} \tag{5.36}$$

The joint Dirichlet probability density function $g(\boldsymbol{b})$ evaluated at the maximizing vector $\boldsymbol{b}_n^* = (b_1^*, b_2^*, ..., b_m^*)$ is:

$$g(\boldsymbol{b}_n^*) = \frac{\Gamma(\alpha_1+\alpha_2+...+\alpha_m)}{\Gamma(\alpha_1)\Gamma(\alpha_2)...\Gamma(\alpha_m)}(b_1^*)^{\alpha_1-1}(b_2^*)^{\alpha_2-1}...(b_m^*)^{\alpha_m-1} \tag{5.37}$$

Thus, another equivalent form of the ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* in (5.25) is:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{\Gamma(\alpha_1+\alpha_1+...+\alpha_m)}{\Gamma(\alpha_1)\Gamma(\alpha_1)..\Gamma(\alpha_m)\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}(b_1^*)^{\alpha_1-1}(b_2^*)^{\alpha_2-1}...(b_m^*)^{\alpha_m-1} \tag{5.38}$$

We run the Cover-Ordentlich algorithm on the real stock market and the approximated DOSES market for a general class of the Dirichlet $(\alpha_1, \alpha_2, ..., \alpha_m)$ distributions to calculate the universal capitals obtained for both markets and make comparisons to determine whether the Laplace approximation (5.25) is satisfied.

In this section, we study the case of a three-stock market with m = 3, where the 2×2 determinant of the sensitivity matrix for a three-stock market is given by:

$$\left|J_n^*\right| = \det \begin{bmatrix} -\dfrac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b}_n^*)}{\partial b_1^2} & -\dfrac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b}_n^*)}{\partial b_1 \partial b_2} \\ -\dfrac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b}_n^*)}{\partial b_1 \partial b_2} & -\dfrac{\partial^2 W_n(\boldsymbol{x}^n, \boldsymbol{b}_n^*)}{\partial b_2^2} \end{bmatrix}$$

The determinant of the sensitivity matrix for a stationary, ergodic DOSES market is:

$$\left|J_n^*\right| = \frac{1}{\left[1 + \sum_{j=1}^{3} \gamma_j\right]^{2(3-1)} \left[\prod_{j=1}^{3} p_j\right]}$$

$$= \frac{1}{p_1 p_2 p_3 (1 + \gamma_1 + \gamma_2 + \gamma_3)^4} \qquad (5.39)$$

for large n, where p_i is the probability of the i^{th} price-relative vector $\gamma_i = \dfrac{c_i}{a_i - c_i}$ for i = 1, 2, 3.

The joint density function for the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ universal portfolio with three-stock market is:

$$g(\boldsymbol{b}_n^*) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} (b_1^*)^{\alpha_1 - 1} (b_2^*)^{\alpha_2 - 1} (b_3^*)^{\alpha_3 - 1} \qquad (5.40)$$

evaluated at $\boldsymbol{b}_n^*$.

The conjectured asymptotic behavior of the ratio of the universal capital, $\hat{S}_n$ to the optimal capital, S_n^* is as follows:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{\Gamma(\alpha_1+\alpha_1+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_1)\Gamma(\alpha_3)\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)(b_1^*)^{\alpha_1-1}(b_2^*)^{\alpha_2-1}(b_3^*)^{\alpha_3-1} \tag{5.41}$$

For the case of uniform Dirichlet (1, 1, …, 1) distribution, we substitute $\alpha_1 = \alpha_1 = \alpha_3 = 1$ to the above conjecture and obtain the following asymptotic behavior:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{\Gamma(3)}{\Gamma(1)\Gamma(1)\Gamma(1)\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)$$

$$= \frac{2}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right) = \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}} \tag{5.42}$$

In this section, for the three-stock market where $m = 3$, we compute the ratio of the universal capital, $\hat{S}_n$ to the optimal capital, S_n^* i.e. $\frac{\hat{S}_n}{S_n^*}$, to compare with

$$\frac{g(\mathbf{b}_n^*)}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right) = \frac{\Gamma(\alpha_1+\alpha_1+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_1)\Gamma(\alpha_3)\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)(b_1^*)^{\alpha_1-1}(b_2^*)^{\alpha_2-1}(b_3^*)^{\alpha_3-1} \tag{5.43}$$

and also compute the determinant of their differences, i.e.

$$\left|\frac{\hat{S}_n}{S_n^*} - \frac{g(\mathbf{b}_n^*)}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)\right| \tag{5.44}$$

to check the asymptotic behavior.

5.4 Computational Results and Discussion

In this section, the computational results of discretizing the stock prices to approximate the real market as a DOSES market are shown as follows:

The price relatives of the three stocks i.e. Resorts, Maybank and KNM are discretized to approximate as a DOSES market for a total of 330 trading days from 2nd May 2006 until 24th August 2007. Each day, there is a dominating stock in price and we shall convert other non-dominating stock prices to a uniform one by some discretization process like averaging as shown in the following example:

By using the discretization process as discussed in Section 5.2.1 with the price relatives of the three stocks Resorts, Maybank and KNM for a total of 330 trading days from 2nd May 2006 until 24th August 2007, we obtain a model of DOSES market discretized with the price-relatives listed as follows:

(1.02, 1.00, 1.00) appears 99 times,

(0.99, 1.01, 0.99) appears 113 times,

(1.00, 1.00, 1.02) appears 118 times.

The period of 330 trading days is given as:

$n = 99 + 113 + 118 = 330$ where $a_1 = 1.02, a_2 = 1.01, a_3 = 1.02$, and

$c_1 = 1.00, c_2 = 0.99, c_3 = 1.00$.

We name these three stocks of DOSES market as Resorts(D), Maybank(D) and KNM(D).

In order to generate a sequence $\{\boldsymbol{b}_n^*\}$ which are positive vectors satisfying the regularity condition in the conjectured behaviour (5.25), we make a further transformation of the DOSES market to another DOSES by appropriate scaling of the price-relatives a_i and c_i. In other words, we multiply or divide a_i and c_i by some constants in order to find a positive, convergent sequence of the maximizing vectors $\boldsymbol{b}_n^*$ through computation.

For example:

$a_1 = 3\times1.02 = 3.0600,\quad a_2 = 2.5\times1.01 = 2.5250,\quad a_3 = 2\times1.02 = 2.0400;$

$c_1 = 1.00\div3 = 0.3333,\quad c_2 = 0.99\div2.5 = 0.3960,\quad c_3 = 1.00\div2 = 0.5000.$

After the transformation, a new DOSES market for Resorts(D), Maybank(D) and KNM(D) with the price-relatives is shown as follows:

(3.0600, 0.3333, 0.3333),

(0.3960, 2.5250, 0.3960),

(0.5000, 0.5000, 2.0400),

which appears 99, 113 and 118 times respectively in the data set. For the transformed DOSES market, it is now possible to find a positive, convergent sequence of the maximizing vectors $\boldsymbol{b}_n^*$. We shall run the Dirichlet universal portfolios on the DOSES market with price relatives (3.0600, 0.3333, 0.3333), (0.3960, 2.5250, 0.3960) and (0.5000, 0.5000, 2.0400) in the next section.

5.4.1 The Dirichlet-Weighted Universal Portfolios and Computed Capitals for the DOSES Market

In this research, we had chosen MATLAB as the platform for the computation of universal portfolios and capitals using the transformed price relatives of the three stocks of Resorts(D), Maybank(D) and KNM(D), for a total of 330 trading days from 2nd May 2006 until 24th August 2007. A MATLAB m-file was written based on the modified algorithm for three-stock universal portfolio discussed in Section 2.1.4. We use different parameters of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution for generating the universal portfolio strategies.

The Dirichlet (1.0, 1.0, 1.0) strategy was first introduced by Cover (1991). Subsequently, Cover and Ordentlich (1996) introduced the general class of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies. But the focus of their study is on the Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies. The aim of this project is to study the parametric class of Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies, especially when the (1.0, 1.0, 1.0) and (0.5, 0.5, 0.5) strategies do not perform well. We observe that by varying the parameters α_1, α_2 and α_3, we can obtain Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ strategies achieving different capitals. For example, refer to Table 5.1 where the investment capitals of 16 different sets of $(\alpha_1, \alpha_2, \alpha_3)$ are calculated, namely for $(\alpha_1, \alpha_2, \alpha_3)$ equals to (0.3, 0.4, 0.6), (0.4, 0.6, 0.9), (0.5, 0.5, 0.5), (0.5, 0.8, 0.6), (0.6, 0.8, 0.8), (0.2, 0.9, 0.7), (0.8, 0.4, 0.5), (0.8, 1.0, 0.3), (0.7, 0.2, 0.3), (0.8, 0.9, 0.7), (1.0, 2.0, 1.0), (2.0, 1.0, 1.0), (1.0, 1.0, 2.0), (1.0, 1.0, 1.0), (3.0, 2.0, 1.0), (2.0, 3.0, 2.0) respectively. Except for the detailed calculations for the Dirichlet (0.3, 0.4, 0.6) universal portfolios listed in Appendix 3, we will not show the detailed calculations for the other 15 universal portfolios listed in Table 5.1.

Table 5.1: The investment capitals (returns) achieved after 330 trading days, with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D)

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	Investment Capital after 330 trading days, $\hat{S}_{330}$ ($\times 10^{13}$)
(0.3, 0.4, 0.6)	1.9973
(0.4, 0.6, 0.9)	2.9520
(0.5, 0.5, 0.5)	2.9720
(0.5, 0.8, 0.6)	3.7868
(0.6, 0.8, 0.8)	4.5422
(0.2, 0.9, 0.7)	1.8930
(0.8, 0.4, 0.5)	3.0309
(0.8, 1.0, 0.3)	3.3386
(0.7, 0.2, 0.3)	1.3953
(0.8, 0.9, 0.7)	5.4356
(1.0, 2.0, 1.0)	7.7702
(2.0, 1.0, 1.0)	7.6685
(1.0, 1.0, 2.0)	5.3923
(1.0, 1.0, 1.0)	6.9437
(3.0, 2.0, 1.0)	1.0534
(2.0, 3.0, 2.0)	1.6107

In Table 5.1, the investment capitals (returns) obtained after these 330 trading days with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D) are ranged from 1.3953×10^{13} units to 1.6107×10^{13} units. The Dirichlet (2.0, 3.0, 2.0) strategy (as shown highlighted in grey colour) achieves the highest capital of $\hat{S}_{330}$ = 1.6107×10^{13} units whereas the Dirichlet (0.7, 0.2, 0.3) achieves the lowest capital of $\hat{S}_{330}$ = 1.3953×10^{13} units. The Dirichlet (1.0, 1.0, 1.0) and Dirichlet (0.5, 0.5, 0.5) strategies introduced by Cover and Ordentlich achieve capitals of $\hat{S}_{330}$ = 6.9437×10^{13} units and $\hat{S}_{330} = 2.9720 \times 10^{13}$ units respectively for this period of study.

5.4.2 The Time-varying Log-optimal Portfolio b_i^* and the Optimal Capital S_n^* for the DOSES Market

In this section we will use the transformed price relatives (3.0600, 0.3333, 0.3333), (0.3960, 2.5250, 0.3960) and (0.5000, 0.5000, 2.0400) of the three stocks of Resorts(D), Maybank(D) and KNM(D) found in Section 5.4, for a total of 330 trading days from 2nd May 2006 until 24th August 2007 to compute the time-varying log-optimal portfolio, b_i^* and the optimal capital, S_n^*. Assume the initial investment capital is one unit and the resulting capital after each trading day is in the corresponding number of units. The investment capital at the end of the *n*th trading day by a constant rebalanced portfolio $\boldsymbol{b} = (b_1, b_2, b_3)$ is given from (5.5) as:

$$S_n(\boldsymbol{x}^n, \boldsymbol{b}) = [(a_1 - c_1)b_1 + c_1]^{n_1} \, [(a_2 - c_2)b_2 + c_2]^{n_2} \, [(a_3 - c_3)b_3 + c_3]^{n_3}$$

It is noted that S_n is maximum over the set of portfolio vectors $\boldsymbol{b}$ if and only if $W_n(\boldsymbol{x}^n, \boldsymbol{b}) = \frac{1}{n} \ln S_n(\boldsymbol{x}^n, \boldsymbol{b})$ is maximum.

From Section 5.2.1, formula (5.19) the optimal capital, S_n^* can be written as:

$$S_n^* = \left[(a_1 - c_1)b_1^* + c_1\right]^{n_1} \left[(a_2 - c_2)b_2^* + c_2\right]^{n_2} \left[(a_3 - c_3)b_3^* + c_3\right]^{n_3}$$

$$= \prod_{i=1}^{3} \left[(a_i - c_i) b_i^* c_i \right]^{n_i}$$

where the time-varying log-optimal portfolio which depends on the time n has the following properties:

$$b_1^* + b_2^* + b_3^* = 1; \quad \boldsymbol{b_n^*} = (b_1^*, b_2^*, 1 - b_1^* - b_2^*)$$

Let $\gamma_i = \dfrac{c_i}{a_i - c_i}$ where $i = 1, 2, 3$ then the time-varying log-optimal portfolio b_i^* from (5.17) can be expressed as:

$$b_i^* = \frac{n_i}{n}\left(1 + \sum_{j=1}^{m} \gamma_j\right) - \gamma_i > 0 \, .$$

5.4.3 The Ratios of the Universal Capitals to the Optimal Capital for the DOSES Market

The investment capitals obtained after 330 trading days with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the transformed price relatives of the three stocks of Resorts(D), Maybank(D) and KNM(D) found in Section 5.4, for a total of 330 trading days from 2nd May 2006 to 24th August 2007 were displayed in Table 5.2. The optimal capital achieved for this period was S_{330}^* = 4.2296E+15 units for the time-varying log-optimal portfolio (0.3736, 0.3750, 0.2514). The ratios of the universal capitals, $\hat{S}_{330}$ to the optimal capital, S_{330}^* achieved i.e. $\dfrac{\hat{S}_{330}}{S_{330}^*}$ are listed in Table 5.2.

Table 5.2: The ratios of the universal capitals to the optimal capital based on the transformed stock price relatives of Resorts(D), Maybank(D) and KNM (D) for a total of 330 trading days

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	Investment Capital after 330 trading days, $\hat{S}_{330}$ $(\times 10^{13})$	$\frac{\hat{S}_{330}}{S^*_{330}}$
(0.3, 0.4, 0.6)	1.9973	0.00555838
(0.4, 0.6, 0.9)	2.9520	0.00821526
(0.5, 0.5, 0.5)	2.9720	0.00827092
(0.5, 0.8, 0.6)	3.7868	0.01053847
(0.6, 0.8, 0.8)	4.5422	0.01264070
(0.2, 0.9, 0.7)	1.8930	0.00526812
(0.8, 0.4, 0.5)	3.0309	0.00843484
(0.8, 1.0, 0.3)	3.3386	0.00929115
(0.7, 0.2, 0.3)	1.3953	0.00388305
(0.8, 0.9, 0.7)	5.4356	0.01512699
(1.0, 2.0, 1.0)	7.7702	0.02162406
(2.0, 1.0, 1.0)	7.6685	0.02134103
(1.0, 1.0, 2.0)	5.3923	0.01500649
(1.0, 1.0, 1.0)	6.9437	0.01932395
(3.0, 2.0, 1.0)	10.534	0.02931557
(2.0, 3.0, 2.0)	16.107	0.04482493

In Table 5.2, it is found that for the 330 trading-day period, the best Cover-Ordentlich universal portfolio is $\hat{\boldsymbol{b}}$ = (2.0, 3.0, 2.0), yielding a resulting optimal universal investment capital of $\hat{S}_{330} = 1.6107 \times 10^{14}$ units. The ratio of the best Cover-Ordentlich universal investment capital, $\hat{S}_{330} = 1.6107 \times 10^{14}$ units to the optimal investment capital, $S^*_{330} = 4.2296 \times 10^{15}$ units is:

$$\frac{\hat{S}_{330}}{S^*_{330}} \approx 0.03808162$$

5.4.4 Computation for the Determinant of the Sensitivity Matrix for the DOSES Market

The transformed price relatives of DOSES market found in Section 5.4 for Resorts(D), Maybank(D) and KNM(D) for a total of 330 trading days from 2nd May 2006 until 24th August 2007 are listed as follows:

(3.0600, 0.3333, 0.3333) appears 99 times,

(0.3960, 2.5250, 0.3960) appears 113 times, and

(0.5000, 0.5000, 2.0400) appears 118 times.

From Section 5.3 formula (5.28), for a three-stock market where m = 3, the determinant of the sensitivity matrix is given by:

$$\left|J_n^*\right| = \frac{1}{\left[1+\sum_{j=1}^{3}\gamma_j\right]^{2(3-1)}\left[\prod_{j=1}^{3}p_j\right]}$$

$$= \frac{1}{p_1 p_2 p_3 (1+\gamma_1+\gamma_2+\gamma_3)^4}$$

for large n, where the probability of the respective price-relative vector is estimated as follows:

For the first price-relative vector, (3.0600, 0.3333, 0.3333) the estimated probability is, $\hat{p}_1 = \dfrac{n_1}{n}$ with the total number of trading days, n_1 = 99 and

$$\gamma_1 = \frac{0.3333}{3.0600-0.3333} = 0.1222;$$

For the second price-relative vector, (0.3960, 2.5250, 0.3960) the estimated probability is, $\hat{p}_2 = \frac{n_2}{n}$ with the total number of trading days, n_2 = 113 and

$$\gamma_2 = \frac{0.3960}{2.5250 - 0.3960} = 0.1860;$$

For the third price-relative vector, (0.5000, 0.5000, 2.0400) the estimated probability is, $\hat{p}_3 = \frac{n_3}{n}$ with the total number of trading days, n_3 = 118 and

$$\gamma_3 = \frac{0.5000}{2.400 - 0.5000} = 0.3247 .$$

The period of 330 trading days is given as: n = 99 + 113 + 118 = 330.

Thus, the determinant of the sensitivity matrix,

$$\left|J_n^*\right| = \frac{1}{p_1 p_2 p_3 (1 + 0.1222 + 0.1860 + 0.3247)^4}$$

$$= \frac{1}{p_1 p_2 p_3 (1.6329)^4}$$

We had chosen MATLAB as the platform to compute the determinant of the sensitivity matrix, $\left|J_n^*\right|$ for the transformed three-stock DOSES market of Resorts(D), Maybank(D) and KNM(D). From the above formula for the determinant of the sensitivity matrix, $\left|J_n^*\right|$, it is realised that $\left|J_n^*\right|$ depends on the probabilities of the transformed price-relative vectors, p_i . We let $\hat{p}_i = \frac{n_i}{n}$ be the estimated value of p_i for i = 1, 2 and 3. The estimated probabilities, $\hat{p}_i$ of the respective transformed price-relative vector, p_i of Resorts(D), Maybank(D) and KNM(D) for a sequence of n trading days starting from n = 230 increasing in 5 days to n = 330 is listed in Table 5.3.

Table 5.3: The estimated probabilities of the respective transformed price-relative vectors of Resorts(D), Maybank(D) and KNM(D), i.e. $\hat{p}_1$, $\hat{p}_2$, and $\hat{p}_3$ for trading days starting from n = 230 increasing in 5 days to n = 330

n	n_1	n_2	n_3	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$
230	68	79	83	0.2957	0.3435	0.3609
235	70	81	84	0.2979	0.3447	0.3574
240	72	81	87	0.3000	0.3375	0.3625
245	72	83	90	0.2939	0.3388	0.3673
250	73	86	91	0.2920	0.3440	0.3640
255	74	90	91	0.2902	0.3529	0.3569
260	75	92	93	0.2885	0.3538	0.3577
265	77	93	95	0.2906	0.3509	0.3585
270	77	95	98	0.2852	0.3519	0.3630
275	79	97	99	0.2873	0.3527	0.3600
280	80	100	100	0.2857	0.3571	0.3571
285	82	103	100	0.2877	0.3614	0.3509
290	84	103	103	0.2897	0.3552	0.3552
295	87	104	104	0.2949	0.3525	0.3525
300	90	104	106	0.3000	0.3467	0.3533
305	92	106	107	0.3016	0.3475	0.3508
310	94	106	110	0.3032	0.3419	0.3548
315	96	107	112	0.3048	0.3397	0.3556
320	98	108	114	0.3063	0.3375	0.3563
325	99	111	115	0.3046	0.3415	0.3538
330	99	113	118	0.3000	0.3424	0.3576

5.4.5 Computation for the Asymptotic Behaviour of the Ratio of the Universal Capitals to the Optimal Capital for the Uniform Dirichlet (1, 1, 1) Distribution

Cover (1991) had proved that for the universal portfolio generated by the uniform Dirichlet (1, 1, …, 1) distribution, the ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* has the following asymptotic behavior:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{(m-1)!}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}$$

where $\left|J_n^*\right|$ denotes the determinant of J_n^*. From (5.42), for the transformed three-stock market m = 3 with the uniform Dirichlet (1, 1, …, 1) distribution, the conjectured asymptotic behavior of the ratio of the universal capital, $\hat{S}_n$ to the optimal capital, S_n^* is given as follows:

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{2\pi\Gamma(3)}{n\Gamma(1)\Gamma(1)\Gamma(1)\left|J_n^*\right|^{\frac{1}{2}}}$$

$$= \frac{2}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)$$

$$= \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}} \qquad \text{Let } Q_n^* = \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}}$$

The transformed price relatives of the three stocks i.e. Resorts(D), Maybank(D) and KNM(D) for a total of 330 trading days from 2nd May 2006 until 24th August 2007 are used. We had chosen MATLAB as the platform for the computation of each of the following:

1. The universal portfolio, $\hat{\mathbf{b}}_n$ and the universal capital, $\hat{S}_n$ on each trading day generated by the Dirichlet (1, 1, 1) distribution.
2. The time-varying log-optimal portfolio, $\mathbf{b}_n^*$ and the optimal capital, S_n^*.
3. The determinant of the sensitivity matrix, $\left|J_n^*\right|$.
4. The ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* i.e. $\frac{\hat{S}_n}{S_n^*}$.
5. The results generated for $Q_n^* = \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}}$.

6. The values of $\left|\frac{\hat{S}_n}{S_n^*} - Q_n^*\right|$ or $\left|\frac{\hat{S}_n}{S_n^*} - \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ generated for determining the closeness of $\frac{\hat{S}_n}{S_n^*}$ to Q_n^*.

We observe that the values of $\left|\frac{\hat{S}_n}{S_n^*} - \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ are small as n gets bigger.

For example, when $n = 330$, $\left|\frac{\hat{S}_{330}}{S_{n330}^*} - \frac{4\pi}{330\left|J_{330}^*\right|^{\frac{1}{2}}}\right| \approx 0.000136$.

As the computation for this section is similar to next section, we will only show the detail of the computation for Section 5.4.6 in Appendix 3.

5.4.6 Computation for the Asymptotic Behaviour of the Ratio of the Universal Capitals to the Optimal Capital for

5.4.6.1 The Dirichlet (0.3, 0.4, 0.6) Distribution

The joint density function for the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution given by (5.40) is as follows:

$$g(\mathbf{b}_n^*) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)}(b_1^*)^{\alpha_1 - 1}(b_2^*)^{\alpha_2 - 1}(b_3^*)^{\alpha_3 - 1}$$

over the region $b_i \geq 0$ for $i = 1, 2, 3$, $\sum_{i=1}^{3} b_i = 1$.

Consider the general Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution for generating the universal portfolio strategies with parameters $\alpha_1 = 0.3,\ \alpha_2 = 0.4$ and $\alpha_3 = 0.6$, i.e. the Dirichlet (0.3, 0.4, 0.6) distribution for the three-stock market $m = 3$, the joint density function at $\mathbf{b}_n^*$ is simplified as:

$$g(\mathbf{b}_n^*) = \frac{\Gamma(1.3)}{\Gamma(0.3)\Gamma(0.4)\Gamma(0.6)}(b_1^*)^{-0.7}(b_2^*)^{-0.6}(b_3^*)^{-0.4}$$

$$= \frac{0.3\Gamma(0.3)}{\Gamma(0.3)\Gamma(0.4)\Gamma(0.6)}(b_1^*)^{-0.7}(b_2^*)^{-0.6}(b_3^*)^{-0.4}$$

$$= \frac{0.3}{\Gamma(0.4)\Gamma(0.6)}(b_1^*)^{-0.7}(b_2^*)^{-0.6}(b_3^*)^{-0.4}$$

Note: $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$

In next computation, we examine the following Laplace approximation conjecture listed in (5.25) which is due to Tan (2002a).

$$\frac{\hat{S}_n(\hat{\mathbf{b}}_n)}{S_n^*(\mathbf{b}_n^*)} \sim \frac{g(\mathbf{b}_n^*)}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}$$

For the transformed three-stock market where $m = 3$, the conjecture can be simplified as: $\frac{\hat{S}_n(\hat{\mathbf{b}}_n)}{S_n^*(\mathbf{b}_n^*)} \sim \left(\frac{2\pi}{n}\right)\frac{\Gamma(\alpha_1+\alpha_2+\alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)\left|J_n^*\right|^{\frac{1}{2}}}(b_1^*)^{\alpha_1-1}(b_2^*)^{\alpha_2-1}(b_3^*)^{\alpha_3-1}$

The asymptotic behavior of the ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* is simplified as: $\frac{\hat{S}_n}{S_n^*} \sim \frac{0.3\Gamma(0.3)}{\Gamma(0.4)\Gamma(0.6)\left|J_n^*\right|^{\frac{1}{2}}(b_1^*)^{0.7}(b_2^*)^{0.6}(b_3^*)^{0.4}}\left(\frac{2\pi}{n}\right)$

Let $G_n^* = \frac{0.3\Gamma(0.3)}{\Gamma(0.4)\Gamma(0.6)\left|J_n^*\right|^{\frac{1}{2}}(b_1^*)^{0.7}(b_2^*)^{0.6}(b_3^*)^{0.4}}\left(\frac{2\pi}{n}\right) = \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$

A MATLAB m-file was written to compute the G_n^* values of the transformed three stocks i.e. Resorts(D), Maybank(D) and KNM(D) for a total of 330 trading days from 2nd May 2006 until 24th August 2007.

To compare the ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* i.e. $\frac{\hat{S}_n}{S_n^*}$, with $G_n^* = \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ again we had chosen MATLAB as the platform for the computation of the values for $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ or $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$.

The items listed below are tabulated in Appendix 3:

1. The transformed price relatives of the three stocks of Resorts(D), Maybank(D) and KNM(D) for a total of 330 trading days from 2nd May 2006 until 24th August 2007.
2. The universal portfolio, $\hat{\mathbf{b}}_n$ and the universal capital, $\hat{S}_n$ on each trading day generated by the Dirichlet (0.3, 0.4, 0.6) distribution.
3. The time-varying log-optimal portfolio, $\mathbf{b}_n^*$ and the optimal capital, S_n^*.
4. The determinant of the sensitivity matrix, $\left|J_n^*\right|$.
5. The ratio of the universal capital $\hat{S}_n$ to the optimal capital S_n^* i.e. $\frac{\hat{S}_n}{S_n^*}$.
6. The results generated for $G_n^* = \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$.

7. The values of $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ generated for determining the asymptotic behaviour, $\frac{\hat{S}_n}{S_n^*} \sim \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$.

In Appendix 3, we observe that the values of $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ are small as n gets bigger. For example, when n = 330, $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right| \approx$ 0.000058.

5.4.6.2 The values of $\boldsymbol{b}_n^*$, $\frac{\hat{S}_n}{S_n^*}$, G_n^*, and $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for the Dirichlet (0.3, 0.4, 0.6) Distribution

From Section 5.2.2, the time-varying log-optimal portfolio $\boldsymbol{b}_n^*$ which depends on the time n satisfies: $b_1^* + b_2^* + b_3^* = 1$; $\boldsymbol{b}_n^* = (b_1^*, b_2^*, 1 - b_1^* - b_2^*)$. In the above, to make the notation simple, we suppress the dependence on n in $b_1^*(n)$, $b_2^*(n)$ and $b_3^*(n)$. Let $\gamma_i = \frac{c_i}{a_i - c_i}$ where i = 1, 2, 3 then the time-varying log-optimal portfolio b_i^* from (5.17) can be expressed as:

$$b_i^* = \frac{n_i}{n}\left(1 + \sum_{j=1}^{m} \gamma_j\right) - \gamma_i > 0, \qquad i = 1, 2, 3$$

This formula is due to Tan and Chan (2008). For a sequence of n trading days starting from n = 200 increasing by 10 trading days to n = 330 based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D) for the Cover and Ordentlich universal portfolio generated by the Dirichlet (0.3, 0.4, 0.6) distribution, the time-varying log-optimal portfolios $\boldsymbol{b}_n^*$, the ratios $\frac{\hat{S}_n}{S_n^*}$, the functions G_n^*, and the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ are listed in Tables 5.4. Note that the sequences $\{\boldsymbol{b}_n^*\}$ are positive, satisfying the regularity conditions for the Laplace approximation in (5.25). The empirical results in the three tables tend to support the asymptotic behaviour $\frac{\hat{S}_n}{S_n^*} \sim G_n^*$ for large n.

