HARMONY SEARCH APPROACH IN THE STRUT AND TIE MODEL TO OPTIMISE THE STRESS DISTRIBUTION IN A CONCRETE BOX GIRDER

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By

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ABSTRACT

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Alice Lim Pei San

Stress evaluation for box girder structure has always been the popular research topic to understand the behaviour of box girder to ensure sufficient ductility capacity provided and good serviceability performance. Out of the many methods introduced by researchers for box girder's behavioural study, strut and tie model (STM) can effectively demonstrate the stress distribution using truss analogy. Good STM construction is important to avoid over-simplified and dense STM that may cause excessive crack width and structure failure. Good STM construction using traditional trial-and-error method can be tedious and time-consuming especially when dealing with complex structure. Optimisation using meta-heuristic algorithm could provide an alternative for more efficient STM construction. This study aims to develop a stress optimisation model using harmony search (HS) algorithm to control and limit cracks in the concrete. Firstly, stresses affecting parameter and critical area at the inner face of the box girder were identified using stress analysis. Secondly, HS optimisation model was developed by constructing the objective function and optimisation procedure. Thirdly, optimisation model validation and efficiency evaluation were performed. Lastly, stress distribution at the inner face of the box girder was optimised. The critical area was located, and stresses recorded ranges from 25.79 MPa to 37.63 MPa. Optimal solution was found to converge at 269^{th} iteration. The optimisation results demonstrated a better stress distribution with reduced total element stress from 9.21 MPa to 8.63 MPa (6.3%) and amount of reinforcement needed reduced by 6.26%. The optimisation results agreed well with the approximated solution from FEM with percentage error of 1.2%. Thus, the optimisation model was efficient in generating results with sufficient accuracy with lesser computational time and simpler computational procedure.

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APPROVAL SHEET

This dissertation entitled "**HARMONY SEARCH APPROACH IN THE STRUT AND TIE MODEL TO OPTIMISE THE STRESS DISTRIBUTION IN A CONCRETE BOX GIRDER**" was prepared by ALICE LIM PEI SAN and submitted as partial fulfillment of the requirements for the degree of Master of Engineering and Science in Civil Engineering at Universiti Tunku Abdul Rahman.

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SUBMISSION OF DISSERTATION

It is hereby certified that **Alice Lim Pei San** (ID No: **18UEM01025**) has completed this dissertation entitled "HARMONY SEARCH APPROACH IN THE STRUT AND TIE MODEL TO OPTIMISE THE STRESS DISTRIBUTION IN A CONCRETE BOX GIRDER" under the supervision of Dr. Lau See Hung (Supervisor) from the Department of Civil Engineering, Lee Kong Chian Faculty of Engineering and Science, and Dr. Ong Chuan Fang (Co-Supervisor) from the Department of Civil Engineering, Lee Kong Chian Faculty of Engineering and Science.

I understand that University will upload softcopy of my dissertation in pdf format into UTAR Institutional Repository, which may be made accessible to UTAR community and public.

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DECLARATION

I Alice Lim Pei San hereby declare that the dissertation is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTAR or other institutions.

(ALICE LIM PEI SAN)

Date 23/11/2021

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LIST OF SYMBOLS / ABBREVIATIONS

$A_{s,required}$	reinforcement area required
A	area
$\boldsymbol{B}$	strain matrix
$\boldsymbol{c}$	material constant matrix
$\boldsymbol{d_e}$	local displacement vector
l_{ij}	Directional cosines of the axial axis of the element in x direction
m_{ij}	Directional cosines of the axial axis of the element in y direction
n_{ij}	Directional cosines of the axial axis of the element in z direction
$\boldsymbol{D_e}$	global displacement vector
DirCos	global directional cosines matrix for each element
E	young modulus
Δf	difference between the newly generated and existing solution
f_{yd}	reinforcement yield strength
f_{ck}	concrete characteristic compressive cylinder strength at 28 days
f_{ctm}	concrete mean value of axial tensile strength
$f_{ct,eff}$	concrete effective tensile strength
$\Delta f_a{}^m$	working average of objective function difference
$f(x^{HMS})$	objective function with variable at the HMS^{th} solution
$\boldsymbol{F}$	vector of nodal forced applied

k_e	local stiffness matrix
K_e	global stiffness matrix
l_e	length of the element
L	differential operator
$\boldsymbol{N}$	shape function
P_B	probability of Boltzmann's distribution
P_f	final acceptance probability
P_s	starting acceptance probability
$\mathbf{T}$	transformation matrix
T	principle tensile force
$\boldsymbol{U}$	displacement vector
ε	strain
u_{ele}	combination matrix of shape function
γ_m	partial safety factor for material
σ	stress
σ_{Approx}	approximated stress for FEM
$\sigma_{ele,strut}$	total element stress for strut member
$\sigma_{Rd,max}$	design compressive strength
x_n^{HMS}	*n* variable at the HMSth solution
AASTHO	American association of state highway and transportation officials
ACI	American concrete institute
ACO	ant colony optimisation

CO_2	carbon dioxide
FE	finite equation
FEM	finite element modelling
FSM	finite strip method
GA	genetic algorithm
GIT	grid independence test
HM	harmony memory
HMCR	harmony memory considering rate
HS	harmony search
MI	maximum iteration
PAR	pitch adjustment rate
PSO	particle swarm optimisation
SA	simulated annealing
SLS	serviceability limit state
STM	strut and tie model
ULS	ultimate limit state

CHAPTER 1

INTRODUCTION

1.1 Research Background

Concrete box girder is commonly used in bridge construction due to its geometry characteristic to resist high flexural moments and torsional stresses (Kamaitis and Kamaitis, 1996; Harish et al., 2017). The analysis of box girder is complicated because of its three-dimensional behaviour, i.e., torsion, distortion, and bending in both longitudinal and transverse directions. Insufficient understanding on the box girder behaviour will lead to underestimation of ductility demand which indirectly induce serviceability problem (Bazant et al., 2008; Recupero et al., 2017; Huang et al., 2018). Congested reinforcement was identified as one of the factors that affect the serviceability performance of the structure (Maree and Sanders, 2015; Lim et al., 2018). Congested reinforcement would cause the redistribution of stresses onto concrete and lead to the formation of cracks on the concrete surface. The cracks formed might be minor, however, the formation of cracks altered the elastic behaviour of the concrete. It is important to avoid or limit the formation of cracks due to reinforcement stress redistribution.

Ezeokpube (2015) has reviewed some of the popular analysis methods that used for stress analysis in the box girder such as Grillage-analogy method, finite strip method (FSM) and finite element method. Finite element method is well-known for the ability to solve more complex structural engineering problems as compared to Grillage-analogy method and FSM but required longer computational time and larger computational effort for huge structure analysis. On the other hand, strut and tie model (STM) can easily and effectively demonstrate the stress flow pattern in the box girder because the construction and evaluation of STM is simpler and more straightforward as compared to other popular analysis methods. In the construction of STM, optimal STM is important to capture the stress distribution of a structure effectively under loadings, and over-simplified or dense STM should be avoided. Optimisation of STM using traditional trial and error method is difficult for complex structure as huge number of STM members are involved during the stress evaluation in each iteration. Optimisation technique using meta-heuristic algorithm can be introduced to improvise the traditional optimisation procedure.

Optimisation technique has been widely implemented in solving engineering problem. Despite the advance in theoretical study, the implementation of optimisation in engineering practice is still lacking (He and Liu, 2010; Zavala et al., 2014; García-Segura et al., 2015; Kaveh, 2016; Huang et al., 2018). In this study, the implementation of optimisation technique to obtain an optimised stress distribution at the inner face of box girder diaphragm using meta-heuristic algorithm was introduced.

HS algorithm is one the popular meta-heuristic algorithms that can be used in solving engineering problem (Fesanghary et al., 2008; Hoang et al., 2014; Alberdi and Khandelwal, 2015; García-Segura et al., 2017). HS algorithm is different from current meta-heuristic algorithms i.e. simulated annealing, evolutionary algorithm, swarm intelligence optimisation that mimic the natural phenomena; HS is developed based on the idea of musical process that seeking for a perfect state of harmony (Lee and Geem, 2005). HS was chosen because it has a better balance between the diversification and intensification due to the characteristic of the HS parameters.

1.2 Problem Statement

The understanding of the behaviour for box girder is important to ensure sufficient ductility capacity is provided. Insufficient concrete ductility capacity provided may lead to the formation of cracks. However, model or analysis method used for behavioural study of box girder structure is often complicated and involved tedious mathematical calculations (Wu et al., 2003; Djelosevic et al., 2012; Yoo et al., 2015).

STM is efficient in providing demonstration on the stress distribution of a structure using truss analogy when loaded, and the construction of STM is relatively simpler as compared to other analysis method such as FEM. The application of STM in real life complex structural analysis is limited because constructing a good STM is highly dependent on the designer’s experience. Construction of a good STM is important to avoid oversimplified or dense STM. An oversimplified STM may lead to under-reinforced design that results in the formation of excessive crack width; dense STM may result in over-reinforced design that leads to structure failure as the ultimate flexural limit state is reached (Ng et al., 2012; Goodchild et al., 2014). Optimal STM can be obtained using traditional trial and error method, but members' stress calculation involved during each iteration can be tedious and time consuming especially when dealing with huge structure such as box girder. Thus, optimisation using meta-heuristic algorithm can be introduced to improve the traditional procedures for a more effective STM construction.

Studies on optimisation using meta-heuristic algorithm were carried out since the past decades (Gandomi et al., 2013; Zavala et al., 2014; Saka et al., 2016). Despite the theoretical advance of optimisation using meta-heuristic algorithm, its implementation in stress optimisation for box girder to control and limit the formation of cracks is still rare.

1.3 Aim and Objectives

The aim of the research is to develop a model for optimising stresses developed at the inner face of a segmental concrete box girder diaphragm. The objectives of this project are:

i. to perform parametric study on the stresses developed on the box girder,
ii. to develop a HS optimisation model for improving the strut and tie model construction in stress evaluation,
iii. to perform case study on stress optimisation of box girder for optimisation model's efficiency evaluation.

1.4 Significance of Study

This study explores the applicability of topology optimisation using meta-heuristic algorithm in box girder design for better stress distribution pattern and optimal reinforcement needed. The implementation of topology optimisation in practical narrows down the gap between the advance in theoretical studies and its usage in structural design practice.

Besides, the optimisation model developed introduced the possibility of using soft-computing technique to obtain an optimised stress distribution with shorter computation time around five to ten minutes and less tedious process.

1.5 Scope and Limitations

The scope of this research is to develop a model that optimise the stress distribution developed at the inner face of the precast segmental concrete box girder diaphragm. Structural analysis involved in this study is limited to the behaviour of the concrete box girder with height of 2.45m and breadth length of 9.8m; also, transverse stress evaluation that considered only axle loading. The optimisation model development is limited to two-dimensional problem. The selection of the HS parameters was not covered in this study. The HS optimisation model's efficiency was evaluated based on the accuracy and the computational complexity only.

1.6 Structure of the Dissertation

This thesis consists of total five chapters, which are: (i) introduction, (ii) literature review, (iii) methodology, (iv) results and discussion, and (v) conclusion and recommendations. Chapter one covers the brief introduction of the research background, the problem statement, research's aim and objectives, significance of the study and lastly is the scope and limitation for the study.

Chapter two reviews the current design practice for box girder and the possible factors that may lead to serviceability problem. The needs of developing an optimisation model were covered. The stress analysis methods were reviewed and the suitability for stress distribution demonstration was discussed. Besides, the five popular meta-heuristic algorithms such as (i) simulated annealing (SA), (ii) ant colony optimisation (ACO), (iii) particle swarm optimisation (PSO), (iv) genetic algorithm (GA), and (v) harmony search (HS) were discussed. The selection of the algorithm to be used in the optimisation process was also covered in this chapter.

Chapter three covers the research framework for optimisation model development. The four stages involved to achieve the aim of this study are: (i) parameter classification, (ii) model development, (iii) model testing and validation, and (iv) case study for box girder. In the first stage, detail stress analysis for stress affecting parameter and critical area identification, as well as grid independence test (GIT) were explained. Stage two presents the formulation of the objective function, static analysis for STM members' design, and the construction of the optimisation procedures. In the third stage, important parameter required for model testing and validation was explained. The last stage covers the explanation of optimisation results comparisons for model's efficiency evaluation.

Chapter four depicts the results accompanied by critical discussion from GIT, stress analysis and optimisation model. GIT determined the suitable mesh element size which was used for stress analysis and results generation for concrete box girder diaphragm. From the results generated during stress analysis, the stresses affecting parameter and the critical area were identified. The stress flow pattern at the inner face of the box girder was determined and used as a reference for initial STM construction. The initial STM constructed was imported to the HS optimisation model developed for optimisation process. Results for STM before and after the optimisation process were recorded. Optimisation model validation was carried out in this chapter by evaluating the percentage of error for the developed optimisation model. Comparisons based on two aspects were also carried out to evaluate the efficiency of the developed model. The two aspects involved for efficiency evaluation are: (i) accuracy of the optimisation results generated and (ii) the computational complexity for the developed model.

Chapter five presents the conclusions drawn from the study and the recommendations for future research to improve the feasibility of the proposed optimisation model to solve a more complex problem.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter presents the general overview of the stress development in concrete box girder, analysis methods that can be used for stress evaluation and the introduction of optimisation method using meta-heuristic algorithms. The possible serviceability problem due to reinforcement stress redistribution, the suitability of analysis method for concrete box girder stress evaluation and the importance of optimisation in structural design were highlighted.

2.2 General Background

The study of reinforced concrete box girder is complicated due to its three-dimensional behaviour that involved both torsional and distortional warping effects on different plane. Fam (1969) commented that when (i) the ratio for live load and dead load is high, (ii) increased number of eccentric loadings, and (iii) structure that has wide and short geometry, the longitudinal and transverse stresses resulted from torsional deformation become significant. Later, Technical Committee CEN (2005) concluded that the torsional effect is considered as insignificant in the design practice of box girder because of the closed box section that possess sufficient torsional strength. On the contrary, Recupero et al., (2017) identified that in the case of intermediate piers of bridge and structure that experienced heavy concentrated loadings, the effect from warping torsion can be significant.

The stresses resulted by factors such as warping, torsion and distortion may be minor when taken individually, but it may result in severe problem when superimposed if ductility demand is underestimated. According to King and Mahamud (2009), underestimating of ductility demand in concrete structure members often manifest in the form of cracks which lead to possible corrosion problem that results in concrete distress. Gergely et al., (1963) concluded that cracks formed modified the elastic behaviour of the concrete even it was assumed to be in elastic in the first place which further complicate the behaviour of the box girder.

