DOUBLE MOVING RANGE CONTROL CHART

By

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ABSTRACT

DOUBLE MOVING RANGE CONTROL CHART

Lai Boon Sing

Control charts is one of the most powerful tools in statistical process contorl (SPC). It can be used to monitor a process over a period of time. In this project, the individual ($\bar{X}$) chart, moving average (MA) chart, double moving average (DMA) and moving range (MR) chart were studied. Then a double moving range control chart was proposed using similar statistic to DMA chart. An example was demonstrated using sample data. The proposed DMR chart is effective in determining the process trend in shifted mean ranges.

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APPROVAL FOR SUBMISSION

I certify that this project report entitled "**DOUBLE MOVING RANGE CONTROL CHART**" was prepared by **LAI BOON SING** has met the required standard for submission in partial fulfilment of the requirements for the award of Master of Mathematics (Coursework) at Universiti Tunku Abdul Rahman.

Approved by,

Signature : ______________________

Supervisor : DR. WONG VOON HEE

Date : 1 SEPTEMBER 2020

FACULTY OF ENGINEERING SCIENCE

UNIVERSITI TUNKU ABDUL RAHMAN

Date: 14 August 2020

SUBMISSION OF FINAL YEAR PROJECT /DISSERTATION/THESIS

It is hereby certified that ***Lai Boon Sing*** (ID No: ***1904324***)has completed this dissertation entitled "DOUBLE MOVING RANGE CONTROL CHART" under the supervision of Dr Wong Voon Hee (Supervisor) from the Department of Mathematics and Actuarial Science, Faculty of Engineering Science.

I understand that University will upload softcopy of my dissertation in pdf format into UTAR Institutional Repository, which may be made accessible to UTAR community and public.

Yours truly,

Boon Sing

(*Lai Boon Sing*)

*Delete whichever not applicable

DECLARATION

I hereby declare that the dissertation is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UTAR or other institutions.

Name Lai Boon Sing

Date 14 August 2020

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Chapter 1

INTRODUCTION

During manufacturing process, one of the most important thing is to examine whether a process is in control. Over the year, an industry standard methodology have been developed to measure and control the quality of a manufacturing. Statistical process control (SPC) is a collection of tools used to reduce the variability and attain the stability of products. In general, there are seven major SPC tools. They are also known as "the magnificent seven".

1. Histogram is a diagram of gap-less vertical bars whose area are relative to the frequency of variable and whose width are equal to the class interval. For discrete data, a stem and leaf plot can sometimes be used in similar fashion: the data values are categorized according to 'stem' (common characteristics) and 'leaf' (fit data values). The distribution and skewness of data are easily seen using histogram and stem-and-left plot; and the central tendencies can also be determined rather quickly.

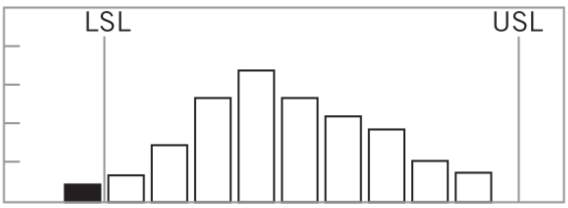


Figure 1.1: A histogram

2. Check sheet is a form that is useful in collecting data in real time at the location where the data is produced. It simplifies data collection and analysis so that one can spot problem region by frequency of location, type and cause. It is sometimes referred as a tally sheet.

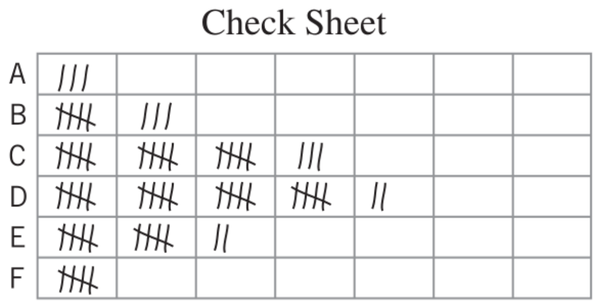


Figure 1.2: A check sheet

3. Pareto chart consists of both bars and a line graph. In this chart, individual values are denoted in a downward order by bars and the cumulative total is represented by the line. One can easily sees the most significant problem with this arrangement, and it is usually needed to be resolved most. This is because by the Pareto principle, roughly 80% of the problems are caused by only 20% of the factors.

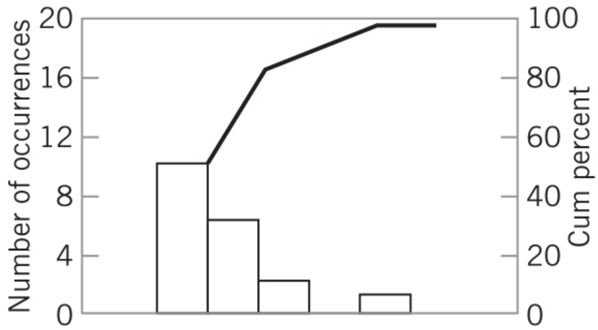


Figure 1.3: A Pareto diagram

4. Cause-and-effect diagram visualized all contributing factors and their relationships. It is a tool to quickly identify problem area and their causes and effects. It is also called Ishikawa diagram or fishbone diagram.