Table 5.4: The values of $\boldsymbol{b}_n^*$, $\frac{\hat{S}_n}{S_n^*}$, G_n^*, and $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for the Dirichlet (0.3, 0.4, 0.6) distribution, based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D) for trading days from n = 200 increasing in 10 days to n = 330

n	The time-varying log-optimal portfolio $\boldsymbol{b}_n^*$			$\frac{\hat{S}_n}{S_n^*}$	G_n^*	$\left\|\frac{\hat{S}_n}{S_n^*} - G_n^*\right\|$
200	0.3431	0.3774	0.2795	0.009270	0.009200	0.000070
210	0.3365	0.3972	0.2663	0.008840	0.008700	0.000140
220	0.3454	0.3929	0.2617	0.008411	0.008300	0.000111
230	0.3605	0.3749	0.2646	0.008011	0.007900	0.000111
240	0.3676	0.3651	0.2673	0.007666	0.007600	0.000066
250	0.3546	0.3757	0.2697	0.007374	0.007300	0.000074
260	0.3488	0.3918	0.2594	0.007099	0.007000	0.000099
270	0.3434	0.3885	0.2680	0.006843	0.006800	0.000043
280	0.3443	0.3972	0.2585	0.006596	0.006500	0.000096
290	0.3507	0.3940	0.2553	0.006356	0.006300	0.000056
300	0.3676	0.3801	0.2523	0.006120	0.006100	0.000020
310	0.3729	0.3723	0.2547	0.005915	0.005900	0.000015
320	0.3778	0.3651	0.2571	0.005725	0.005700	0.000025
330	0.3676	0.3731	0.2592	0.005558	0.005500	0.000058

5.4.6.3 The General Class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution

The ratio of the universal capital, $\hat{S}_n$ to the optimal capital, S_n^* i.e. $\frac{\hat{S}_n}{S_n^*}$, obtained after 330 trading days with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D) from 2nd May 2006 until 24th August 2007; the function G_n^* for the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution. In Table 5.5, for the 16 selected values of the parametric vector $(\alpha_1, \alpha_2, \alpha_3)$, the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ or $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ where $G_n^* = \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ for some large *n*, say *n* = 330 are displayed to show the validity of the asymptotic behaviour, $\frac{\hat{S}_n}{S_n^*} \sim \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ for large *n*. The empirical results here support the Laplace approximation conjecture (5.25) for different $\boldsymbol{\alpha}$ values of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ universal portfolios where there exist convergent positive sequences $\{\boldsymbol{b}_n^*\}$.

Table 5.5: The differences between the ratios of the universal capitals to the optimal capital and G^*_{330} for the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution, based on the transformed stock price data of Resorts(D), Maybank(D) and KNM(D)

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution	$\frac{\hat{S}_{330}}{S^*_{330}}$	G^*_{330}	$\left\lvert \frac{\hat{S}_{330}}{S^*_{330}} - G^*_{330} \right\rvert$
(0.3, 0.4, 0.6)	0.00555838	0.00552000	0.00003838
(0.4, 0.6, 0.9)	0.00821526	0.00820400	0.00001126
(0.5, 0.5, 0.5)	0.00827092	0.00821200	0.00005892
(0.5, 0.8, 0.6)	0.01053847	0.01049700	0.00004147
(0.6, 0.8, 0.8)	0.01264070	0.01264500	0.00000430
(0.2, 0.9, 0.7)	0.00526812	0.00524300	0.00002512
(0.8, 0.4, 0.5)	0.00843484	0.00837200	0.00006284
(0.8, 1.0, 0.3)	0.00929115	0.00918900	0.00010215
(0.7, 0.2, 0.3)	0.00388305	0.00383100	0.00005205
(0.8, 0.9, 0.7)	0.01512699	0.01512700	0.00000001
(1.0, 2.0, 1.0)	0.02162406	0.02178400	0.00015994
(2.0, 1.0, 1.0)	0.02134103	0.02146300	0.00012197
(1.0, 1.0, 2.0)	0.01500649	0.01513300	0.00012651
(1.0, 1.0, 1.0)	0.01932395	0.01945996	0.00013601
(3.0, 2.0, 1.0)	0.02931557	0.02944300	0.00012743
(2.0, 3.0, 2.0)	0.04482493	0.04647900	0.00165407

5.4.7 The Ratios of the Universal Capitals to the Optimal Capital in Longer Trading Period with Different Data Sets of DOSES Market

In this section, we consider a longer trading period with different sets of the price relatives for the DOSES market. We use different sets of the stock-price data from those used in Section 5.4 to test if the DOSES market is applicable for any sets of the stock-price data. The stock-price data of Aeon, CIMB and Daibochi were chosen as the first set of three-stock portfolio for investment for a total of 500 trading days from 18th July 2005 until 30th April 2009, this included both the non-volatile period, from 2nd May 2006 until 24th August 2007 and the volatile period, from 24th December 2007 until 30th April 2009 that we have studied in separate sections earlier. The opening and closing prices for each stock were observed for each trading day and the corresponding price relatives were calculated. In this study, Aeon was chosen as the first constituent stock in the three-stock universal portfolio.

We use the same method as applied in Section 5.4 to discretize the price relatives of the three stocks i.e. Aeon, CIMB and Daibochi to approximate as a DOSES market for a total of 500 trading days from 18th July 2005 until 30th April 2009. By using the discretization process, we obtain a model of DOSES market discretized with the price-relatives listed as follows:

(0.9962, 0.9962, 1.0369) appears 236 times,

(0.9963, 1.0162, 0.9963) appears 148 times, and

(1.0052, 0.9930, 0.9930) appears 116 times.

The period of 500 trading days is given as:

$n = 236 + 148 + 116 = 500$ where $a_1 = 1.0052,\ a_2 = 1.0162,\ a_3 = 1.0369,$ and $c_1 = 0.9930,\ c_2 = 0.9963,\ c_3 = 0.9962$.

We name these three stocks of DOSES market as Aeon(D), CIMB(D) and Daibochi(D). In order to obtain a positive sequence $\{\boldsymbol{b}_n^*\}$, this model of DOSES market is transformed either by a multiple or division of their initial price-relatives respectively as follows:

$a_1 = 1.6 \times 1.0052 = 1.6084, \qquad a_2 = 1.8 \times 1.0369 = 1.8291,$

$a_3 = 2.2 \times 1.0369 = 2.2812; \qquad c_1 = 0.9930 \div 1.6 = 0.6206,$

$c_2 = 0.9963 \div 1.7 = 0.5860, \qquad c_3 = 0.9962 \div 1.8 = 0.5534.$

By using the transformation process, a transformed set of DOSES market for Aeon(D), CIMB(D) and Daibochi(D) with the price-relatives is obtained as shown below:

(0.5534, 0.5534, 2.2812) appears 236 times,

(0.5860, 1.8291, 0.5860) appears 148 times, and

(1.6084, 0.6206, 0.6206) appears 116 times.

The investment capitals obtained after 500 trading days with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the above transformed price relatives of the three stocks Aeon(D), CIMB(D) and Daibochi(D), for a total of 500 trading days from 2nd May 2006 until 30th April 2009 were displayed in Table 5.6. The optimal capital achieved for this period was S^*_{500} = 16379.065198 units for the time-varying log-optimal portfolio (0.4103, 0.4051, 0.1846). The ratios of the universal capitals, $\hat{S}_{500}$ to the optimal capital, S^*_{330} achieved i.e. $\frac{\hat{S}_{500}}{S^*_{500}}$ are listed in Table 5.6.

Table 5.6: The ratios of the universal capitals to the optimal capital based on the transformed stock price relatives of Aeon(D), CIMB(D) and Daibochi(D) for a total of 500 trading days

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution	Investment Capital after 500 trading days, $\hat{S}_{500}$	$\frac{\hat{S}_{500}}{S^*_{500}}$
(0.3, 0.4, 0.6)	19.2900	0.00803965
(0.4, 0.6, 0.9)	26.4330	0.01101669
(0.5, 0.5, 0.5)	29.5710	0.01232454
(0.5, 0.8, 0.6)	32.7976	0.01366932
(0.6, 0.8, 0.8)	40.1028	0.01671397
(0.2, 0.9, 0.7)	14.1098	0.00588066
(0.8, 0.4, 0.5)	34.8751	0.01453518
(0.8, 1.0, 0.3)	29.9565	0.01248521
(0.7, 0.2, 0.3)	17.2355	0.00718338
(0.8, 0.9, 0.7)	49.4535	0.02061113
(1.0, 2.0, 1.0)	46.6039	0.01942348
(2.0, 1.0, 1.0)	97.2076	0.04051400
(1.0, 1.0, 2.0)	46.0640	0.01919846
(1.0, 1.0, 1.0)	63.2919	0.02637868
(3.0, 2.0, 1.0)	120.3422	0.05015599
(2.0, 3.0, 2.0)	83.3018	0.03471836

Table 5.7: The estimated probabilities of the respective transformed price-relative vectors of Aeon(D), CIMB(D) and Daibochi(D), i.e. $\hat{p}_1$, $\hat{p}_2$, and $\hat{p}_3$ for trading days starting from n = 230 increasing in 5 days to n = 500

n	n_1	n_2	n_3	$\hat{p}_1$	$\hat{p}_2$	$\hat{p}_3$
230	110	71	49	0.4783	0.3087	0.2130
235	110	72	53	0.4681	0.3064	0.2255
240	112	72	56	0.4667	0.3000	0.2333
245	113	74	58	0.4612	0.3020	0.2367
250	115	74	61	0.4600	0.2960	0.2440
255	119	75	61	0.4667	0.2941	0.2392
260	122	76	62	0.4692	0.2923	0.2385
265	124	77	64	0.4679	0.2906	0.2415
270	124	80	66	0.4593	0.2963	0.2444
275	127	81	67	0.4618	0.2945	0.2436
280	128	84	68	0.4571	0.3000	0.2429
285	130	86	69	0.4561	0.3018	0.2421
290	131	87	72	0.4517	0.3000	0.2483
295	133	89	73	0.4508	0.3017	0.2475
300	134	91	75	0.4467	0.3033	0.2500
305	136	93	76	0.4459	0.3049	0.2492
310	137	94	79	0.4419	0.3032	0.2548
315	137	96	82	0.4349	0.3048	0.2603
320	139	97	84	0.4344	0.3031	0.2625
325	141	98	86	0.4338	0.3015	0.2646
330	145	99	86	0.4394	0.3000	0.2606
335	149	99	87	0.4448	0.2955	0.2597
340	154	99	87	0.4529	0.2912	0.2559
345	157	100	88	0.4551	0.2899	0.2551
350	160	102	88	0.4571	0.2914	0.2514
355	162	104	89	0.4563	0.2930	0.2507
360	166	105	89	0.4611	0.2917	0.2472
365	170	106	89	0.4658	0.2904	0.2438
370	174	107	89	0.4703	0.2892	0.2405
375	176	109	90	0.4693	0.2907	0.2400
380	180	110	90	0.4737	0.2895	0.2368
385	184	111	90	0.4779	0.2883	0.2338
390	185	114	91	0.4744	0.2923	0.2333
395	188	114	93	0.4759	0.2886	0.2354
400	191	115	94	0.4775	0.2875	0.2350
405	193	115	97	0.4765	0.2840	0.2395
410	196	116	98	0.4780	0.2829	0.2390
415	198	118	99	0.4771	0.2843	0.2386
420	200	120	100	0.4762	0.2857	0.2381
425	203	120	102	0.4776	0.2824	0.2400
430	204	122	104	0.4744	0.2837	0.2419
435	207	124	104	0.4759	0.2851	0.2391
440	211	125	104	0.4795	0.2841	0.2364
445	212	128	105	0.4764	0.2876	0.2360
450	214	128	108	0.4756	0.2844	0.2400
455	217	129	109	0.4769	0.2835	0.2396
460	220	130	110	0.4783	0.2826	0.2391
465	223	132	110	0.4796	0.2839	0.2366
470	225	135	110	0.4787	0.2872	0.2340
475	228	136	111	0.4800	0.2863	0.2337
480	229	138	113	0.4771	0.2875	0.2354
485	231	140	114	0.4763	0.2887	0.2351
490	233	142	115	0.4755	0.2898	0.2347
495	235	145	115	0.4747	0.2929	0.2323
500	236	148	116	0.4720	0.2960	0.2320

5.4.8 The Values of b_n^*, $\frac{\hat{S}_n}{S_n^*}$, G_n^*, and $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for the Dirichlet (1.0, 1.0, 2.0) Distribution in Longer Trading Period

For a sequence of n trading days starting from n = 200 increasing by 10 trading days to n = 500 based on the transformed stock price data of Aeon(D), CIMB(D) and Daibochi(D) from 2nd May 2006 until 30th April 2009 inclusive of the non-volatile and volatile trading days, for the universal portfolio generated by the Dirichlet (1.0, 1.0, 2.0) distribution, the time-varying log-optimal portfolios b_n^*, the ratios $\frac{\hat{S}_n}{S_n^*}$, the functions G_n^*, and the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ are listed in Tables 5.8. Note that sequences $\{\boldsymbol{b}_n^*\}$ are positive, satisfying the regularity conditions for the Laplace approximation in (5.25). By comparing the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for n = 330 in Tables 5.4 and the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for n = 500 in Tables 5.8, we observe that for a longer trading period of trading, the values obtained for $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ are comparatively smaller. Thus, the empirical result in Tables 5.8 tends to support the asymptotic behaviour $\frac{\hat{S}_n}{S_n^*} \sim G_n^*$ for large n.

Table 5.8: The values of $\boldsymbol{b}_n^*$, $\frac{\hat{S}_n}{S_n^*}$, G_n^* and $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ for the Dirichlet (1.0, 1.0, 2.0) distribution, based on the transformed stock price data of Aeon(D), CIMB(D) and Daibochi(D) for trading days from n = 200 increasing in 10 days to n = 500

n	The time-varying log-optimal portfolio $\boldsymbol{b}_n^*$			$\frac{\hat{S}_n}{S_n^*}$	G_n^*	$\left\lvert\frac{\hat{S}_n}{S_n^*} - G_n^*\right\rvert$
200	0.5575	0.2909	0.1516	0.029668	0.029033	0.000635
210	0.5587	0.2892	0.1522	0.028326	0.027754	0.000572
220	0.5487	0.2766	0.1747	0.031166	0.030742	0.000424
230	0.5291	0.2756	0.1953	0.033552	0.033241	0.000311
240	0.5010	0.2546	0.2444	0.040737	0.040628	0.000109
250	0.4849	0.2449	0.2702	0.043502	0.043489	0.000013
260	0.5073	0.2360	0.2568	0.039485	0.039432	0.000053
270	0.4831	0.2456	0.2713	0.040462	0.040452	0.000010
280	0.4780	0.2546	0.2674	0.038500	0.038480	0.000020
290	0.4649	0.2546	0.2805	0.039158	0.039173	0.000015
300	0.4527	0.2627	0.2847	0.038536	0.038560	0.000024
310	0.4412	0.2624	0.2964	0.038960	0.039009	0.000049
320	0.4229	0.2621	0.3149	0.040310	0.040396	0.000086
330	0.4351	0.2546	0.3104	0.038414	0.038484	0.000070
340	0.4678	0.2332	0.2989	0.035614	0.035658	0.000044
350	0.4780	0.2338	0.2882	0.033244	0.033266	0.000022
360	0.4876	0.2344	0.2780	0.031080	0.031084	0.000004
370	0.5098	0.2284	0.2618	0.028275	0.028254	0.000021
380	0.5180	0.2291	0.2529	0.026507	0.026474	0.000033
390	0.5197	0.2360	0.2444	0.024925	0.024882	0.000043
400	0.5273	0.2243	0.2484	0.024661	0.024625	0.000036
410	0.5286	0.2133	0.2581	0.025014	0.024992	0.000022
420	0.5241	0.2200	0.2559	0.024233	0.024209	0.000024
430	0.5198	0.2152	0.2650	0.024561	0.024549	0.000012
440	0.5322	0.2161	0.2517	0.022689	0.022663	0.000026
450	0.5226	0.2169	0.2605	0.023043	0.023027	0.000016
460	0.5291	0.2125	0.2584	0.022313	0.022295	0.000018
470	0.5302	0.2237	0.2461	0.020765	0.020737	0.000028
480	0.5263	0.2243	0.2494	0.020638	0.020613	0.000025
490	0.5225	0.2299	0.2477	0.020092	0.020067	0.000025
500	0.5140	0.2449	0.2411	0.019198	0.019169	0.000029

5.4.9 Computation for the Asymptotic Behaviour of the Ratio of the Universal Capitals to the Optimal Capital for the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution for Aeon(D), CIMB(D) and Daibochi(D)

The values of $\frac{\hat{S}_n}{S_n^*}$ obtained after 500 trading days with 16 universal portfolio strategies generated by the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution based on the transformed stock price data of Aeon(D), CIMB(D) and Daibochi(D) from 2nd May 2006 until 30th April 2009 inclusive of the non-volatile and volatile trading days, the function G_n^* and the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ are listed in Table 5.9 to test the validity of the asymptotic behaviour, $\frac{\hat{S}_n}{S_n^*} \sim \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$.

Table 5.9: The differences between the ratios of the universal capitals to the optimal capital and the density function for the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution, based on the transformed stock price data of Aeon(D), CIMB(D) and Daibochi(D) for a total of 500 trading days

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution	$\frac{\hat{S}_{500}}{S_{500}^*}$	G_{500}^*	$\left\|\frac{\hat{S}_{500}}{S_{500}^*} - G_{500}^*\right\|$
(0.3, 0.4, 0.6)	0.00803965	0.00787800	0.00016165
(0.4, 0.6, 0.9)	0.01101669	0.01116200	0.00014531
(0.5, 0.5, 0.5)	0.01232454	0.01210400	0.00022054
(0.5, 0.8, 0.6)	0.01366932	0.01353700	0.00013232
(0.6, 0.8, 0.8)	0.01671397	0.01662000	0.00009397
(0.2, 0.9, 0.7)	0.00588066	0.00582100	0.00005966
(0.8, 0.4, 0.5)	0.01453518	0.01423200	0.00030318
(0.8, 1.0, 0.3)	0.01248521	0.01230900	0.00017621
(0.7, 0.2, 0.3)	0.00718338	0.00695000	0.00023338
(0.8, 0.9, 0.7)	0.02061113	0.02053300	0.00007813
(1.0, 2.0, 1.0)	0.01942348	0.01946900	0.00004552
(2.0, 1.0, 1.0)	0.04051400	0.04085700	0.00034300
(1.0, 1.0, 2.0)	0.01919846	0.01916900	0.00002946
(1.0, 1.0, 1.0)	0.02637868	0.02649800	0.00011932
(3.0, 2.0, 1.0)	0.05015599	0.05142700	0.00127101
(2.0, 3.0, 2.0)	0.03471836	0.03545500	0.00073664

5.4.10 Comparison of the Asymptotic Behaviour of the Ratio of the Universal Capitals to the Optimal Capital for the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution for the Three Data Sets of the DOSES Market in Longer Trading Period of 500 Trading Days

The values of $\left| \frac{\hat{S}_{500}}{S^*_{500}} - G^*_{500} \right|$ after 500 trading days for a second and third data sets of the DOSES market, namely Astro(D), Digi(D) and Nestle(D); Scientex(D), Alliance(D) and Measat(D) are also computed by using the same procedure as the first data set of the DOSES market, i.e.. Aeon(D), CIMB(D) and Daibochi(D). These three sets are listed as follows:

1st set: Aeon(D), CIMB(D) and Daibochi(D), abbreviated as ACD(D)

2nd set: Astro(D), Digi(D) and Nestle(D), abbreviated as ADN(D)

3rd set: Scientex(D), Alliance(D) and Measat(D), abbreviated as SAM(D)

The ratio of the universal capitals to the optimal capital for the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution together with the values of $\left| \frac{\hat{S}_{500}}{S^*_{500}} - G^*_{500} \right|$ for the respective three data sets of the DOSES market are listed in Table 5.10 for comparison.

Table 5.10: The comparison of the asymptotic behaviour of the ratio of the universal capitals to the optimal capital for the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution for the three data sets of the DOSES market in longer trading period of 500 trading days

Cover and Ordentlich Universal Portfolio generated by Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ Distribution	$\left\|\frac{\hat{S}_{500}}{S^*_{500}} - G^*_{500}\right\|$ ACD(D)	$\left\|\frac{\hat{S}_{500}}{S^*_{500}} - G^*_{500}\right\|$ ADN(D)	$\left\|\frac{\hat{S}_{500}}{S^*_{500}} - G^*_{500}\right\|$ SAM(D)
(0.3, 0.4, 0.6)	0.00016165	0.00005289	0.00001093
(0.4, 0.6, 0.9)	0.00014531	0.00008652	0.00000475
(0.5, 0.5, 0.5)	0.00022054	0.00008317	0.00008317
(0.5, 0.8, 0.6)	0.00013232	0.00011957	0.00000411
(0.6, 0.8, 0.8)	0.00009397	0.00015496	0.00000564
(0.2, 0.9, 0.7)	0.00005966	0.00005331	0.00000490
(0.8, 0.4, 0.5)	0.00030318	0.00008997	0.00001414
(0.8, 1.0, 0.3)	0.00017621	0.00008662	0.00001030
(0.7, 0.2, 0.3)	0.00023338	0.00002939	0.00001480
(0.8, 0.9, 0.7)	0.00007813	0.00020160	0.00001288
(1.0, 2.0, 1.0)	0.00004552	0.00035241	0.00002960
(2.0, 1.0, 1.0)	0.00034300	0.00040376	0.00009585
(1.0, 1.0, 2.0)	0.00002946	0.00008758	0.00002289
(1.0, 1.0, 1.0)	0.00011932	0.00028092	0.00004968
(3.0, 2.0, 1.0)	0.00127101	0.00086410	0.00019655
(2.0, 3.0, 2.0)	0.00073664	0.00081570	0.00025250

From the above Table 5.10, for the three data sets of the DOSES market i.e. ACD(D), ADN(D) and SAM(D) in a longer trading period of 500 trading days for the 16 selected values of the parametric vector $(\alpha_1, \alpha_2, \alpha_3)$, the values of $\left|\frac{\hat{S}_n}{S^*_n} - G^*_n\right|$ where $G^*_n = \frac{2\pi\, g(\mathbf{b}^*_n)}{n\left|J^*_n\right|^{\frac{1}{2}}}$ for some large n, here n = 500 are displayed to show the validity of the asymptotic behaviour, $\frac{\hat{S}_n}{S^*_n} \sim \frac{2\pi\, g(\mathbf{b}^*_n)}{n\left|J^*_n\right|^{\frac{1}{2}}}$ for large n. The empirical results here support the Laplace approximation conjecture (5.25) for different $\boldsymbol{\alpha}$ values of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ universal portfolios where there exist convergent positive sequences $\{\boldsymbol{b}^*_n\}$.

5.5 Conclusion

- We began by discretizing and transforming the price relatives of stock price data from the KLSE to a new set of data namely **DOSES** market means **Dominating Stock with Equal Subordinates.** We model a real 3-stock market from KLSE as an approximate DOSES market by transformation, i.e. the initial price relatives of each stock data set are converted to a DOSES market by a discretization procedure, these price relatives of the first DOSES market are scaled by appropriate constants to become a second DOSES market where the ergodic structure of the first market is preserved. The first approximation by a DOSES market entails a problem in the sequence of maximizing vectors $\{\boldsymbol{b}_n^*\}$ for the optimal investment capitals, S_n^* where $\boldsymbol{b}_n^*$ lie on the boundary of the simplex of portfolio vectors, i.e. there is a zero component in $\boldsymbol{b}_n^*$. This does not satisfy the regularity condition for a positive, convergent sequence of $\{\boldsymbol{b}_n^*\}$ to verify the asymptotic behaviour of $\frac{\hat{S}_n(\hat{\boldsymbol{b}}_n)}{S_n^*(\boldsymbol{b}_n^*)}$.

- We study the Laplace approximation conjecture of the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution. Subject to certain regularity conditions on the sequence $\{\boldsymbol{b}_n^*\}$ and $\{|J_n^*|\}$, the ratio of the universal capital $\hat{S}_n(\hat{\boldsymbol{b}}_n)$ to the optimal capital $S_n^*(\boldsymbol{b}_n^*)$ is conjectured to have the following asymptotic behavior:

$$\frac{\hat{S}_n(\hat{\boldsymbol{b}}_n)}{S_n^*(\boldsymbol{b}_n^*)} \sim \frac{g(\boldsymbol{b}_n^*)}{\left|J_n^*(\boldsymbol{b}_n^*)\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}$$

This Laplace approximation conjecture is due to Tan (2002a).

- With MATLAB as the platform for the computation, we run the Cover-Ordentlich algorithm in the DOSES market for 4 selected stock data sets (where 1 data set consist of 330 trading days and 3 data sets consist of 500 trading days) to calculate the universal capitals obtained and also the optimal capitals to study the asymptotic behavior of $\frac{\hat{S}_n}{S_n^*}$ for the general Dirichlet $(\alpha_1, \alpha_2, \ldots\ldots, \alpha_m)$ universal portfolio.

- We used the transformed price relatives of stock price data from the KLSE, i.e. different data sets of the DOSES market to calculate the universal capitals obtained and also the optimal capitals to examine the following asymptotic behaviour,

$$\frac{\hat{S}_n}{S_n^*} \sim \frac{(m-1)!}{\left|J_n^*\right|^{\frac{1}{2}}}\left(\frac{2\pi}{n}\right)^{\frac{m-1}{2}}$$

for the uniform Dirichlet (1, 1, 1) distribution, where $\left|J_n^*\right|$ denotes the determinant of J_n^*. We observed that for the three-stock DOSES market where m = 3, the values of $\left|\frac{\hat{S}_n}{S_n^*} - \frac{4\pi}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ are small as n gets bigger.

- The results of the values of the time-varying log-optimal portfolios b_n^*, where $b_i^* = \frac{n_i}{n}\left(1+\sum_{j=1}^{m}\gamma_j\right)-\gamma_i > 0$, $i = 1,2,3$; the ratios $\frac{\hat{S}_n}{S_n^*}$, the functions G_n^*, and the values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ where $G_n^* = \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ and $\left|J_n^*\right|$ denotes the determinant of J_n^* for a sequence of trading days obtained where the sequences $\{\boldsymbol{b}_n^*\}$ are positive, satisfying the regularity conditions for Laplace approximation in (5.25). The empirical results listed in the tables in this chapter tend to support the asymptotic behaviour $\frac{\hat{S}_n}{S_n^*} \sim G_n^*$ for large *n*.

- We used different data sets of the transformed price relatives of three-stock-price data from the KLSE, i.e. different data sets of the **DOSES** market with MATLAB as the platform for the computation, to calculate the universal capitals obtained and also the optimal capitals for the general class of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ distribution. The values of $\left|\frac{\hat{S}_n}{S_n^*} - G_n^*\right|$ or $\left|\frac{\hat{S}_n}{S_n^*} - \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}\right|$ where $G_n^* = \frac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ for some large *n*, where $\left|J_n^*\right|$ denotes the determinant of J_n^* and

$$g(\boldsymbol{b}_n^*) = \frac{\Gamma(\alpha_1 + \alpha_2 + ... + \alpha_m)}{\Gamma(\alpha_1)\Gamma(\alpha_2)...\Gamma(\alpha_m)}(b_1^*)^{\alpha_1 - 1}(b_2^*)^{\alpha_2 - 1}...(b_m^*)^{\alpha_m - 1}$$

is the joint Dirichlet probability density function of the maximizing vector $\boldsymbol{b}_n^* = \left(b_1^*, b_2^*, \ldots\ldots, b_m^*\right)$ are displayed to show the validity of the asymptotic behaviour, $\dfrac{\hat{S}_n}{S_n^*} \sim \dfrac{2\pi\, g(\mathbf{b}_n^*)}{n\left|J_n^*\right|^{\frac{1}{2}}}$ for large n. The empirical results obtained support the Laplace approximation conjecture (5.25) for different $\boldsymbol{\alpha}$ values of the Dirichlet $(\alpha_1, \alpha_2, \alpha_3)$ universal portfolios where there exist convergent positive sequences $\{\boldsymbol{b}_n^*\}$.