Various studies had been carried out to examine the behaviour of the box girder, i.e. the effect from temperature variance, time-dependent effect, dimensional parameters, and loading conditions (Debbarma and Saha, 2011; Guo et al., 2012; Bobade and Varghese, 2016; Reyaz and Fathima, 2018; Lee et al., 2018). Despite all the studies carried out, study on serviceability problem due to congested reinforcement provided in concrete structure is still lacking. Congested reinforcement provided in the concrete structure may result in the redistribution of stresses to the concrete surface which brings significant effect to the serviceability performance of the structure (Maree and Sanders, 2015; Lim et al., 2018). Stress redistribution on the concrete surface induce cracks when the flexural tensile stress experienced exceeded the effective tensile strength of the concrete (Technical Committee CEN, 2004).

Lim et al. (2018) concluded that the concrete box girder diaphragm experienced distortional warping stress when loaded and the inner face of the box girder diaphragm is most likely to be in tension state which will cause the formation of cracks. Thus, sufficient amount of reinforcement must be provided to resist the tensile stress. In the design process, the reinforcement is evaluated based on the ultimate limit states (ULS), where the final design is safe but cannot guarantee to be optimal. Optimisation method can be adopted to obtain the optimal reinforcement configuration.

Zavala et al. (2014) conducted a study to review the application of optimisation method on structural design since the 1970s. The application of optimisation method can be classified into two main categories, which are: (i) bar or element design and (ii) topological design. Bar or element design covered the optimisation problem related to trusses and frames that aimed to optimise the shape and sizes of the elements in the structure. Topological design is related to the optimisation problem for the entire layout of a structure that focused on the optimal distribution of internal elements or the external shape. Besides, most of the optimisation problems were focused on obtaining the optimal weight of a structure (Galante, 1996; Liang et al., 2000; Savković et al., 2017), optimal cost that involved (Kaveh and Mahmud, 2010; García segura et al., 2017) and optimal solutions that satisfying the environmental objectives (García-Segura and Yepes, 2016; García segura et al., 2017). Despite all the studies that carried out, there is insufficient study on (i) the optimisation for stress development in concrete box girder and (ii) the most relevant design objectives under the specification of Eurocodes.

2.3 Stress Analysis Methods for Concrete Box Girder

Stress analysis studies the stresses and strains behaviour in a structure when subjected to loading. It is an important part in designing a structure. The structural behaviour of the box girder can be classified into three main types: (i) longitudinal and transverse bending, (ii) torsion and distortion, (iii) warping. Longitudinal bending resulted from self-weight of the structure and causes flexural stress in the longitudinal direction (Kumar, 1997). The stresses due to transverse bending of a box section is mainly resulted from the bending moments of the frame effect and cross section distortion as shown in Figure 2.1.

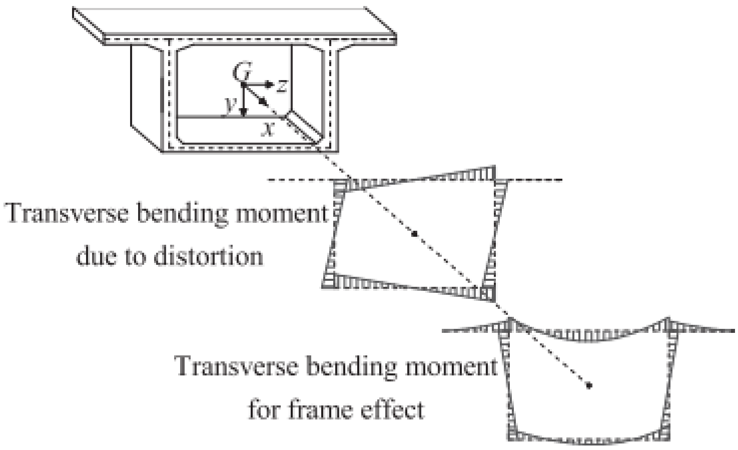


Figure 2.1: Stress State due to Transverse Bending Moment from Distortion and Frame Effect (Recupero et al., 2017)

Torsion involves the rotation of the section about the longitudinal axis of the box girder, but its effect is normally considered as insignificant (Beeby and Narayanan, 2009). Distortion of box girder is resulted from the shear force that developed across the box section and caused the flanges and webs to deform out of plane and the transverse moment due to the deformation are as shown in Figure 2.2.

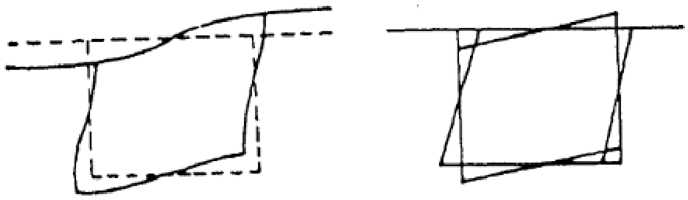


Figure 2.2: Deformation (Left) and Transverse Moment (Right) due to Distortion of Box Section (Kumar, 1997)

Warping demonstrates the out of plane deformation of the box section in the longitudinal direction (Kumar, 1997). Warping can be classified into two types namely torsional warping and distortional warping and the stresses involved are as shown in Figure 2.3.

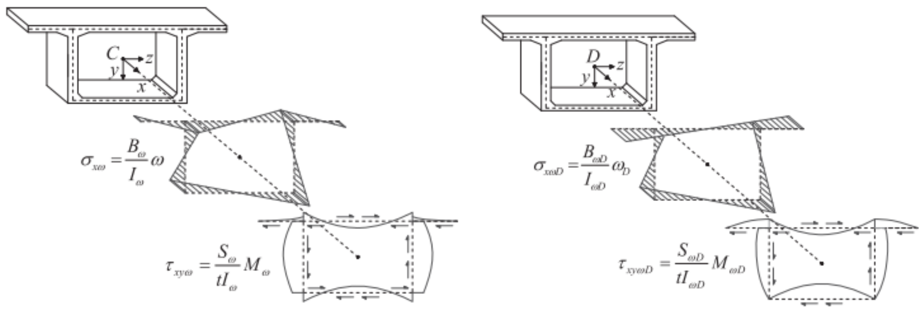


Figure 2.3: Stress State due to Torsional Warping (Left) and Distortional Warping (Right) (Recupero et al., 2017)

Stress evaluation for box girder structure can be carried out using both experimental and analytical method. Experimental study provides concise measurement to the behaviour of a structure during actual condition; but it is often constraint by experiment environment, material and equipment and its consistency. Analytical study is more convenient as compared to experimental study because analytical study can be carried out easily on a digital computer and huge number of variables can be involved during the study. Despite the convenience of the analytical method, higher computational memory may involve for complex problem analysis and longer computational time will be needed.

Some of the popular used analytical method for stress analysis are Grillage-analogy method, finite strip method (FSM) and finite element method (FEM) (Sennah and Kennedy, 2002; Ezeokpube, 2015). Grillage-analogy method idealised bridge deck structure into a network of rigidly connected equivalent longitudinal and transverse beams (Jaeger and Bakht, 1982). The application of Grillage-analogy method is popular because of: (i) its ability to solve complex problem, (ii) the analysis and design can be easily performed on a digital computer, and (iii) its ability to distribute and share the loadings to the support.

In FSM, the structure is idealised as the combination of bending and plane-stress plates that divided into finite number of simply supported strips (Ramana, 2013). FSM is popular because of its efficiency that requires lesser computing effort and time as smaller number of degrees of freedom is involved. FSM is normally analysed as one-dimensional problem; modifications on the equations used must be carried out to include the second dimension.

Both Grillage-analogy method and FSM are popular for bridge loading assessment and require shorter computational time as compared to FEM. However, Grillage-analogy method and FSM analyse the behaviour for the overall bridge span instead of the behaviour at a single transverse segment i.e.

behaviour of a segmental box girder (Shreedhar and Kharde, 2013; Jamali et al., 2017). FEM is capable to solve complicated structural engineering problem with multiple constraints such as the displacement constraint, temperature constraint and etc (Liu and Quek, 2003). FEM can be used for complex structural analysis but for the analysis of box girder structure huge amount of computational time and effort are usually involved. STM is another effective method that able to demonstrate the stress distribution using truss analogy which is suitable to be used in both transverse and longitudinal evaluation with relatively shorter amount of time as compared to FEM (Goodchild et al., 2014).

2.3.1 Finite Element Method (FEM)

FEM is a numerical method that provide approximate solution to a problem that is difficult to solve analytically (Jamali et al., 2017). The basic procedures involved in FEM are as shown in Figure 2.4. FEM discretises structures into different element connecting together based on equilibrium and/or compatibility conditions (Liu and Quek, 2003).

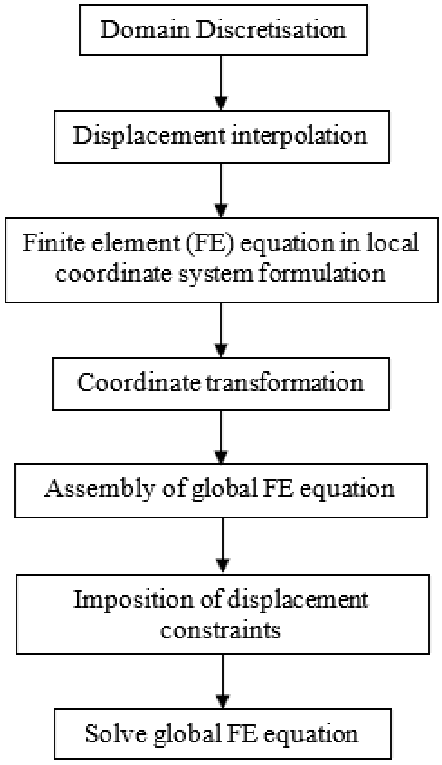


Figure 2.4: FEM Procedure

After domain discretization, displacement interpolation involves the definition of shape function as shown in Equation (2.1).

$$\boldsymbol{N}(x,y,z) = \left[[\boldsymbol{N}_1(x,y,z) \quad \boldsymbol{N}_2(x,y,z) \quad \cdots \quad \boldsymbol{N}_{nd}(x,y,z)]\right] \tag{2.1}$$

where,

$\boldsymbol{N}$ is the matrix of shape functions

$\boldsymbol{nd}$ is the node number

$\boldsymbol{x}$ is the x-direction

$\boldsymbol{y}$ is the y-direction

$\boldsymbol{z}$ is the z-direction

Shape function from displacement interpolation in Eq. (2.1) is used to form the strain-displacement that finally obtain the local stiffness matrix as shown in Equation (2.2).

$$\boldsymbol{k}_e = \int_{V_e} \boldsymbol{B}^T \boldsymbol{c} \boldsymbol{B} \, dV \tag{2.2}$$

where,

$\boldsymbol{k_e}$ is the local stiffness matrix

$\boldsymbol{B}$ is the strain matrix

$\boldsymbol{c}$ is the matrix of material constant

All local coordinate element equation is then assembled to form a global FE equation, Equation (2.3) using transformation matrix **T**, Equation (2.4)**.**

$$\boldsymbol{F} = \boldsymbol{KD} \tag{2.3}$$

$$\mathbf{T} = \begin{bmatrix} l_{ij} & m_{ij} & n_{ij} & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{ij} & m_{ij} & n_{ij} \end{bmatrix} \tag{2.4}$$

where,

$\boldsymbol{F}$ is the force vector in global coordinate system

$\boldsymbol{K}$ is the global stiffness matrix

$\boldsymbol{D}$ is the displacement vector in global coordinate system

l_{ij}, m_{ij} and n_{ij} are the direction cosines of the axial axis of the element

Constraints or support can be defined by deleting rows and columns respective to the constrained nodal displacement. Nodal displacement is obtained by solving Equation (2.3). Strain for the member can be calculated using Equation (2.5). Stress is then calculated using Hooke's law.

$$\boldsymbol{\varepsilon} = \boldsymbol{L}\boldsymbol{U} \tag{2.5}$$

where,

$\boldsymbol{\varepsilon}$ is the strain for member

$\boldsymbol{L}$ is the differential operator

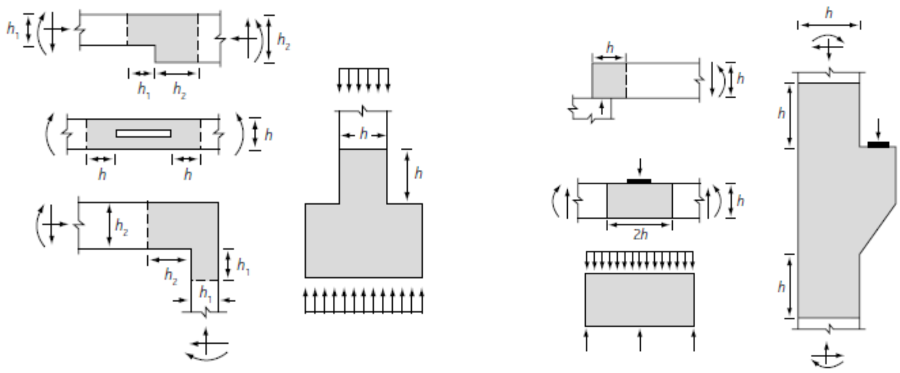
$\boldsymbol{U}$ is the displacement vector

The main advantages of FEM are (i) able to treat arbitrary loadings, (ii) suitable for complex boundary conditions and (iii) suitable for irregular material and dimensional properties. However, FEM requires greater amount of computer time and a refined mesh size to achieve accurate results in the vicinity of steep gradients.

2.3.2 Strut and Tie Model (STM)

STM is a simple method that demonstrate stress distribution effectively using truss analogy that connects compressive strut and tension tie together to form the stress flow pattern (Williams et al., 2012). STM is constructed based on two important principles, which the truss model developed must be in equilibrium when loaded and STM members constructed must have sufficient ductility capacity to resist the forces in members (Goodchild et al., 2014).

The three basic steps involved are (i) D-region definition, (ii) STM members construction and (iii) STM members' design. D-region is the disturbed region in the structure which plane section does not remain plane. Two common examples of D-region are region that experienced geometrical or loading discontinuity as shown in Figure 2.5.



(a) Geometrical discontinuity (b) Loading discontinuity

Figure 2.5: D-region Definition (Shaded Region) (ACI Committee 318, 2014)

In STM construction for complex structure, FEM is useful in providing the initial clue on the stress flow pattern. Eurocode provides a guidance on the STM members construction for box girder shape in the Eurocode as shown in Figure 2.6. Once the truss model was constructed, STM members' design to be carried out. STM design involved the members' force calculation, and strut effective size calculation which is important for the evaluation of nominal compressive strength for a strut member. For the strength checking, ACI Building Code mentioned three important strength checking which is necessary to be carried out, (i) strength for struts, (ii) strength for ties, and (iii) strength for nodal zone.