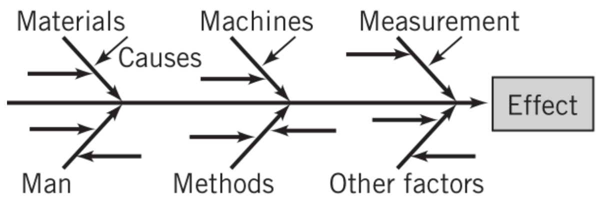


Figure 1.4: A cause-and-effect diagram

5. Defect concentration diagram is a graphical tool that is beneficial in studying the causes of the product or part defects. It is also known as problem concentration diagram. Dark shades represent identified defects.

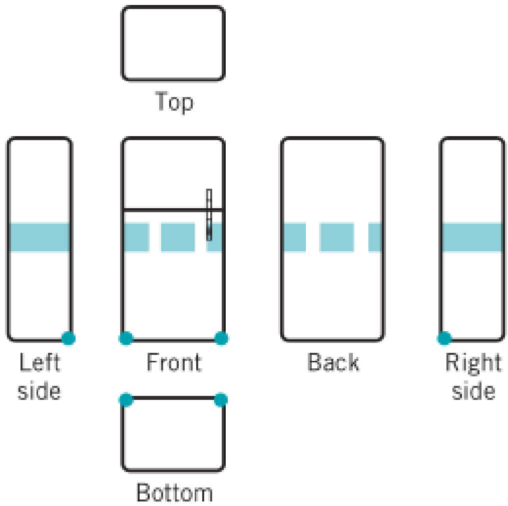


Figure 1.5: A defect concentration diagram.

6. Scatter diagram is useful to identify the relationship between two factors among many of them. These two variables are plotted along two axes and the pattern of the resulting plot of points reveals whether there is positive, negative or no correlation at all between the two factors.

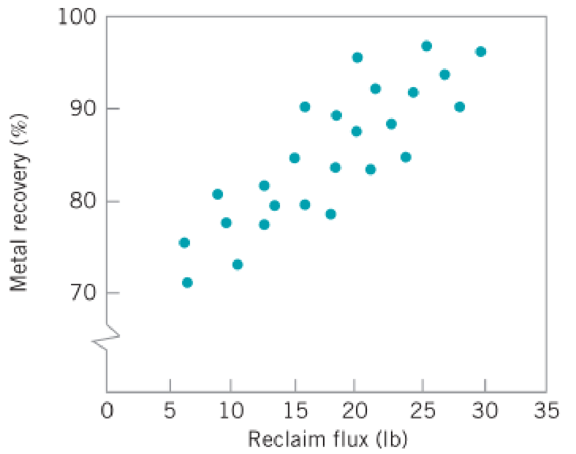


Figure 1.6: A scatter diagram.

7. A control chart is used to study and observe how a process changes over time. Since performance are monitored over time, behaviours such as trends and out-of-control are immediately reflected in the graph¿ This allows process corrections to be injected quickly. This subsequently helps to reduce variability of products.

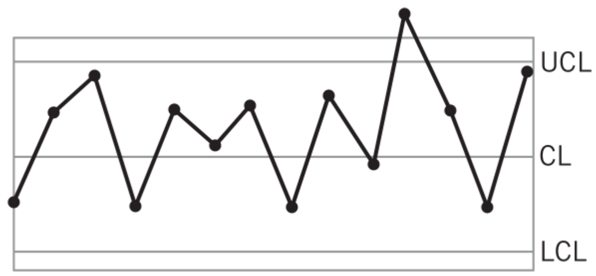


Figure 1.7: A control chart.

Among these seven tools, the control chart which was invented by Dr Walter A. Shewhart in 1924, turns out to be the most technically sophisticated tool. In simple words, a control chart is very much similar to a patient's temperature chart which tells the doctor the progress of a patient's condition. Likewise, a control chart gives a picture of the continuing story of the state of 3 qualities of the manufactured items. They are very good method for problem solving and to improve quality based on the analysis and results obtained as they are used for the purpose of detecting assignable causes that affect process stability. Apart from that, they are also excellent decision making tools in deciding if the process is a good one, a poor one or has no effect based on the pattern of the plotted points. The control charts are widely used in the industries due to five reasons:

1. proven technique for improving productivity,
2. effective in defect prevention,
3. prevent unnecessary process adjustment,
4. provide diagnostic information, and
5. provide information about process capability.