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LIST OF APPENDICES

Appendix 1

The Investment Capitals Achieved by Using the Cover and Ordentlich Universal Portfolio for Digi and LPI

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
1	27/10/2005	1.03	1.01	0.5714	0.4286	1.01980482	0.4167	0.5833	1.01617713	0.5000	0.5000	1.01813050
2	28/10/2005	1.01	1.01	0.5747	0.4253	1.02964799	0.4192	0.5808	1.02687929	0.5012	0.4988	1.02838133
3	31/10/2005	1.00	1.00	0.5739	0.4261	1.02964799	0.4186	0.5814	1.02687929	0.5009	0.4991	1.02838133
4	1/11/2005	1.00	1.00	0.5739	0.4261	1.02964799	0.4186	0.5814	1.02687929	0.5009	0.4991	1.02838133
5	2/11/2005	1.00	1.01	0.5739	0.4261	1.03527231	0.4186	0.5814	1.03453355	0.5009	0.4991	1.03496204
6	3/11/2005	1.00	1.00	0.5721	0.4279	1.03527231	0.4172	0.5828	1.03453355	0.5002	0.4998	1.03496204
7	4/11/2005	1.00	1.00	0.5721	0.4279	1.03527231	0.4172	0.5828	1.03453355	0.5002	0.4998	1.03496204
8	7/11/2005	1.04	1.00	0.5721	0.4279	1.05657737	0.4172	0.5828	1.05005851	0.5002	0.4998	1.05358518
9	8/11/2005	0.98	1.01	0.5772	0.4228	1.04680569	0.4211	0.5789	1.04478169	0.5020	0.4980	1.04598435
10	9/11/2005	0.99	1.01	0.5733	0.4267	1.04542501	0.4181	0.5819	1.04557800	0.5006	0.4994	1.04562351
11	10/11/2005	1.00	1.00	0.5713	0.4287	1.04542501	0.4166	0.5834	1.04557800	0.5000	0.5000	1.04562351
12	11/11/2005	1.02	1.01	0.5713	0.4287	1.06387855	0.4166	0.5834	1.06266625	0.5000	0.5000	1.06344959
13	14/11/2005	1.01	0.99	0.5725	0.4275	1.06942295	0.4175	0.5825	1.06486079	0.5004	0.4996	1.06743444
14	15/11/2005	1.05	0.99	0.5754	0.4246	1.09577838	0.4198	0.5802	1.08213387	0.5014	0.4986	1.08946438
15	16/11/2005	0.97	1.01	0.5830	0.4170	1.08217116	0.4257	0.5743	1.07423177	0.5040	0.4960	1.07873257
16	17/11/2005	0.99	1.01	0.5783	0.4217	1.08392530	0.4220	0.5780	1.07927296	0.5024	0.4976	1.08209059
17	18/11/2005	1.01	1.01	0.5755	0.4245	1.09817974	0.4198	0.5802	1.09339395	0.5014	0.4986	1.09628642
18	21/11/2005	1.00	1.01	0.5756	0.4244	1.10114855	0.4199	0.5801	1.09743399	0.5014	0.4986	1.09976768
19	22/11/2005	1.00	1.01	0.5747	0.4253	1.10413178	0.4192	0.5808	1.10149388	0.5011	0.4989	1.10326223
20	23/11/2005	1.00	1.01	0.5737	0.4263	1.10712952	0.4185	0.5815	1.10557370	0.5008	0.4992	1.10677010
21	24/11/2005	1.00	1.00	0.5728	0.4272	1.10712952	0.4178	0.5822	1.10557370	0.5005	0.4995	1.10677010
22	25/11/2005	1.00	1.00	0.5728	0.4272	1.10712952	0.4178	0.5822	1.10557370	0.5005	0.4995	1.10677010
23	28/11/2005	1.01	0.99	0.5728	0.4272	1.10834518	0.4178	0.5822	1.10455310	0.5005	0.4995	1.10694171
24	29/11/2005	0.99	1.01	0.5747	0.4253	1.10714848	0.4192	0.5808	1.10559863	0.5011	0.4989	1.10680771
25	30/11/2005	1.00	1.01	0.5728	0.4272	1.11016085	0.4178	0.5822	1.10969859	0.5005	0.4995	1.11032910
26	1/12/2005	1.00	1.02	0.5719	0.4281	1.11918446	0.4171	0.5829	1.12198076	0.5002	0.4998	1.12086649

Appendix 1 continued

	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
27	2/12/2005	1.00	1.01	0.5692	0.4308	1.12524914	0.4150	0.5850	1.13023672	0.4992	0.5008	1.12792674
28	5/12/2005	1.03	1.01	0.5674	0.4326	1.14509323	0.4136	0.5864	1.14668150	0.4986	0.5014	1.14626121
29	6/12/2005	1.00	1.02	0.5702	0.4298	1.15426295	0.4158	0.5842	1.15916373	0.4996	0.5004	1.15694925
30	7/12/2005	1.01	1.02	0.5676	0.4324	1.17179249	0.4138	0.5862	1.17774330	0.4987	0.5013	1.17495583
31	8/12/2005	1.01	0.99	0.5668	0.4332	1.17716015	0.4132	0.5868	1.17976465	0.4984	0.5016	1.17883950
32	9/12/2005	0.99	1.01	0.5695	0.4305	1.17907615	0.4152	0.5848	1.18501484	0.4993	0.5007	1.18227142
33	12/12/2005	1.00	1.00	0.5669	0.4331	1.17907615	0.4132	0.5868	1.18501484	0.4984	0.5016	1.18227142
34	13/12/2005	0.98	1.01	0.5669	0.4331	1.16958438	0.4132	0.5868	1.17997236	0.4984	0.5016	1.17475236
35	14/12/2005	1.01	1.01	0.5633	0.4367	1.17682093	0.4105	0.5895	1.18719801	0.4972	0.5028	1.18198851
36	15/12/2005	1.00	1.01	0.5633	0.4367	1.17987988	0.4105	0.5895	1.19136354	0.4972	0.5028	1.18552603
37	16/12/2005	1.00	1.01	0.5625	0.4375	1.18606256	0.4099	0.5901	1.19978318	0.4969	0.5031	1.19266894
38	19/12/2005	1.00	1.00	0.5607	0.4393	1.18606256	0.4086	0.5914	1.19978318	0.4963	0.5037	1.19266894
39	20/12/2005	1.00	1.01	0.5607	0.4393	1.18918235	0.4086	0.5914	1.20403213	0.4963	0.5037	1.19626616
40	21/12/2005	0.98	1.00	0.5599	0.4401	1.17662055	0.4079	0.5921	1.19476499	0.4960	0.5040	1.18507067
41	22/12/2005	0.97	0.99	0.5571	0.4429	1.15350079	0.4058	0.5942	1.17380079	0.4951	0.5049	1.16280685
42	23/12/2005	1.01	1.00	0.5550	0.4450	1.15774072	0.4043	0.5957	1.17694363	0.4944	0.5056	1.16661370
43	26/12/2005	1.00	1.00	0.5560	0.4440	1.15774072	0.4050	0.5950	1.17694363	0.4947	0.5053	1.16661370
44	27/12/2005	1.01	1.00	0.5560	0.4440	1.16626647	0.4050	0.5950	1.18325736	0.4947	0.5053	1.17425743
45	28/12/2005	1.01	1.01	0.5579	0.4421	1.17364404	0.4065	0.5935	1.19065723	0.4953	0.5047	1.18165062
46	29/12/2005	1.01	1.01	0.5580	0.4420	1.18104052	0.4065	0.5935	1.19808287	0.4954	0.5046	1.18906554
47	30/12/2005	1.00	1.00	0.5580	0.4420	1.18104052	0.4066	0.5934	1.19808287	0.4954	0.5046	1.18906554
48	2/1/2006	1.00	1.00	0.5580	0.4420	1.18104052	0.4066	0.5934	1.19808287	0.4954	0.5046	1.18906554
49	3/1/2006	0.99	1.01	0.5580	0.4420	1.17577381	0.4066	0.5934	1.19617346	0.4954	0.5046	1.18517237
50	4/1/2006	1.01	1.02	0.5553	0.4447	1.18952048	0.4045	0.5955	1.21226685	0.4944	0.5056	1.19987172
51	5/1/2006	1.01	1.00	0.5536	0.4464	1.19376894	0.4032	0.5968	1.21542053	0.4939	0.5061	1.20369473
52	6/1/2006	1.01	1.00	0.5545	0.4455	1.19801240	0.4039	0.5961	1.21856760	0.4942	0.5058	1.20750781
53	9/1/2006	1.02	1.00	0.5555	0.4445	1.21072794	0.4046	0.5954	1.22798920	0.4945	0.5055	1.21891758
54	10/1/2006	1.00	1.00	0.5582	0.4418	1.21072794	0.4067	0.5933	1.22798920	0.4954	0.5046	1.21891758
55	11/1/2006	1.00	0.99	0.5582	0.4418	1.20443515	0.4067	0.5933	1.21941775	0.4954	0.5046	1.21168215
56	12/1/2006	1.00	0.99	0.5599	0.4401	1.20128016	0.4080	0.5920	1.21512068	0.4960	0.5040	1.20804736
57	13/1/2006	0.99	1.00	0.5608	0.4392	1.19275268	0.4086	0.5914	1.20883523	0.4963	0.5037	1.20045754
58	16/1/2006	1.00	0.99	0.5589	0.4411	1.18962132	0.4072	0.5928	1.20457012	0.4957	0.5043	1.19685401
59	17/1/2006	0.99	0.99	0.5598	0.4402	1.18224384	0.4079	0.5921	1.19716972	0.4960	0.5040	1.18946081
60	18/1/2006	1.00	1.00	0.5598	0.4402	1.18224384	0.4079	0.5921	1.19716972	0.4960	0.5040	1.18946081

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
61	19/1/2006	0.99	1.01	0.5598	0.4402	1.18113713	0.4079	0.5921	1.19831016	0.4960	0.5040	1.18929065
62	20/1/2006	1.02	0.99	0.5579	0.4421	1.19082977	0.4065	0.5935	1.20351509	0.4954	0.5046	1.19715157
63	23/1/2006	0.99	1.01	0.5616	0.4384	1.18973372	0.4093	0.5907	1.20468645	0.4966	0.5034	1.19701699
64	24/1/2006	1.00	1.01	0.5598	0.4402	1.19288851	0.4079	0.5921	1.20898330	0.4960	0.5040	1.20065130
65	25/1/2006	0.99	1.00	0.5589	0.4411	1.18866849	0.4073	0.5927	1.20586704	0.4957	0.5043	1.19688445
66	26/1/2006	0.99	1.01	0.5580	0.4420	1.18763514	0.4066	0.5934	1.20707498	0.4954	0.5046	1.19677018
67	27/1/2006	1.00	1.00	0.5562	0.4438	1.18763514	0.4052	0.5948	1.20707498	0.4948	0.5052	1.19677018
68	30/1/2006	1.00	1.00	0.5562	0.4438	1.18763514	0.4052	0.5948	1.20707498	0.4948	0.5052	1.19677018
69	31/1/2006	1.00	1.00	0.5562	0.4438	1.18763514	0.4052	0.5948	1.20707498	0.4948	0.5052	1.19677018
70	1/2/2006	1.00	1.00	0.5562	0.4438	1.18763514	0.4052	0.5948	1.20707498	0.4948	0.5052	1.19677018
71	2/2/2006	1.00	1.00	0.5562	0.4438	1.18763514	0.4052	0.5948	1.20707498	0.4948	0.5052	1.19677018
72	3/2/2006	0.97	1.00	0.5562	0.4438	1.16673010	0.4052	0.5948	1.19159634	0.4948	0.5052	1.17803218
73	6/2/2006	1.01	0.99	0.5515	0.4485	1.17193536	0.4017	0.5983	1.19351836	0.4932	0.5068	1.18198024
74	7/2/2006	1.01	0.99	0.5543	0.4457	1.17296079	0.4038	0.5962	1.19231479	0.4941	0.5059	1.18212415
75	8/2/2006	1.01	0.99	0.5561	0.4439	1.17083141	0.4051	0.5949	1.18681430	0.4947	0.5053	1.17863320
76	9/2/2006	1.01	1.01	0.5588	0.4412	1.18129815	0.4072	0.5928	1.19847242	0.4957	0.5043	1.18960340
77	10/2/2006	0.99	0.97	0.5580	0.4420	1.16399049	0.4065	0.5935	1.17749586	0.4954	0.5046	1.17077158
78	13/2/2006	1.00	1.00	0.5608	0.4392	1.16399049	0.4086	0.5914	1.17749586	0.4963	0.5037	1.17077158
79	14/2/2006	1.00	1.00	0.5608	0.4392	1.16399049	0.4086	0.5914	1.17749586	0.4963	0.5037	1.17077158
80	15/2/2006	0.99	1.00	0.5608	0.4392	1.15985933	0.4086	0.5914	1.17445046	0.4963	0.5037	1.16709383
81	16/2/2006	1.04	0.98	0.5598	0.4402	1.17880972	0.4079	0.5921	1.18230499	0.4960	0.5040	1.18142923
82	17/2/2006	1.01	0.99	0.5689	0.4311	1.18029595	0.4149	0.5851	1.17920680	0.4991	0.5009	1.18084223
83	20/2/2006	1.01	0.99	0.5725	0.4275	1.17759136	0.4176	0.5824	1.17298188	0.5004	0.4996	1.17649360
84	21/2/2006	1.00	1.01	0.5753	0.4247	1.18097039	0.4198	0.5802	1.17758045	0.5014	0.4986	1.18045733
85	22/2/2006	1.01	1.01	0.5744	0.4256	1.19237121	0.4190	0.5810	1.18801156	0.5010	0.4990	1.19140981
86	23/2/2006	1.04	0.99	0.5751	0.4249	1.21303098	0.4196	0.5804	1.20087457	0.5013	0.4987	1.20837796
87	24/2/2006	0.99	1.01	0.5810	0.4190	1.20850201	0.4242	0.5758	1.19979113	0.5033	0.4967	1.20556025
88	27/2/2006	1.00	0.99	0.5784	0.4216	1.20166293	0.4222	0.5778	1.19048534	0.5024	0.4976	1.19750880
89	28/2/2006	1.00	1.01	0.5803	0.4197	1.20511708	0.4237	0.5763	1.19518478	0.5031	0.4969	1.20158427
90	1/3/2006	0.98	1.01	0.5794	0.4206	1.19662032	0.4229	0.5771	1.19124405	0.5028	0.4972	1.19531987
91	2/3/2006	1.02	1.01	0.5759	0.4241	1.21211595	0.4202	0.5798	1.20470592	0.5016	0.4984	1.20985767
92	3/3/2006	0.90	1.00	0.5774	0.4226	1.14012951	0.4214	0.5786	1.15248965	0.5021	0.4979	1.14737545
93	6/3/2006	1.04	0.99	0.5617	0.4383	1.16100378	0.4095	0.5905	1.16571692	0.4967	0.5033	1.16503357
94	7/3/2006	1.00	0.99	0.5681	0.4319	1.15752163	0.4143	0.5857	1.16097582	0.4989	0.5011	1.16097930

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
95	8/3/2006	1.00	1.00	0.5691	0.4309	1.15752163	0.4151	0.5849	1.16097582	0.4992	0.5008	1.16097930
96	9/3/2006	1.01	1.01	0.5691	0.4309	1.16505095	0.4151	0.5849	1.16868102	0.4992	0.5008	1.16860072
97	10/3/2006	1.02	1.01	0.5690	0.4310	1.18415149	0.4150	0.5850	1.18704860	0.4992	0.5008	1.18740025
98	13/3/2006	0.99	1.01	0.5696	0.4304	1.17956425	0.4155	0.5845	1.18589433	0.4994	0.5006	1.18436977
99	14/3/2006	1.01	1.00	0.5669	0.4331	1.18361687	0.4134	0.5866	1.18886561	0.4985	0.5015	1.18794774
100	15/3/2006	1.01	1.01	0.5678	0.4322	1.19114504	0.4141	0.5859	1.19658453	0.4988	0.5012	1.19557405
101	16/3/2006	1.01	1.00	0.5676	0.4324	1.19919433	0.4140	0.5860	1.20248172	0.4987	0.5013	1.20267239
102	17/3/2006	0.99	1.00	0.5693	0.4307	1.19520161	0.4153	0.5847	1.19956141	0.4993	0.5007	1.19916063
103	20/3/2006	1.01	0.99	0.5685	0.4315	1.19566610	0.4146	0.5854	1.19767778	0.4990	0.5010	1.19856592
104	21/3/2006	1.02	0.99	0.5703	0.4297	1.20807466	0.4160	0.5840	1.20451020	0.4997	0.5003	1.20843883
105	22/3/2006	1.00	1.00	0.5747	0.4253	1.20807466	0.4194	0.5806	1.20451020	0.5012	0.4988	1.20843883
106	23/3/2006	1.02	0.99	0.5747	0.4253	1.21633958	0.4194	0.5806	1.20826354	0.5012	0.4988	1.21457563
107	24/3/2006	1.00	1.01	0.5781	0.4219	1.22351731	0.4220	0.5780	1.21803087	0.5024	0.4976	1.22302924
108	27/3/2006	0.99	0.99	0.5761	0.4239	1.20853685	0.4205	0.5795	1.20263892	0.5017	0.4983	1.20782488
109	28/3/2006	0.99	0.99	0.5764	0.4236	1.20109284	0.4208	0.5792	1.19499184	0.5018	0.4982	1.20026996
110	29/3/2006	0.99	0.99	0.5766	0.4234	1.18976005	0.4209	0.5791	1.18453600	0.5018	0.4982	1.18934014
111	30/3/2006	0.98	1.00	0.5760	0.4240	1.17801211	0.4204	0.5796	1.17599911	0.5016	0.4984	1.17911263
112	31/3/2006	0.99	0.99	0.5735	0.4265	1.16664542	0.4185	0.5815	1.16552677	0.5008	0.4992	1.16814701
113	3/4/2006	1.01	0.99	0.5728	0.4272	1.16713940	0.4180	0.5820	1.16371380	0.5005	0.4995	1.16756242
114	4/4/2006	1.03	1.00	0.5747	0.4253	1.18675051	0.4194	0.5806	1.17798367	0.5012	0.4988	1.18467164
115	5/4/2006	1.01	1.01	0.5788	0.4212	1.19412457	0.4226	0.5774	1.18553560	0.5026	0.4974	1.19214674
116	6/4/2006	1.04	0.99	0.5786	0.4214	1.22119219	0.4224	0.5776	1.20294829	0.5025	0.4975	1.21453616
117	7/4/2006	1.00	1.01	0.5858	0.4142	1.22816982	0.4280	0.5720	1.21243892	0.5050	0.4950	1.22282783
118	10/4/2006	1.01	1.00	0.5838	0.4162	1.23196351	0.4265	0.5735	1.21517485	0.5044	0.4956	1.22609101
119	11/4/2006	1.00	1.00	0.5846	0.4154	1.23196351	0.4271	0.5729	1.21517485	0.5046	0.4954	1.22609101
120	12/4/2006	1.01	1.00	0.5846	0.4154	1.23575375	0.4271	0.5729	1.21790628	0.5046	0.4954	1.22934738
121	13/4/2006	1.00	0.99	0.5853	0.4147	1.23226756	0.4277	0.5723	1.21316440	0.5049	0.4951	1.22520676
122	14/4/2006	0.99	1.00	0.5863	0.4137	1.22848515	0.4284	0.5716	1.21044322	0.5052	0.4948	1.22196591
123	17/4/2006	0.99	1.00	0.5855	0.4145	1.22469934	0.4278	0.5722	1.20771758	0.5050	0.4950	1.21871830
124	18/4/2006	1.03	0.98	0.5848	0.4152	1.23309665	0.4272	0.5728	1.20708296	0.5047	0.4953	1.22250129
125	19/4/2006	1.02	1.01	0.5914	0.4086	1.24779067	0.4325	0.5675	1.21983866	0.5070	0.4930	1.23619361
126	20/4/2006	1.00	1.00	0.5926	0.4074	1.24779067	0.4334	0.5666	1.21983866	0.5074	0.4926	1.23619361
127	21/4/2006	1.04	1.02	0.5926	0.4074	1.28431841	0.4334	0.5666	1.25273539	0.5074	0.4926	1.27085732
128	24/4/2006	1.05	0.99	0.5946	0.4054	1.31748864	0.4350	0.5650	1.27411852	0.5082	0.4918	1.29765271

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
129	25/4/2006	1.00	1.01	0.6021	0.3979	1.32107912	0.4410	0.5590	1.27899658	0.5108	0.4892	1.30200042
130	26/4/2006	1.04	1.00	0.6012	0.3988	1.35022324	0.4403	0.5597	1.29966052	0.5105	0.4895	1.32639174
131	27/4/2006	0.97	1.00	0.6062	0.3938	1.32849356	0.4443	0.5557	1.28432963	0.5123	0.4877	1.30835196
132	28/4/2006	1.00	1.01	0.6024	0.3976	1.33213612	0.4413	0.5587	1.28927835	0.5109	0.4891	1.31276473
133	1/5/2006	1.00	1.00	0.6015	0.3985	1.33213612	0.4405	0.5595	1.28927835	0.5106	0.4894	1.31276473
134	2/5/2006	1.01	0.99	0.6015	0.3985	1.33565371	0.4405	0.5595	1.28940878	0.5106	0.4894	1.31434920
135	3/5/2006	0.99	1.00	0.6037	0.3963	1.32851837	0.4423	0.5577	1.28436191	0.5114	0.4886	1.30840099
136	4/5/2006	1.02	1.01	0.6024	0.3976	1.34620199	0.4413	0.5587	1.29925427	0.5109	0.4891	1.32454244
137	5/5/2006	0.97	0.99	0.6039	0.3961	1.31451637	0.4425	0.5575	1.27446906	0.5115	0.4885	1.29674946
138	8/5/2006	1.02	1.01	0.5999	0.4001	1.33195404	0.4393	0.5607	1.28918601	0.5101	0.4899	1.31270497
139	9/5/2006	0.98	0.99	0.6014	0.3986	1.31364404	0.4405	0.5595	1.27385935	0.5106	0.4894	1.29603627
140	10/5/2006	1.01	1.01	0.5998	0.4002	1.32820953	0.4392	0.5608	1.28887426	0.5100	0.4900	1.31091301
141	11/5/2006	0.99	1.01	0.5992	0.4008	1.32446217	0.4387	0.5613	1.28856022	0.5098	0.4902	1.30909662
142	12/5/2006	1.00	1.00	0.5969	0.4031	1.32446217	0.4369	0.5631	1.28856022	0.5090	0.4910	1.30909662
143	15/5/2006	0.96	0.99	0.5969	0.4031	1.29099637	0.4369	0.5631	1.26238018	0.5090	0.4910	1.27957984
144	16/5/2006	0.99	0.99	0.5925	0.4075	1.27991857	0.4334	0.5666	1.25215072	0.5074	0.4926	1.26892660
145	17/5/2006	1.01	1.00	0.5920	0.4080	1.28742110	0.4330	0.5670	1.25751907	0.5073	0.4927	1.27529966
146	18/5/2006	0.99	0.99	0.5934	0.4066	1.27627170	0.4341	0.5659	1.24723998	0.5078	0.4922	1.26458865
147	19/5/2006	1.01	1.00	0.5930	0.4070	1.28383987	0.4338	0.5662	1.25265020	0.5076	0.4924	1.27100771
148	22/5/2006	0.99	0.98	0.5944	0.4056	1.26551058	0.4349	0.5651	1.23261059	0.5081	0.4919	1.25167827
149	23/5/2006	1.01	1.01	0.5959	0.4041	1.27287619	0.4361	0.5639	1.24017238	0.5086	0.4914	1.25917840
150	24/5/2006	1.01	1.01	0.5957	0.4043	1.28403234	0.4359	0.5641	1.25043649	0.5085	0.4915	1.26987932
151	25/5/2006	1.00	1.01	0.5961	0.4039	1.29123562	0.4362	0.5638	1.26022745	0.5087	0.4913	1.27854453
152	26/5/2006	1.01	1.01	0.5941	0.4059	1.30611000	0.4347	0.5653	1.27554599	0.5080	0.4920	1.29371191
153	29/5/2006	1.00	0.99	0.5936	0.4064	1.30247427	0.4343	0.5657	1.27060327	0.5078	0.4922	1.28935061
154	30/5/2006	1.01	0.99	0.5946	0.4054	1.30639918	0.4350	0.5650	1.27103713	0.5082	0.4918	1.29137015
155	31/5/2006	0.98	1.01	0.5969	0.4031	1.29862229	0.4369	0.5631	1.27026454	0.5090	0.4910	1.28747536
156	1/6/2006	1.01	1.01	0.5922	0.4078	1.30983952	0.4331	0.5669	1.28065915	0.5073	0.4927	1.29828386
157	2/6/2006	1.02	1.01	0.5926	0.4074	1.32847219	0.4335	0.5665	1.29637304	0.5075	0.4925	1.31539428
158	5/6/2006	1.00	1.00	0.5943	0.4057	1.32847219	0.4348	0.5652	1.29637304	0.5081	0.4919	1.31539428
159	6/6/2006	0.99	1.00	0.5943	0.4057	1.32088077	0.4348	0.5652	1.29095290	0.5081	0.4919	1.30896827
160	7/6/2006	1.00	0.99	0.5929	0.4071	1.31714684	0.4337	0.5663	1.28587644	0.5076	0.4924	1.30449216
161	8/6/2006	1.01	1.01	0.5939	0.4061	1.32850845	0.4345	0.5655	1.29642178	0.5079	0.4921	1.31544552
162	9/6/2006	1.01	1.00	0.5943	0.4057	1.33617369	0.4348	0.5652	1.30189468	0.5081	0.4919	1.32193414

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
163	12/6/2006	1.00	0.99	0.5957	0.4043	1.33242175	0.4359	0.5641	1.29679470	0.5085	0.4915	1.31742253
164	13/6/2006	0.99	1.00	0.5966	0.4034	1.32470364	0.4367	0.5633	1.29129677	0.5089	0.4911	1.31091354
165	14/6/2006	1.00	1.01	0.5953	0.4047	1.32847944	0.4356	0.5644	1.29642932	0.5084	0.4916	1.31545184
166	15/6/2006	0.98	0.99	0.5943	0.4057	1.30536031	0.4348	0.5652	1.27748930	0.5081	0.4919	1.29454602
167	16/6/2006	1.00	1.01	0.5918	0.4082	1.31281353	0.4328	0.5672	1.28762321	0.5072	0.4928	1.30346902
168	19/6/2006	0.99	0.99	0.5898	0.4102	1.30520866	0.4313	0.5687	1.27978226	0.5065	0.4935	1.29571505
169	20/6/2006	1.00	1.01	0.5901	0.4099	1.30892437	0.4315	0.5685	1.28483497	0.5066	0.4934	1.30015496
170	21/6/2006	1.01	1.01	0.5891	0.4109	1.32048755	0.4307	0.5693	1.29553259	0.5062	0.4938	1.31129511
171	22/6/2006	1.00	1.00	0.5895	0.4105	1.32048755	0.4311	0.5689	1.29553259	0.5064	0.4936	1.31129511
172	23/6/2006	1.03	1.00	0.5895	0.4105	1.34004692	0.4311	0.5689	1.30956392	0.5064	0.4936	1.32797891
173	26/6/2006	1.01	0.99	0.5930	0.4070	1.34407697	0.4338	0.5662	1.31002046	0.5076	0.4924	1.33007835
174	27/6/2006	1.02	1.00	0.5954	0.4046	1.35976800	0.4357	0.5643	1.32121208	0.5085	0.4915	1.34333874
175	28/6/2006	1.01	1.00	0.5981	0.4019	1.36766407	0.4379	0.5621	1.32682883	0.5094	0.4906	1.34998266
176	29/6/2006	1.02	1.02	0.5995	0.4005	1.39461431	0.4390	0.5610	1.35332410	0.5099	0.4901	1.37678302
177	30/6/2006	1.02	1.01	0.5992	0.4008	1.41401148	0.4388	0.5612	1.36955520	0.5098	0.4902	1.39446283
178	3/7/2006	1.01	1.02	0.6009	0.3991	1.43316918	0.4401	0.5599	1.39052411	0.5104	0.4896	1.41473858
179	4/7/2006	1.00	1.01	0.5994	0.4006	1.44072386	0.4389	0.5611	1.40079040	0.5099	0.4901	1.42386234
180	5/7/2006	0.99	0.99	0.5975	0.4025	1.42920292	0.4374	0.5626	1.39015301	0.5092	0.4908	1.41279260
181	6/7/2006	1.01	1.01	0.5972	0.4028	1.44082072	0.4371	0.5629	1.40087593	0.5091	0.4909	1.42395398
182	7/7/2006	1.02	0.99	0.5976	0.4024	1.45265975	0.4374	0.5626	1.40683261	0.5092	0.4908	1.43254004
183	10/7/2006	0.99	0.99	0.6010	0.3990	1.43723822	0.4402	0.5598	1.39094078	0.5104	0.4896	1.41678347
184	11/7/2006	1.00	1.00	0.6016	0.3984	1.43723822	0.4407	0.5593	1.39094078	0.5107	0.4893	1.41678347
185	12/7/2006	1.01	1.01	0.6016	0.3984	1.44891591	0.4407	0.5593	1.40169966	0.5107	0.4893	1.42798262
186	13/7/2006	1.00	1.00	0.6019	0.3981	1.44891591	0.4409	0.5591	1.40169966	0.5108	0.4892	1.42798262
187	14/7/2006	1.01	1.00	0.6019	0.3981	1.45684459	0.4409	0.5591	1.40731848	0.5108	0.4892	1.43461341
188	17/7/2006	1.00	0.99	0.6032	0.3968	1.45299075	0.4420	0.5580	1.40208288	0.5112	0.4888	1.42993879
189	18/7/2006	1.01	1.01	0.6041	0.3959	1.46869143	0.4427	0.5573	1.41821388	0.5116	0.4884	1.44596377
190	19/7/2006	1.01	0.99	0.6035	0.3965	1.47289351	0.4422	0.5578	1.41867686	0.5114	0.4886	1.44800630
191	20/7/2006	1.03	1.01	0.6057	0.3943	1.50063668	0.4440	0.5560	1.44077266	0.5121	0.4879	1.47254825
192	21/7/2006	1.05	1.01	0.6085	0.3915	1.55484766	0.4462	0.5538	1.48403111	0.5131	0.4869	1.52046452
193	24/7/2006	1.03	1.01	0.6135	0.3865	1.58264920	0.4503	0.5497	1.50610581	0.5149	0.4851	1.54489024
194	25/7/2006	1.00	1.01	0.6160	0.3840	1.58662121	0.4524	0.5476	1.51149631	0.5158	0.4842	1.54977895
195	26/7/2006	1.00	1.00	0.6151	0.3849	1.58662121	0.4517	0.5483	1.51149631	0.5155	0.4845	1.54977895
196	27/7/2006	1.03	1.00	0.6151	0.3849	1.61888421	0.4517	0.5483	1.53406441	0.5155	0.4845	1.57619007