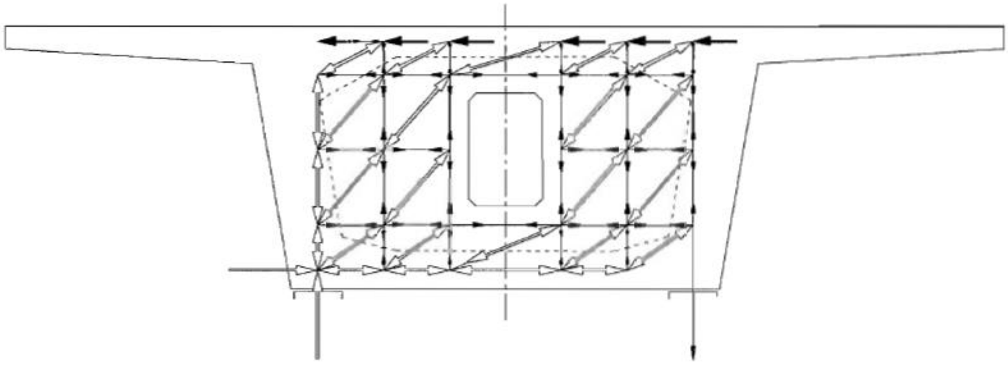


Figure 2.6: STM for a Solid Type Diaphragm with Manhole (Technical Committee CEN, 2005)

The construction steps involved in STM is simple, however, reasonable experience is needed for engineers to construct an efficient STM for a structure (Yun et al., 2018). Optimisation of STM is important to avoid over-simplified or dense STM. Over simplified STM may result in insufficient SLS demand and results in excessive crack width (Ng et al., 2012; Goodchild et al., 2014). Contrary, dense STM may result in over-reinforced design and hence caused structure to fail when the ultimate flexural limit state is reached.

Traditionally, optimisation of STM can be done using trial and error method and is easy for a simple structure such as simply support beam. It is not the case when dealing with a more complex structure such as box girder structure that involved greater amount of strut and tie members. Thus, the application of STM on complex structure design is difficult, also not practical to be applied in real life complex design when design time is one of the main concerns.

STM is relatively simpler in demonstrating the stress distribution in the structure due to the construction and design of STM members are quite straightforward. Besides, STM also poses the ability to predict and evaluate the strength of the structure under compression and tension easily from STM members' strength calculation (ACI Committee 318, 2014). Besides, obtaining an optimised STM using traditional trial and error method is not as efficient as using optimisation technique when computational time and effort are the important concern in structural design. Hence, optimisation technique can be introduced to improvise the traditional STM optimisation procedure that enable efficient STM construction with reduced computational time and effort.

2.4 Optimisation Method

Optimisation method optimises scalar objective function/s that subjected to a number of defined constraint/s. (Bendsoe and Sigmund, 2007). Optimisation can be carried out through mathematical approach and using meta-heuristic algorithm. Complicated mathematical programming techniques are usually involved in the optimisation using the mathematical approach (Simões and Negrão, 2000; Saka et al., 2016; Picelli et al., 2018). While, optimisation using meta-heuristic algorithm is relatively simpler and preferable in solving the engineering design problems (Andersson, 2014).

Most of the studies that carried out in the past were about obtaining the optimum cost, CO_2 emission, safety, and structure's weight (Felix, 1981; Chang et al., 2012; Taylor et al., 2013; Kaveh, 2016; García-Segura and Yepes, 2016;

García segura et al., 2017; Elrehim et al., 2019). Kwak and Noh (2006) studied on the implementation of meta-heuristic algorithms (Evolutional Structural Optimisation, ESO) in the construction of STM, but it's limited to simple concrete structure such as deep beam, corbel structure or etc. On the other hand, theoretical knowledge on meta-heuristic algorithms optimisation has advanced since the past decades, its implementation in structural optimisation for huge structure such as concrete box girder is rare.

Different types of meta-heuristic algorithms have been implemented in structural optimisation since the past decades. Some of the popular used meta-heuristic algorithms in structural optimisation are genetic algorithms (GA), simulated annealing (SA), harmony search (HS) and swarm-intelligence-based algorithms (Saka et al., 2016). The details of respective algorithms were discussed in the following sections.

2.4.1 Simulated Annealing (SA)

SA is a nature-inspired meta-heuristic algorithm that inspired by the natural phenomena of annealing of solids (Zavala et al., 2014). In the annealing process, solid was heated to provide mobility to the atoms, and atoms were arranged into crystalline pattern (optimal solution) as temperature decreases until thermal equilibrium is reached with minimum internal energy (Alberdi and Khandelwal, 2015; Geem et al., 2001). SA is considered as a popular local search algorithm in structural optimisation and was first adopted for steel frame design optimisation in the year 1991 (Balling, 1991; García Segura, 2016).

Parameters' initialisation involved in SA are by setting the starting (P_s) and final acceptance probability (P_f), and number of temperature cycles (N); which defined the cooling schedule of SA algorithm. The initial defined parameters were used for starting and final temperature calculation. The summarised steps involved in SA algorithm is as shown in Table 2.1. Initially, random guess will start with higher energy and recorded as the current best solution. A new random solution was generated and recorded for objective function evaluation, which is a similar process as molecules with given mobility will move to other locations randomly with slightly reduced in energy. Boltzmann's distribution is an important factor in SA that determines the probability of acceptance in the iteration process. After the number of iterations has reached the predefined number of iterations per cycle, temperature was adjusted based on the cooling factor calculated using the starting and final acceptance probability. The equation used for probability of acceptance evaluation is as shown in Equation (2.6) (Alberdi and Khandelwal, 2015). Alberdi and Khandelwal, (2015) stated that the convergence properties of SA are very much affected by temperature parameter, which means at higher temperature, the model tends to accept a newly generated solution even it has a poor performance as the probability of acceptance is high at higher temperature. Therefore, temperature is the key in controlling the balance between diversification and intensification.

Table 2.1: Steps involved in Simulated Annealing (Alberdi and Khandelwal, 2015)

Simulated annealing	
1.	Assign parameters: P_s, P_f, N
2.	Generate and evaluate random design
3.	Start temperature cycle
4.	Determine random group and mutate
5.	Calculate $\Delta f^k = f_p(\boldsymbol{x}^{k+1}) - f_p(\boldsymbol{x}^k)$
6.	Accept or reject based on metropolis algorithm
7.	Repeat steps 4–6 for all member groups
8.	Perform iterations for temperature cycle
9.	Update temperature
10.	Repeat steps 4–9 until convergence

$$P_B = \min\left\{1, \exp\left(-\frac{\Delta f}{\Delta {f_a}^m}\frac{1}{T}\right)\right\} \quad (2.6)$$

where,

P_B is the probability of Boltzmann distribution

Δf is the difference between the newly generated solution and the existing solution

$\Delta {f_a}^m$ is the working average of objective function difference

T is the temperature at current iteration

Past researches concluded that the convergence in SA is strongly affected by the initial parameters that chose (Geem et al., 2001; Yang, 2009; García Segura, 2016). This is because the solution acceptance, initial temperature during iteration and the cooling schedule of SA algorithm is very much affected by the initial parameters defined. Hence, longer computational time will be needed if the initial set variable range is large. Comparison studies were carried out and concluded that, SA has difficulties in providing good optimal solution due to its nature in restricted diversification (Geem et al., 2001; Andersson, 2014; Zavala et al., 2014; Alberdi and Khandelwal, 2015).

2.4.2 Swarm-intelligence-based Algorithms

Swarm-intelligence-based algorithms adopted the collective behaviour of animal or insect. The most widely used swarm algorithms such as ant colony optimisation (ACO) and particle swarm optimisation (PSO) were discussed briefly below.

2.4.2.1 Ant Colony Optimisation (ACO)

ACO is developed based on the behaviour of social ant, which the algorithms mimic the foraging behaviour of real-life ant colonies. The shortest route found between food source and their nest will be the optimal solution. Initialisation, solution construction and pheromone updating are the main steps involved in the ACO (Yang, 2009; Gandomi et al., 2013). The summarised procedures involved in ACO is as shown in Table 2.2.

Table 2.2: Procedures involved in ACO (Alberdi and Khandelwal, 2015)

Ant colony optimization	
1.	Assign parameters
2.	Calculate τ_0 and initialize pheromone matrix
3.	Assign a random starting member to all ants
4.	Add the rest of the member groups sequentially
5.	Evaluate objective function and rank ants
6.	Check for convergence
7.	Update pheromone matrix using the top 15% of ants
8.	Repeat steps 3–7 until convergence

Pheromone trail initialisation and placing number of ants arbitrarily on randomly chose nodes using probability spinner operator are involved in the initialisation process. The assignment of ants on randomly chose nodes covers the diversification characteristic of ACO which enables the algorithm to explore into a wider search space. Each ant that travels through node was subjected to penalised objective function evaluation. Pheromone at each route was reduced based on the evaporation rate, while route that rank higher in the pheromone matrix was rewarded with additional pheromone. The higher the state of pheromone, increases the probability that the route will be selected in the future. Thus, the intensification is manifested through the pheromone updating.

A comparative study showed that ACO required longer time to region into a fitter search space (Alberdi and Khandelwal, 2015). This is due to the nature of ACO that have an imbalance relationship between diversification and intensification. Stronger diversification characteristic can be seen during the early stage when random solution was assigned to each ant, while stronger intensification characteristic during the later stage when pheromone updating taken place.

2.4.2.2 Particle Swarm Optimisation (PSO)

In PSO the idea of social sharing among members is used. Population of random potential solution is initialized. The summarised procedures involved in PSO is as shown in Table 2.3. Each particle in the population will moves across the search space. Particles will be attracted to the best position that achieved by the particle itself previously (local best) or by the neighbouring particles (global best) (Yang, 2009; Saka et al., 2016).

Table 2.3: Procedures involved in PSO

Particle swarm optimization	
1.	Assign parameters
2.	Assign initial position and velocity to all particles
3.	Evaluate objective functions
4.	Sort particles by objective function value
5.	Check for convergence
6.	Store best design as swarm best
7.	Update position and velocity for all particles
8.	Evaluate objective function for each particle
9.	If objective function improves, store as particle best
10.	Repeat 4–9 until convergence

Particle swarm optimisation uses two equations in controlling the flying speed of the particles and updating the best particle's position in the search space. Combination of random vector in terms of velocity and updated position controls the system's diversification. Besides, the intensification of the system is represented by the updated best position. Inertia factor is included to control the trade-off between global exploration and local exploitation.

Alberdi and Khandelwal, (2015) mentioned the main assumption in PSO is that the distance between particles is well-defined. Hence, the initialised values have strong effect on the convergence performance. According to study, global search ability of the system will be lost at the end of the computation and trapped in local optima (Gandomi et al., 2013).

2.4.3 Genetic Algorithm (GA)

GA is one of the popular algorithms among all evolutionary algorithms (EA) that adopt the idea of natural selection and the survival of fittest (Galante, 1996). The diversification and intensification of GA is control by three main mechanisms named reproduction, cross-over and mutation (Geem et al., 2001; Carr, 2014). The general steps involved in GA was summarised as shown in Table 2.4. The first step in GA is by generating an initial set of solutions randomly according to the predefined population size and was tested by the penalised objective function. The initial set of solution is named parent population.

Table 2.4: Steps involved in GA (Alberdi and Khandelwal, 2015)

Genetic algorithms	
1.	Assign parameters
2.	Randomly generate binary strings for population
3.	Convert binary strings to sections for all member groups
4.	Evaluate objective functions
5.	Store best design and check for convergence
6.	Perform reproduction, crossover, mutation
7.	Repeat 3–6 until convergence

New set of population or solution was generated based on the three mechanisms mentioned before. Reproduction mechanism controls the intensification characteristic in GA, where an entire new set of solution was generated by choosing variables with probability. The fitter the design, higher the probability for it to be chosen for reproduction. The reproduction mechanism guides the algorithm to converge to optimal solution, however Alberdi and Khandelwal, (2015) discussed that the nature of reproduction mechanism that generating an entire new solution each time will have the probability to introduce a poor design into the population pool because the acceptance of a new population is regardless of its fitness (Alberdi and Khandelwal, 2015).

Diversification of GA is controlled by the remaining mechanism, crossover, and mutation. The fittest population is selected for crossover and/or mutation to produce a new set of population. Crossover involves the combination of binary substrings fitter parents which enable the algorithm to explore to a wider search space. Next, mutation involves an occasional random flipping of bit values to generate non-recursive offspring which enable the algorithm to explore into new search space in the population (Geem et al., 2001; Carr, 2014). Alberdi and Khandelwal, (2015) commented that the performance of GA is independent of the size of the variable space, thus GA can deal with complex problem, parallelism, and various types of optimisations. However, GA generates new solution by the crossover process that considers only two parents' strings at one time; and creates an entirely new populations each time. In view of this, GA parameter has limited the exploration search space and has the possibility to generate poor design during each iteration.

Hence, researchers concluded that GA able to perform better than other meta-heuristic algorithms (i.e. SA and Swarm-intelligence-based algorithms) that able to explore in a wider search space and provide optimum solution; but in a less effective way as compare HS algorithms (Geem et al., 2001; Manjarres et al., 2013; Alberdi and Khandelwal, 2015).

2.4.4 Harmony Search (HS)

HS is a similar process that seek for best state in musical performance that determined by aesthetic estimation as the optimisation algorithms seek for global optimal solution that determined by the objective function evaluation. Harmony memory considering rate (HMCR) and pitch adjusting rate (PAR) are two important parameters in HS. HMCR enable HS to search for good solution randomly without considering harmony memory (HM). While PAR helps to improve diversity of the solution and avoid the system to trap in local optima (Geem et al., 2001; Geem, 2010).

Diversification is controlled by the pitch adjustment and randomisation. Randomisation (1 – HMCR) can be observed in the process of initialisation of HM that allows model to explore into wider search space (Geem et al., 2001; Manjarres et al., 2013). While, PAR is a refinement process for local solution that adjust the pitch to the neighbouring value relative to the existing solution from HM (Yang, 2009; Saka et al., 2016). PAR further enhance diversification and control intensification of the system because it produced a completely random design from the existing search space. Both parameters help the system to retain good local solutions while delve into the global search space more efficiently. On the other hand, intensification of the system is also controlled by HMCR where it will decide the selection of new solutions from HM. Besides, HMCR also poses some diversification properties as forming new solution from existing design in HM explores a new region in the existing best search space. Another distinctive feature of HS is the acceptance of new solution in the algorithm, the nature of HS results the algorithm to have sets of best existing solution throughout the optimisation process as only the better solution will be accepted and recorded in HM (Alberdi and Khandelwal, 2015).

Various studies were carried out to compare the efficiency and robustness of HS algorithms with other popular meta-heuristic algorithms (Geem et al., 2001; Manjarres et al., 2013; Alberdi and Khandelwal, 2015; Saka et al., 2016). HS has better balance between diversification and intensification as compared to others due to the characteristics of the improvisation operators (Akin and Saka, 2015; García segura et al., 2017). HS generate new solutions by considering all the existing solutions in HM which allows the model to search for better solution in a wider range (diversification). In view of this, its diversification properties guide the model towards the solution space with better fitness (intensification) and HS is not strongly affected by the initialized value. Thus, among all meta-heuristic algorithms, HS is selected in this study to optimise the STM at the inner face of concrete box girder end diaphragm for more economical design and optimal reinforcement configuration due to its efficiency and robustness in structural optimisation problem.