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
197	28/7/2006	0.98	1.02	0.6196	0.3804	1.61470378	0.4553	0.5447	1.53907460	0.5171	0.4829	1.57786960
198	31/7/2006	0.99	1.01	0.6147	0.3853	1.61455996	0.4513	0.5487	1.54418461	0.5154	0.4846	1.58099969
199	1/8/2006	1.00	1.00	0.6118	0.3882	1.61455996	0.4490	0.5510	1.54418461	0.5143	0.4857	1.58099969
200	2/8/2006	0.98	0.99	0.6118	0.3882	1.59426572	0.4490	0.5510	1.52733977	0.5143	0.4857	1.56269938
201	3/8/2006	1.00	1.01	0.6104	0.3896	1.60249282	0.4478	0.5522	1.53851038	0.5138	0.4862	1.57276248
202	4/8/2006	1.00	0.99	0.6086	0.3914	1.59836601	0.4463	0.5537	1.53290623	0.5132	0.4868	1.56772506
203	7/8/2006	1.01	0.99	0.6095	0.3905	1.60231045	0.4471	0.5529	1.53299379	0.5135	0.4865	1.56936010
204	8/8/2006	1.00	0.99	0.6115	0.3885	1.59821539	0.4487	0.5513	1.52743411	0.5142	0.4858	1.56434460
205	9/8/2006	1.02	1.00	0.6124	0.3876	1.61480549	0.4495	0.5505	1.53907084	0.5146	0.4854	1.57798765
206	10/8/2006	1.02	1.00	0.6148	0.3852	1.63135077	0.4514	0.5486	1.55064950	0.5154	0.4846	1.59154232
207	11/8/2006	0.99	1.00	0.6170	0.3830	1.62303181	0.4533	0.5467	1.54484087	0.5162	0.4838	1.58475243
208	14/8/2006	1.00	1.00	0.6159	0.3841	1.62303181	0.4523	0.5477	1.54484087	0.5158	0.4842	1.58475243
209	15/8/2006	0.99	0.99	0.6159	0.3841	1.61060027	0.4523	0.5477	1.53345154	0.5158	0.4842	1.57289237
210	16/8/2006	1.01	1.00	0.6156	0.3844	1.61893272	0.4521	0.5479	1.53927763	0.5157	0.4843	1.57970884
211	17/8/2006	1.00	1.00	0.6168	0.3832	1.61893272	0.4531	0.5469	1.53927763	0.5161	0.4839	1.57970884
212	18/8/2006	0.99	1.01	0.6168	0.3832	1.61462296	0.4531	0.5469	1.53895586	0.5161	0.4839	1.57788608
213	21/8/2006	1.01	1.00	0.6147	0.3853	1.62303452	0.4514	0.5486	1.54484266	0.5154	0.4846	1.58477773
214	22/8/2006	0.99	1.00	0.6159	0.3841	1.61463435	0.4523	0.5477	1.53897062	0.5158	0.4842	1.57790855
215	23/8/2006	0.99	1.00	0.6147	0.3853	1.60622274	0.4514	0.5486	1.53308377	0.5154	0.4846	1.57101680
216	24/8/2006	1.00	1.01	0.6136	0.3864	1.61438994	0.4504	0.5496	1.54417016	0.5150	0.4850	1.58104323
217	25/8/2006	1.00	1.00	0.6118	0.3882	1.61438994	0.4489	0.5511	1.54417016	0.5143	0.4857	1.58104323
218	28/8/2006	1.00	1.01	0.6118	0.3882	1.61845993	0.4489	0.5511	1.54969572	0.5143	0.4857	1.58602960
219	29/8/2006	1.00	1.01	0.6109	0.3891	1.62658648	0.4482	0.5518	1.56072944	0.5140	0.4860	1.59597585
220	30/8/2006	1.00	1.01	0.6091	0.3909	1.63063654	0.4468	0.5532	1.56622917	0.5133	0.4867	1.60092291
221	31/8/2006	1.00	1.00	0.6082	0.3918	1.63063654	0.4460	0.5540	1.56622917	0.5130	0.4870	1.60092291
222	1/9/2006	0.99	1.01	0.6082	0.3918	1.62627535	0.4460	0.5540	1.56580003	0.5130	0.4870	1.59889675
223	4/9/2006	0.99	1.00	0.6061	0.3939	1.61777747	0.4444	0.5556	1.55980172	0.5123	0.4877	1.59183554
224	5/9/2006	1.01	1.00	0.6049	0.3951	1.62628749	0.4434	0.5566	1.56581580	0.5119	0.4881	1.59892073
225	6/9/2006	1.01	0.99	0.6061	0.3939	1.63075696	0.4444	0.5556	1.56634246	0.5123	0.4877	1.60107759
226	7/9/2006	1.01	0.99	0.6082	0.3918	1.63521593	0.4461	0.5539	1.56685700	0.5130	0.4870	1.60319455
227	8/9/2006	1.01	1.00	0.6103	0.3897	1.64360176	0.4477	0.5523	1.57275214	0.5138	0.4862	1.61011619
228	11/9/2006	1.00	1.01	0.6114	0.3886	1.64764396	0.4487	0.5513	1.57824015	0.5142	0.4858	1.61506691
229	12/9/2006	0.99	1.00	0.6105	0.3895	1.63933018	0.4480	0.5520	1.57239730	0.5139	0.4861	1.60820790
230	13/9/2006	1.00	1.01	0.6094	0.3906	1.64340867	0.4470	0.5530	1.57793552	0.5135	0.4865	1.61319170

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
231	14/9/2006	1.01	1.00	0.6085	0.3915	1.65174238	0.4463	0.5537	1.58380421	0.5131	0.4869	1.62009003
232	15/9/2006	0.99	1.00	0.6097	0.3903	1.64341993	0.4472	0.5528	1.57795012	0.5136	0.4864	1.61321392
233	18/9/2006	1.00	1.01	0.6085	0.3915	1.64746628	0.4463	0.5537	1.58344509	0.5131	0.4869	1.61815356
234	19/9/2006	1.00	1.01	0.6076	0.3924	1.65150620	0.4456	0.5544	1.58893171	0.5128	0.4872	1.62308052
235	20/9/2006	1.00	1.03	0.6068	0.3932	1.66774126	0.4449	0.5551	1.61098213	0.5125	0.4875	1.64286096
236	21/9/2006	1.00	1.01	0.6033	0.3967	1.67575977	0.4421	0.5579	1.62187583	0.5113	0.4887	1.65259287
237	22/9/2006	1.00	1.01	0.6017	0.3983	1.68375414	0.4408	0.5592	1.63273813	0.5107	0.4893	1.66227707
238	25/9/2006	1.01	1.00	0.6000	0.4000	1.69224350	0.4394	0.5606	1.63876737	0.5101	0.4899	1.66940246
239	26/9/2006	1.02	1.01	0.6012	0.3988	1.71324017	0.4404	0.5596	1.65628679	0.5105	0.4895	1.68849977
240	27/9/2006	0.99	1.00	0.6026	0.3974	1.70477738	0.4416	0.5584	1.65029213	0.5110	0.4890	1.68142690
241	28/9/2006	1.00	1.01	0.6015	0.3985	1.70887003	0.4406	0.5594	1.65585307	0.5106	0.4894	1.68638378
242	29/9/2006	1.02	1.01	0.6006	0.3994	1.72969588	0.4400	0.5600	1.67331902	0.5103	0.4897	1.70538215
243	2/10/2006	1.06	1.01	0.6021	0.3979	1.79313266	0.4411	0.5589	1.72099222	0.5108	0.4892	1.76001649
244	3/10/2006	0.98	1.00	0.6089	0.3911	1.76793589	0.4466	0.5534	1.70325350	0.5133	0.4867	1.73916888
245	4/10/2006	1.01	1.01	0.6057	0.3943	1.78065933	0.4440	0.5560	1.71499495	0.5121	0.4879	1.75138011
246	5/10/2006	0.99	1.01	0.6059	0.3941	1.77641649	0.4442	0.5558	1.71477274	0.5122	0.4878	1.74949394
247	6/10/2006	0.98	1.01	0.6040	0.3960	1.76367556	0.4427	0.5573	1.70855160	0.5115	0.4885	1.74049998
248	9/10/2006	0.99	0.99	0.6009	0.3991	1.75093128	0.4402	0.5598	1.69673817	0.5104	0.4896	1.72822856
249	10/10/2006	1.01	1.01	0.6006	0.3994	1.76360838	0.4400	0.5600	1.70850431	0.5103	0.4897	1.74044441
250	11/10/2006	1.01	1.00	0.6009	0.3991	1.77201892	0.4402	0.5598	1.71447289	0.5104	0.4896	1.74749486
251	12/10/2006	0.99	1.00	0.6020	0.3980	1.76327515	0.4411	0.5589	1.70827461	0.5108	0.4892	1.74017811
252	13/10/2006	1.00	1.01	0.6008	0.3992	1.76756678	0.4401	0.5599	1.71410630	0.5104	0.4896	1.74537314
253	16/10/2006	1.00	1.00	0.6000	0.4000	1.76756678	0.4395	0.5605	1.71410630	0.5101	0.4899	1.74537314
254	17/10/2006	1.02	1.00	0.6000	0.4000	1.78524213	0.4395	0.5605	1.72666088	0.5101	0.4899	1.76021172
255	18/10/2006	1.00	1.00	0.6023	0.3977	1.78524213	0.4413	0.5587	1.72666088	0.5109	0.4891	1.76021172
256	19/10/2006	1.01	1.00	0.6023	0.3977	1.79398402	0.4413	0.5587	1.73285599	0.5109	0.4891	1.76752338
257	20/10/2006	1.00	1.01	0.6034	0.3966	1.79829576	0.4422	0.5578	1.73871385	0.5113	0.4887	1.77275817
258	23/10/2006	1.00	1.00	0.6026	0.3974	1.79829576	0.4415	0.5585	1.73871385	0.5110	0.4890	1.77275817
259	24/10/2006	1.00	1.00	0.6026	0.3974	1.79829576	0.4415	0.5585	1.73871385	0.5110	0.4890	1.77275817
260	25/10/2006	1.00	1.00	0.6026	0.3974	1.79829576	0.4415	0.5585	1.73871385	0.5110	0.4890	1.77275817
261	26/10/2006	0.98	1.00	0.6026	0.3974	1.77207890	0.4415	0.5585	1.72014000	0.5110	0.4890	1.75084061
262	27/10/2006	0.99	1.01	0.5992	0.4008	1.77186218	0.4388	0.5612	1.72553275	0.5098	0.4902	1.75380426
263	30/10/2006	0.99	1.01	0.5963	0.4037	1.77157264	0.4365	0.5635	1.73083069	0.5088	0.4912	1.75662391
264	31/10/2006	1.00	1.01	0.5935	0.4065	1.78004569	0.4343	0.5657	1.74235057	0.5078	0.4922	1.76679607

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
265	1/11/2006	1.01	1.01	0.5918	0.4082	1.79322234	0.4330	0.5670	1.75452119	0.5072	0.4928	1.77948199
266	2/11/2006	1.02	1.00	0.5922	0.4078	1.81122083	0.4333	0.5667	1.76740504	0.5073	0.4927	1.79478354
267	3/11/2006	1.00	1.02	0.5945	0.4055	1.82849988	0.4351	0.5649	1.79089570	0.5082	0.4918	1.81555355
268	6/11/2006	0.99	1.02	0.5913	0.4087	1.83229792	0.4325	0.5675	1.80195439	0.5070	0.4930	1.82327368
269	7/11/2006	1.01	1.03	0.5876	0.4124	1.86725939	0.4297	0.5703	1.84360753	0.5057	0.4943	1.86187712
270	8/11/2006	1.00	1.00	0.5841	0.4159	1.86725939	0.4269	0.5731	1.84360753	0.5045	0.4955	1.86187712
271	9/11/2006	1.03	0.99	0.5841	0.4159	1.89960387	0.4269	0.5731	1.86419160	0.5045	0.4955	1.88832417
272	10/11/2006	0.99	1.00	0.5895	0.4105	1.89049892	0.4312	0.5688	1.85765644	0.5064	0.4936	1.88054967
273	13/11/2006	1.01	1.00	0.5884	0.4116	1.89969193	0.4303	0.5697	1.86426236	0.5060	0.4940	1.88841381
274	14/11/2006	0.99	1.00	0.5896	0.4104	1.89051179	0.4312	0.5688	1.85767326	0.5064	0.4936	1.88057516
275	15/11/2006	1.02	0.99	0.5884	0.4116	1.91374378	0.4303	0.5697	1.87157865	0.5060	0.4940	1.89897795
276	16/11/2006	1.05	0.99	0.5926	0.4074	1.95986370	0.4336	0.5664	1.89893965	0.5075	0.4925	1.93510193
277	17/11/2006	1.02	1.02	0.6009	0.3991	2.00481781	0.4402	0.5598	1.94238966	0.5104	0.4896	1.97942687
278	20/11/2006	1.00	1.00	0.6009	0.3991	2.00481781	0.4402	0.5598	1.94238966	0.5104	0.4896	1.97942687
279	21/11/2006	0.99	1.01	0.6009	0.3991	2.00025479	0.4402	0.5598	1.94206824	0.5104	0.4896	1.97727423
280	22/11/2006	1.02	0.99	0.5991	0.4009	2.02300829	0.4388	0.5612	1.95534579	0.5098	0.4902	1.99476771
281	23/11/2006	1.00	1.01	0.6030	0.3970	2.02752038	0.4419	0.5581	1.96147626	0.5112	0.4888	2.00024571
282	24/11/2006	1.00	1.00	0.6022	0.3978	2.02752038	0.4413	0.5587	1.96147626	0.5109	0.4891	2.00024571
283	27/11/2006	1.00	1.01	0.6022	0.3978	2.03653189	0.4413	0.5587	1.97372073	0.5109	0.4891	2.01117665
284	28/11/2006	1.01	0.99	0.6007	0.3993	2.04109976	0.4401	0.5599	1.97404837	0.5103	0.4897	2.01333883
285	29/11/2006	1.00	1.00	0.6025	0.3975	2.04109976	0.4415	0.5585	1.97404837	0.5110	0.4890	2.01333883
286	30/11/2006	1.00	1.00	0.6025	0.3975	2.04109976	0.4415	0.5585	1.97404837	0.5110	0.4890	2.01333883
287	1/12/2006	1.03	1.01	0.6025	0.3975	2.08179994	0.4415	0.5585	2.00584134	0.5110	0.4890	2.04909780
288	4/12/2006	1.06	0.99	0.6057	0.3943	2.15830381	0.4441	0.5559	2.05691701	0.5122	0.4878	2.11100951
289	5/12/2006	0.98	1.00	0.6151	0.3849	2.13157431	0.4518	0.5482	2.03820608	0.5155	0.4845	2.08909749
290	6/12/2006	0.98	0.99	0.6123	0.3877	2.10032176	0.4495	0.5505	2.01324023	0.5145	0.4855	2.06149506
291	7/12/2006	0.97	1.01	0.6102	0.3898	2.06931965	0.4478	0.5522	1.99444264	0.5138	0.4862	2.03770565
292	8/12/2006	1.02	1.00	0.6055	0.3945	2.09617019	0.4440	0.5560	2.01341881	0.5121	0.4879	2.06006619
293	11/12/2006	1.02	0.99	0.6085	0.3915	2.11380891	0.4464	0.5536	2.01988966	0.5131	0.4869	2.07109960
294	12/12/2006	1.01	1.00	0.6129	0.3871	2.12268230	0.4500	0.5500	2.02611536	0.5147	0.4853	2.07840145
295	13/12/2006	1.02	0.98	0.6138	0.3862	2.13074808	0.4508	0.5492	2.01960633	0.5151	0.4849	2.07747248
296	14/12/2006	1.01	1.19	0.6197	0.3803	2.30374890	0.4556	0.5444	2.24271828	0.5172	0.4828	2.28414906
297	15/12/2006	1.02	1.03	0.5972	0.4028	2.35754552	0.4375	0.5625	2.29841841	0.5091	0.4909	2.33935647
298	18/12/2006	1.04	1.00	0.5959	0.4041	2.42058292	0.4364	0.5636	2.34343125	0.5087	0.4913	2.39275461

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
299	19/12/2006	0.96	0.97	0.6020	0.3980	2.33957432	0.4414	0.5586	2.26816745	0.5109	0.4891	2.31450953
300	20/12/2006	0.96	0.99	0.6008	0.3992	2.27226047	0.4404	0.5596	2.21681148	0.5104	0.4896	2.25590916
301	21/12/2006	1.01	1.03	0.5953	0.4047	2.30804721	0.4360	0.5640	2.25972745	0.5085	0.4915	2.29587680
302	22/12/2006	0.99	0.99	0.5922	0.4078	2.28483268	0.4335	0.5665	2.23964834	0.5074	0.4926	2.27422338
303	25/12/2006	1.00	1.00	0.5911	0.4089	2.28483268	0.4327	0.5673	2.23964834	0.5070	0.4930	2.27422338
304	26/12/2006	1.00	1.03	0.5911	0.4089	2.31152423	0.4327	0.5673	2.27595000	0.5070	0.4930	2.30625652
305	27/12/2006	1.03	1.02	0.5871	0.4129	2.36362155	0.4296	0.5704	2.32365912	0.5056	0.4944	2.35635482
306	28/12/2006	1.03	1.01	0.5885	0.4115	2.40503854	0.4307	0.5693	2.35684109	0.5061	0.4939	2.39365513
307	29/12/2006	0.96	1.01	0.5914	0.4086	2.36165134	0.4329	0.5671	2.33254500	0.5071	0.4929	2.36031490
308	1/1/2007	1.00	1.00	0.5844	0.4156	2.36165134	0.4274	0.5726	2.33254500	0.5046	0.4954	2.36031490
309	2/1/2007	1.00	1.00	0.5844	0.4156	2.36165134	0.4274	0.5726	2.33254500	0.5046	0.4954	2.36031490
310	3/1/2007	1.05	1.01	0.5844	0.4156	2.44472955	0.4274	0.5726	2.39922456	0.5046	0.4954	2.43544341
311	4/1/2007	1.00	1.01	0.5901	0.4099	2.45495415	0.4319	0.5681	2.41313167	0.5067	0.4933	2.44770341
312	5/1/2007	1.00	1.02	0.5887	0.4113	2.47535265	0.4308	0.5692	2.44087964	0.5062	0.4938	2.47212292
313	8/1/2007	0.98	1.03	0.5859	0.4141	2.47894616	0.4286	0.5714	2.46293490	0.5052	0.4948	2.48533159
314	9/1/2007	0.97	1.00	0.5790	0.4210	2.44283556	0.4233	0.5767	2.43670837	0.5028	0.4972	2.45389583
315	10/1/2007	1.00	0.99	0.5754	0.4246	2.43286204	0.4205	0.5795	2.42312990	0.5015	0.4985	2.44213387
316	11/1/2007	1.00	1.01	0.5768	0.4232	2.44295665	0.4215	0.5785	2.43687211	0.5020	0.4980	2.45405745
317	12/1/2007	1.03	1.00	0.5754	0.4246	2.47923085	0.4205	0.5795	2.46331320	0.5015	0.4985	2.48581802
318	15/1/2007	1.01	1.00	0.5790	0.4210	2.48831640	0.4233	0.5767	2.46991225	0.5028	0.4972	2.49372817
319	16/1/2007	0.99	1.00	0.5799	0.4201	2.47027869	0.4240	0.5760	2.45682264	0.5031	0.4969	2.47804611
320	17/1/2007	0.99	1.00	0.5781	0.4219	2.46118238	0.4226	0.5774	2.45020990	0.5025	0.4975	2.47011539
321	18/1/2007	1.02	1.04	0.5772	0.4228	2.52971245	0.4219	0.5781	2.52618863	0.5021	0.4979	2.54267181
322	19/1/2007	1.01	0.99	0.5744	0.4256	2.52871296	0.4197	0.5803	2.51897990	0.5012	0.4988	2.53870832
323	22/1/2007	0.99	1.03	0.5767	0.4233	2.54113525	0.4214	0.5786	2.54758107	0.5020	0.4980	2.55905102
324	23/1/2007	0.99	1.02	0.5708	0.4292	2.54292940	0.4169	0.5831	2.56172977	0.4999	0.5001	2.56657128
325	24/1/2007	1.00	1.02	0.5662	0.4338	2.56335622	0.4135	0.5865	2.58955462	0.4983	0.5017	2.59041497
326	25/1/2007	0.99	1.00	0.5636	0.4364	2.55409555	0.4115	0.5885	2.58272456	0.4974	0.5026	2.58215517
327	26/1/2007	0.99	0.98	0.5627	0.4373	2.52445443	0.4108	0.5892	2.54816564	0.4971	0.5029	2.55020992
328	29/1/2007	1.01	0.99	0.5644	0.4356	2.52361635	0.4120	0.5880	2.54123871	0.4977	0.5023	2.54669933
329	30/1/2007	0.99	1.00	0.5666	0.4334	2.51445009	0.4138	0.5862	2.53449841	0.4985	0.5015	2.53856178
330	31/1/2007	0.99	1.02	0.5657	0.4343	2.52549609	0.4131	0.5869	2.55529206	0.4982	0.5018	2.55399525
331	1/2/2007	1.00	1.00	0.5621	0.4379	2.52549609	0.4104	0.5896	2.55529206	0.4969	0.5031	2.55399525

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
332	2/2/2007	1.01	0.99	0.5621	0.4379	2.52466095	0.4104	0.5896	2.54840339	0.4969	0.5031	2.55055560
333	5/2/2007	1.01	1.00	0.5644	0.4356	2.54304588	0.4121	0.5879	2.56195294	0.4977	0.5023	2.56693486
334	6/2/2007	0.97	0.98	0.5662	0.4338	2.48537834	0.4135	0.5865	2.50623821	0.4983	0.5017	2.50978602
335	7/2/2007	1.01	1.01	0.5653	0.4347	2.51420426	0.4128	0.5872	2.53415834	0.4980	0.5020	2.53838796
336	8/2/2007	1.01	0.98	0.5657	0.4343	2.50164411	0.4131	0.5869	2.51137523	0.4982	0.5018	2.52121780
337	9/2/2007	1.00	1.00	0.5695	0.4305	2.50164411	0.4160	0.5840	2.51137523	0.4995	0.5005	2.52121780
338	12/2/2007	0.99	1.01	0.5695	0.4305	2.49809296	0.4160	0.5840	2.51220787	0.4995	0.5005	2.51965322
339	13/2/2007	1.01	0.99	0.5679	0.4321	2.51071314	0.4147	0.5853	2.51805526	0.4989	0.5011	2.52929047
340	14/2/2007	0.99	1.01	0.5704	0.4296	2.50720886	0.4167	0.5833	2.51891551	0.4998	0.5002	2.52777939
341	15/2/2007	1.00	1.01	0.5688	0.4312	2.51266884	0.4155	0.5845	2.52635199	0.4992	0.5008	2.53417247
342	16/2/2007	1.00	1.01	0.5681	0.4319	2.52363096	0.4149	0.5851	2.54128298	0.4990	0.5010	2.54699740
343	19/2/2007	1.00	1.00	0.5666	0.4334	2.52363096	0.4138	0.5862	2.54128298	0.4985	0.5015	2.54699740
344	20/2/2007	1.00	1.00	0.5666	0.4334	2.52363096	0.4138	0.5862	2.54128298	0.4985	0.5015	2.54699740
345	21/2/2007	1.04	1.00	0.5666	0.4334	2.58251313	0.4138	0.5862	2.58458324	0.4985	0.5015	2.59927625
346	22/2/2007	1.02	1.00	0.5724	0.4276	2.60756942	0.4182	0.5818	2.60290478	0.5005	0.4995	2.62132560
347	23/2/2007	0.99	1.01	0.5748	0.4252	2.61017199	0.4201	0.5799	2.61174109	0.5013	0.4987	2.62692646
348	26/2/2007	1.01	1.00	0.5726	0.4274	2.61856894	0.4184	0.5816	2.61787999	0.5006	0.4994	2.63431369
349	27/2/2007	0.99	0.97	0.5734	0.4266	2.56861337	0.4190	0.5810	2.56044663	0.5008	0.4992	2.58051420
350	28/2/2007	1.00	0.98	0.5761	0.4239	2.55160179	0.4211	0.5789	2.53728635	0.5018	0.4982	2.56042563
351	1/3/2007	1.01	0.99	0.5784	0.4216	2.54902607	0.4228	0.5772	2.52830582	0.5026	0.4974	2.55468341
352	2/3/2007	0.97	0.98	0.5807	0.4193	2.48273122	0.4246	0.5754	2.46591696	0.5034	0.4966	2.48992728
353	5/3/2007	0.96	0.94	0.5795	0.4205	2.36735677	0.4237	0.5763	2.34249853	0.5029	0.4971	2.36984128
354	6/3/2007	1.02	1.06	0.5829	0.4171	2.45080187	0.4263	0.5737	2.43925757	0.5041	0.4959	2.46059598
355	7/3/2007	1.06	1.03	0.5776	0.4224	2.55821727	0.4222	0.5778	2.53533714	0.5023	0.4977	2.56314574
356	8/3/2007	0.99	1.05	0.5815	0.4185	2.60672004	0.4253	0.5747	2.60691730	0.5037	0.4963	2.62358502
357	9/3/2007	0.99	0.99	0.5732	0.4268	2.58355084	0.4189	0.5811	2.58645789	0.5008	0.4992	2.60154675
358	12/3/2007	1.03	0.99	0.5722	0.4278	2.62220360	0.4181	0.5819	2.61119699	0.5004	0.4996	2.63392786
359	13/3/2007	1.00	0.98	0.5772	0.4228	2.60002882	0.4219	0.5781	2.58100834	0.5021	0.4979	2.60770167
360	14/3/2007	1.03	1.02	0.5801	0.4199	2.66871730	0.4242	0.5758	2.64547264	0.5032	0.4968	2.67473783
361	15/3/2007	1.01	1.01	0.5813	0.4187	2.68948180	0.4252	0.5748	2.66792954	0.5036	0.4964	2.69649196
362	16/3/2007	1.00	1.01	0.5807	0.4193	2.70122875	0.4247	0.5753	2.683918381	0.5034	0.4966	2.71044132
363	19/3/2007	0.99	0.99	0.5792	0.4208	2.67714642	0.4235	0.5765	2.662684175	0.5029	0.4971	2.68761107
364	20/3/2007	1.00	1.00	0.5783	0.4217	2.67714642	0.4228	0.5772	2.662684175	0.5025	0.4975	2.68761107

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
365	21/3/2007	1.01	0.99	0.5783	0.4217	2.68932901	0.4228	0.5772	2.667852902	0.5025	0.4975	2.69642436
366	22/3/2007	1.01	1.01	0.5807	0.4193	2.71876080	0.4247	0.5753	2.696633045	0.5034	0.4966	2.72572516
367	23/3/2007	1.05	1.01	0.5808	0.4192	2.80403160	0.4248	0.5752	2.762221257	0.5034	0.4966	2.80169468
368	26/3/2007	0.98	1.00	0.5870	0.4130	2.76919355	0.4296	0.5704	2.737104169	0.5056	0.4944	2.77171477
369	27/3/2007	1.00	1.02	0.5840	0.4160	2.78691586	0.4273	0.5727	2.761221261	0.5045	0.4955	2.79284204
370	28/3/2007	1.01	0.99	0.5818	0.4182	2.78392158	0.4256	0.5744	2.751598489	0.5038	0.4962	2.78647818
371	29/3/2007	1.01	1.01	0.5840	0.4160	2.81334275	0.4273	0.5727	2.780486823	0.5046	0.4954	2.81582815
372	30/3/2007	1.00	1.01	0.5841	0.4159	2.81931247	0.4273	0.5727	2.788610586	0.5046	0.4954	2.82294571
373	2/4/2007	0.99	1.02	0.5834	0.4166	2.81997909	0.4268	0.5732	2.800668478	0.5043	0.4957	2.82943871
374	3/4/2007	1.00	1.01	0.5797	0.4203	2.82602663	0.4239	0.5761	2.808900454	0.5030	0.4970	2.83661303
375	4/4/2007	1.02	1.00	0.5789	0.4211	2.85227449	0.4233	0.5767	2.827977084	0.5028	0.4972	2.85949264
376	5/4/2007	0.99	1.04	0.5812	0.4188	2.88594725	0.4251	0.5749	2.879386249	0.5036	0.4964	2.90231526
377	6/4/2007	1.01	1.00	0.5755	0.4245	2.90361509	0.4207	0.5793	2.892271757	0.5016	0.4984	2.91780127
378	9/4/2007	0.99	1.01	0.5770	0.4230	2.90695875	0.4218	0.5782	2.902407286	0.5021	0.4979	2.92447509
379	10/4/2007	0.99	1.01	0.5748	0.4252	2.91035507	0.4202	0.5798	2.912617817	0.5013	0.4987	2.93115675
380	11/4/2007	0.99	1.00	0.5726	0.4274	2.90153702	0.4185	0.5815	2.906168622	0.5006	0.4994	2.92339339
381	12/4/2007	0.98	1.00	0.5719	0.4281	2.87505797	0.4179	0.5821	2.886788268	0.5003	0.4997	2.90005375
382	13/4/2007	1.00	1.00	0.5696	0.4304	2.87505797	0.4161	0.5839	2.886788268	0.4995	0.5005	2.90005375
383	16/4/2007	0.99	0.99	0.5696	0.4304	2.84531745	0.4161	0.5839	2.857346668	0.4995	0.5005	2.87024745
384	17/4/2007	0.99	1.01	0.5694	0.4306	2.84868884	0.4160	0.5840	2.867441788	0.4995	0.5005	2.87672261
385	18/4/2007	1.00	1.01	0.5672	0.4328	2.86077494	0.4144	0.5856	2.883905496	0.4987	0.5013	2.89086067
386	19/4/2007	0.99	0.98	0.5658	0.4342	2.82158307	0.4133	0.5867	2.835991738	0.4982	0.5018	2.84752206
387	20/4/2007	0.99	1.00	0.5686	0.4314	2.81291043	0.4154	0.5846	2.829623544	0.4992	0.5008	2.83983858
388	23/4/2007	0.99	1.01	0.5679	0.4321	2.81626497	0.4148	0.5852	2.839638677	0.4989	0.5011	2.84622743
389	24/4/2007	1.01	1.00	0.5656	0.4344	2.83367500	0.4131	0.5869	2.852460332	0.4982	0.5018	2.86172319
390	25/4/2007	1.02	1.00	0.5672	0.4328	2.85973928	0.4143	0.5857	2.871625916	0.4987	0.5013	2.88486583
391	26/4/2007	1.00	1.00	0.5695	0.4305	2.85973928	0.4161	0.5839	2.871625916	0.4995	0.5005	2.88486583
392	27/4/2007	1.03	1.00	0.5695	0.4305	2.90305552	0.4161	0.5839	2.903404933	0.4995	0.5005	2.92318977
393	30/4/2007	1.06	0.99	0.5733	0.4267	2.98576617	0.4190	0.5810	2.956201071	0.5008	0.4992	2.99232049
394	1/5/2007	1.00	1.00	0.5826	0.4174	2.98576617	0.4262	0.5738	2.956201071	0.5041	0.4959	2.99232049
395	2/5/2007	1.00	1.00	0.5826	0.4174	2.98576617	0.4262	0.5738	2.956201071	0.5041	0.4959	2.99232049
396	3/5/2007	1.04	1.02	0.5826	0.4174	3.07742925	0.4262	0.5738	3.038155277	0.5041	0.4959	3.07971034
397	4/5/2007	0.98	0.99	0.5852	0.4148	3.02384921	0.4283	0.5717	2.991610388	0.5050	0.4950	3.02938235

Appendix 1 continued

No	Date	Price Relative		U.P. Generated by the Dirichlet (0.4, 0.3) Dist.		Investment Capital	U.P. Generated by the Dirichlet (0.5, 0.7) Dist.		Investment Capital	U.P. Generated by the Dirichlet (2.0, 2.0) Dist.		Investment Capital
		Digi	LPI	Digi	LPI		Digi	LPI		Digi	LPI	
398	7/5/2007	0.98	1.02	0.5833	0.4167	3.01511335	0.4268	0.5732	3.000724880	0.5043	0.4957	3.02970539
399	8/5/2007	1.01	1.00	0.5779	0.4221	3.03170765	0.4226	0.5774	3.012801448	0.5024	0.4976	3.04420211
400	9/5/2007	0.98	1.03	0.5792	0.4208	3.03470464	0.4236	0.5764	3.038105310	0.5029	0.4971	3.05828051
401	10/5/2007	1.02	1.00	0.5725	0.4275	3.07707661	0.4184	0.5816	3.069109137	0.5005	0.4995	3.09561568
402	11/5/2007	1.02	1.00	0.5759	0.4241	3.12029882	0.4211	0.5789	3.100628247	0.5017	0.4983	3.13349716
403	14/5/2007	1.00	1.03	0.5793	0.4207	3.16474567	0.4237	0.5763	3.155645775	0.5029	0.4971	3.18351720
404	15/5/2007	1.00	1.00	0.5762	0.4238	3.15606244	0.4213	0.5787	3.149315225	0.5018	0.4982	3.17590980
405	16/5/2007	1.00	1.01	0.5755	0.4245	3.15922801	0.4208	0.5792	3.159118659	0.5016	0.4984	3.18229610
406	17/5/2007	0.98	1.00	0.5736	0.4264	3.12438197	0.4193	0.5807	3.133647880	0.5009	0.4991	3.15164166
407	18/5/2007	0.99	0.99	0.5708	0.4292	3.09532233	0.4171	0.5829	3.105012648	0.4999	0.5001	3.12256521
408	21/5/2007	1.00	0.99	0.5706	0.4294	3.09226137	0.4170	0.5830	3.095371416	0.4999	0.5001	3.11643471
409	22/5/2007	1.02	1.00	0.5726	0.4274	3.12697941	0.4185	0.5815	3.120773022	0.5006	0.4994	3.14702302
410	23/5/2007	0.99	1.02	0.5754	0.4246	3.12495387	0.4207	0.5793	3.134414745	0.5015	0.4985	3.15252256

Appendix 2

The Investment Capitals Achieved by Using the Cover and Ordentlich Universal Portfolio for Daiboci, KNM and Topglov

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
1	27/10/2005	1.00	0.99	1.02	0.1579	0.3684	0.4737	1.00722721	0.4000	0.4000	0.2000	1.00156098
2	28/10/2005	1.02	0.98	1.00	0.1575	0.3667	0.4758	1.00148182	0.3999	0.3995	0.2006	1.00020333
3	31/10/2005	1.00	0.99	1.01	0.1590	0.3643	0.4767	1.00155594	0.4014	0.3979	0.2007	0.99717667
4	1/11/2005	1.00	1.00	1.00	0.1590	0.3628	0.4783	1.00155594	0.4016	0.3973	0.2011	0.99717667
5	2/11/2005	1.00	1.01	1.05	0.1590	0.3628	0.4783	1.02918481	0.4016	0.3973	0.2011	1.01183198
6	3/11/2005	1.00	1.00	1.00	0.1575	0.3609	0.4816	1.02918481	0.4006	0.3972	0.2022	1.01183198
7	4/11/2005	1.00	1.00	1.00	0.1575	0.3609	0.4816	1.02918481	0.4006	0.3972	0.2022	1.01183198
8	7/11/2005	1.00	0.99	1.03	0.1575	0.3609	0.4816	1.04063045	0.4006	0.3972	0.2022	1.01500319
9	8/11/2005	1.00	1.00	1.01	0.1569	0.3588	0.4843	1.04517085	0.4004	0.3966	0.2030	1.01685969
10	9/11/2005	1.00	1.01	1.00	0.1566	0.3583	0.4851	1.04991079	0.4003	0.3964	0.2033	1.02196241
11	10/11/2005	0.98	0.99	1.01	0.1564	0.3593	0.4843	1.04864044	0.4000	0.3969	0.2031	1.01260016
12	11/11/2005	1.02	1.01	1.00	0.1553	0.3586	0.4860	1.05457674	0.3992	0.3971	0.2037	1.02394800
13	14/11/2005	1.00	0.99	0.98	0.1562	0.3587	0.4851	1.04301656	0.3998	0.3968	0.2033	1.01762530
14	15/11/2005	1.00	1.02	1.00	0.1568	0.3593	0.4839	1.05036472	0.4003	0.3968	0.2029	1.02554276
15	16/11/2005	1.00	1.00	0.98	0.1564	0.3608	0.4828	1.04122825	0.3997	0.3976	0.2027	1.02179757
16	17/11/2005	1.00	1.05	1.00	0.1569	0.3619	0.4812	1.05824725	0.4000	0.3978	0.2022	1.04015504
17	18/11/2005	1.00	0.99	1.00	0.1560	0.3655	0.4785	1.05347270	0.3988	0.3996	0.2016	1.03502381
18	21/11/2005	1.00	1.00	1.00	0.1563	0.3645	0.4793	1.05347270	0.3992	0.3991	0.2018	1.03502381
19	22/11/2005	0.98	1.01	1.00	0.1563	0.3645	0.4793	1.05461396	0.3992	0.3991	0.2018	1.03100646
20	23/11/2005	1.00	1.07	1.00	0.1550	0.3659	0.4791	1.08081511	0.3979	0.4002	0.2019	1.05902171
21	24/11/2005	1.00	0.97	0.99	0.1537	0.3712	0.4751	1.06450955	0.3962	0.4028	0.2010	1.04473939
22	25/11/2005	0.98	1.00	1.00	0.1545	0.3694	0.4761	1.06077078	0.3971	0.4018	0.2011	1.03531110
23	28/11/2005	1.02	1.01	1.01	0.1535	0.3698	0.4767	1.07143189	0.3962	0.4024	0.2015	1.04901129
24	29/11/2005	1.04	1.00	1.00	0.1542	0.3693	0.4765	1.07861397	0.3968	0.4019	0.2013	1.06710826
25	30/11/2005	1.06	0.96	1.01	0.1561	0.3685	0.4754	1.07756082	0.3985	0.4008	0.2007	1.07848402
26	1/12/2005	1.00	0.99	1.01	0.1596	0.3633	0.4771	1.07977553	0.4020	0.3973	0.2007	1.07778890
27	2/12/2005	1.04	1.01	1.02	0.1595	0.3623	0.4782	1.10109166	0.4020	0.3970	0.2010	1.10509315

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
28	5/12/2005	1.04	1.00	1.01	0.1606	0.3614	0.4779	1.11278301	0.4031	0.3961	0.2008	1.12485723
29	6/12/2005	1.04	1.02	0.99	0.1623	0.3601	0.4776	1.12257359	0.4045	0.3950	0.2005	1.14867733
30	7/12/2005	1.02	1.01	1.02	0.1639	0.3613	0.4747	1.14027766	0.4057	0.3948	0.1995	1.16685801
31	8/12/2005	1.00	1.01	1.00	0.1641	0.3609	0.4750	1.14275670	0.4059	0.3946	0.1995	1.16963164
32	9/12/2005	0.98	1.02	1.01	0.1640	0.3614	0.4747	1.15146687	0.4057	0.3948	0.1995	1.17121848
33	12/12/2005	1.00	1.00	1.01	0.1625	0.3627	0.4749	1.15622157	0.4044	0.3959	0.1997	1.17325249
34	13/12/2005	1.04	1.01	1.03	0.1622	0.3622	0.4756	1.18012336	0.4042	0.3958	0.1999	1.20001752
35	14/12/2005	0.96	0.99	1.02	0.1632	0.3604	0.4765	1.17764792	0.4052	0.3947	0.2001	1.18083081
36	15/12/2005	1.02	1.02	1.03	0.1612	0.3592	0.4796	1.20318221	0.4038	0.3950	0.2012	1.20441911
37	16/12/2005	1.00	1.02	1.02	0.1611	0.3587	0.4802	1.22043927	0.4038	0.3949	0.2013	1.21692030
38	19/12/2005	1.00	1.02	1.01	0.1603	0.3592	0.4805	1.23546073	0.4031	0.3954	0.2015	1.23016886
39	20/12/2005	1.00	1.00	1.00	0.1596	0.3605	0.4799	1.23546073	0.4024	0.3962	0.2015	1.23016886
40	21/12/2005	1.00	1.01	1.02	0.1596	0.3605	0.4799	1.25236394	0.4024	0.3962	0.2015	1.23896570
41	22/12/2005	0.96	1.01	1.03	0.1589	0.3595	0.4816	1.26529715	0.4019	0.3961	0.2020	1.22802113
42	23/12/2005	0.92	1.01	1.01	0.1559	0.3590	0.4851	1.25614993	0.3995	0.3971	0.2034	1.19184353
43	26/12/2005	1.00	1.00	1.00	0.1518	0.3606	0.4876	1.25614993	0.3958	0.3994	0.2047	1.19184353
44	27/12/2005	1.00	0.99	1.00	0.1518	0.3606	0.4876	1.25356186	0.3958	0.3994	0.2047	1.18912316
45	28/12/2005	1.02	1.04	1.00	0.1519	0.3601	0.4879	1.27595492	0.3960	0.3992	0.2048	1.21868464
46	29/12/2005	1.00	1.00	1.02	0.1522	0.3628	0.4850	1.28533110	0.3958	0.4002	0.2040	1.22245077
47	30/12/2005	0.98	0.98	1.02	0.1518	0.3619	0.4863	1.28732092	0.3956	0.4000	0.2044	1.20923660
48	2/1/2006	1.00	1.00	1.00	0.1506	0.3597	0.4898	1.28732092	0.3949	0.3996	0.2055	1.20923660
49	3/1/2006	1.00	1.03	0.99	0.1506	0.3597	0.4898	1.29565141	0.3949	0.3996	0.2055	1.22092027
50	4/1/2006	1.00	1.03	1.01	0.1503	0.3623	0.4875	1.31301619	0.3942	0.4008	0.2049	1.23599975
51	5/1/2006	1.00	0.98	0.99	0.1496	0.3640	0.4865	1.30081372	0.3934	0.4018	0.2048	1.22633872
52	6/1/2006	1.00	1.06	0.98	0.1501	0.3632	0.4868	1.31380610	0.3940	0.4012	0.2048	1.24894916
53	9/1/2006	1.00	0.99	1.01	0.1496	0.3690	0.4814	1.31386793	0.3928	0.4038	0.2034	1.24595341
54	10/1/2006	1.00	1.00	1.00	0.1496	0.3678	0.4827	1.31386793	0.3929	0.4033	0.2037	1.24595341
55	11/1/2006	1.07	1.00	1.02	0.1496	0.3678	0.4827	1.33857762	0.3929	0.4033	0.2037	1.28415179
56	12/1/2006	1.00	0.99	1.04	0.1519	0.3660	0.4821	1.35772077	0.3951	0.4016	0.2032	1.28881061
57	13/1/2006	1.00	1.00	1.03	0.1511	0.3631	0.4858	1.37926775	0.3949	0.4007	0.2044	1.29898963
58	16/1/2006	0.98	0.97	1.01	0.1503	0.3617	0.4880	1.36486080	0.3944	0.4005	0.2051	1.27443985
59	17/1/2006	1.09	1.00	1.00	0.1497	0.3593	0.4910	1.38344046	0.3942	0.3998	0.2060	1.32010805

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
60	18/1/2006	1.00	1.02	1.01	0.1536	0.3577	0.4887	1.40063493	0.3976	0.3975	0.2048	1.33196231
61	19/1/2006	1.00	1.01	1.01	0.1529	0.3580	0.4890	1.41293967	0.3971	0.3979	0.2050	1.33852650
62	20/1/2006	1.00	1.00	1.01	0.1525	0.3576	0.4899	1.42036280	0.3967	0.3979	0.2053	1.34312566
63	23/1/2006	0.92	1.01	1.01	0.1522	0.3575	0.4902	1.41228701	0.3965	0.3980	0.2054	1.30596963
64	24/1/2006	1.00	1.02	1.00	0.1482	0.3594	0.4924	1.42219097	0.3927	0.4006	0.2067	1.31617730
65	25/1/2006	1.00	1.00	1.00	0.1478	0.3609	0.4912	1.41973494	0.3922	0.4014	0.2064	1.31364978
66	26/1/2006	1.00	1.01	1.01	0.1479	0.3606	0.4915	1.43206701	0.3924	0.4012	0.2065	1.32317418
67	27/1/2006	1.00	1.01	1.01	0.1475	0.3612	0.4912	1.44694924	0.3919	0.4016	0.2065	1.33208654
68	30/1/2006	1.00	1.00	1.00	0.1470	0.3611	0.4919	1.44694924	0.3915	0.4018	0.2067	1.33208654
69	31/1/2006	1.00	1.00	1.00	0.1470	0.3611	0.4919	1.44694924	0.3915	0.4018	0.2067	1.33208654
70	1/2/2006	1.00	1.00	1.00	0.1470	0.3611	0.4919	1.44694924	0.3915	0.4018	0.2067	1.33208654
71	2/2/2006	1.00	1.00	1.00	0.1470	0.3611	0.4919	1.44694924	0.3915	0.4018	0.2067	1.33208654
72	3/2/2006	1.00	1.03	1.06	0.1470	0.3611	0.4919	1.50169635	0.3915	0.4018	0.2067	1.36260769
73	6/2/2006	1.02	0.99	1.01	0.1452	0.3599	0.4949	1.50881781	0.3900	0.4021	0.2079	1.36985039
74	7/2/2006	1.04	1.00	1.00	0.1459	0.3577	0.4964	1.51612902	0.3910	0.4009	0.2082	1.39113445
75	8/2/2006	1.00	1.00	1.00	0.1479	0.3566	0.4956	1.51863182	0.3928	0.3996	0.2076	1.39370785
76	9/2/2006	1.00	1.02	1.02	0.1478	0.3569	0.4953	1.54381395	0.3927	0.3998	0.2076	1.40984955
77	10/2/2006	0.98	1.01	0.99	0.1470	0.3571	0.4959	1.53892900	0.3919	0.4002	0.2079	1.40128112
78	13/2/2006	1.00	1.01	1.01	0.1461	0.3586	0.4953	1.55150279	0.3910	0.4012	0.2078	1.41083315
79	14/2/2006	1.00	1.00	1.01	0.1457	0.3593	0.4950	1.55419084	0.3905	0.4017	0.2078	1.41029359
80	15/2/2006	1.00	1.04	1.00	0.1456	0.3585	0.4959	1.57638071	0.3905	0.4014	0.2081	1.43283607
81	16/2/2006	1.02	1.00	1.00	0.1449	0.3616	0.4934	1.58377328	0.3895	0.4030	0.2075	1.44742641
82	17/2/2006	1.00	1.00	1.01	0.1458	0.3616	0.4927	1.59146166	0.3903	0.4026	0.2072	1.45191102
83	20/2/2006	0.98	1.00	0.99	0.1455	0.3615	0.4930	1.57600316	0.3901	0.4026	0.2073	1.43582120
84	21/2/2006	1.02	1.00	1.01	0.1450	0.3627	0.4923	1.59152633	0.3894	0.4034	0.2072	1.45202375
85	22/2/2006	1.00	1.01	1.01	0.1455	0.3615	0.4930	1.60162678	0.3901	0.4026	0.2073	1.45895676
86	23/2/2006	1.02	1.00	1.01	0.1452	0.3617	0.4930	1.61437154	0.3898	0.4029	0.2074	1.47579493
87	24/2/2006	1.00	1.02	1.01	0.1459	0.3613	0.4928	1.63706074	0.3904	0.4024	0.2072	1.49221679
88	27/2/2006	1.00	1.00	1.00	0.1452	0.3621	0.4927	1.63464115	0.3897	0.4030	0.2073	1.48976203
89	28/2/2006	1.00	1.00	1.01	0.1453	0.3618	0.4929	1.63994229	0.3898	0.4029	0.2073	1.49179394
90	1/3/2006	1.00	1.00	1.00	0.1451	0.3614	0.4935	1.63994229	0.3897	0.4028	0.2075	1.49179394
91	2/3/2006	1.00	1.00	1.00	0.1451	0.3614	0.4935	1.63746261	0.3897	0.4028	0.2075	1.48927988

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
92	3/3/2006	1.00	1.00	1.01	0.1452	0.3610	0.4938	1.64278180	0.3898	0.4026	0.2076	1.49131352
93	6/3/2006	1.00	1.00	1.01	0.1450	0.3606	0.4943	1.65346694	0.3897	0.4025	0.2077	1.49538996
94	7/3/2006	1.00	1.01	1.01	0.1447	0.3598	0.4955	1.66920417	0.3896	0.4023	0.2081	1.50449838
95	8/3/2006	1.00	0.99	1.00	0.1443	0.3597	0.4961	1.66424242	0.3892	0.4025	0.2084	1.49949412
96	9/3/2006	1.00	1.02	1.01	0.1444	0.3590	0.4966	1.68209179	0.3894	0.4021	0.2085	1.51409989
97	10/3/2006	1.00	0.99	1.01	0.1439	0.3602	0.4959	1.68257635	0.3888	0.4029	0.2084	1.51116194
98	13/3/2006	1.00	1.00	1.01	0.1439	0.3592	0.4969	1.68555410	0.3889	0.4024	0.2087	1.51072015
99	14/3/2006	1.02	1.00	1.01	0.1438	0.3585	0.4977	1.69654595	0.3889	0.4022	0.2089	1.52613581
100	15/3/2006	1.00	1.00	1.01	0.1446	0.3577	0.4978	1.69957843	0.3897	0.4015	0.2088	1.52570699
101	16/3/2006	1.00	1.01	1.01	0.1445	0.3570	0.4986	1.71566907	0.3897	0.4013	0.2090	1.53493528
102	17/3/2006	1.00	1.00	1.00	0.1440	0.3568	0.4992	1.71566907	0.3893	0.4014	0.2093	1.53493528
103	20/3/2006	0.98	1.00	1.01	0.1440	0.3568	0.4992	1.71827467	0.3893	0.4014	0.2093	1.52626969
104	21/3/2006	1.00	1.01	1.00	0.1429	0.3571	0.5000	1.72572690	0.3883	0.4020	0.2097	1.53372259
105	22/3/2006	1.00	1.00	1.00	0.1427	0.3580	0.4993	1.72325543	0.3879	0.4025	0.2095	1.53125315
106	23/3/2006	1.00	1.01	1.01	0.1427	0.3577	0.4995	1.73758356	0.3880	0.4024	0.2096	1.54202488
107	24/3/2006	1.00	0.99	1.00	0.1424	0.3584	0.4992	1.73141757	0.3876	0.4028	0.2096	1.53587466
108	27/3/2006	1.00	1.00	1.00	0.1425	0.3576	0.4998	1.73141757	0.3879	0.4024	0.2097	1.53587466
109	28/3/2006	1.00	1.00	1.01	0.1425	0.3576	0.4998	1.73956083	0.3879	0.4024	0.2097	1.54046210
110	29/3/2006	1.00	1.01	1.00	0.1423	0.3575	0.5002	1.74578038	0.3877	0.4025	0.2098	1.54666239
111	30/3/2006	1.00	0.99	1.00	0.1421	0.3583	0.4995	1.73958685	0.3874	0.4029	0.2097	1.54049266
112	31/3/2006	1.00	1.00	1.01	0.1423	0.3575	0.5002	1.74527358	0.3877	0.4025	0.2098	1.54260544
113	3/4/2006	1.00	1.00	1.00	0.1421	0.3571	0.5007	1.74777675	0.3876	0.4024	0.2100	1.54509839
114	4/4/2006	1.02	1.00	1.00	0.1421	0.3574	0.5005	1.75545929	0.3875	0.4026	0.2100	1.56006898
115	5/4/2006	1.00	1.00	1.01	0.1429	0.3574	0.4997	1.76685214	0.3882	0.4022	0.2096	1.56431610
116	6/4/2006	1.04	1.01	1.01	0.1425	0.3566	0.5008	1.79457206	0.3880	0.4020	0.2100	1.59909456
117	7/4/2006	1.16	1.02	1.01	0.1437	0.3560	0.5004	1.86068505	0.3891	0.4012	0.2097	1.71746397
118	10/4/2006	1.04	0.99	1.03	0.1494	0.3541	0.4964	1.88858505	0.3944	0.3979	0.2076	1.74683203
119	11/4/2006	1.00	1.00	1.00	0.1507	0.3512	0.4981	1.88858505	0.3959	0.3962	0.2079	1.74683203
120	12/4/2006	1.06	1.00	1.01	0.1507	0.3512	0.4981	1.90781022	0.3959	0.3962	0.2079	1.78543681
121	13/4/2006	0.98	1.00	1.01	0.1530	0.3495	0.4975	1.91106291	0.3981	0.3945	0.2074	1.77783059
122	14/4/2006	0.98	0.99	1.01	0.1520	0.3498	0.4982	1.90483578	0.3972	0.3951	0.2077	1.76003801
123	17/4/2006	1.00	1.00	1.02	0.1512	0.3490	0.4998	1.92777720	0.3966	0.3951	0.2083	1.76887269

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
124	18/4/2006	1.00	1.01	1.03	0.1506	0.3476	0.5019	1.96565088	0.3963	0.3947	0.2090	1.78955952
125	19/4/2006	1.00	1.00	1.01	0.1496	0.3469	0.5035	1.97696233	0.3955	0.3949	0.2096	1.79384647
126	20/4/2006	1.00	1.00	1.01	0.1493	0.3462	0.5045	1.98259726	0.3954	0.3947	0.2099	1.79597407
127	21/4/2006	1.00	1.04	1.04	0.1491	0.3459	0.5050	2.05475220	0.3953	0.3946	0.2101	1.84100077
128	24/4/2006	1.00	1.00	1.01	0.1474	0.3462	0.5064	2.06037692	0.3937	0.3955	0.2108	1.84309838
129	25/4/2006	0.98	1.00	1.01	0.1472	0.3459	0.5069	2.06535575	0.3936	0.3955	0.2109	1.83245134
130	26/4/2006	1.02	1.00	1.02	0.1461	0.3456	0.5083	2.09361055	0.3926	0.3958	0.2115	1.85504511
131	27/4/2006	1.06	0.99	1.01	0.1464	0.3440	0.5096	2.11025256	0.3931	0.3950	0.2119	1.89297949
132	28/4/2006	1.09	1.01	1.01	0.1488	0.3420	0.5091	2.15723998	0.3956	0.3931	0.2113	1.97354982
133	1/5/2006	1.00	1.00	1.00	0.1523	0.3407	0.5071	2.15723998	0.3987	0.3911	0.2102	1.97354982
134	2/5/2006	1.02	1.02	1.00	0.1523	0.3407	0.5071	2.17690166	0.3987	0.3911	0.2102	2.00181870
135	3/5/2006	1.00	1.02	1.01	0.1527	0.3418	0.5055	2.19626631	0.3989	0.3914	0.2097	2.01845911
136	4/5/2006	0.98	1.00	1.01	0.1523	0.3429	0.5048	2.19612845	0.3984	0.3921	0.2096	2.00674258
137	5/5/2006	1.04	1.03	1.01	0.1514	0.3429	0.5057	2.23393994	0.3976	0.3924	0.2100	2.05834450
138	8/5/2006	1.00	1.05	1.01	0.1523	0.3441	0.5036	2.28600215	0.3982	0.3925	0.2092	2.10555322
139	9/5/2006	0.98	0.99	1.00	0.1511	0.3474	0.5014	2.27366328	0.3968	0.3945	0.2088	2.08487890
140	10/5/2006	1.00	1.00	1.00	0.1506	0.3471	0.5023	2.27366328	0.3963	0.3945	0.2091	2.08487890
141	11/5/2006	0.97	1.01	1.00	0.1506	0.3471	0.5023	2.26901435	0.3963	0.3945	0.2091	2.06464280
142	12/5/2006	1.00	1.00	1.00	0.1490	0.3483	0.5027	2.26901435	0.3948	0.3957	0.2095	2.06464280
143	15/5/2006	0.93	1.01	0.98	0.1490	0.3483	0.5027	2.22410254	0.3948	0.3957	0.2095	2.00458904
144	16/5/2006	1.00	0.97	0.98	0.1465	0.3517	0.5018	2.17525477	0.3921	0.3982	0.2096	1.96940221
145	17/5/2006	1.00	1.02	1.01	0.1476	0.3503	0.5021	2.19408025	0.3933	0.3972	0.2095	1.98503810
146	18/5/2006	0.95	0.99	1.00	0.1472	0.3513	0.5015	2.16981907	0.3928	0.3978	0.2094	1.93570499
147	19/5/2006	1.02	1.03	1.01	0.1450	0.3515	0.5034	2.20145856	0.3908	0.3989	0.2104	1.97251391
148	22/5/2006	0.98	0.98	0.99	0.1453	0.3529	0.5018	2.17074885	0.3908	0.3993	0.2099	1.94030661
149	23/5/2006	1.02	1.00	0.98	0.1450	0.3525	0.5024	2.15396183	0.3906	0.3993	0.2101	1.94683845
150	24/5/2006	1.02	1.00	0.95	0.1464	0.3535	0.5001	2.10222904	0.3917	0.3991	0.2092	1.93989360
151	25/5/2006	1.04	1.01	0.97	0.1486	0.3565	0.4949	2.08614592	0.3932	0.3994	0.2074	1.96416813
152	26/5/2006	1.00	1.02	1.04	0.1510	0.3586	0.4903	2.14114500	0.3951	0.3991	0.2058	1.99446936
153	29/5/2006	1.00	0.99	1.02	0.1497	0.3576	0.4927	2.15813754	0.3941	0.3993	0.2067	1.99674686
154	30/5/2006	1.00	1.03	1.02	0.1493	0.3555	0.4952	2.20181512	0.3940	0.3986	0.2074	2.02679435
155	31/5/2006	1.00	1.01	1.00	0.1483	0.3562	0.4955	2.20857652	0.3930	0.3993	0.2077	2.03377164