2.5 Concluding Remarks

Box girder structure is widely used in bridge construction however the behaviour of the box girder is complicated. Insufficient understanding of box girder behaviour may result in underestimation of the ductility demand and hence results in severe serviceability problem. Studies show that, one of the factors that affect the serviceability performance for a structure is the redistribution of the reinforcement stress on concrete surface due to over-reinforced design. Stress redistribution to the concrete may induce cracks which further complicates the behaviour of the box girder. As the results, adequate reinforcement provided is important. Current design practice evaluates the amount of reinforcement based on ULS, the final design is safe but may not be optimal. Thus, study on optimising the stress distribution can be carried out to obtain the optimal amount of reinforcement. Analysis model for stress pattern demonstration should be selected prior to the optimisation of the stress distribution for a structure. Current studies demonstrate the importance of optimal design in controlling the serviceability performance of the structure, but study about the approach to obtain an optimal design is insufficient.

Stress analysis can be used to study the stresses and strains behaviour of a structure. The analysis method that are popular in stress analysis for box girder structure were discussed. Grillage-analogy method and FSM required shorter computational time as compared to FEM. However, Grillage-analogy method and FSM are suitable in behaviour analysis for the overall bridge span instead of the behaviour at a single transverse segment. FEM can be adopted to solve complicated engineering problem, but process is much tedious and required longer computational time. Apart from the three methods, stress distribution of the structure can be effectively demonstrated using STM.

STM is a simple method that demonstrate the stress distribution effectively using truss analogy. STM is chosen in this study because the construction and design of STM member is simple and straightforward.

Optimisation of STM is important to avoid over-simplified or dense STM which may lead to insufficient SLS demand or structure failure when the ultimate flexural limit state is reached. Even though the construction of STM is relatively simpler, optimisation of STM could be tedious especially for complex structure when the optimisation process is carried out using traditional trial and error method. Various studies were carried out to study the method in optimising the stress analysis procedure. However, most of them are either time-consuming or tedious mathematical calculations are involved. Studies of implementation of meta-heuristic algorithm in optimising the stress analysis procedure are limited. Hence, optimisation using meta-heuristic algorithm can be introduced to replace the traditional trial and error method.

Optimisation using meta-heuristic algorithm is preferable as compared to solving optimisation problem using mathematical programming approach. The application of optimisation technique was mostly on obtaining the optimal cost, weight of a structure and so on. However, its application on complex structure design problem is still lacking. Furthermore, the study on optimisation of concrete box girder structure in stress development is still immature. Thus, this research intended to introduce the implementation of soft computing technique to obtain the optimised stress distribution for segmental box girder diaphragm using meta-heuristic algorithm.

The five popular meta-heuristic algorithms were discussed and the summary comparison of the five algorithms were listed as shown in Table 2.5. Firstly, Both SA and PSO were found to be very dependent on the initial parameter chose. ACO has imbalance diversification and intensification properties, which caused the algorithm to converge much slower. GA was found to perform better as compared to other algorithms but less effective as compared to HS. Lasty, HS was found to have better balance between diversification and intensification properties. Besides, the characteristic of the HS parameters allowed HS to obtain the optimal solution effectively as compared to other algorithms. Hence, HS is selected in this study for stress optimisation.

Table 2.5: Comparison of Algorithms

	SA	ACO	PSO	GA	HS
Initial Design	Random design	Using probability spinner	Random swarm population	Random population	Random HM
Acceptance	Better solution or passes the Boltzmann test	Using probability spinner	New position	New population always accepted	Better solution than the worst solution in HM
Intensification	Accept better solution, start new neighbourhood for search	Pheromone updating	Swarm and particle best positions	Entire new set of population generated with probability	HMCR for new solution from existing HM
Diversification	Worse solution has the probability to be accepted	Probability spinner (worse solution has the probability to be assigned)	Velocity and updated position of the particle	Crossover and mutation	HMCR for random solution and PAR for mutation
Termination	Best design or max iterations	Best solution or max iterations	All members of same design or max iterations	Fittest population or max iterations	Best solution in HM or max iterations

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter provides systematic research framework for developing an optimisation model to optimise the stress distribution of the segmental box girder diaphragm using meta-heuristic algorithm. Parameter classification was carried out to perform stress analysis using finite element modelling (FEM) software ANSYS Workbench for concrete box girder diaphragm. Next, a single objective harmony search (HS) optimisation model was developed to optimise the stress distribution of the box girder diaphragm. The model was then validated by comparing the optimisation results with the FEM results obtained. The validated optimisation model was used to optimise the stress distribution at the inner face of the segmental concrete box girder diaphragm.

3.2 Research Framework

The main objective of this study is to develop an optimisation model using HS algorithm for stresses development at the inner face of the box girder, as well as to perform parametric study on the stresses developed in the box girder. A series of research activities were systematically planned based on the theory of isotropic elasticity and the optimisation model was developed to demonstrate two-dimensional stress distribution at the inner face of the box girder diaphragm. Figure 3.1 shows the framework of the research activities.

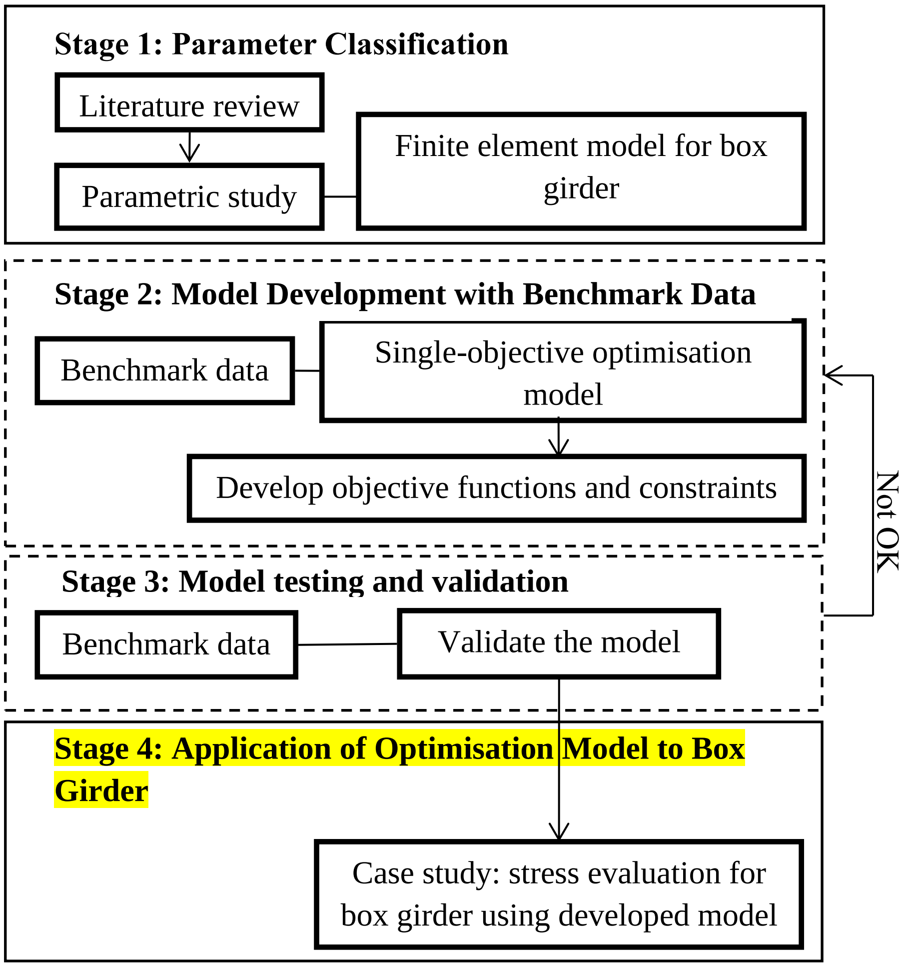


Figure 3.1: Framework of the Research

3.2.1 Stage 1: Parameter Classification

Parameter classification is the first stage involved in this research project to identify the parameters that affect the stresses development in the concrete segmental box girder diaphragm. Parameter classification it is important to justify weather the stresses developed at the inner face of the box girder is due to two-dimensional or three-dimensional effect and hence justification on the validity of the optimisation results generated can be done. Parameters considers during the FEM includes the geometry (i.e., breadth and height) and the shape (i.e., presence of web and flanges) of the box girder.

The two main parts that involved in this stage as shown in Figure 3.2, are: (i) stresses affecting parameters identification and (ii) critical area identification. The identification of the stresses affecting parameters for box girder was carried out by performing stress analysis, and the FEM results obtained were validated through grid independence test (GIT). Later, areas that affected by the parameters obtained were located. Stresses comparison was carried out to locate the critical area at the inner face of the box girder.

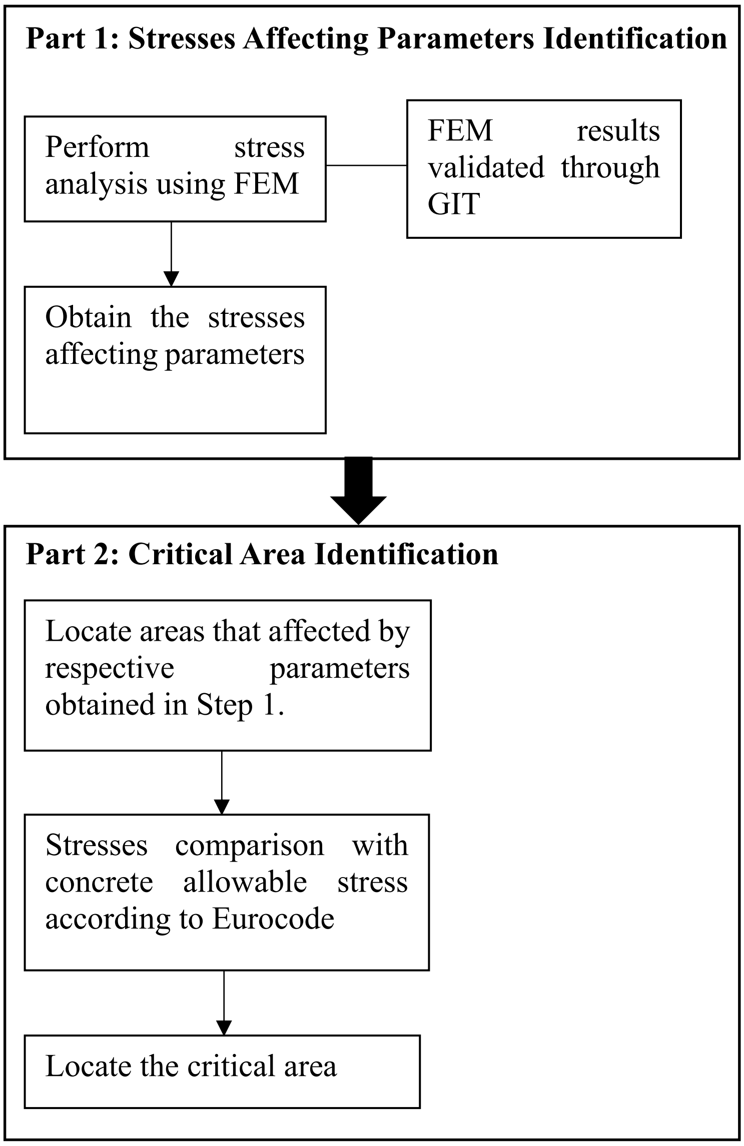


Figure 3.2: Concepts involved in Stage 1 Parameter Classification

3.2.1.1 Stress Analysis using FEM

Stress analysis is the first part that involved in the parameter classification to observe the stresses development of the box girder diaphragm when loaded. The stress analysis was performed using the FEM software ANSYS Workbench. The parameters that affect the stress distribution at the inner face of the box girder can be justified from the FEM results obtained.

The stress analysis was carried out based on the theory of isotropic elasticity and the steps involved are: (i) geometry definition, (ii) mesh definition, (iii) static structure analysis definition and (iv) results evaluation. The summarised steps are as shown in Figure 3.3.

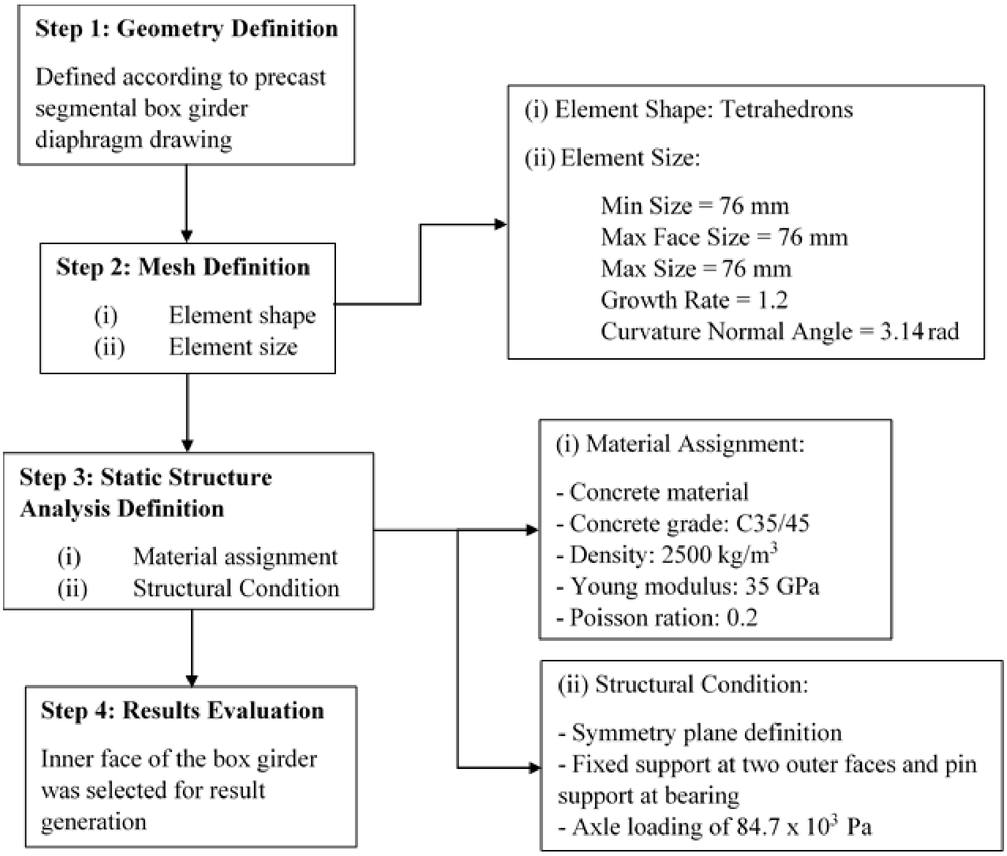


Figure 3.3: FEM Flowchart

Geometry definition is the first step involved in stress analysis that carried out using ANSYS SpaceClaim. Geometry of the segmental box girder diaphragm constructed is as shown in Figure 3.4.

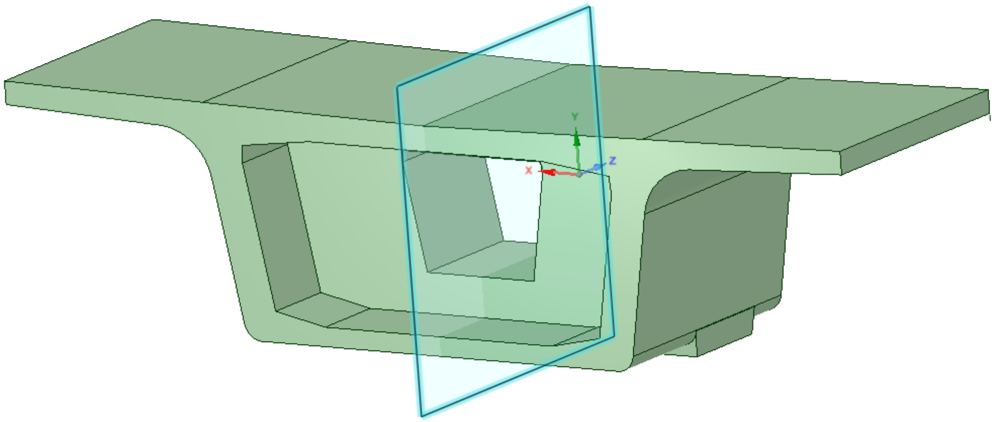


Figure 3.4: Segmental Box Girder Diaphragm using ANSYS SpaceClaim

Mesh definition is the second step involved in FEM after the whole constructed structure was imported from SpaceClaim. Details such as element shape and sizes were defined under this step. Element shape of tetrahedrons was selected because of its suitability in complex structure analysis. Mesh element size was parameterised for GIT where varied range of mesh element sizes were tested to obtain the suitable mesh size to be used for the analysis. Details explanation on GIT was discussed in the next subsection. On the other hand, growth rate and the curvature normal angle were kept unchanged with the value of 1.2 and 3.14 rad respectively.

After mesh definition, material assignment and structural condition definition were carried out in step 3. Material assignment involved the definition of concrete properties for the structure. In this study, concrete grade of C35/45 with the density of 2500 kg/m^3 was used. The concrete young modulus and Poisson ratio were defined as 35 GPa and 0.2 respectively. Under structural condition, plane symmetry was adopted as shown in Figure 3.5 so that the design domain covered only half of the structure, this is to reduce the computational time and simplify the computational process. Fixed supports were defined at two outer faces and pin support at bearing of the box girder as shown in Figure 3.5. The axle loading was obtained by the construction of influence line for trains that commonly used in Malaysia and then applied on box girder as line pressure in the ANSYS Workbench (Siemmens AG., 2016). In the last step of FEM, equivalent stress at the inner face of the box girder was generated to determine if the concrete will yield when subjected to loading.

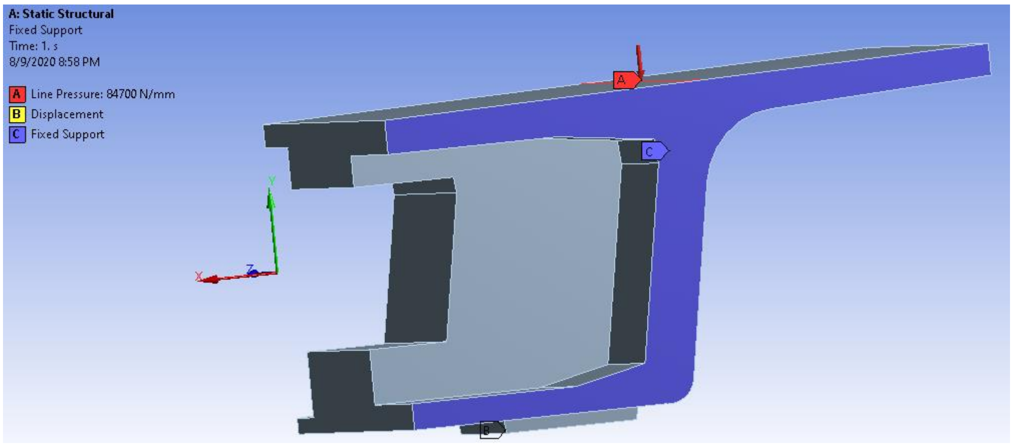


Figure 3.5: Loading and Supports Definition

3.2.1.2 Grid Independence Test (GIT)

Grid independence test (GIT) is a test that carried out to obtain the mesh element size that able to generate acceptable results that is independent to the defined element size. GIT was carried out in ANSYS Workbench and the steps involved are summarised as shown in Figure 3.6. The young modulus was kept constant throughout the whole GIT test with the value of 35 GPa.

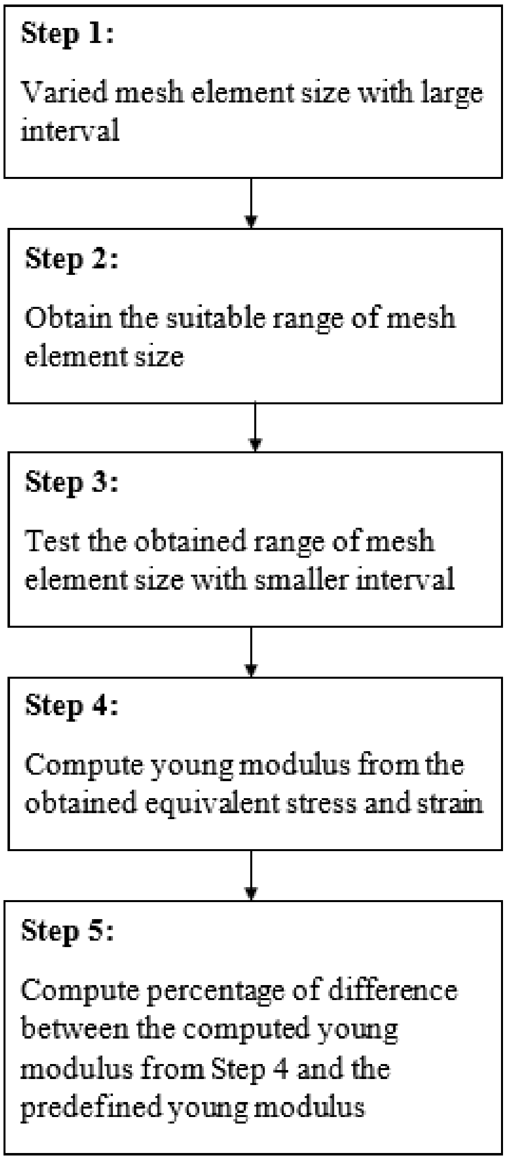


Figure 3.6: GIT Flowchart

The GIT evaluated the mesh element size ranged from 40 to 200 mm with the interval of 10 mm. Young modulus for each design point was computed from the obtained equivalent stress and strain. Step two involved the selection of a smaller range of mesh element size by comparing the computed young modulus with the constant young modulus defined earlier. From the comparison results obtained, a smaller range of mesh element size ranged from 40 to 80 mm was selected for further testing. In step three, the mesh element size obtained from step two was re-evaluated with smaller interval of 2 mm. Respective young modulus was computed at each design points in step four. Lastly, percentage of difference between the computed young modulus and the constant young modulus was evaluated to identify the suitable mesh element size to be used in the stress analysis.

3.2.1.3 Critical Area Identification

This section described the second part that involved in the parameter classification to locate the critical area at the inner face of the box girder diaphragm. The areas that affected by the stresses affecting parameters obtained from section 3.2.1.1 were identified. Stresses comparison was then carried out between the stresses recorded at the affected area and the concrete allowable stress according to the specification of Eurocode.

In stresses comparison, the two characteristics of concrete for strength evaluation are: (i) the concrete characteristic compressive cylinder strength at 28 days, f_{ck} and (ii) the concrete mean value of axial tensile strength, f_{ctm}. Concrete characteristic compressive cylinder strength, f_{ck} indicates the concrete compressive strength before the concrete behaves as a plastic material. According to EN 1992-1-1:2004 cl.7.1(2), the first crack formed when the flexural tensile stress exceeded the effective tensile strength of the concrete. The effective tensile strength of the concrete, $f_{ct,eff}$ may refer to mean axial tensile strength of the concrete, f_{ctm} as stated under the same clause (Technical Committee CEN, 2004; Beeby and Narayanan, 2009). According to EN 1992-1-

1:2004, Table 3.1, the f_{ck} and f_{ctm} for concrete grade C35/45 are 35 MPa and 3.2 MPa respectively.

Through stress comparison, the area that probable to the formation of cracks was identified as the critical area. Stress recorded at the critical area was benchmarked for optimisation model validation.

3.2.2 Stage 2: Optimisation Model Development

The idea of STM is adopted to demonstrate the stress distribution pattern of the box girder according to Eurocode specification. The objective function of the optimisation problem was formulated first before optimisation model development. The equations required for member force calculation are demonstrated in the section 3.2.2.1 and 3.2.2.2.

3.2.2.1 Optimisation Problem Formulation

Single objective optimisation model is developed to optimise stress distribution at the inner face of the box girder by minimising the element stresses for the STM developed. Study focused on the behaviour of the concrete before it undergoes plastic failure where concrete's deformation will disappear when unloaded. Hence, problem formulation is based on the theory of isotropic elasticity. Objective function used for this study is as stated in Equation (3.1).

Objective function *f* can be solved by solving the strain variable in the STM that subjected to the constraint as stated in Equation (3.2).

$$\min(f) = E(\varepsilon_n^{HMS})_{max} \tag{3.1}$$

subject to:

$$\sigma_{ele,strut} < \sigma_{Rd,max} \tag{3.2}$$

where,

f is the objective function

E is the young modulus

ε is the strain variables

n is the number of variables

$\sigma_{ele,strut}$ is the total element stress for strut members

$\sigma_{Rd,max}$ is the design compressive strength

To solve for the strain variables, Equation (3.3), static finite element analysis was carried out where strain matrix, global stiffness matrix and global displacement vector need to be computed. Detailed explanation for solving the strain variables was discussed in the next section.

$$\varepsilon = \frac{\delta u_{ele}}{\delta x} \tag{3.3}$$

The approximate displacement component of the STM member, u_{ele} is the combination matrix of shape function, $\boldsymbol{N}(\boldsymbol{x})$ as discussed in section 2.2.1 and local displacement vector, $\boldsymbol{d}_e$. The local displacement vector is then transformed to global coordinate system using the transformation matrix, **T**. Substitution of shape function and global displacement vector, $\boldsymbol{D}_e$ into approximate displacement, equation becomes $\varepsilon = L\boldsymbol{NTD}_e$ as stated in Equation (3.6).

3.2.2.2 Static Analysis

Static analysis involved the solving of the global finite element equation $\boldsymbol{K}_e\boldsymbol{D}_e$ = **F**, where $\boldsymbol{K}_e$ is the global stiffness matrix and **F** is the vector of nodal forces applied on the model. To generate global stiffness matrix, $\boldsymbol{K}_e$ the assembly of local stiffness matrix, $\boldsymbol{k}_e$ for each element is necessary using Equation (3.4).

$$\boldsymbol{K}_e = T^T \boldsymbol{k}_e T \qquad (3.4)$$

The simplified

$$\boldsymbol{K}_e = \frac{AE}{l_e}[\boldsymbol{DirCos}] \qquad (3.5)$$

where,

$\boldsymbol{k}_e = \int_{Ve} B^T cB \, dV$ derived from strain energy equation

$\boldsymbol{T}$ is the transformation matrix

$\boldsymbol{DirCos}$ is the global directional cosines matrix for each element

Constraints or support were defined by deleting rows and columns respective to the constrained nodal displacement. Nodal displacement was obtained by solving the global finite element equation. Strain and stress for the member was then calculated using Equation (3.6) and (3.7) after obtaining the nodal displacement, $\boldsymbol{D}_e$.

$$\varepsilon = L\boldsymbol{N}\boldsymbol{T}\boldsymbol{D}_e \tag{3.6}$$

$$\sigma = E\boldsymbol{\varepsilon} \tag{3.7}$$

where,

L is the differential operator

N is the shape function

E is the young modulus

3.2.2.3 Topology Optimisation Procedures

Optimisation procedures were developed using MATLAB after the construction of objective function and finite element analysis formulas. The general flow of the optimisation procedure is shown in Figure 3.7. The first step in the optimisation process is general and HS parameters definition. General parameters include the STM coordinates, concrete grade, loadings and constraints involved in the model. Firstly, STM coordinates can be defined after the construction of the initial STM. Concrete grade was selected based on the commonly used concrete grade in concrete box girder bridge construction.

Loading was defined based on the weight of the train. Lastly, the constraints were defined based on the location of the supporting bearing of the box girder. HS parameters include harmony memory size (HMS), maximum improvisation (MI), harmony memory considering rate (HMCR) and pitch adjustment rate (PAR). HMS refers to the number of solutions that will be involved in the algorithms. MI represents the number of iterations that will be carried out during the improvisation process. HMCR is the probability for the model to pick a solution from the memory. PAR is the probability for the model to adjust the picked solution to the neighbouring solution in the memory. The selection of the HS parameters was based on the parameters implemented in the structural optimisation problem which carried out by other researcher (Lee, et al., 2011).