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
156	1/6/2006	0.98	1.01	1.01	0.1482	0.3569	0.4949	2.21489576	0.3928	0.3997	0.2075	2.02740915
157	2/6/2006	1.00	1.02	1.01	0.1471	0.3576	0.4954	2.24029544	0.3917	0.4004	0.2078	2.04584261
158	5/6/2006	1.00	1.02	1.01	0.1465	0.3582	0.4953	2.26582852	0.3911	0.4010	0.2079	2.06431970
159	6/6/2006	1.00	1.03	1.00	0.1460	0.3589	0.4951	2.28668034	0.3906	0.4015	0.2079	2.08557119
160	7/6/2006	1.00	0.99	1.00	0.1455	0.3609	0.4936	2.27974521	0.3899	0.4025	0.2076	2.07851692
161	8/6/2006	1.00	0.99	1.00	0.1457	0.3602	0.4941	2.27278538	0.3901	0.4022	0.2077	2.07143289
162	9/6/2006	1.00	1.00	1.01	0.1458	0.3596	0.4946	2.27876492	0.3903	0.4018	0.2078	2.07372278
163	12/6/2006	1.00	1.03	1.01	0.1457	0.3592	0.4951	2.31193800	0.3903	0.4018	0.2080	2.09985723
164	13/6/2006	1.00	0.97	0.99	0.1450	0.3606	0.4944	2.27164537	0.3895	0.4026	0.2079	2.06660236
165	14/6/2006	1.00	1.01	0.98	0.1458	0.3585	0.4956	2.25470539	0.3905	0.4014	0.2081	2.06476027
166	15/6/2006	1.00	1.00	0.98	0.1462	0.3606	0.4932	2.23026556	0.3906	0.4021	0.2074	2.05534999
167	16/6/2006	1.00	1.04	1.03	0.1468	0.3620	0.4913	2.29573279	0.3909	0.4024	0.2068	2.09877379
168	19/6/2006	1.00	0.99	0.99	0.1454	0.3626	0.4920	2.27633320	0.3895	0.4033	0.2072	2.08684859
169	20/6/2006	1.00	0.99	0.99	0.1458	0.3626	0.4916	2.26309541	0.3899	0.4031	0.2070	2.07727214
170	21/6/2006	1.00	1.00	1.00	0.1461	0.3623	0.4916	2.26309541	0.3902	0.4028	0.2070	2.07727214
171	22/6/2006	1.00	0.99	1.01	0.1461	0.3623	0.4916	2.26844151	0.3902	0.4028	0.2070	2.07493130
172	23/6/2006	1.00	1.00	1.01	0.1459	0.3609	0.4931	2.27458775	0.3903	0.4023	0.2074	2.07729581
173	26/6/2006	0.98	0.99	1.01	0.1458	0.3606	0.4936	2.26717261	0.3902	0.4023	0.2076	2.05637256
174	27/6/2006	1.00	1.01	1.00	0.1450	0.3599	0.4951	2.27414726	0.3895	0.4024	0.2081	2.06344428
175	28/6/2006	1.00	1.00	0.99	0.1448	0.3606	0.4946	2.26800145	0.3893	0.4027	0.2080	2.06109915
176	29/6/2006	1.00	1.01	1.00	0.1450	0.3609	0.4941	2.27493897	0.3894	0.4028	0.2078	2.06813439
177	30/6/2006	1.00	1.01	0.99	0.1448	0.3616	0.4936	2.27568272	0.3892	0.4031	0.2077	2.07277995
178	3/7/2006	1.00	1.00	1.00	0.1448	0.3626	0.4926	2.27568272	0.3890	0.4035	0.2074	2.07277995
179	4/7/2006	1.00	1.01	1.01	0.1448	0.3626	0.4926	2.28875236	0.3890	0.4035	0.2074	2.08212565
180	5/7/2006	1.02	1.02	1.00	0.1445	0.3629	0.4925	2.31596403	0.3888	0.4038	0.2075	2.11915862
181	6/7/2006	1.00	0.99	1.00	0.1449	0.3645	0.4905	2.30915543	0.3889	0.4043	0.2069	2.11224992
182	7/7/2006	1.00	1.01	1.00	0.1451	0.3639	0.4910	2.31593203	0.3891	0.4039	0.2070	2.11913056
183	10/7/2006	1.00	1.06	1.01	0.1449	0.3645	0.4905	2.36952096	0.3889	0.4043	0.2069	2.16953891
184	11/7/2006	1.00	0.99	1.01	0.1439	0.3686	0.4876	2.36290528	0.3874	0.4064	0.2062	2.15886570
185	12/7/2006	1.00	0.98	1.00	0.1440	0.3670	0.4890	2.34976590	0.3877	0.4057	0.2066	2.14559569
186	13/7/2006	1.00	1.00	1.00	0.1443	0.3658	0.4900	2.34976590	0.3881	0.4051	0.2068	2.14559569
187	14/7/2006	0.98	0.99	1.00	0.1443	0.3658	0.4900	2.33613332	0.3881	0.4051	0.2068	2.12181080

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
188	17/7/2006	1.00	1.01	0.99	0.1436	0.3655	0.4909	2.33670396	0.3875	0.4053	0.2072	2.12635979
189	18/7/2006	1.02	1.01	1.01	0.1435	0.3665	0.4899	2.35685173	0.3874	0.4057	0.2069	2.15258519
190	19/7/2006	1.00	1.01	1.01	0.1441	0.3665	0.4894	2.37011498	0.3879	0.4054	0.2067	2.16198311
191	20/7/2006	1.00	0.99	1.00	0.1438	0.3667	0.4894	2.36337689	0.3876	0.4057	0.2067	2.15518456
192	21/7/2006	1.02	1.00	1.00	0.1440	0.3661	0.4899	2.37032060	0.3878	0.4053	0.2068	2.17224191
193	24/7/2006	1.00	1.00	1.00	0.1448	0.3658	0.4894	2.37032060	0.3886	0.4048	0.2066	2.17224191
194	25/7/2006	1.00	1.00	1.00	0.1448	0.3658	0.4894	2.37032060	0.3886	0.4048	0.2066	2.17224191
195	26/7/2006	1.02	1.02	0.99	0.1448	0.3658	0.4894	2.39091588	0.3886	0.4048	0.2066	2.20748716
196	27/7/2006	1.00	1.01	1.01	0.1454	0.3676	0.4871	2.41046626	0.3889	0.4053	0.2058	2.22335019
197	28/7/2006	1.00	1.02	1.01	0.1450	0.3684	0.4866	2.43649983	0.3884	0.4058	0.2058	2.24576563
198	31/7/2006	1.00	1.01	0.99	0.1445	0.3698	0.4857	2.43651526	0.3878	0.4066	0.2056	2.24981158
199	1/8/2006	1.00	1.01	1.00	0.1445	0.3707	0.4848	2.44941942	0.3877	0.4070	0.2054	2.26289127
200	2/8/2006	1.02	0.99	0.99	0.1442	0.3719	0.4839	2.44417208	0.3873	0.4075	0.2051	2.27289998
201	3/8/2006	1.02	1.00	0.99	0.1454	0.3713	0.4834	2.43874896	0.3884	0.4068	0.2048	2.28656839
202	4/8/2006	1.00	1.01	0.99	0.1465	0.3716	0.4819	2.44520106	0.3894	0.4064	0.2042	2.29726251
203	7/8/2006	1.00	1.01	0.99	0.1464	0.3731	0.4806	2.44509702	0.3891	0.4070	0.2039	2.30124520
204	8/8/2006	1.00	1.01	1.00	0.1464	0.3740	0.4796	2.45149155	0.3890	0.4074	0.2036	2.30780106
205	9/8/2006	1.00	1.01	0.99	0.1463	0.3745	0.4792	2.44466827	0.3888	0.4077	0.2035	2.30905679
206	10/8/2006	1.00	1.00	1.01	0.1464	0.3758	0.4778	2.45794171	0.3888	0.4081	0.2031	2.31438672
207	11/8/2006	1.00	1.03	0.99	0.1461	0.3751	0.4788	2.48316004	0.3886	0.4079	0.2034	2.34431284
208	14/8/2006	1.00	1.03	1.01	0.1456	0.3782	0.4762	2.51501506	0.3878	0.4094	0.2028	2.37274884
209	15/8/2006	1.00	1.01	1.01	0.1450	0.3800	0.4750	2.52793526	0.3870	0.4104	0.2026	2.38179845
210	16/8/2006	1.00	1.00	0.97	0.1448	0.3802	0.4750	2.49420352	0.3868	0.4106	0.2026	2.36824132
211	17/8/2006	1.00	1.00	1.01	0.1454	0.3820	0.4726	2.50101700	0.3872	0.4110	0.2019	2.37100476
212	18/8/2006	1.00	1.00	1.00	0.1453	0.3816	0.4731	2.50101700	0.3871	0.4109	0.2020	2.37100476
213	21/8/2006	1.00	1.01	1.00	0.1453	0.3816	0.4731	2.50729643	0.3871	0.4109	0.2020	2.37741414
214	22/8/2006	1.00	1.01	1.01	0.1452	0.3822	0.4727	2.52041150	0.3869	0.4112	0.2019	2.38660405
215	23/8/2006	1.00	1.02	0.99	0.1449	0.3823	0.4727	2.53253693	0.3867	0.4113	0.2020	2.40311418
216	24/8/2006	1.00	1.05	0.99	0.1447	0.3843	0.4710	2.56764385	0.3862	0.4122	0.2016	2.44713926
217	25/8/2006	1.00	1.02	1.00	0.1440	0.3891	0.4669	2.58527245	0.3851	0.4144	0.2006	2.46503330
218	28/8/2006	1.00	1.01	1.01	0.1437	0.3905	0.4658	2.59810309	0.3846	0.4151	0.2003	2.47383620
219	29/8/2006	1.00	0.99	1.00	0.1434	0.3906	0.4659	2.59220278	0.3844	0.4152	0.2004	2.46786411

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
220	30/8/2006	1.00	1.00	0.99	0.1436	0.3901	0.4663	2.58525571	0.3845	0.4150	0.2005	2.46502080
221	31/8/2006	1.00	1.00	1.00	0.1437	0.3905	0.4658	2.58525571	0.3846	0.4151	0.2003	2.46502080
222	1/9/2006	1.00	1.01	1.00	0.1437	0.3905	0.4658	2.59699455	0.3846	0.4151	0.2003	2.47691797
223	4/9/2006	1.00	1.00	1.01	0.1435	0.3915	0.4651	2.60385719	0.3843	0.4155	0.2002	2.47973480
224	5/9/2006	1.00	0.99	0.99	0.1433	0.3911	0.4656	2.59122747	0.3842	0.4155	0.2003	2.47105731
225	6/9/2006	1.00	1.03	1.00	0.1436	0.3910	0.4655	2.62034004	0.3845	0.4153	0.2002	2.50054718
226	7/9/2006	1.00	1.01	1.00	0.1430	0.3933	0.4636	2.63179186	0.3837	0.4165	0.1998	2.51211811
227	8/9/2006	1.00	1.00	0.99	0.1428	0.3943	0.4629	2.62490859	0.3834	0.4169	0.1997	2.50928419
228	11/9/2006	1.09	0.98	0.99	0.1429	0.3946	0.4624	2.62822766	0.3835	0.4170	0.1995	2.57385836
229	12/9/2006	1.00	0.97	1.01	0.1472	0.3922	0.4606	2.60634929	0.3875	0.4141	0.1984	2.54700075
230	13/9/2006	1.00	1.01	1.01	0.1476	0.3895	0.4629	2.62498037	0.3882	0.4129	0.1989	2.56205423
231	14/9/2006	1.00	1.03	1.00	0.1472	0.3901	0.4626	2.65423996	0.3878	0.4133	0.1989	2.59230880
232	15/9/2006	1.00	1.00	0.99	0.1467	0.3925	0.4609	2.64725017	0.3870	0.4145	0.1985	2.58936822
233	18/9/2006	1.00	0.96	1.00	0.1468	0.3928	0.4604	2.60531805	0.3871	0.4145	0.1984	2.54608705
234	19/9/2006	1.00	1.00	1.00	0.1476	0.3894	0.4630	2.60531805	0.3882	0.4129	0.1989	2.54608705
235	20/9/2006	1.00	0.98	1.00	0.1476	0.3894	0.4630	2.58812161	0.3882	0.4129	0.1989	2.52827028
236	21/9/2006	1.00	1.01	1.00	0.1479	0.3880	0.4640	2.59677891	0.3887	0.4122	0.1992	2.53725376
237	22/9/2006	1.00	0.99	1.01	0.1478	0.3887	0.4635	2.59506830	0.3884	0.4125	0.1991	2.53121041
238	25/9/2006	1.00	1.00	0.99	0.1478	0.3877	0.4645	2.58821910	0.3886	0.4121	0.1993	2.52834378
239	26/9/2006	1.14	0.99	1.01	0.1479	0.3880	0.4640	2.63878756	0.3887	0.4122	0.1992	2.65638861
240	27/9/2006	0.98	1.03	1.00	0.1536	0.3844	0.4620	2.66465081	0.3939	0.4083	0.1978	2.67072474
241	28/9/2006	1.00	1.01	1.00	0.1520	0.3876	0.4604	2.67318611	0.3921	0.4103	0.1976	2.67978020
242	29/9/2006	1.00	0.99	1.00	0.1518	0.3883	0.4599	2.66467879	0.3919	0.4106	0.1975	2.67076120
243	2/10/2006	1.00	0.98	1.00	0.1520	0.3876	0.4604	2.64760799	0.3921	0.4103	0.1976	2.65265002
244	3/10/2006	1.00	1.00	1.00	0.1523	0.3862	0.4615	2.64760799	0.3926	0.4096	0.1978	2.65265002
245	4/10/2006	1.00	1.02	0.99	0.1523	0.3862	0.4615	2.65789076	0.3926	0.4096	0.1978	2.66794577
246	5/10/2006	1.07	1.01	1.00	0.1521	0.3879	0.4600	2.69336492	0.3922	0.4103	0.1975	2.74674972
247	6/10/2006	0.96	0.99	1.01	0.1548	0.3873	0.4579	2.67451889	0.3945	0.4089	0.1965	2.69553253
248	9/10/2006	1.00	1.01	1.01	0.1530	0.3871	0.4599	2.69683496	0.3930	0.4097	0.1973	2.71052081
249	10/10/2006	1.05	1.05	1.03	0.1526	0.3871	0.4604	2.80766223	0.3926	0.4098	0.1975	2.83059419
250	11/10/2006	1.00	1.02	1.00	0.1528	0.3880	0.4592	2.83280361	0.3927	0.4101	0.1972	2.85738309
251	12/10/2006	1.00	0.99	1.01	0.1523	0.3899	0.4578	2.83143365	0.3921	0.4110	0.1969	2.85156068

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
252	13/10/2006	0.96	1.01	1.03	0.1523	0.3890	0.4587	2.86241418	0.3922	0.4107	0.1971	2.82967524
253	16/10/2006	1.04	0.96	1.01	0.1496	0.3885	0.4619	2.84703502	0.3899	0.4117	0.1984	2.83844195
254	17/10/2006	1.00	1.01	1.01	0.1521	0.3842	0.4637	2.86892036	0.3926	0.4090	0.1985	2.85312012
255	18/10/2006	1.02	1.01	1.00	0.1517	0.3842	0.4641	2.88679596	0.3922	0.4091	0.1987	2.88635804
256	19/10/2006	1.00	1.01	1.01	0.1525	0.3844	0.4631	2.90857175	0.3929	0.4089	0.1983	2.90102119
257	20/10/2006	0.98	1.01	1.01	0.1521	0.3844	0.4635	2.92091311	0.3925	0.4090	0.1984	2.89141345
258	23/10/2006	1.00	1.00	1.00	0.1508	0.3848	0.4644	2.92091311	0.3914	0.4098	0.1989	2.89141345
259	24/10/2006	1.00	1.00	1.00	0.1508	0.3848	0.4644	2.92091311	0.3914	0.4098	0.1989	2.89141345
260	25/10/2006	1.00	1.00	1.00	0.1508	0.3848	0.4644	2.92091311	0.3914	0.4098	0.1989	2.89141345
261	26/10/2006	1.00	1.02	1.02	0.1508	0.3848	0.4644	2.97293789	0.3914	0.4098	0.1989	2.92932479
262	27/10/2006	1.00	1.00	1.01	0.1499	0.3854	0.4647	2.98634959	0.3905	0.4104	0.1991	2.93498660
263	30/10/2006	1.00	1.00	1.02	0.1497	0.3848	0.4655	3.01308235	0.3904	0.4103	0.1993	2.94623749
264	31/10/2006	1.00	0.99	1.01	0.1492	0.3836	0.4671	3.00973392	0.3902	0.4100	0.1998	2.93433895
265	1/11/2006	1.00	0.99	1.01	0.1493	0.3819	0.4688	3.00614460	0.3904	0.4093	0.2003	2.92222310
266	2/11/2006	1.04	0.99	1.01	0.1494	0.3801	0.4706	3.03079563	0.3907	0.4085	0.2008	2.96954016
267	3/11/2006	1.00	1.01	0.99	0.1512	0.3780	0.4708	3.02631992	0.3925	0.4070	0.2005	2.97316295
268	6/11/2006	1.05	0.99	1.00	0.1512	0.3792	0.4695	3.02999674	0.3924	0.4074	0.2002	3.00811778
269	7/11/2006	0.98	1.00	1.03	0.1535	0.3771	0.4694	3.05924415	0.3946	0.4056	0.1998	2.99940744
270	8/11/2006	1.04	0.99	1.05	0.1519	0.3759	0.4722	3.13899477	0.3934	0.4058	0.2008	3.06452543
271	9/11/2006	1.04	1.03	1.01	0.1528	0.3707	0.4765	3.20757205	0.3948	0.4034	0.2019	3.15973761
272	10/11/2006	1.02	1.00	1.01	0.1539	0.3717	0.4744	3.23043029	0.3956	0.4033	0.2011	3.19102794
273	13/11/2006	1.02	0.99	1.00	0.1546	0.3708	0.4746	3.22300491	0.3963	0.4027	0.2011	3.19793753
274	14/11/2006	1.00	1.02	1.00	0.1558	0.3693	0.4749	3.24985008	0.3975	0.4015	0.2010	3.22690241
275	15/11/2006	1.02	1.04	1.00	0.1554	0.3711	0.4736	3.31402584	0.3969	0.4024	0.2007	3.31203436
276	16/11/2006	1.00	1.08	1.00	0.1555	0.3741	0.4704	3.41006726	0.3966	0.4036	0.1998	3.41558305
277	17/11/2006	0.98	0.99	0.99	0.1540	0.3802	0.4659	3.36914715	0.3946	0.4066	0.1988	3.36342252
278	20/11/2006	1.04	1.02	1.00	0.1535	0.3801	0.4665	3.41625702	0.3942	0.4068	0.1991	3.44666432
279	21/11/2006	1.33	1.01	1.02	0.1549	0.3808	0.4643	3.63558398	0.3953	0.4064	0.1983	3.92994478
280	22/11/2006	0.87	1.04	1.01	0.1683	0.3744	0.4573	3.62683445	0.4064	0.3989	0.1948	3.79143594
281	23/11/2006	0.98	1.00	1.01	0.1605	0.3805	0.4590	3.62958894	0.3995	0.4043	0.1962	3.76943367
282	24/11/2006	1.04	0.99	1.02	0.1595	0.3804	0.4602	3.67643900	0.3987	0.4047	0.1966	3.82746914
283	27/11/2006	1.02	1.00	1.02	0.1608	0.3772	0.4620	3.71456461	0.4002	0.4029	0.1969	3.86789309

Appendix 2 continued

No	Date	Price Relative			Universal Portfolio Generated by the Dirichlet (0.3, 0.7, 0.9) Distribution			Investment Capital	Universal Portfolio Generated by the Dirichlet (2.0, 2.0, 1.0) Distribution			Investment Capital
		Daibocu	KNM	Topglov	Daibocu	KNM	Topglov		Daibocu	KNM	Topglov	
284	28/11/2006	1.02	0.98	1.01	0.1613	0.3758	0.4629	3.70655804	0.4007	0.4022	0.1971	3.86653500
285	29/11/2006	0.96	1.03	1.01	0.1625	0.3731	0.4645	3.73893317	0.4020	0.4006	0.1974	3.86050797
286	30/11/2006	0.96	1.01	1.00	0.1599	0.3758	0.4643	3.73353378	0.3996	0.4028	0.1977	3.82249741
287	1/12/2006	1.06	1.00	1.02	0.1580	0.3775	0.4645	3.79444720	0.3978	0.4042	0.1980	3.92195493
288	4/12/2006	1.06	0.99	1.00	0.1602	0.3754	0.4644	3.82132120	0.3998	0.4025	0.1977	4.00334665
289	5/12/2006	1.06	1.02	1.02	0.1629	0.3738	0.4633	3.90756480	0.4022	0.4008	0.1970	4.13250477
290	6/12/2006	0.98	1.01	1.02	0.1647	0.3731	0.4622	3.94534731	0.4037	0.3998	0.1965	4.13128659
291	7/12/2006	0.98	1.00	1.01	0.1632	0.3726	0.4642	3.96078665	0.4026	0.4002	0.1972	4.11400368
292	8/12/2006	0.95	1.00	0.99	0.1620	0.3721	0.4659	3.91250817	0.4017	0.4005	0.1978	4.01801075
293	11/12/2006	1.00	1.01	0.99	0.1596	0.3737	0.4667	3.90209461	0.3996	0.4021	0.1984	4.02432406
294	12/12/2006	0.96	0.99	1.01	0.1598	0.3754	0.4648	3.89119214	0.3995	0.4027	0.1979	3.96199500
295	13/12/2006	1.00	0.99	0.99	0.1580	0.3744	0.4676	3.85624859	0.3981	0.4030	0.1989	3.94143839
296	14/12/2006	0.98	1.00	0.99	0.1585	0.3748	0.4667	3.81785623	0.3985	0.4030	0.1985	3.90012469
297	15/12/2006	1.00	1.02	1.00	0.1580	0.3761	0.4659	3.84205774	0.3979	0.4037	0.1984	3.92665965
298	18/12/2006	1.00	0.95	1.00	0.1576	0.3775	0.4649	3.76994526	0.3975	0.4044	0.1982	3.84771091
299	19/12/2006	1.00	0.98	1.01	0.1587	0.3734	0.4680	3.75900346	0.3988	0.4023	0.1989	3.82670736
300	20/12/2006	1.00	1.01	1.02	0.1588	0.3715	0.4697	3.80219459	0.3992	0.4015	0.1993	3.85621661
301	21/12/2006	1.00	1.00	1.01	0.1582	0.3715	0.4703	3.81563809	0.3987	0.4018	0.1995	3.86200187
302	22/12/2006	1.00	0.99	1.01	0.1580	0.3711	0.4709	3.82090949	0.3986	0.4017	0.1997	3.85884288
303	26/12/2006	1.00	1.01	1.00	0.1580	0.3702	0.4719	3.82913249	0.3986	0.4013	0.2000	3.86784722
304	27/12/2006	1.04	0.98	1.00	0.1578	0.3706	0.4715	3.82704987	0.3985	0.4016	0.1999	3.89815208
305	28/12/2006	1.00	1.00	1.01	0.1599	0.3685	0.4716	3.84051942	0.4004	0.3999	0.1997	3.90396078

Appendix 3

The Asymptotic Behaviour of the Ratio of Capitals for the Dirichlet (0.3, 0.4, 0.6) Distribution

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\left\|J_n^*\right\|$	G_n^*	$\left\|\frac{\hat{S}_n}{S_n^*}-G_n^*\right\|$
1	0.5000	0.5000	2.0400	0.2308	0.3077	0.4615	1.210800	1.0000	0.0000	0.0000	3.060000	0.395686	Inf	0.000000	0.395686
2	0.5000	0.5000	2.0400	0.1719	0.2292	0.5990	1.722200	-0.1222	-0.1860	1.3082	6.323700	0.272341	Inf	0.000000	0.272341
3	0.3960	2.5250	0.3960	0.1275	0.1700	0.7024	1.305400	-0.1222	0.3583	0.7639	3.256900	0.400811	Inf	0.000000	0.400811
4	3.0600	0.3333	0.3333	0.1118	0.2997	0.5885	0.833110	0.2860	0.2222	0.4918	1.529400	0.544730	4.501000	0.528900	0.015830
5	0.5000	0.5000	2.0400	0.2373	0.2595	0.5032	1.062200	0.2043	0.1406	0.6551	2.126700	0.499459	5.860700	0.550700	0.051241
6	0.3960	2.5250	0.3960	0.1874	0.2063	0.6063	0.887110	0.1499	0.3583	0.4918	1.980800	0.447854	5.063700	0.392300	0.055554
7	0.3960	2.5250	0.3960	0.1610	0.3294	0.5096	0.973450	0.1110	0.5138	0.3751	2.633400	0.369655	5.360600	0.362000	0.007655
8	0.5000	0.5000	2.0400	0.1330	0.4588	0.4082	1.098700	0.0819	0.4263	0.4918	3.082000	0.356489	6.001400	0.371800	0.015311
9	0.3960	2.5250	0.3960	0.1148	0.3854	0.4998	1.336600	0.0592	0.5397	0.4011	4.399400	0.303814	6.408700	0.378000	0.074186
10	3.0600	0.3333	0.3333	0.0975	0.4946	0.4079	0.800970	0.2043	0.4672	0.3285	3.035500	0.263868	4.395500	0.203800	0.060068
11	0.5000	0.5000	2.0400	0.1785	0.4514	0.3702	0.857100	0.1747	0.4078	0.4176	3.265900	0.262439	4.680400	0.197600	0.064839
12	0.3960	2.5250	0.3960	0.1567	0.3948	0.4486	1.059800	0.1499	0.4944	0.3557	4.435100	0.238957	4.861100	0.187900	0.051057
13	0.5000	0.5000	2.0400	0.1348	0.4855	0.3797	1.149600	0.1290	0.4420	0.4290	4.901800	0.234526	5.150400	0.185700	0.048826
14	3.0600	0.3333	0.3333	0.1199	0.4318	0.4483	0.758900	0.2277	0.3972	0.3751	4.015400	0.188997	4.288500	0.142900	0.046097
15	0.3960	2.5250	0.3960	0.1991	0.3947	0.4061	0.938260	0.2043	0.4672	0.3285	5.288700	0.177408	4.395500	0.135900	0.041508
16	0.3960	2.5250	0.3960	0.1750	0.4712	0.3538	1.312800	0.1839	0.5284	0.2877	7.704000	0.170405	4.572500	0.131700	0.038705
17	0.5000	0.5000	2.0400	0.1546	0.5356	0.3098	1.282700	0.1659	0.4864	0.3477	7.625500	0.168212	4.701000	0.128000	0.040212
18	3.0600	0.3333	0.3333	0.1402	0.4936	0.3662	0.917940	0.2406	0.4490	0.3103	6.768200	0.135625	4.185300	0.108400	0.027225
19	3.0600	0.3333	0.3333	0.2130	0.4559	0.3310	0.839110	0.3075	0.4156	0.2769	7.315900	0.114697	3.937800	0.097800	0.016897
20	0.5000	0.5000	2.0400	0.2894	0.4159	0.2947	0.800370	0.2860	0.3855	0.3285	7.071400	0.113184	4.018800	0.094500	0.018684
21	0.5000	0.5000	2.0400	0.2680	0.3861	0.3458	0.826460	0.2666	0.3583	0.3751	7.369600	0.112144	4.135300	0.092400	0.019744
22	3.0600	0.3333	0.3333	0.2475	0.3574	0.3951	0.833180	0.3231	0.3336	0.3433	8.383200	0.099387	3.962200	0.085200	0.014187
23	0.5000	0.5000	2.0400	0.3144	0.3276	0.3580	0.875890	0.3037	0.3110	0.3853	8.897500	0.098442	4.074700	0.083500	0.014942
24	0.5000	0.5000	2.0400	0.2936	0.3034	0.4030	0.981520	0.2860	0.2903	0.4237	9.994100	0.098210	4.208800	0.082400	0.015810
25	0.5000	0.5000	2.0400	0.2741	0.2810	0.4448	1.163100	0.2697	0.2712	0.4591	11.79380	0.098620	4.360600	0.081700	0.016920
26	0.3960	2.5250	0.3960	0.2562	0.2605	0.4833	1.105700	0.2546	0.3164	0.4290	12.05090	0.091752	4.292000	0.077200	0.014552
27	0.3960	2.5250	0.3960	0.2386	0.3121	0.4493	1.172400	0.2406	0.3583	0.4011	13.43090	0.087291	4.272500	0.073900	0.013391
28	0.3960	2.5250	0.3960	0.2224	0.3596	0.4180	1.361900	0.2277	0.3972	0.3751	16.12350	0.084467	4.288500	0.071400	0.013067