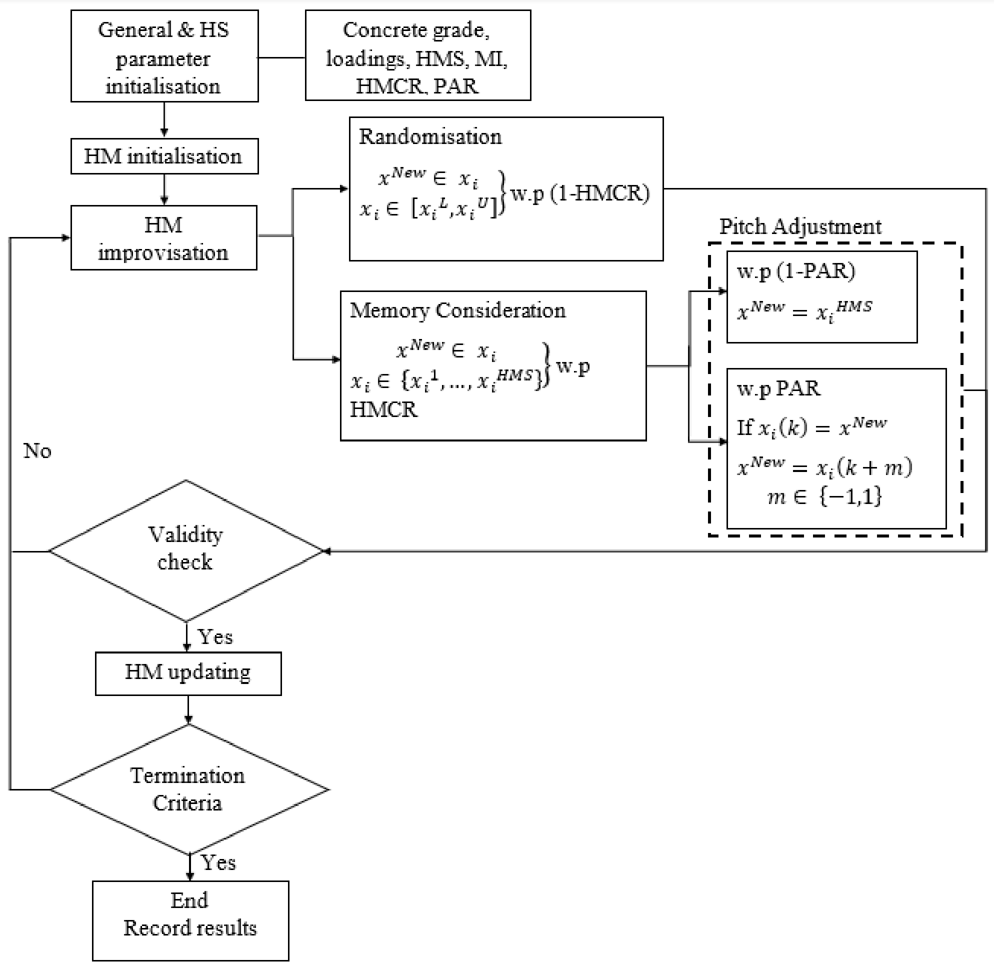


Figure 3.7: General Flow of the Optimisation Procedure

HM initialisation was carried out after the definition of general and HS parameters. This step involves a random possible solutions generation by randomly removing member/s automatically by the model. The possible solution generation that passes the validity tests and constraint checking will be added to the harmony memory until the predefined HMS reached. After the HM initialization, the first set of solution will be saved and present in matrix form as shown in Equation (3.8).

$$HM = \left(\begin{array}{ccc|c} x_1^1 & \cdots & x_n^1 & f(x^1) \\ \vdots & \ddots & \vdots & \vdots \\ x_1^{HMS} & \cdots & x_n^{HMS} & f(x^{HMS}) \end{array} \right) \tag{3.8}$$

where,

x is the variables in the objective function

n is the number of variables

f is the objective function

The three core concepts involved at the improvisation stage are random selection, memory consideration and pitch adjustment which (1-HMCR), HMCR and PAR are involved respectively for the model to explore and widen search space. Random selection involved picking a trial solution with the probability of (1 – HMCR) without considering HM. All generated possible solutions must be tested before updating to HM.

Constraint checking involved in the optimisation procedure is as shown in Equation (3.2), where the element stress for the strut member should not be exceeding the design compressive stress. Validity tests that control the connection validity must be satisfied when randomly removing member/s i.e., (i) must have at least two members connected to one node and (ii) must have member/s connected to the support and loading acting nodes.

HM updating justified the new solution found in the previous step with reference to the objective function. The solution generated with lesser stress compared to the existing solutions stored in HS will be identified as the better solution. If the new solution found is better, then the worst solution in the HM will be replaced by the better solution found. Computation will be terminated if the HS satisfies the termination criteria, otherwise, HS will improvise another new harmony.

3.2.3 Stage 3: Optimisation Model Testing and Validation

STM was constructed to demonstrate the stress distribution at the inner face of the box girder. The optimisation model developed from stage 2 was used to optimise the STM constructed. Critical area at the inner face of the box girder was located from the stress analysis carried out from stage 1, parameter classification and the results were benchmarked for optimisation model validation. The stress for the critical member was recorded and validated with the benchmarked data obtained from stage 1, parameter classification.

Percentage difference of stresses between benchmarked data and the results obtained after optimisation process for justification

3.2.4 Stage 4: Application of Optimisation Model on Box Girder

Case study of segmental box girder diaphragm using the HS optimisation model developed is the last stage of this study. Validated optimisation model was used to optimise stress distribution at the inner face of the segmental box girder diaphragm. Element stresses before and after the optimisation process were recorded. Reinforcement needed before and after the optimisation process was evaluated and recorded. The efficiency of the HS optimisation model was discussed by carried out comparisons regarding the computational complexity and accuracy between the optimisation model and FEM.

3.3 Concluding Remarks

This chapter illustrated the research framework proposed for this study. Parameter classification was first carried out by performing stress analysis. Steps involved in stress analysis as well as in GIT to validate the stress analysis results were discussed. Next, a single objective optimisation model was developed in MATLAB. The optimisation model was developed based on the theory of isotropic elasticity and the optimisation technique adopted was HS algorithm. The explanation involved in the optimisation model development covers optimisation problem formulation, static analysis for members' forces calculation and the optimisation procedures using HS algorithm. After optimisation model development, the necessary evaluations involved in the optimisation model testing and validation were discussed. Lastly, case study for box girder was carried out to evaluate the difference between the stress distribution before and after optimisation. Evaluation on the robustness of the developed optimisation model was also covered.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

This chapter presents the results for stress analysis, validation, and performance evaluation for the HS optimisation model. The GIT was first discussed to identify the most suitable mesh element size for stress analysis and results generation. Stress analysis was carried out using FEM software ANSYS to evaluate the stresses at the inner face of the box girder. Stresses affecting parameter was identified. Comparison was carried out between the obtained stresses and the concrete strength to determine the critical area at the inner face of the box girder diaphragm. From the stress analysis results, the stress flow pattern at the inner face was determined and used as a reference for the initial STM construction.

The constructed initial STM was imported to the HS optimisation model for stress optimisation. Static analysis was performed during the optimisation process to obtain the element stresses. Element stresses before and after the optimisation process were recorded for amount of reinforcement computation. Validation of the HS optimisation model developed was carried out. Lastly, efficiency evaluation for HS optimisation model was discussed based on the accuracy of the optimisation results generated and the computational complexity.

4.2 Stress Analysis for Parameter Classification

Parameter classification was carried out by performing stress analysis to obtain the stresses affecting parameter as discussed in section 3.2.1. Stress analysis was carried out using the FEM software ANSYS Workbench to observe the stress distribution of the box girder and the results were presented in the following section. The box girder was modelled with height of 2450 mm and breadth length of 9800 mm as shown in Figure 4.1. The FEM results obtained were verified by conducting GIT. In GIT, suitable mesh element size was selected for analysis and FEM results generation. From the FEM results generated, results evaluation was carried out to obtain the important stresses affecting parameter in the box girder, and to identify the critical area at the inner face of the box girder.

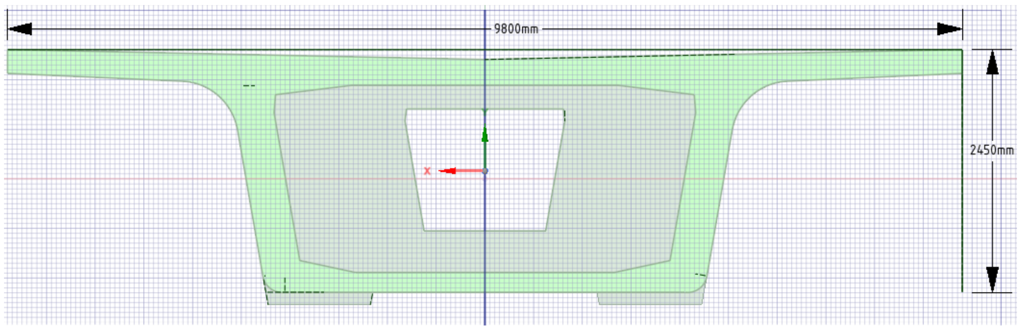


Figure 4.1: Layout of Full Box Girder

4.2.1 Grid Independence Test (GIT)

Grid independence test (GIT) is an important test to obtain the mesh element size that able to generate results with acceptable accuracy for reasonable computing time and memory. The inner face of the box girder was selected for results generation. Maximum equivalent stress and the equivalent elastic strain at the inner face of the box girder was recorded to observe the mesh convergence properties. The GIT was carried out in the ANSYS Workbench under parameter set as shown in Figure 4.2.

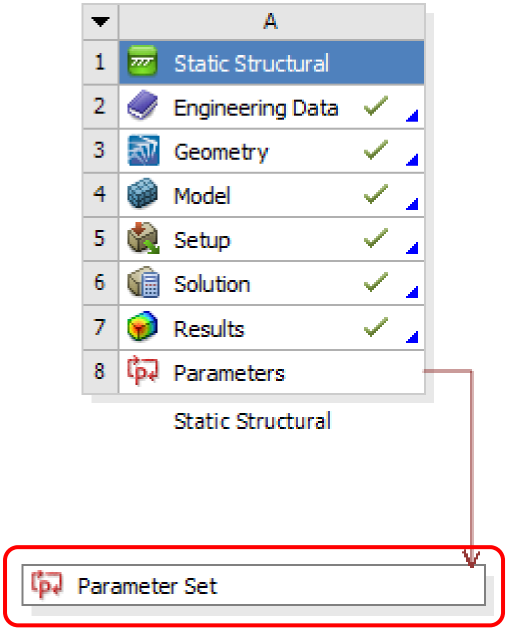


Figure 4.2: Parameter Set for GIT

The mesh element sizes, maximum equivalent stress and elastic strain generated at each design point were recorded as shown in Figure 4.3. In this study, GIT was carried out with varied mesh element size ranges from 40 mm to 200 mm. Young modulus was kept constant as 35 MPa for each design point as shown at parameter P15 in Figure 4.3. The maximum equivalent stress and strain at parameter P13 and P14 respectively were generated using respective mesh size at P10. Results generated in Figure 4.3 were imported to Excel for new young modulus computation using P13 and P14. The computed young modulus for each design point was compared with P15 for percentage of difference evaluation.

Table of Design Points

	A	B	E	G	H	I
1	Name	P10 - Mesh Min Size	P15 - Young's Modulus	P8 - Mesh Elements	P13 - Equivalent Stress 3 Maximum	P14 - Equivalent Elastic Strain 3 Maximum
2	Units	mm	Pa		MPa	mm mm^-1
16	DP 16	80	3.5E+10	1.8344E+05	121.01	0.003649
17	DP 17	78	3.5E+10	1.983E+05	118.75	0.0035704
18	DP 18	76	3.5E+10	2.1379E+05	121.63	0.0036642
19	DP 19	74	3.5E+10	2.3158E+05	122.45	0.0037026
20	DP 20	72	3.5E+10	2.5163E+05	132.49	0.003978
21	DP 21	70	3.5E+10	2.7359E+05	123.71	0.0037257
22	DP 22	69	3.5E+10	2.856E+05	123.28	0.0037038
23	DP 23	66	3.5E+10	3.2658E+05	129.92	0.0039128
24	DP 24	64	3.5E+10	3.5686E+05	127.12	0.0038142
25	DP 25	62	3.5E+10	3.9147E+05	127.57	0.0038451
26	DP 26	60	3.5E+10	4.3209E+05	133.62	0.0040485
27	DP 27	58	3.5E+10	4.7845E+05	131.9	0.0040899
28	DP 28	56	3.5E+10	5.3144E+05	136.49	0.0041206
29	DP 29	54	3.5E+10	5.9233E+05	136.96	0.0041102
30	DP 30	52	3.5E+10	6.6306E+05	137.05	0.0041359
31	DP 31	50	3.5E+10	7.4396E+05	139.85	0.0041985
32	DP 32	48	3.5E+10	8.4028E+05	144.92	0.0043872
33	DP 33	46	3.5E+10	9.548E+05	144.11	0.0045245
34	DP 34	44	3.5E+10	1.0901E+06	149.86	0.0046022
35	DP 35	42	3.5E+10	1.2515E+06	145.27	0.0043836
36	DP 36	40	3.5E+10	1.4492E+06	156.26	0.0049289

Figure 4.3: Varied Mesh Element Size and the Respective Equivalent Stress and Strain with Constant Young Modulus

According to past researches, percentage difference that commonly used for GIT in FEM were 2% (Lei et al., 2008), 3% (Eiamsa-ard and Promvonge, 2008) and 5% (Kulkarni et al., 2016). The comparison results and the percentage of difference was recorded as shown in Table 4.1. Design point 16 to 19 show a higher percentage of difference that close to or more than 5%. DB24 (4.78%) and DB20 (4.84%) was found to have lower percentage of difference that less than 5%. Despite the lower percentage of difference of 4.78% at DB 24, the accuracy difference between DB24 and DB20 is not significant, but longer computational time and larger memory are required for DB24 as compared to DB20 as the mesh element size become finer. Considering both computational time and the solution accuracy, results generated using mesh element size of 72 mm (DB20) were selected with percentage difference of 4.84%.

Table 4.1: Computed Young Modulus and the Percentage of Difference

Design Points	P10- Mesh Size (mm)	P15- Constant Young Modulus (Pa)	Computed Young Modulus (Pa)	Percentage of Difference (%)
DP 16	80	3.5E+10	3.32E+10	5.25
DP 17	78	3.5E+10	3.33E+10	4.97
DP 18	76	3.5E+10	3.32E+10	5.16
DP 19	74	3.5E+10	3.31E+10	5.51
DP 20	72	3.5E+10	3.33E+10	4.84
DP 21	70	3.5E+10	3.32E+10	5.13
DP 22	69	3.5E+10	3.33E+10	4.90
DP 23	66	3.5E+10	3.32E+10	5.13
DP 24	64	3.5E+10	3.33E+10	4.78
DP 25	62	3.5E+10	3.32E+10	5.20
DP 26	60	3.5E+10	3.3E+10	5.70
DP 27	58	3.5E+10	3.23E+10	7.86
DP 28	56	3.5E+10	3.31E+10	5.36
DP 29	54	3.5E+10	3.33E+10	4.80
DP 30	52	3.5E+10	3.31E+10	5.32
DP 31	50	3.5E+10	3.33E+10	4.83
DP 32	48	3.5E+10	3.3E+10	5.62
DP 33	46	3.5E+10	3.19E+10	8.99
DP 34	44	3.5E+10	3.26E+10	6.96
DP 35	42	3.5E+10	3.31E+10	5.32
DP 36	40	3.5E+10	3.17E+10	9.42

4.2.2 Finite Element Modelling for Box Girder

Stress analysis was carried out for box girder and the results were generated using 72 mm mesh element size that determined from GIT. The FEM results obtained is crucial in providing three important information for: (i) the stresses affecting parameter in the box girder, (ii) the identification of critical area at the inner face of the box girder, and (iii) the construction of the initial STM for optimisation.

4.2.2.1 Stresses Affecting Parameter for Box Girder

Stress evaluation at the inner face of the box girder was carried out and the result was generated as shown in Figure 4.4. From the modelling result obtained, the edge area that connected to web and flanges was found to experience higher stress as compared to other area at the diaphragm.