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
29	3.0600	0.3333	0.3333	0.2078	0.4029	0.3894	1.225500	0.2719	0.3771	0.3510	16.35760	0.074919	4.083900	0.066100	0.008819
30	0.3960	2.5250	0.3960	0.2602	0.3791	0.3607	1.474300	0.2588	0.4127	0.3285	20.23410	0.072862	4.110100	0.064200	0.008662
31	0.5000	0.5000	2.0400	0.2455	0.4181	0.3364	1.500900	0.2465	0.3934	0.3601	20.84880	0.071990	4.186100	0.063100	0.008890
32	0.5000	0.5000	2.0400	0.2319	0.3977	0.3704	1.606700	0.2350	0.3753	0.3897	22.46500	0.071520	4.275600	0.062400	0.009120
33	3.0600	0.3333	0.3333	0.2191	0.3785	0.4024	1.495300	0.2736	0.3583	0.3681	23.06650	0.064826	4.102900	0.058400	0.006426
34	0.3960	2.5250	0.3960	0.2645	0.3585	0.3769	1.733600	0.2620	0.3903	0.3477	27.52020	0.062994	4.113400	0.056700	0.006294
35	3.0600	0.3333	0.3333	0.2516	0.3934	0.3550	1.767300	0.2977	0.3739	0.3285	30.17290	0.058572	3.988500	0.053700	0.004872
36	3.0600	0.3333	0.3333	0.2923	0.3748	0.3329	1.997500	0.3314	0.3583	0.3103	35.93570	0.055585	3.906200	0.051400	0.004185
35	3.0600	0.3333	0.3333	0.2516	0.3934	0.3550	1.767300	0.2977	0.3739	0.3285	30.17290	0.058572	3.988500	0.053700	0.004872
36	3.0600	0.3333	0.3333	0.2923	0.3748	0.3329	1.997500	0.3314	0.3583	0.3103	35.93570	0.055585	3.906200	0.051400	0.004185
37	3.0600	0.3333	0.3333	0.3293	0.3578	0.3129	2.459200	0.3632	0.3436	0.2932	46.01220	0.053447	3.855400	0.049500	0.003947
38	0.3960	2.5250	0.3960	0.3634	0.3421	0.2945	2.764900	0.3504	0.3726	0.2769	53.30740	0.051867	3.855200	0.048200	0.003667
39	0.3960	2.5250	0.3960	0.3498	0.3732	0.2769	3.292000	0.3383	0.4002	0.2615	64.96990	0.050670	3.870000	0.047100	0.003570
40	0.5000	0.5000	2.0400	0.3371	0.4024	0.2606	2.967000	0.3268	0.3855	0.2877	59.96430	0.049479	3.897000	0.046100	0.003379
41	0.5000	0.5000	2.0400	0.3250	0.3870	0.2880	2.799600	0.3159	0.3716	0.3126	57.70180	0.048518	3.934300	0.045300	0.003218
42	3.0600	0.3333	0.3333	0.3133	0.3722	0.3145	3.325100	0.3443	0.3583	0.2974	71.17260	0.046719	3.876900	0.043800	0.002919
43	0.5000	0.5000	2.0400	0.3436	0.3582	0.2982	3.189400	0.3335	0.3456	0.3209	69.47610	0.045906	3.915700	0.043100	0.002806
44	0.5000	0.5000	2.0400	0.3322	0.3447	0.3232	3.182000	0.3231	0.3336	0.3433	70.27790	0.045277	3.962200	0.042600	0.002677
45	0.5000	0.5000	2.0400	0.3213	0.3318	0.3470	3.291200	0.3132	0.3220	0.3648	73.46340	0.044801	4.015500	0.042100	0.002701
46	0.5000	0.5000	2.0400	0.3109	0.3195	0.3696	3.518900	0.3037	0.3110	0.3853	79.16610	0.044450	4.074700	0.041800	0.002650
47	0.3960	2.5250	0.3960	0.3010	0.3078	0.3912	3.699500	0.2947	0.3351	0.3702	85.80560	0.043115	4.056500	0.040700	0.002415
48	0.5000	0.5000	2.0400	0.2914	0.3337	0.3750	3.986200	0.2860	0.3243	0.3897	93.11610	0.042809	4.115200	0.040300	0.002509
49	0.5000	0.5000	2.0400	0.2823	0.3223	0.3955	4.420600	0.2777	0.3139	0.4085	1.038E+02	0.042590	4.178800	0.040100	0.002490
50	0.3960	2.5250	0.3960	0.2736	0.3114	0.4150	4.681300	0.2697	0.3365	0.3938	1.130E+02	0.041438	4.162400	0.039100	0.002338
51	0.3960	2.5250	0.3960	0.2650	0.3355	0.3995	5.197200	0.2620	0.3583	0.3797	1.283E+02	0.040507	4.157400	0.038200	0.002307
52	3.0600	0.3333	0.3333	0.2569	0.3583	0.3848	5.372500	0.2860	0.3478	0.3662	1.386E+02	0.038756	4.067800	0.036800	0.001956
53	0.5000	0.5000	2.0400	0.2826	0.3472	0.3702	5.749100	0.2783	0.3378	0.3839	1.494E+02	0.038484	4.119700	0.036500	0.001984
54	0.5000	0.5000	2.0400	0.2746	0.3367	0.3887	6.315700	0.2709	0.3281	0.4011	1.650E+02	0.038277	4.175800	0.036300	0.001977
55	0.5000	0.5000	2.0400	0.2669	0.3267	0.4064	7.111000	0.2637	0.3187	0.4176	1.865E+02	0.038125	4.235600	0.036100	0.002025
56	0.5000	0.5000	2.0400	0.2595	0.3170	0.4235	8.193400	0.2568	0.3097	0.4335	2.155E+02	0.038022	4.298900	0.035900	0.002122
57	0.3960	2.5250	0.3960	0.2523	0.3077	0.4400	8.612400	0.2502	0.3297	0.4202	2.320E+02	0.037121	4.281500	0.035100	0.002021
58	0.3960	2.5250	0.3960	0.2452	0.3287	0.4261	9.437600	0.2438	0.3489	0.4073	2.595E+02	0.036372	4.273400	0.034400	0.001972
59	0.5000	0.5000	2.0400	0.2384	0.3488	0.4128	10.71800	0.2376	0.3399	0.4226	2.956E+02	0.036263	4.331700	0.034300	0.001963
60	0.5000	0.5000	2.0400	0.2319	0.3395	0.4286	12.43300	0.2316	0.3311	0.4374	3.435E+02	0.036195	4.393000	0.034100	0.002095

Appendix 3 continued

Day n	Transformed Price Relatives			U. P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
61	0.5000	0.5000	2.0400	0.2256	0.3306	0.4438	14.71400	0.2258	0.3226	0.4516	4.069E+02	0.036161	4.457100	0.034000	0.002161
62	0.5000	0.5000	2.0400	0.2195	0.3220	0.4585	17.74600	0.2201	0.3144	0.4654	4.908E+02	0.036158	4.524000	0.033900	0.002258
63	3.0600	0.3333	0.3333	0.2136	0.3136	0.4728	16.25100	0.2406	0.3065	0.4529	4.719E+02	0.034436	4.407400	0.032600	0.001836
64	0.5000	0.5000	2.0400	0.2359	0.3049	0.4592	19.61700	0.2350	0.2988	0.4663	5.700E+02	0.034417	4.471600	0.032600	0.001817
65	0.3960	2.5250	0.3960	0.2301	0.2970	0.4729	20.17300	0.2295	0.3164	0.4541	5.990E+02	0.033678	4.450200	0.031900	0.001778
66	0.3960	2.5250	0.3960	0.2242	0.3155	0.4603	21.53800	0.2241	0.3336	0.4423	6.517E+02	0.033050	4.436900	0.031300	0.001750
67	0.3960	2.5250	0.3960	0.2185	0.3333	0.4482	23.81300	0.2190	0.3502	0.4309	7.324E+02	0.032513	4.430700	0.030800	0.001713
68	3.0600	0.3333	0.3333	0.2130	0.3505	0.4365	21.76800	0.2380	0.3423	0.4197	7.004E+02	0.031080	4.323300	0.029700	0.001380
69	0.5000	0.5000	2.0400	0.2335	0.3420	0.4245	25.11400	0.2327	0.3346	0.4326	8.099E+02	0.031009	4.375700	0.029600	0.001409
70	3.0600	0.3333	0.3333	0.2281	0.3342	0.4377	23.99100	0.2510	0.3272	0.4218	8.041E+02	0.029836	4.283100	0.028600	0.001236
71	0.5000	0.5000	2.0400	0.2475	0.3263	0.4262	27.74300	0.2457	0.3200	0.4343	9.321E+02	0.029763	4.333900	0.028500	0.001263
72	0.3960	2.5250	0.3960	0.2421	0.3189	0.4390	29.82400	0.2406	0.3356	0.4237	1.020E+03	0.029249	4.323100	0.028000	0.001249
73	0.5000	0.5000	2.0400	0.2367	0.3351	0.4282	34.57800	0.2357	0.3285	0.4359	1.185E+03	0.029185	4.373200	0.028000	0.001185
74	0.5000	0.5000	2.0400	0.2316	0.3278	0.4406	40.74900	0.2308	0.3215	0.4476	1.398E+03	0.029139	4.425300	0.027900	0.001239
75	0.5000	0.5000	2.0400	0.2266	0.3208	0.4526	48.77800	0.2261	0.3148	0.4591	1.676E+03	0.029110	4.479100	0.027800	0.001310
76	0.5000	0.5000	2.0400	0.2217	0.3139	0.4643	59.26900	0.2215	0.3082	0.4703	2.037E+03	0.029095	4.534700	0.027700	0.001395
77	0.3960	2.5250	0.3960	0.2170	0.3072	0.4758	62.23900	0.2171	0.3230	0.4600	2.175E+03	0.028612	4.519600	0.027300	0.001312
78	3.0600	0.3333	0.3333	0.2123	0.3225	0.4652	56.77100	0.2337	0.3164	0.4499	2.062E+03	0.027535	4.421600	0.026400	0.001135
79	3.0600	0.3333	0.3333	0.2299	0.3155	0.4545	54.51400	0.2498	0.3101	0.4401	2.046E+03	0.026641	4.338700	0.025700	0.000941
80	3.0600	0.3333	0.3333	0.2469	0.3088	0.4443	54.86400	0.2656	0.3039	0.4305	2.120E+03	0.025881	4.268400	0.025000	0.000881
81	0.5000	0.5000	2.0400	0.2632	0.3024	0.4344	64.13400	0.2608	0.2978	0.4414	2.483E+03	0.025827	4.313900	0.024900	0.000927
82	0.5000	0.5000	2.0400	0.2584	0.2962	0.4454	76.06000	0.2561	0.2919	0.4520	2.950E+03	0.025785	4.360900	0.024900	0.000885
83	0.5000	0.5000	2.0400	0.2536	0.2902	0.4562	91.46400	0.2516	0.2862	0.4623	3.551E+03	0.025754	4.409300	0.024800	0.000954
84	0.3960	2.5250	0.3960	0.2490	0.2843	0.4667	91.58800	0.2471	0.3000	0.4529	3.622E+03	0.025283	4.387800	0.024400	0.000883
85	0.3960	2.5250	0.3960	0.2443	0.2986	0.4570	94.49600	0.2428	0.3135	0.4438	3.800E+03	0.024866	4.371500	0.024000	0.000866
86	0.3960	2.5250	0.3960	0.2398	0.3125	0.4477	100.2900	0.2385	0.3267	0.4348	4.094E+03	0.024494	4.359900	0.023700	0.000794
87	0.5000	0.5000	2.0400	0.2355	0.3260	0.4385	117.8700	0.2344	0.3208	0.4449	4.821E+03	0.024451	4.403700	0.023600	0.000851
88	3.0600	0.3333	0.3333	0.2312	0.3200	0.4488	113.6000	0.2489	0.3150	0.4361	4.783E+03	0.023751	4.329400	0.023000	0.000751
89	0.3960	2.5250	0.3960	0.2463	0.3140	0.4397	120.9400	0.2447	0.3277	0.4276	5.167E+03	0.023408	4.318800	0.022700	0.000708
90	0.5000	0.5000	2.0400	0.2420	0.3271	0.4310	140.7300	0.2406	0.3220	0.4374	6.024E+03	0.023361	4.359700	0.022600	0.000761
91	0.3960	2.5250	0.3960	0.2378	0.3213	0.4409	151.9900	0.2366	0.3344	0.4290	6.595E+03	0.023046	4.351200	0.022300	0.000746
92	0.5000	0.5000	2.0400	0.2337	0.3339	0.4324	177.1900	0.2327	0.3287	0.4385	7.703E+03	0.023003	4.391700	0.022300	0.000703
93	3.0600	0.3333	0.3333	0.2297	0.3282	0.4421	170.0600	0.2465	0.3232	0.4303	7.600E+03	0.022375	4.320400	0.021700	0.000675

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
94	0.3960	2.5250	0.3960	0.2440	0.3224	0.4336	184.0900	0.2426	0.3351	0.4223	8.336E+03	0.022084	4.312600	0.021400	0.000684
95	0.5000	0.5000	2.0400	0.2400	0.3346	0.4254	212.6400	0.2387	0.3297	0.4316	9.649E+03	0.022037	4.350500	0.021400	0.000637
96	0.5000	0.5000	2.0400	0.2360	0.3291	0.4349	248.7200	0.2350	0.3243	0.4408	1.131E+04	0.021998	4.389600	0.021300	0.000698
97	0.3960	2.5250	0.3960	0.2322	0.3237	0.4441	269.8900	0.2313	0.3359	0.4329	1.242E+04	0.021725	4.382100	0.021100	0.000625
98	0.3960	2.5250	0.3960	0.2284	0.3355	0.4361	299.6400	0.2277	0.3472	0.4251	1.395E+04	0.021476	4.377800	0.020800	0.000676
99	0.5000	0.5000	2.0400	0.2247	0.3470	0.4283	347.4400	0.2241	0.3418	0.4341	1.621E+04	0.021438	4.415100	0.020800	0.000638
100	0.3960	2.5250	0.3960	0.2211	0.3416	0.4373	390.2600	0.2207	0.3529	0.4265	1.840E+04	0.021206	4.412400	0.020600	0.000606
101	0.5000	0.5000	2.0400	0.2175	0.3528	0.4296	453.3200	0.2173	0.3475	0.4352	2.141E+04	0.021172	4.449300	0.020500	0.000672
102	0.3960	2.5250	0.3960	0.2141	0.3475	0.4385	514.8600	0.2140	0.3583	0.4277	2.457E+04	0.020957	4.448000	0.020300	0.000657
103	0.3960	2.5250	0.3960	0.2106	0.3584	0.4309	596.7700	0.2107	0.3689	0.4204	2.875E+04	0.020760	4.449300	0.020100	0.000660
104	0.3960	2.5250	0.3960	0.2073	0.3692	0.4235	705.3700	0.2075	0.3792	0.4133	3.428E+04	0.020579	4.452900	0.019900	0.000679
105	0.5000	0.5000	2.0400	0.2040	0.3797	0.4163	804.8900	0.2043	0.3739	0.4218	3.918E+04	0.020544	4.487100	0.019900	0.000644
106	0.5000	0.5000	2.0400	0.2008	0.3743	0.4249	929.1800	0.2013	0.3686	0.4302	4.529E+04	0.020516	4.522300	0.019800	0.000716
107	0.3960	2.5250	0.3960	0.1976	0.3690	0.4334	1097.900	0.1982	0.3786	0.4231	5.396E+04	0.020346	4.525800	0.019700	0.000646
108	0.3960	2.5250	0.3960	0.1945	0.3792	0.4263	1321.100	0.1953	0.3885	0.4162	6.544E+04	0.020189	4.531400	0.019500	0.000689
109	0.3960	2.5250	0.3960	0.1914	0.3893	0.4193	1618.000	0.1924	0.3983	0.4094	8.072E+04	0.020044	4.539000	0.019300	0.000744
110	3.0600	0.3333	0.3333	0.1884	0.3991	0.4125	1370.500	0.2043	0.3929	0.4027	7.056E+04	0.019422	4.453000	0.018800	0.000622
111	0.3960	2.5250	0.3960	0.2010	0.3935	0.4055	1691.000	0.2014	0.4024	0.3962	8.769E+04	0.019284	4.461200	0.018700	0.000584
112	0.3960	2.5250	0.3960	0.1980	0.4032	0.3988	2121.100	0.1985	0.4118	0.3897	1.107E+05	0.019156	4.471100	0.018600	0.000556
113	0.5000	0.5000	2.0400	0.1950	0.4126	0.3924	2342.200	0.1957	0.4065	0.3979	1.225E+05	0.019118	4.500100	0.018500	0.000618
114	0.5000	0.5000	2.0400	0.1921	0.4073	0.4006	2616.100	0.1929	0.4013	0.4058	1.371E+05	0.019085	4.530000	0.018500	0.000585
115	3.0600	0.3333	0.3333	0.1892	0.4021	0.4087	2221.600	0.2043	0.3962	0.3995	1.199E+05	0.018525	4.448100	0.018000	0.000525
116	3.0600	0.3333	0.3333	0.2012	0.3968	0.4020	1959.300	0.2156	0.3911	0.3932	1.086E+05	0.018036	4.374900	0.017600	0.000436
117	0.3960	2.5250	0.3960	0.2129	0.3916	0.3956	2409.200	0.2127	0.4002	0.3871	1.345E+05	0.017909	4.382200	0.017400	0.000509
118	0.3960	2.5250	0.3960	0.2099	0.4007	0.3894	3009.300	0.2099	0.4090	0.3811	1.692E+05	0.017790	4.390900	0.017300	0.000490
119	0.5000	0.5000	2.0400	0.2070	0.4097	0.3833	3280.900	0.2071	0.4040	0.3889	1.849E+05	0.017748	4.416900	0.017300	0.000448
120	3.0600	0.3333	0.3333	0.2042	0.4046	0.3912	2920.100	0.2180	0.3991	0.3829	1.687E+05	0.017305	4.348000	0.016900	0.000405
121	3.0600	0.3333	0.3333	0.2154	0.3996	0.3850	2688.500	0.2286	0.3943	0.3771	1.590E+05	0.016911	4.286200	0.016500	0.000411
122	0.5000	0.5000	2.0400	0.2264	0.3946	0.3790	2913.300	0.2258	0.3895	0.3847	1.727E+05	0.016864	4.310500	0.016500	0.000364
123	0.5000	0.5000	2.0400	0.2235	0.3898	0.3867	3191.600	0.2229	0.3849	0.3922	1.897E+05	0.016823	4.335500	0.016400	0.000423
124	3.0600	0.3333	0.3333	0.2206	0.3851	0.3943	2983.500	0.2333	0.3802	0.3864	1.812E+05	0.016461	4.277600	0.016100	0.000361
125	3.0600	0.3333	0.3333	0.2313	0.3804	0.3883	2875.700	0.2435	0.3757	0.3807	1.782E+05	0.016134	4.225400	0.015800	0.000334
126	0.3960	2.5250	0.3960	0.2417	0.3758	0.3825	3439.400	0.2406	0.3842	0.3751	2.148E+05	0.016014	4.229300	0.015700	0.000314
127	0.3960	2.5250	0.3960	0.2388	0.3844	0.3769	4176.600	0.2378	0.3926	0.3696	2.627E+05	0.015902	4.234600	0.015600	0.000302

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
128	3.0600	0.3333	0.3333	0.2359	0.3928	0.3713	4078.300	0.2477	0.3881	0.3642	2.614E+05	0.015602	4.185900	0.015300	0.000302
129	0.3960	2.5250	0.3960	0.2460	0.3882	0.3657	4985.900	0.2449	0.3963	0.3589	3.218E+05	0.015496	4.191600	0.015200	0.000296
130	3.0600	0.3333	0.3333	0.2431	0.3965	0.3603	4967.200	0.2546	0.3918	0.3536	3.263E+05	0.015221	4.146900	0.014900	0.000321
131	3.0600	0.3333	0.3333	0.2531	0.3920	0.3550	5083.000	0.2642	0.3874	0.3484	3.396E+05	0.014969	4.106400	0.014700	0.000269
132	3.0600	0.3333	0.3333	0.2628	0.3875	0.3497	5336.700	0.2736	0.3830	0.3433	3.622E+05	0.014735	4.069900	0.014500	0.000235
133	0.5000	0.5000	2.0400	0.2724	0.3831	0.3445	5499.500	0.2706	0.3788	0.3506	3.746E+05	0.014681	4.087400	0.014400	0.000281
134	0.3960	2.5250	0.3960	0.2694	0.3788	0.3518	6612.800	0.2677	0.3867	0.3456	4.536E+05	0.014579	4.091300	0.014300	0.000279
135	3.0600	0.3333	0.3333	0.2664	0.3868	0.3467	7008.200	0.2769	0.3825	0.3406	4.880E+05	0.014361	4.056800	0.014100	0.000261
136	0.5000	0.5000	2.0400	0.2758	0.3825	0.3417	7191.600	0.2740	0.3783	0.3477	5.026E+05	0.014308	4.073600	0.014100	0.000208
137	0.5000	0.5000	2.0400	0.2728	0.3783	0.3489	7459.500	0.2711	0.3742	0.3547	5.232E+05	0.014258	4.091100	0.014000	0.000258
138	0.5000	0.5000	2.0400	0.2699	0.3742	0.3560	7818.800	0.2682	0.3701	0.3616	5.502E+05	0.014212	4.109200	0.014000	0.000212
139	3.0600	0.3333	0.3333	0.2670	0.3701	0.3629	8298.300	0.2772	0.3661	0.3567	5.925E+05	0.014006	4.075700	0.013800	0.000206
140	0.5000	0.5000	2.0400	0.2761	0.3660	0.3579	8723.100	0.2743	0.3622	0.3635	6.248E+05	0.013962	4.093700	0.013700	0.000262
141	3.0600	0.3333	0.3333	0.2732	0.3620	0.3648	9405.100	0.2831	0.3583	0.3586	6.831E+05	0.013769	4.062600	0.013600	0.000169
142	0.5000	0.5000	2.0400	0.2821	0.3581	0.3598	9914.200	0.2802	0.3545	0.3653	7.223E+05	0.013726	4.080500	0.013500	0.000226
143	0.3960	2.5250	0.3960	0.2792	0.3542	0.3666	11403.00	0.2774	0.3621	0.3605	8.370E+05	0.013624	4.080500	0.013400	0.000224
144	3.0600	0.3333	0.3333	0.2763	0.3620	0.3617	12393.00	0.2860	0.3583	0.3557	9.219E+05	0.013443	4.050900	0.013200	0.000243
145	3.0600	0.3333	0.3333	0.2850	0.3581	0.3569	13762.00	0.2944	0.3545	0.3510	1.037E+06	0.013274	4.024100	0.013100	0.000174
146	0.3960	2.5250	0.3960	0.2936	0.3543	0.3521	15831.00	0.2916	0.3620	0.3464	1.201E+06	0.013177	4.024100	0.013000	0.000177
147	0.5000	0.5000	2.0400	0.2907	0.3619	0.3474	16386.00	0.2888	0.3583	0.3529	1.248E+06	0.013133	4.040000	0.012900	0.000233
148	0.3960	2.5250	0.3960	0.2879	0.3581	0.3540	18982.00	0.2860	0.3657	0.3483	1.456E+06	0.013041	4.040600	0.012800	0.000241
149	3.0600	0.3333	0.3333	0.2851	0.3655	0.3494	21080.00	0.2942	0.3620	0.3438	1.637E+06	0.012881	4.014600	0.012700	0.000181
150	0.3960	2.5250	0.3960	0.2934	0.3618	0.3448	24586.00	0.2914	0.3692	0.3394	1.922E+06	0.012793	4.015600	0.012600	0.000193
151	0.5000	0.5000	2.0400	0.2906	0.3691	0.3403	25178.00	0.2887	0.3655	0.3458	1.975E+06	0.012749	4.030400	0.012600	0.000149
152	0.5000	0.5000	2.0400	0.2878	0.3654	0.3468	26036.00	0.2860	0.3619	0.3521	2.049E+06	0.012708	4.045700	0.012500	0.000208
153	0.3960	2.5250	0.3960	0.2851	0.3617	0.3532	30361.00	0.2833	0.3690	0.3477	2.405E+06	0.012623	4.046800	0.012400	0.000223
154	0.3960	2.5250	0.3960	0.2824	0.3689	0.3487	35867.00	0.2807	0.3760	0.3433	2.860E+06	0.012542	4.048800	0.012400	0.000142
155	3.0600	0.3333	0.3333	0.2797	0.3760	0.3443	39313.00	0.2886	0.3723	0.3390	3.173E+06	0.012390	4.022300	0.012200	0.000190
156	3.0600	0.3333	0.3333	0.2878	0.3723	0.3399	43951.00	0.2965	0.3688	0.3348	3.589E+06	0.012248	3.998100	0.012100	0.000148
157	0.5000	0.5000	2.0400	0.2957	0.3687	0.3356	44692.00	0.2938	0.3652	0.3410	3.661E+06	0.012206	4.011900	0.012000	0.000206
158	0.3960	2.5250	0.3960	0.2930	0.3651	0.3419	52439.00	0.2912	0.3721	0.3368	4.324E+06	0.012128	4.013300	0.012000	0.000128
159	0.3960	2.5250	0.3960	0.2903	0.3720	0.3376	62300.00	0.2886	0.3788	0.3326	5.168E+06	0.012054	4.015600	0.011900	0.000154
160	3.0600	0.3333	0.3333	0.2877	0.3788	0.3334	69642.00	0.2962	0.3753	0.3285	5.843E+06	0.011919	3.992000	0.011800	0.000119
161	3.0600	0.3333	0.3333	0.2955	0.3753	0.3293	79316.00	0.3037	0.3718	0.3244	6.727E+06	0.011791	3.970500	0.011600	0.000191

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
162	0.3960	2.5250	0.3960	0.3031	0.3718	0.3251	94189.00	0.3011	0.3785	0.3204	8.036E+06	0.011720	3.972700	0.011600	0.000120
163	3.0600	0.3333	0.3333	0.3004	0.3785	0.3211	1.09E+05	0.3085	0.3750	0.3165	9.358E+06	0.011599	3.952600	0.011400	0.000199
164	0.3960	2.5250	0.3960	0.3079	0.3750	0.3171	1.30E+05	0.3059	0.3815	0.3126	1.124E+07	0.011532	3.955200	0.011400	0.000132
165	3.0600	0.3333	0.3333	0.3053	0.3816	0.3131	1.51E+05	0.3132	0.3781	0.3087	1.324E+07	0.011418	3.936500	0.011300	0.000118
166	0.3960	2.5250	0.3960	0.3127	0.3781	0.3092	1.82E+05	0.3106	0.3845	0.3049	1.599E+07	0.011354	3.939400	0.011200	0.000154
167	0.3960	2.5250	0.3960	0.3100	0.3846	0.3054	2.21E+05	0.3080	0.3909	0.3011	1.953E+07	0.011293	3.943000	0.011100	0.000193
168	0.3960	2.5250	0.3960	0.3074	0.3910	0.3015	2.71E+05	0.3054	0.3972	0.2974	2.412E+07	0.011234	3.947300	0.011100	0.000134
169	0.5000	0.5000	2.0400	0.3049	0.3974	0.2978	2.60E+05	0.3029	0.3937	0.3034	2.321E+07	0.011187	3.956400	0.011000	0.000187
170	0.3960	2.5250	0.3960	0.3023	0.3939	0.3038	3.21E+05	0.3004	0.3999	0.2997	2.880E+07	0.011130	3.961100	0.011000	0.000130
171	3.0600	0.3333	0.3333	0.2998	0.4001	0.3001	3.69E+05	0.3075	0.3965	0.2960	3.347E+07	0.011022	3.941800	0.010900	0.000122
172	0.5000	0.5000	2.0400	0.3069	0.3967	0.2964	3.53E+05	0.3050	0.3931	0.3019	3.215E+07	0.010976	3.950600	0.010800	0.000176
173	0.5000	0.5000	2.0400	0.3044	0.3933	0.3023	3.41E+05	0.3025	0.3898	0.3077	3.116E+07	0.010933	3.959900	0.010800	0.000133
174	3.0600	0.3333	0.3333	0.3019	0.3899	0.3082	3.94E+05	0.3095	0.3865	0.3041	3.639E+07	0.010829	3.941400	0.010700	0.000129
175	3.0600	0.3333	0.3333	0.3089	0.3865	0.3045	4.63E+05	0.3163	0.3832	0.3005	4.317E+07	0.010730	3.924400	0.010600	0.000130
176	3.0600	0.3333	0.3333	0.3158	0.3833	0.3009	5.53E+05	0.3231	0.3800	0.2969	5.203E+07	0.010636	3.908900	0.010500	0.000136
177	3.0600	0.3333	0.3333	0.3227	0.3800	0.2973	6.71E+05	0.3298	0.3768	0.2934	6.365E+07	0.010546	3.894800	0.010400	0.000146
178	3.0600	0.3333	0.3333	0.3294	0.3768	0.2938	8.27E+05	0.3364	0.3736	0.2900	7.903E+07	0.010461	3.881900	0.010300	0.000161
179	3.0600	0.3333	0.3333	0.3361	0.3736	0.2902	1.03E+06	0.3430	0.3705	0.2865	9.955E+07	0.010379	3.870300	0.010300	0.000079
180	0.3960	2.5250	0.3960	0.3427	0.3705	0.2868	1.22E+06	0.3404	0.3764	0.2831	1.186E+08	0.010324	3.872100	0.010200	0.000124
181	3.0600	0.3333	0.3333	0.3402	0.3765	0.2833	1.54E+06	0.3469	0.3733	0.2798	1.506E+08	0.010247	3.861200	0.010100	0.000147
182	0.5000	0.5000	2.0400	0.3467	0.3734	0.2799	1.44E+06	0.3443	0.3703	0.2854	1.409E+08	0.010201	3.867900	0.010100	0.000101
183	3.0600	0.3333	0.3333	0.3441	0.3703	0.2856	1.83E+06	0.3507	0.3672	0.2821	1.804E+08	0.010127	3.857800	0.010000	0.000127
184	0.3960	2.5250	0.3960	0.3505	0.3672	0.2823	2.15E+06	0.3481	0.3731	0.2788	2.136E+08	0.010074	3.859100	0.010000	0.000074
185	0.3960	2.5250	0.3960	0.3479	0.3732	0.2789	2.56E+06	0.3456	0.3789	0.2755	2.556E+08	0.010023	3.861100	0.009900	0.000123
186	0.5000	0.5000	2.0400	0.3454	0.3790	0.2756	2.37E+06	0.3431	0.3759	0.2811	2.374E+08	0.009978	3.867200	0.009900	0.000078
187	0.3960	2.5250	0.3960	0.3428	0.3759	0.2812	2.83E+06	0.3406	0.3816	0.2778	2.854E+08	0.009929	3.869400	0.009800	0.000129
188	0.3960	2.5250	0.3960	0.3403	0.3817	0.2780	3.43E+06	0.3381	0.3873	0.2746	3.466E+08	0.009882	3.872300	0.009800	0.000082
189	0.5000	0.5000	2.0400	0.3379	0.3874	0.2747	3.16E+06	0.3357	0.3842	0.2801	3.214E+08	0.009838	3.878200	0.009700	0.000138
190	0.3960	2.5250	0.3960	0.3354	0.3844	0.2803	3.84E+06	0.3333	0.3898	0.2769	3.920E+08	0.009792	3.881300	0.009700	0.000092
191	0.3960	2.5250	0.3960	0.3330	0.3900	0.2770	4.71E+06	0.3309	0.3953	0.2738	4.829E+08	0.009748	3.884900	0.009600	0.000148
192	0.5000	0.5000	2.0400	0.3306	0.3956	0.2739	4.34E+06	0.3285	0.3923	0.2792	4.471E+08	0.009706	3.890700	0.009600	0.000106
193	3.0600	0.3333	0.3333	0.3282	0.3925	0.2793	5.33E+06	0.3346	0.3893	0.2760	5.532E+08	0.009634	3.878600	0.009500	0.000134
194	0.5000	0.5000	2.0400	0.3344	0.3895	0.2761	4.93E+06	0.3323	0.3864	0.2814	5.140E+08	0.009593	3.884500	0.009500	0.000093
195	0.3960	2.5250	0.3960	0.3320	0.3865	0.2815	6.01E+06	0.3300	0.3918	0.2782	6.294E+08	0.009550	3.887700	0.009400	0.000150