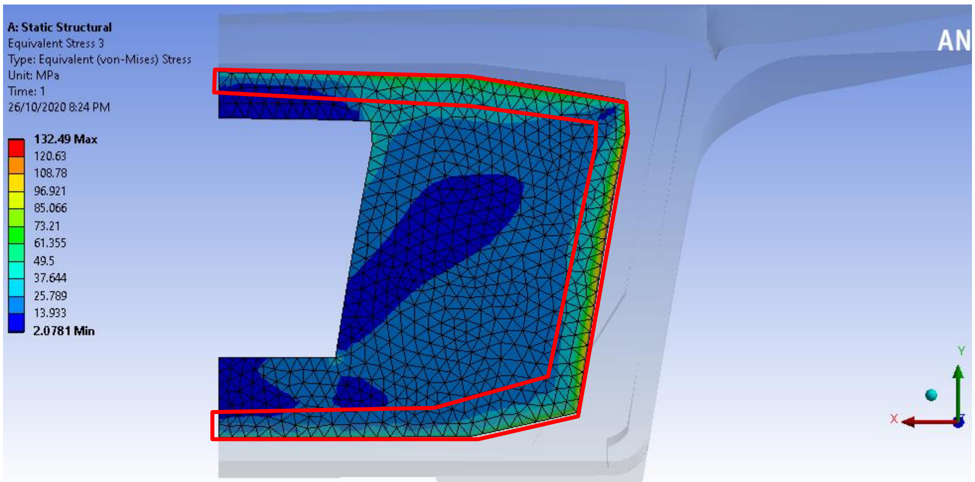
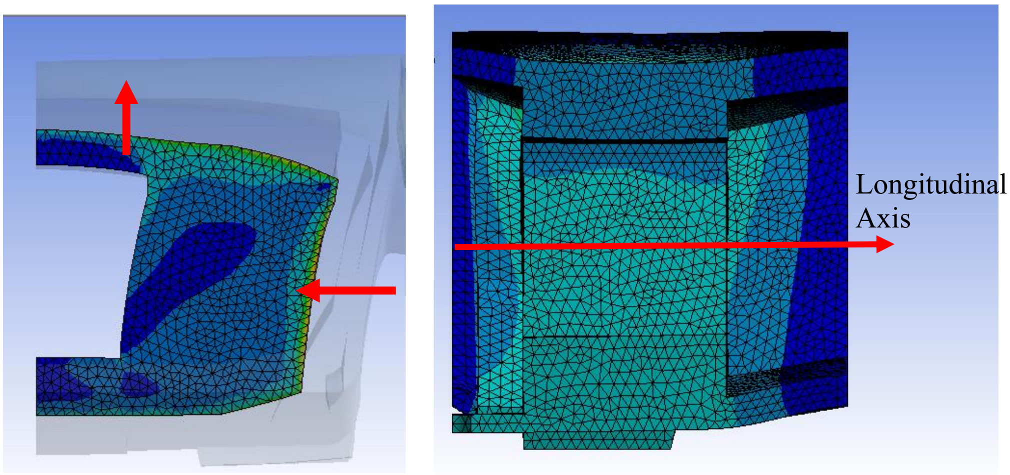


Figure 4.4: Equivalent Stress at the Inner Face of the Box Girder

The high stress recorded at the edge area is due to the deformation of the web and flanges. The web of the box girder bended inwards while the flanges deformed upwards as the box girder was loaded with axle loading as shown in Figure 4.5 (a). The deformed web and flanges induced distortional warping stress that acted in the longitudinal direction. Total deformation for the box girder was generated to capture the distortional warping effect that acted in the longitudinal direction as shown in Figure 4.5 (b). Thus, affecting parameter that contributed to high stress formation at the edge area was identified as the deformation of the web and flanges.



(a) Deformation of Web and Flanges (Transverse Plane)

(b) Distortional Wrapping Effect in the Box Girder (Longitudinal Plane)

Figure 4.5: Deformation and the Distortional Warping Effect with Adjusted Deformation Scale Factor

4.2.2.2 Critical Area Identification

This section presented the critical area identification at the inner face of the box girder. Stress recorded at the critical area is useful for optimisation model validation in Stage 3. This study focused on two-dimensional stress evaluation that considered only the stress effect from the transverse direction. From the FEM results obtained in section 4.2.2.1, the high stress recorded at the edge area was resulted from the distortional warping effect that acted in the longitudinal direction. Therefore, the remaining area A at the inner face of the box girder diaphragm should be the focus of this study as shown in Figure 4.6.

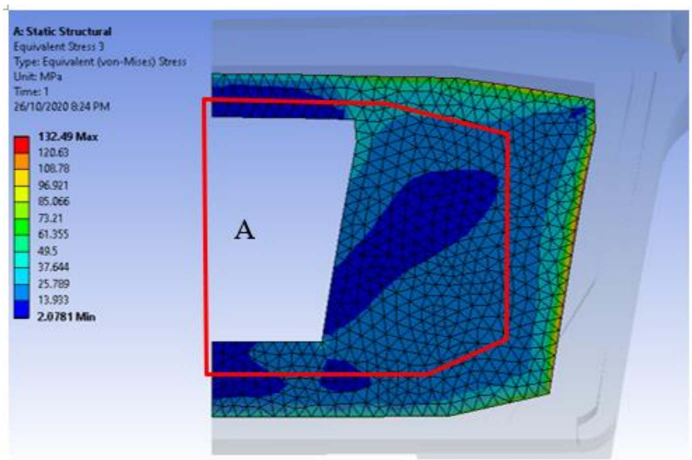


Figure 4.6: Area that Affected by Stress Effect from Transverse Direction

According to Technical Committee CEN (2004), the concrete f_{ck} and f_{ctm} for grade C35/45 are 35 MPa and 3.2 MPa respectively. Highest stresses recorded within area A was located at the top right corner of the diaphragm opening that ranges from 25.79 MPa to 37.64 MPa as shown at the red circled area in Figure 4.7. The stress recorded at the circled area exceeded both the concrete f_{ck} and f_{ctm}. Cracks due to flexure and excessive compressive stress were expected to form at the circled area therefore the circled area was considered as the critical area at the inner face of the box girder.

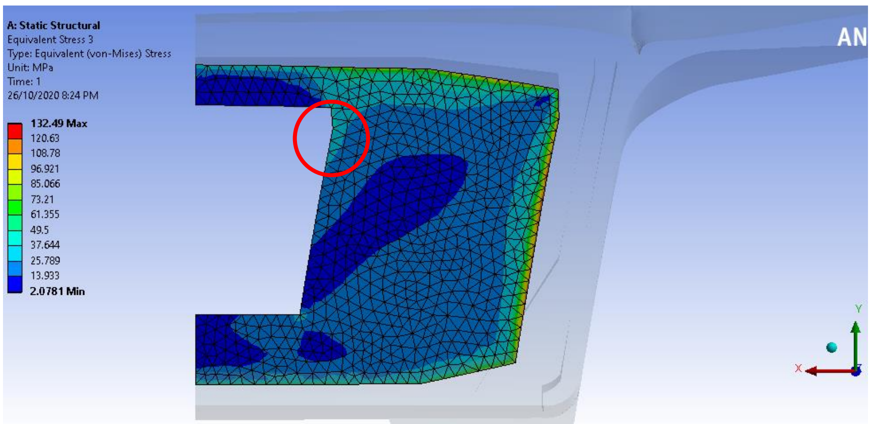
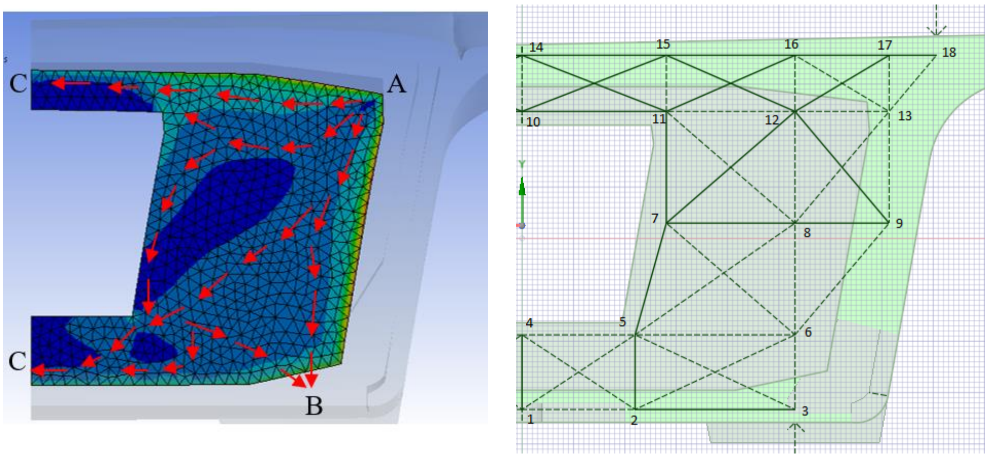


Figure 4.7: Critical Area at the Inner Face of the Box Girder

Optimal amount of reinforcement is needed to resist the tensile stress and the excessive compressive stress that formed at the inner face of the box girder. Congested reinforcement provided may result in extra reinforcement stress redistribution to the concrete surface, which further complicate the stress behaviour for the box girder and bring significant effects to its serviceability performance. Hence, optimisation of the stress distribution was carried out by optimising the STM connection at the inner face of the box girder.

4.2.2.3 Construction of Initial STM

The initial STM was constructed based on the typical STM connection for a solid type of diaphragm with manhole from the specification of Eurocode to demonstrate the stress distribution at the inner face of the box girder before optimisation (Technical Committee CEN, 2005). The FEM results obtained in section 4.2.2.1 and 4.2.2.2 can be useful in the construction of the initial STM. From the FEM result recorded at the inner face of the box girder, basic stress flow pattern can be observed as shown in Figure 4.8 (a).



(a) The Stress Flow Pattern (b) The Initital STM Constructed

Figure 4.8: Stress Flow Pattern and the Constructed STM at the Inner Face of the Box Girder

STM was constructed to transfer the load from A through the diaphragm area to the support at B as well as towards the other half of the girder from C. The initial STM was constructed as shown in Figure 4.8 (b) was based on the stress flow pattern demonstrated. The coordinates and the connectivity of the initial STM constructed was then imported to the HS optimisation model developed for stress optimisation.

4.3 Topology Optimisation

The HS optimisation model developed was used to optimise the stress distribution at the inner face of the box girder diaphragm which aimed to provide an optimised stress distribution and optimised amount of reinforcement used. The stress distribution for the inner face of the box girder diaphragm was demonstrated using STM construction according to Eurocode. General and HS parameters were predefined before the optimisation process started. The proposed and the optimised STM were discussed in the following subsections.

4.3.1 Proposed STM

STM was constructed based on the stress analysis results obtained in the previous section 4.2.2 and the STM example proposed for solid type diaphragm with manhole in Eurocode (Technical Committee CEN, 2005). The proposed STM connection before optimisation is as shown in Figure 4.9 with a total of 18 nodes and 43 members.

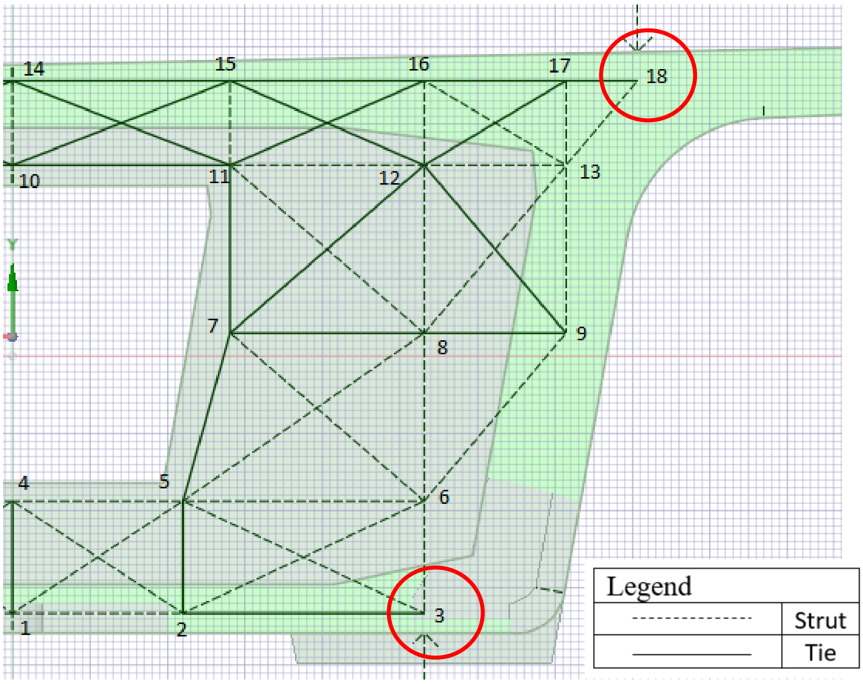


Figure 4.9: Proposed STM Connections before Optimisation Process

The proposed STM constructed allowed the vertical load of 84.7kN that acted at node 18 to transfer through the diaphragm body to the support at node 3. Symmetry axis was defined with zero displacement in the x-direction. First static analysis was carried out in the HS optimisation model for members stress calculation. From the static analysis, total element stress experienced in the proposed model was found to be 9.21 MPa with the combination of 5.88 MPa and 3.33 MPa for 24 struts (dashed line) and 19 ties (solid line) respectively. The stresses and strains results obtained in the proposed STM were recorded and acted as the first solution in the HM. Before the HM initialisation was started, the HS parameters were defined as shown in Table 4.2.

Table 4.2: HS parameters defined before HM Initialisation

HMS	MI	HMCR	PAR
40	800	0.68	0.3

4.3.2 Optimisation of STM

In the HM initialisation, remaining of 39 possible solutions were generated by randomly removing member/s and were recorded in HM before proceeding to HM improvisation. Tuning was carried out during the HM improvisation and the optimisation process ends after no better solution was discovered. During the improvisation process, further iteration does not contribute to significant increase of accuracy, thus the improvisation process stopped at 269th iteration. The convergence graph is as shown in Figure 4.10.

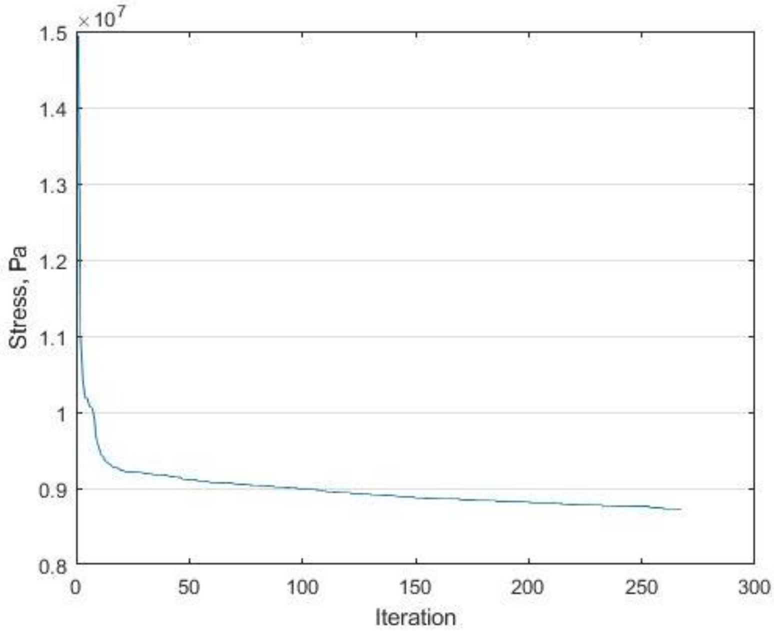


Figure 4.10: Convergence of the Optimisation Model after 269^{th} Iteration

The stresses and strains for the optimised model were recorded and the optimised STM was plotted as shown in Figure 4.11. Total of seven members were removed during the optimisation process. From the optimisation results, the total element stress experienced by the optimised STM was 8.63MPa. Total element compressive stress and tensile stress experienced were found to be 5.49 MPa and 3.14 MPa respectively with both equal number of struts and ties of 18 nos. Since this study is limited to the transverse stress evaluation that considered the axle loading and the concrete self-weight alone, the total element stress is expected to be much lower as compared to the concrete allowable compressive stress of 35MPa. As inspected from the obtained optimised STM connection, the load distribution path is clearer as compared to the proposed STM in section 4.3.1. The STM experienced most of the tensile stress at the upper half of the girder. The tensile stress resulted from the load that acted at node 18 distributed through node seven and split two ways to the top and the bottom of the diaphragm as shown in Figure 4.11.