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
196	3.0600	0.3333	0.3333	0.3297	0.3920	0.2784	7.41E+06	0.3360	0.3888	0.2752	7.812E+08	0.009480	3.876000	0.009400	0.000080
197	0.5000	0.5000	2.0400	0.3357	0.3890	0.2753	6.84E+06	0.3337	0.3859	0.2804	7.248E+08	0.009440	3.881700	0.009300	0.000140
198	0.5000	0.5000	2.0400	0.3334	0.3861	0.2806	6.38E+06	0.3314	0.3830	0.2856	6.783E+08	0.009402	3.887900	0.009300	0.000102
199	3.0600	0.3333	0.3333	0.3310	0.3831	0.2858	7.88E+06	0.3373	0.3802	0.2825	8.444E+08	0.009335	3.876600	0.009200	0.000135
200	3.0600	0.3333	0.3333	0.3370	0.3803	0.2827	9.87E+06	0.3431	0.3774	0.2795	1.065E+09	0.009270	3.866300	0.009200	0.000070
201	0.3960	2.5250	0.3960	0.3429	0.3774	0.2796	1.18E+07	0.3408	0.3827	0.2765	1.283E+09	0.009228	3.868500	0.009100	0.000128
202	0.3960	2.5250	0.3960	0.3406	0.3828	0.2766	1.43E+07	0.3385	0.3879	0.2735	1.561E+09	0.009187	3.871200	0.009100	0.000087
203	3.0600	0.3333	0.3333	0.3383	0.3881	0.2736	1.80E+07	0.3443	0.3851	0.2706	1.973E+09	0.009126	3.861300	0.009000	0.000126
204	0.5000	0.5000	2.0400	0.3441	0.3853	0.2706	1.65E+07	0.3420	0.3823	0.2757	1.816E+09	0.009087	3.866400	0.009000	0.000087
205	0.3960	2.5250	0.3960	0.3418	0.3824	0.2758	2.00E+07	0.3398	0.3875	0.2727	2.208E+09	0.009048	3.869000	0.008900	0.000148
206	0.3960	2.5250	0.3960	0.3395	0.3877	0.2728	2.44E+07	0.3375	0.3927	0.2698	2.708E+09	0.009010	3.872100	0.008900	0.000110
207	0.3960	2.5250	0.3960	0.3373	0.3928	0.2699	3.01E+07	0.3353	0.3977	0.2670	3.351E+09	0.008973	3.875700	0.008900	0.000073
208	3.0600	0.3333	0.3333	0.3351	0.3980	0.2670	3.75E+07	0.3409	0.3949	0.2641	4.206E+09	0.008914	3.865500	0.008800	0.000114
209	0.3960	2.5250	0.3960	0.3408	0.3952	0.2641	4.64E+07	0.3387	0.4000	0.2613	5.224E+09	0.008878	3.869200	0.008800	0.000078
210	0.5000	0.5000	2.0400	0.3385	0.4002	0.2612	4.19E+07	0.3365	0.3972	0.2663	4.735E+09	0.008840	3.873400	0.008700	0.000140
211	0.5000	0.5000	2.0400	0.3363	0.3974	0.2663	3.81E+07	0.3344	0.3944	0.2712	4.327E+09	0.008803	3.878000	0.008700	0.000103
212	0.5000	0.5000	2.0400	0.3341	0.3946	0.2713	3.50E+07	0.3322	0.3917	0.2761	3.987E+09	0.008768	3.882900	0.008700	0.000068
213	3.0600	0.3333	0.3333	0.3319	0.3918	0.2762	4.33E+07	0.3377	0.3890	0.2733	4.971E+09	0.008710	3.872500	0.008600	0.000110
214	3.0600	0.3333	0.3333	0.3375	0.3891	0.2734	5.43E+07	0.3432	0.3863	0.2705	6.271E+09	0.008654	3.862900	0.008600	0.000054
215	0.3960	2.5250	0.3960	0.3430	0.3864	0.2705	6.61E+07	0.3411	0.3912	0.2677	7.674E+09	0.008619	3.865800	0.008500	0.000119
216	0.5000	0.5000	2.0400	0.3409	0.3914	0.2677	6.03E+07	0.3389	0.3885	0.2725	7.029E+09	0.008584	3.870400	0.008500	0.000084
217	0.3960	2.5250	0.3960	0.3387	0.3887	0.2726	7.38E+07	0.3368	0.3934	0.2698	8.635E+09	0.008550	3.873400	0.008500	0.000050
218	0.3960	2.5250	0.3960	0.3366	0.3936	0.2698	9.11E+07	0.3347	0.3983	0.2671	1.070E+10	0.008517	3.876900	0.008400	0.000117
219	3.0600	0.3333	0.3333	0.3345	0.3985	0.2671	1.13E+08	0.3401	0.3956	0.2644	1.340E+10	0.008464	3.867100	0.008400	0.000064
220	3.0600	0.3333	0.3333	0.3399	0.3958	0.2643	1.43E+08	0.3454	0.3929	0.2617	1.699E+10	0.008411	3.858000	0.008300	0.000111
221	0.3960	2.5250	0.3960	0.3452	0.3931	0.2616	1.76E+08	0.3433	0.3977	0.2590	2.103E+10	0.008380	3.861400	0.008300	0.000080
222	3.0600	0.3333	0.3333	0.3431	0.3979	0.2590	2.24E+08	0.3485	0.3951	0.2564	2.685E+10	0.008330	3.852900	0.008200	0.000130
223	0.5000	0.5000	2.0400	0.3484	0.3953	0.2563	2.00E+08	0.3464	0.3925	0.2611	2.412E+10	0.008295	3.856400	0.008200	0.000095
224	3.0600	0.3333	0.3333	0.3463	0.3927	0.2611	2.56E+08	0.3516	0.3899	0.2585	3.099E+10	0.008246	3.848400	0.008200	0.000046
225	0.5000	0.5000	2.0400	0.3515	0.3901	0.2584	2.30E+08	0.3495	0.3873	0.2632	2.795E+10	0.008212	3.852000	0.008100	0.000112
226	3.0600	0.3333	0.3333	0.3494	0.3875	0.2631	2.95E+08	0.3546	0.3848	0.2606	3.614E+10	0.008165	3.844400	0.008100	0.000065
227	3.0600	0.3333	0.3333	0.3545	0.3850	0.2605	3.84E+08	0.3597	0.3823	0.2580	4.725E+10	0.008120	3.837500	0.008000	0.000120
228	3.0600	0.3333	0.3333	0.3597	0.3824	0.2579	5.04E+08	0.3648	0.3798	0.2554	6.242E+10	0.008077	3.831300	0.008000	0.000077
229	0.5000	0.5000	2.0400	0.3647	0.3799	0.2553	4.50E+08	0.3626	0.3773	0.2600	5.598E+10	0.008043	3.834600	0.008000	0.000043

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
230	0.5000	0.5000	2.0400	0.3626	0.3774	0.2600	4.05E+08	0.3605	0.3749	0.2646	5.061E+10	0.008011	3.838200	0.007900	0.000111
231	0.5000	0.5000	2.0400	0.3605	0.3750	0.2646	3.68E+08	0.3584	0.3724	0.2691	4.610E+10	0.007980	3.842200	0.007900	0.000080
232	0.3960	2.5250	0.3960	0.3583	0.3725	0.2691	4.37E+08	0.3564	0.3771	0.2666	5.504E+10	0.007948	3.843700	0.007900	0.000048
233	3.0600	0.3333	0.3333	0.3563	0.3772	0.2666	5.71E+08	0.3613	0.3747	0.2640	7.220E+10	0.007906	3.837200	0.007800	0.000106
234	3.0600	0.3333	0.3333	0.3612	0.3748	0.2640	7.52E+08	0.3662	0.3723	0.2615	9.568E+10	0.007864	3.831300	0.007800	0.000064
235	0.3960	2.5250	0.3960	0.3662	0.3724	0.2615	8.94E+08	0.3642	0.3768	0.2590	1.142E+11	0.007834	3.832700	0.007800	0.000034
236	0.5000	0.5000	2.0400	0.3641	0.3770	0.2589	8.04E+08	0.3621	0.3744	0.2635	1.030E+11	0.007803	3.836200	0.007700	0.000103
237	0.5000	0.5000	2.0400	0.3620	0.3745	0.2634	7.28E+08	0.3601	0.3721	0.2679	9.366E+10	0.007773	3.839900	0.007700	0.000073
238	0.5000	0.5000	2.0400	0.3600	0.3722	0.2679	6.64E+08	0.3580	0.3697	0.2722	8.579E+10	0.007745	3.844100	0.007700	0.000045
239	3.0600	0.3333	0.3333	0.3579	0.3698	0.2723	8.70E+08	0.3629	0.3674	0.2697	1.129E+11	0.007705	3.837900	0.007600	0.000105
240	3.0600	0.3333	0.3333	0.3628	0.3675	0.2698	1.15E+09	0.3676	0.3651	0.2673	1.501E+11	0.007666	3.832300	0.007600	0.000066
241	0.3960	2.5250	0.3960	0.3676	0.3651	0.2673	1.35E+09	0.3656	0.3696	0.2648	1.768E+11	0.007635	3.833100	0.007600	0.000035
242	0.3960	2.5250	0.3960	0.3655	0.3697	0.2648	1.60E+09	0.3636	0.3740	0.2624	2.100E+11	0.007605	3.834200	0.007500	0.000105
243	0.5000	0.5000	2.0400	0.3635	0.3741	0.2623	1.44E+09	0.3616	0.3717	0.2667	1.905E+11	0.007577	3.837800	0.007500	0.000077
244	0.5000	0.5000	2.0400	0.3615	0.3718	0.2667	1.31E+09	0.3596	0.3695	0.2709	1.741E+11	0.007550	3.841800	0.007500	0.000050
245	0.5000	0.5000	2.0400	0.3595	0.3695	0.2710	1.21E+09	0.3576	0.3672	0.2752	1.603E+11	0.007524	3.846000	0.007500	0.000024
246	0.5000	0.5000	2.0400	0.3575	0.3672	0.2753	1.11E+09	0.3557	0.3649	0.2794	1.486E+11	0.007499	3.850500	0.007400	0.000099
247	0.3960	2.5250	0.3960	0.3556	0.3649	0.2795	1.31E+09	0.3538	0.3693	0.2769	1.750E+11	0.007469	3.851200	0.007400	0.000069
248	0.3960	2.5250	0.3960	0.3536	0.3694	0.2770	1.55E+09	0.3518	0.3737	0.2745	2.077E+11	0.007440	3.852300	0.007400	0.000040
249	3.0600	0.3333	0.3333	0.3517	0.3737	0.2746	2.00E+09	0.3565	0.3714	0.2721	2.698E+11	0.007402	3.845700	0.007300	0.000102
250	0.3960	2.5250	0.3960	0.3564	0.3715	0.2722	2.37E+09	0.3546	0.3757	0.2697	3.214E+11	0.007374	3.847000	0.007300	0.000074
251	0.3960	2.5250	0.3960	0.3545	0.3758	0.2697	2.83E+09	0.3527	0.3800	0.2673	3.858E+11	0.007347	3.848600	0.007300	0.000047
252	3.0600	0.3333	0.3333	0.3526	0.3801	0.2674	3.67E+09	0.3573	0.3777	0.2650	5.020E+11	0.007310	3.842100	0.007200	0.000110
253	0.3960	2.5250	0.3960	0.3572	0.3778	0.2650	4.41E+09	0.3554	0.3820	0.2627	6.048E+11	0.007284	3.843900	0.007200	0.000084
254	0.3960	2.5250	0.3960	0.3553	0.3821	0.2626	5.33E+09	0.3535	0.3862	0.2603	7.340E+11	0.007259	3.845900	0.007200	0.000059
255	0.3960	2.5250	0.3960	0.3534	0.3863	0.2603	6.49E+09	0.3516	0.3903	0.2581	8.974E+11	0.007234	3.848300	0.007200	0.000034
256	0.3960	2.5250	0.3960	0.3515	0.3905	0.2580	7.97E+09	0.3498	0.3944	0.2558	1.105E+12	0.007210	3.850900	0.007100	0.000110
257	0.3960	2.5250	0.3960	0.3497	0.3946	0.2557	9.85E+09	0.3479	0.3985	0.2535	1.370E+12	0.007188	3.853900	0.007100	0.000088
258	0.5000	0.5000	2.0400	0.3478	0.3988	0.2534	8.77E+09	0.3461	0.3963	0.2576	1.225E+12	0.007161	3.856700	0.007100	0.000061
259	3.0600	0.3333	0.3333	0.3460	0.3965	0.2575	1.12E+10	0.3506	0.3940	0.2554	1.571E+12	0.007124	3.849700	0.007100	0.000024
260	0.5000	0.5000	2.0400	0.3505	0.3942	0.2553	1.00E+10	0.3488	0.3918	0.2594	1.408E+12	0.007099	3.852600	0.007000	0.000099
261	3.0600	0.3333	0.3333	0.3487	0.3920	0.2594	1.28E+10	0.3532	0.3896	0.2572	1.818E+12	0.007064	3.845900	0.007000	0.000064
262	0.3960	2.5250	0.3960	0.3532	0.3897	0.2571	1.57E+10	0.3514	0.3936	0.2549	2.235E+12	0.007041	3.848500	0.007000	0.000041
263	0.5000	0.5000	2.0400	0.3513	0.3938	0.2549	1.40E+10	0.3496	0.3914	0.2590	2.002E+12	0.007015	3.851300	0.007000	0.000015

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
264	0.5000	0.5000	2.0400	0.3495	0.3916	0.2589	1.26E+10	0.3478	0.3892	0.2629	1.806E+12	0.006991	3.854400	0.006900	0.000091
265	3.0600	0.3333	0.3333	0.3477	0.3894	0.2629	1.62E+10	0.3522	0.3871	0.2607	2.325E+12	0.006957	3.847700	0.006900	0.000057
266	0.5000	0.5000	2.0400	0.3521	0.3872	0.2607	1.46E+10	0.3504	0.3849	0.2646	2.103E+12	0.006933	3.850900	0.006900	0.000033
267	0.5000	0.5000	2.0400	0.3503	0.3850	0.2646	1.32E+10	0.3487	0.3828	0.2686	1.915E+12	0.006910	3.854300	0.006900	0.000010
268	0.5000	0.5000	2.0400	0.3485	0.3829	0.2686	1.21E+10	0.3469	0.3806	0.2724	1.755E+12	0.006888	3.858000	0.006800	0.000088
269	0.3960	2.5250	0.3960	0.3468	0.3807	0.2725	1.46E+10	0.3452	0.3846	0.2702	2.125E+12	0.006865	3.859900	0.006800	0.000065
270	0.3960	2.5250	0.3960	0.3450	0.3847	0.2703	1.77E+10	0.3434	0.3885	0.2680	2.590E+12	0.006843	3.862000	0.006800	0.000043
271	3.0600	0.3333	0.3333	0.3433	0.3887	0.2680	2.25E+10	0.3478	0.3864	0.2658	3.304E+12	0.006809	3.855000	0.006800	0.000009
272	0.3960	2.5250	0.3960	0.3476	0.3865	0.2658	2.74E+10	0.3460	0.3903	0.2637	4.041E+12	0.006787	3.857200	0.006700	0.000087
273	0.3960	2.5250	0.3960	0.3459	0.3905	0.2636	3.37E+10	0.3443	0.3942	0.2615	4.974E+12	0.006766	3.859700	0.006700	0.000066
274	3.0600	0.3333	0.3333	0.3442	0.3943	0.2615	4.28E+10	0.3486	0.3921	0.2594	6.357E+12	0.006733	3.852900	0.006700	0.000033
275	0.5000	0.5000	2.0400	0.3485	0.3922	0.2593	3.85E+10	0.3469	0.3900	0.2632	5.736E+12	0.006711	3.855900	0.006700	0.000011
276	0.3960	2.5250	0.3960	0.3467	0.3901	0.2632	4.72E+10	0.3452	0.3938	0.2610	7.058E+12	0.006690	3.858300	0.006600	0.000090
277	0.3960	2.5250	0.3960	0.3450	0.3940	0.2610	5.83E+10	0.3435	0.3976	0.2589	8.740E+12	0.006670	3.861000	0.006600	0.000070
278	0.5000	0.5000	2.0400	0.3433	0.3978	0.2589	5.24E+10	0.3418	0.3955	0.2627	7.881E+12	0.006648	3.864000	0.006600	0.000048
279	0.3960	2.5250	0.3960	0.3417	0.3957	0.2627	6.49E+10	0.3401	0.3993	0.2606	9.788E+12	0.006628	3.866800	0.006600	0.000028
280	3.0600	0.3333	0.3333	0.3400	0.3994	0.2606	8.18E+10	0.3443	0.3972	0.2585	1.240E+13	0.006596	3.859600	0.006500	0.000096
281	0.3960	2.5250	0.3960	0.3442	0.3974	0.2585	1.02E+11	0.3427	0.4009	0.2564	1.544E+13	0.006578	3.862500	0.006500	0.000078
282	3.0600	0.3333	0.3333	0.3425	0.4011	0.2564	1.29E+11	0.3468	0.3988	0.2544	1.966E+13	0.006546	3.855700	0.006500	0.000046
283	3.0600	0.3333	0.3333	0.3467	0.3990	0.2543	1.65E+11	0.3509	0.3968	0.2523	2.525E+13	0.006516	3.849300	0.006500	0.000016
284	0.3960	2.5250	0.3960	0.3508	0.3970	0.2522	2.04E+11	0.3492	0.4005	0.2503	3.143E+13	0.006498	3.852100	0.006400	0.000098
285	0.3960	2.5250	0.3960	0.3492	0.4007	0.2502	2.55E+11	0.3476	0.4041	0.2483	3.937E+13	0.006480	3.855200	0.006400	0.000080
286	0.5000	0.5000	2.0400	0.3475	0.4044	0.2481	2.25E+11	0.3459	0.4021	0.2520	3.485E+13	0.006457	3.857300	0.006400	0.000057
287	0.5000	0.5000	2.0400	0.3458	0.4023	0.2519	2.00E+11	0.3443	0.4000	0.2557	3.105E+13	0.006436	3.859700	0.006400	0.000036
288	0.5000	0.5000	2.0400	0.3442	0.4002	0.2556	1.79E+11	0.3427	0.3980	0.2593	2.783E+13	0.006414	3.862300	0.006400	0.000014
289	3.0600	0.3333	0.3333	0.3426	0.3982	0.2593	2.26E+11	0.3467	0.3960	0.2573	3.544E+13	0.006385	3.855700	0.006300	0.000085
290	3.0600	0.3333	0.3333	0.3466	0.3961	0.2572	2.89E+11	0.3507	0.3940	0.2553	4.551E+13	0.006356	3.849500	0.006300	0.000056
291	3.0600	0.3333	0.3333	0.3507	0.3941	0.2552	3.73E+11	0.3547	0.3920	0.2533	5.894E+13	0.006329	3.843700	0.006300	0.000029
292	3.0600	0.3333	0.3333	0.3547	0.3921	0.2532	4.85E+11	0.3587	0.3900	0.2513	7.698E+13	0.006301	3.838300	0.006200	0.000101
293	0.3960	2.5250	0.3960	0.3586	0.3902	0.2512	5.95E+11	0.3570	0.3936	0.2494	9.469E+13	0.006284	3.840600	0.006200	0.000084
294	3.0600	0.3333	0.3333	0.3570	0.3938	0.2492	7.77E+11	0.3610	0.3916	0.2474	1.243E+14	0.006257	3.835400	0.006200	0.000057
295	0.5000	0.5000	2.0400	0.3609	0.3918	0.2473	6.85E+11	0.3593	0.3897	0.2510	1.098E+14	0.006236	3.837400	0.006200	0.000036
296	0.5000	0.5000	2.0400	0.3593	0.3898	0.2509	6.07E+11	0.3577	0.3877	0.2546	9.765E+13	0.006215	3.839700	0.006200	0.000015
297	0.5000	0.5000	2.0400	0.3576	0.3879	0.2545	5.41E+11	0.3561	0.3858	0.2581	8.738E+13	0.006196	3.842100	0.006100	0.000096

Appendix 3 continued

Day n	Transformed Price Relatives			U.P. Generated by the Dirichlet (.3, .4, .6) Distr. $\hat{b}_n$			Universal Capital $\hat{S}_n$	Time-Varying Log-Optimal Portfolio, b_n^*			Optimal Capital S_n^*	$\frac{\hat{S}_n}{S_n^*}$	$\lvert J_n^* \rvert$	G_n^*	$\left\lvert \frac{\hat{S}_n}{S_n^*} - G_n^* \right\rvert$
298	3.0600	0.3333	0.3333	0.3560	0.3859	0.2581	7.06E+11	0.3600	0.3839	0.2562	1.144E+14	0.006170	3.837000	0.006100	0.000070
299	3.0600	0.3333	0.3333	0.3599	0.3840	0.2561	9.28E+11	0.3638	0.3820	0.2542	1.510E+14	0.006144	3.832200	0.006100	0.000044
300	3.0600	0.3333	0.3333	0.3638	0.3821	0.2541	1.23E+12	0.3676	0.3801	0.2523	2.010E+14	0.006120	3.827800	0.006100	0.000020
301	0.3960	2.5250	0.3960	0.3676	0.3802	0.2522	1.48E+12	0.3660	0.3836	0.2504	2.430E+14	0.006102	3.829300	0.006100	0.000002
302	0.5000	0.5000	2.0400	0.3660	0.3838	0.2503	1.31E+12	0.3644	0.3817	0.2539	2.158E+14	0.006082	3.831500	0.006000	0.000082
303	0.3960	2.5250	0.3960	0.3644	0.3819	0.2538	1.59E+12	0.3628	0.3852	0.2520	2.617E+14	0.006065	3.833200	0.006000	0.000065
304	3.0600	0.3333	0.3333	0.3627	0.3854	0.2519	2.10E+12	0.3666	0.3834	0.2501	3.474E+14	0.006041	3.828700	0.006000	0.000041
305	3.0600	0.3333	0.3333	0.3665	0.3835	0.2500	2.80E+12	0.3703	0.3815	0.2482	4.648E+14	0.006017	3.824600	0.006000	0.000017
306	3.0600	0.3333	0.3333	0.3703	0.3816	0.2481	3.76E+12	0.3740	0.3796	0.2463	6.266E+14	0.005994	3.820800	0.005900	0.000094
307	0.5000	0.5000	2.0400	0.3740	0.3798	0.2462	3.30E+12	0.3724	0.3778	0.2498	5.527E+14	0.005975	3.822600	0.005900	0.000075
308	0.5000	0.5000	2.0400	0.3724	0.3779	0.2497	2.92E+12	0.3708	0.3760	0.2532	4.904E+14	0.005956	3.824700	0.005900	0.000056
309	0.5000	0.5000	2.0400	0.3708	0.3761	0.2531	2.60E+12	0.3692	0.3742	0.2566	4.377E+14	0.005938	3.827000	0.005900	0.000038
310	3.0600	0.3333	0.3333	0.3692	0.3742	0.2566	3.48E+12	0.3729	0.3723	0.2547	5.888E+14	0.005915	3.823100	0.005900	0.000015
311	0.5000	0.5000	2.0400	0.3729	0.3724	0.2547	3.11E+12	0.3713	0.3706	0.2581	5.269E+14	0.005897	3.825500	0.005900	0.000003
312	0.5000	0.5000	2.0400	0.3713	0.3706	0.2581	2.79E+12	0.3697	0.3688	0.2615	4.743E+14	0.005880	3.828000	0.005800	0.000080
313	0.3960	2.5250	0.3960	0.3697	0.3688	0.2615	3.29E+12	0.3682	0.3722	0.2596	5.619E+14	0.005862	3.828800	0.005800	0.000062
314	3.0600	0.3333	0.3333	0.3681	0.3723	0.2596	4.40E+12	0.3718	0.3704	0.2578	7.541E+14	0.005840	3.824900	0.005800	0.000040
315	3.0600	0.3333	0.3333	0.3718	0.3705	0.2577	5.93E+12	0.3754	0.3687	0.2559	1.020E+15	0.005818	3.821400	0.005800	0.000018
316	0.5000	0.5000	2.0400	0.3754	0.3687	0.2559	5.30E+12	0.3738	0.3669	0.2592	9.142E+14	0.005801	3.823700	0.005800	0.000001
317	0.3960	2.5250	0.3960	0.3738	0.3670	0.2592	6.24E+12	0.3723	0.3703	0.2574	1.080E+15	0.005784	3.824400	0.005700	0.000084
318	3.0600	0.3333	0.3333	0.3722	0.3704	0.2574	8.42E+12	0.3759	0.3686	0.2556	1.461E+15	0.005763	3.820900	0.005700	0.000063
319	0.5000	0.5000	2.0400	0.3758	0.3686	0.2555	7.52E+12	0.3743	0.3668	0.2589	1.309E+15	0.005746	3.823300	0.005700	0.000046
320	3.0600	0.3333	0.3333	0.3743	0.3669	0.2588	1.02E+13	0.3778	0.3651	0.2571	1.779E+15	0.005725	3.819900	0.005700	0.000025
321	0.3960	2.5250	0.3960	0.3778	0.3652	0.2570	1.19E+13	0.3763	0.3685	0.2552	2.093E+15	0.005708	3.820500	0.005700	0.000008
322	0.5000	0.5000	2.0400	0.3763	0.3685	0.2552	1.07E+13	0.3747	0.3668	0.2585	1.875E+15	0.005691	3.822800	0.005700	0.000009
323	0.3960	2.5250	0.3960	0.3747	0.3668	0.2585	1.26E+13	0.3732	0.3701	0.2567	2.213E+15	0.005675	3.823400	0.005600	0.000075
324	0.3960	2.5250	0.3960	0.3732	0.3702	0.2567	1.49E+13	0.3717	0.3734	0.2549	2.628E+15	0.005658	3.824300	0.005600	0.000058
325	3.0600	0.3333	0.3333	0.3716	0.3735	0.2548	2.00E+13	0.3752	0.3717	0.2531	3.552E+15	0.005638	3.820800	0.005600	0.000038
326	0.3960	2.5250	0.3960	0.3752	0.3718	0.2531	2.38E+13	0.3736	0.3750	0.2514	4.230E+15	0.005622	3.821800	0.005600	0.000022
327	0.5000	0.5000	2.0400	0.3736	0.3751	0.2513	2.11E+13	0.3721	0.3733	0.2546	3.763E+15	0.005606	3.823800	0.005600	0.000006
328	0.5000	0.5000	2.0400	0.3721	0.3734	0.2545	1.88E+13	0.3706	0.3716	0.2578	3.366E+15	0.005590	3.826000	0.005600	0.000010
329	0.5000	0.5000	2.0400	0.3706	0.3716	0.2578	1.69E+13	0.3691	0.3699	0.2610	3.027E+15	0.005574	3.828400	0.005500	0.000074
330	0.3960	2.5250	0.3960	0.3691	0.3699	0.2610	2.00E+13	0.3676	0.373	0.2592	3.593E+15	0.005558	3.829200	0.005500	0.000058