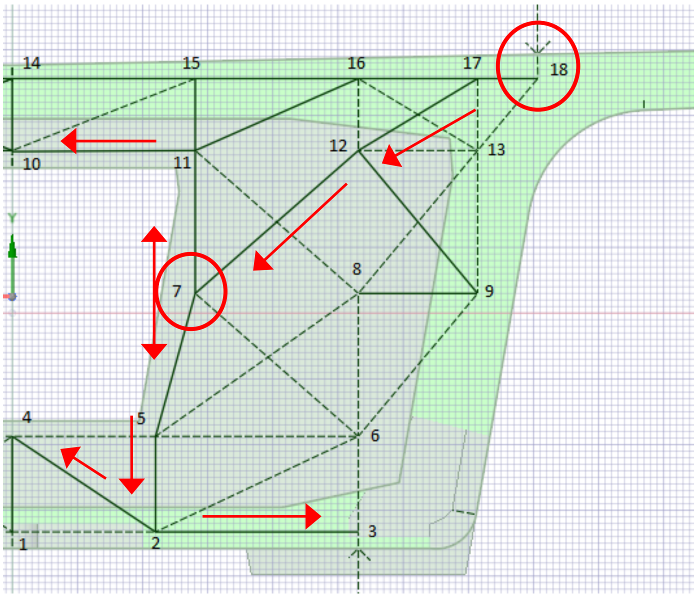


Figure 4.11: Optimised STM Connections after the Optimisation Process

4.3.3 Stresses and Reinforcement Comparisons Before and After Optimisation

The total element stress and the amount of reinforcement needed before and after optimisation were recorded in Table 4.3. The total element stress experienced at the inner face of the box girder after optimisation reduced from 9.21 MPa to 8.63 MPa, which was a reduction of 6.3% as compared to the original proposed STM.

Table 4.3: Total Element Stress and Amount of Reinforcement Needed Before and After Optimisation

	Before Optimisation	After Optimisation	Percentage of Difference
Total Element Stress	9.21 MPa	8.63 MPa	6.3%
Total Element Tensile Stress	3.33 MPa	3.14 MPa	5.71%
Total Element Compressive Stress	5.88 MPa	5.49 MPa	6.63%
Total Reinforcement Needed	*3849.7 mm^2	[1]*3608.53 mm^2	6.26%

*1390.79 mm^2 + 2458.91 mm^2 = 3849.70 mm^2
[1]*1312.71 mm^2 + 2295.82 mm^2 = 3608.53 mm^2

The total element tensile stress in the optimised STM was found to reduce by 5.71% which is the reduction from 3.33 MPa to 3.14 MPa. Reinforcement area needed for 3.33 MPa total element tensile stress in the proposed model was found to be 1390.79 mm^2 with reinforcement yield strength of 275 MPa calculated using Equation (4.1).

$$A_{s,required} = T/\left({}^{f_{yd}}/_{\gamma_m}\right) \tag{4.1}$$

where,

T is the element tensile force

f_{yd} is the reinforcement yield strength

γ_m is the partial safety factor for material

After the optimisation process, total element tensile stress in the optimised STM reduced to 3.14 MPa which contributed to the reinforcement area of 1312.71 mm^2. Apart from the reinforcement needed to resist the element tensile stress, ACI suggested that for concrete grade less than 40 MPa, reinforcement should be provided to resist the transverse tensile force resulting from the compression force spreading in the strut member (ACI Committee 318, 2014). The total compressive stress in the proposed model was recorded as 5.88 MPa, and the reinforcement needed was computed as 2458.91 mm^2. In the optimised STM, the reinforcement area needed was reduced to 2295.82 MPa for total compressive stress of 5.49 MPa. Total reinforcement area was computed by combining the reinforcement area evaluated from both ties and struts as shown in Table 4.3. From the computed results recorded in Table 4.3, total reinforcement area needed at the inner face of the box girder after optimisation was found to reduce by 6.26%.

From the comparisons results, the optimised STM was found to have a better stress distribution as compared to the proposed STM with reduced element stresses. The reduction of element stresses in the STM indicated a reduced amount of reinforcement needed at the inner face of the box girder and which lowered the possibility of over-reinforced design in the structure.

4.3.4 Optimisation Model Validation and Efficiency Evaluation

Optimisation model validation is an important step involved for model efficiency evaluation. The optimisation model efficiency can be discussed from two important aspects which are (i) the accuracy of the optimisation results generated and (ii) the computational complexity for the developed model. The two aspects were discussed in the following subsections and the summarised comparison between FEM and the developed optimisation model is as shown in Table 4.4.

Table 4.4: Comparisons between FEM and HS Optimisation Model

	FEM	**Optimisation Model**
Accuracy	Approximate solution	Percentage error (critical member) = 1.2%
Computational Complexity		
- Computational procedure (Input Parameters)	Geometry development, GIT, mesh, material, and structural conditions definitions	General (STM coordinates, concrete grade, loadings, constraints) and HS parameters
- Computational Time	Half an hour or more	Five to ten minutes

4.3.4.1 The Accuracy of the Optimisation Results Generated

The HS optimisation model was verified by evaluating the accuracy of the results generated. Percentage error for the critical member was computed by comparing the optimisation results with the FEM results. From the analysis, the critical member in STM was identified as member 7-11 as shown in Figure 4.12.

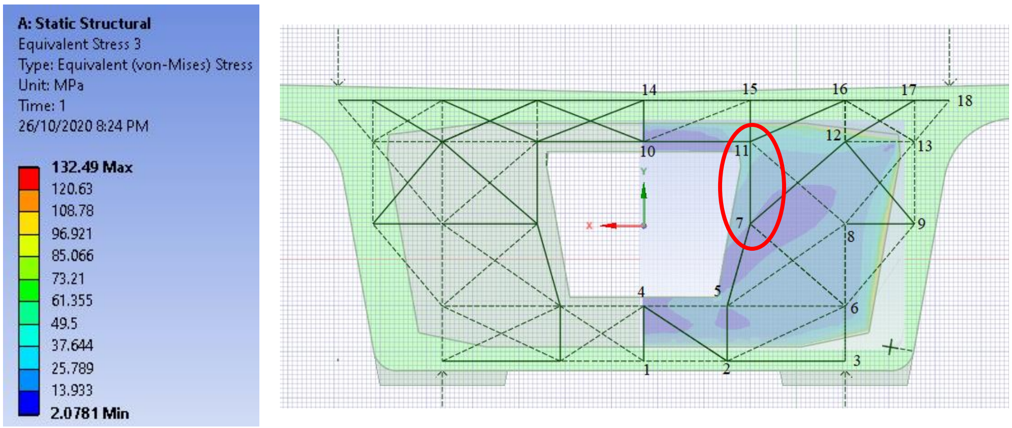


Figure 4.12: Combination of STM and Stress Distribution Result from ANSYS

The stresses recorded from FEM at the critical area was 25.79MPa, while the stress recorded for critical member 7-11 was 25.58MPa. Percentage of error for the critical member was computed as 1.2% as shown in Table 4.5. Other members in the STM may be affected by the distortional warping effect from the longitudinal direction hence higher percentage error is expected as stated in Table 4.5. Despite that, Wollmann et al., (2000) concluded that the developed STM with the percentage error of 18.45% for concrete diaphragm was safe and conservative. Hence percentage error of 16.01% for member 5-7 is still acceptable. Another study was carried out by He and Liu, (2010) to obtain the optimal STM for anchorage diaphragm; study discussed that the developed STM has a good agreement between the approximate value and the calculated value from STM with the percentage error of 5.59%. In this study, the calculated optimised value for STM is compared with the approximate value obtained from FEM. Hence, when referred to the optimised STM obtained in this study, results indicated that the optimisation model developed can generate acceptable results with the percentage error of 1.2%.

Table 4.5: Comparison of Stress for Respective Member and Percentage of Error

Member		Length (mm)	Force (kN)	Stress, $\boldsymbol{\sigma}$ (MPa)	Stress from FEM, $\boldsymbol{\sigma_{Approx}}$ (MPa)	Percentage of Error (%)
7	11	695.00	8.00 (T)	25.48	25.58	1.20
10	11	898.78	10.31 (T)	12.82	13.93	7.97
5	7	721.78	12.99 (T)	16.16	13.93	16.01

4.3.4.2 The Computational Complexity for the HS Optimisation Model

The computational complexity of a model was evaluated based on two factors which are (i) the computational procedure (input parameters) and (ii) computational time. Computational procedure represented the necessary parameters definitions involved in the computation process until the results is generated.

The input parameters required for the HS optimisation model and FEM are as stated in Table 4.4. The main difference between the HS optimisation model and FEM is the present of geometry development and GIT. Geometry development involved in FEM requires actual structure geometry construction in a separate design interface, while geometry development is not required in the developed optimisation model. GIT is necessary in FEM because the FEM results generated is highly dependent on the mesh element size; while results generation in the developed optimisation model is not dependent on the mesh element size.

Next, computational time is another important factor for model computational complexity evaluation (Goldreich, 2008). Computational time required in FEM are normally half an hour or more especially when dealing with complex structure. Geometry development, GIT, mesh generation, structural analysis and results generation are the most time-consuming steps involved in FEM. It is another case for the HS optimisation model as geometry development, GIT and mesh generation are not required in the optimisation process; and the construction of the initial STM is based on the reference provided by Eurocode thus, the optimisation process takes only five.

4.4 Concluding Remarks

Stress distribution at the inner face of the box girder diaphragm was modelled with 72 mm mesh element size. The box girder experienced distortional warping stress that acted in longitudinal direction resulted from the deformation of the web and flanges. Therefore, the stress recorded at the edge area near to web and flanges were neglected. The critical area was then identified at the top right corner of the diaphragm opening. Stress recorded at the critical area ranges from 25.79 MPa to 37.64 MPa. Cracks were expected to form at the critical area resulted from the excessive compressive stress and the tensile stress developed. Adequate amount of reinforcement is needed at the area to avoid reinforcement stress redistribution due to congested reinforcement provided. Optimal amount of reinforcement can be obtained by optimising the stress distribution of the box girder. The stress flow pattern at the inner face of the box girder was determined from the FEM results obtained. From the determined stress flow pattern, initial STM was constructed for the optimisation process.

The optimisation model using HS algorithm was developed in MATLAB and was used to optimise the stress distribution at the inner face of the box girder diaphragm. The optimised results were found to have the percentage error of 1.2% which is acceptable and indicated a good agreement between the predicted value and the exact value obtained from FEM. Efficiency evaluation for the developed optimisation model was also carried out by comparing the computational complexity and accuracy for FEM and developed model. Comparison results show that the HS optimisation model can efficiently provide an optimal stress distribution with shorter computational time, simpler computational procedure and yet results generated are acceptable.

Thus, development of the HS optimisation model was found to be efficient and could be practical for actual structural design, also it introduced the implementation of soft computing technique to easily obtain an optimised stress distribution as compared to using conventional method or FEM. Advance study could be carried out based on the HS optimisation model developed to widen the application of stress optimisation in structural design.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion of Research Work

Conclusions can be drawn to address the three main objectives of this study as outlined in Chapter one.

i. From the stress analysis results obtained, the box girder was found to experienced distortional warping stress that acted in the longitudinal direction due to the deformation of the web and flanges. The top right corner of the diaphragm opening was identified as the critical area and experienced stress that ranges from 25.79 MPa to 37.64 MPa. Cracks are expected to form at the critical area due to the excessive compressive stress and tensile stress developed. Stress flow pattern at the inner face of the box girder was identified for initial STM construction.

ii. HS optimisation model was developed to optimise the stress development at the inner face of the box girder. During the optimisation process, the solution was found to converge at 269^{th} iteration and the critical member in the STM was identified as member 7-11. The results generated from the optimisation model agreed well with approximated value obtained from FEM with the percentage error of 1.2%. The optimisation process took about five to ten minutes which is lesser as compared to FEM. The HS optimisation model required lesser computation effort as geometry construction and mesh element size are not important for the HS optimisation model. Thus, the HS optimisation model developed can generate good results with shorter computational time and simpler computational procedure.

iii. Stress optimisation at the inner face of the box girder was carried out, total of seven members were removed during the optimisation process. From the optimised STM connection, the stress distribution path was found to be clearer as compared to the initial STM before optimisation process. The axle load was found to distribute through the centre part of the diaphragm and split two ways at node 7 to the upper half and lower half of the box girder, the upper half of the STM was found to experience more tensile stress as compared to the lower half of the STM. The total element tensile and compressive stresses experienced in the STM were found to reduce by 5.71% and 6.63% respectively after optimisation. Reduction in the element tensile and compressive stress resulted in the reduction of the total element stress from 9.21 MPa to 8.63 MPa. The amount of reinforcement needed before and after the optimisation process was computed and found to reduce by 6.26% from 3849.7 mm^2 to 3608.53 mm^2. The inner face of the box girder obtained a better stress distribution after the optimisation process which contributed to lesser amount reinforcement needed and indirectly reduced the possibility for the formation of cracks due to stress redistribution from the reinforcement provided.

5.2 Recommendations for Future Research

Future research could be carried out on three-dimensional optimisation model to include the effect from the third direction such as the effect from prestress loading. The extension of the study on three-dimensional optimisation model could explores the availability of the developed model to a more complex structural engineering problem.

Besides, application of the developed optimisation model on other type of reinforced concrete structures could be carried out in the future for a more throughout feasibility evaluation. On the other hand, experiment or actual field test could be carried out to verify the results obtained from the developed optimisation model. This is to establish a more rational and complete design approach. Study on the selection of HS parameters' value to be used in solving stress optimisation problem for concrete box girder should be carried out to justify if the existing HS parameters selected is sufficient in generating optimal solution.

Finally, study on the development of solutions regardless of the construction of the initial STM could be carried out. This future work could improve the robustness and the flexibility of the developed model, which enables the model to be applied on wider range of structural engineering applications.

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