1.1 Objectives

The research objectives of this project are as follows:

1. To study the following control charts used for detecting shifts in the process mean.
    (a) Moving average (MA) chart
    (b) Moving range (MR) chart
    (c) Double moving average (DMA) chart
2. To proposed a new control chart namely double moving range (DMR) control chart .
3. To compare the performance of proposed chart with the moving range chart.

1.2 Problem Statement

The main question of this research topic is that whether the proposed double moving range chart can be used as an alternative of current control charts, and whether it provides some advantages over the others such as better performance, simpler implementation or higher sensitivity.

1.3 Literature Review

Among the applications of moving range chart, Wang, Zheng and Chen [19] used moving range chart to monitor the resistance sport welding processing based on electrode displacement curve. A case study is used to demonstrates the validity and effectiveness of the methodology.

It is arguable whether individual and moving range (X-MR) chart is more necessary than using individual value chart (X-chart) alone. Amin and Ethridge [5] showed the disadvantage in using the X-MR procedure based on average run length values does not exist, however they recommended X-MR procedure to Shewhart individual measurement chart.

After examining five dissimilar non-standard conditions, including both independent and identically distributed (iid) and non-iid circumstances, Rahardja [16] further added that using X-MR chart generally not more helpful than X-chart alone for detecting iid departures from standard conditions. However, it may be advantageous in detecting some non-iid conditions.

Khaliq et al. [9] compared Tukey's control chart (TCC) with X-MR chart, and concluded that TCC is more efficient in most cases, except when the observations has Student's t-distribution with small degree of freedom and when the observations has gamma distribution.

Hassan et al. [8] applied logistic transformation on moving range chart (MRC) to become logistic moving range chart (LMRC), which is used to reduced outlier effects from the data. Both charts are considered for the detection of significant gene-compound interaction.

Gildeh and Shafiee [7] discussed the construction of fuzzy control charts for autocorrelated fuzzy observations in the case where vague data are generated with uncertain values due to environmental conditions and other factors. They calculated the variance, covariance, and autocorrelation coefficient of the defined $D_{p,q}$-distance between fuzzy numbers. The autocorrelation coefficient is used in order to modify the limit of X-MR control chart.

The exponentially weighted moving average (EWMA) control chart and cumulative sum (CUSUM) control chart are useful to detect small parameter shifts, since information is accumulated over time to determine the state of statistical control. Shamma et al. [17] extended the EWMA statistics to create a double exponentially weighted moving average (DEWMA) chart. Mahmoud and Woodall [13] compared the performance of DEWMA chart with EWMA chart based on worst-case average run length (ARL) and zero-state measures. When the past observations is given weight closer in EWMA than those in DEWMA chart, the smoothing constant will result in loss of advantage for DEWMA chart.When small smoothing constants are used, EWMA chart tends to performed better in terms of worst-case ARL values.

Khoo and Wong [11] extended the technique from double exponentially weighted moving average (DEWMA) to become the basis of double moving average statistics This is accomplished by computing the moving average twice. Abbas et al. [1] proposed a double progressive mean (DPM) chart, using similar technique as in Khoo and Wong [11], which is an enhancement of progressive mean (PM) chart in terms of performance.

However, for the same purpose, Aslam et al. [6] claimed that the alternative control chart of DMA chart combined with EWMA statistic is more efficient in detecting process shifts when different combinations of the moving average span, the shift constant, the smoothing parameter of

EWMA chart, and the target in-control average run lengths were examined.

Khan et al. [10] proposed an EWMA control chart to monitor exponential distributed characteristics. Since the sample data is required to assumed normal, it is transformed in the first place. The moving average statistic is then calculated for each subgroup. In the next step, the EWMA statistic is constructed based on the current and previous MA statistics. The mean and variance of EWMA statistics are used to derive the upper and lower control limits. The in-control and out-of-control average run lengths are derived and tabulated according to process shift parameters and smoothing constants. At last, they claimed that for all shift parameters, the proposed control chart outperforms the MA control chart.

Similarly, Ahmad et al. [3] combined DMA and EWMA chart for exponentially distributed data. This chart shows the quickest detection of the shifted process.

Several modification were made to the DEWMA chart proposed by Shamma et al. Adeoti [2] proposes a new DEWMA chart using repetitive sampling (RS-DEWMA). The proposed chart consistently gives smaller ARL values and quickly detects the process shift. However, the performance relatively deteriorates for large smoothing constants. Teh et al. [18] proposed a sum of squares double exponentially weighted moving average (SS-DEWMA) chart by using sum of squares statistic to monitor the mean and variance of the process in a single chart at the same time. Although there is a better performance of zero state average run length (ARL) and standard deviation of the run length (SDRL) in SS-DEWMA chart than the optimal SS-EWMA chart, this is not the case in cyclical steady state ARLs and SDRLs.

Li et al. [12] developed a double CUSUM (DCUSUM) algorithm to determine the effect of interference data on the classification of industrial data streams. Its advantages are ability to block misclassification problems, greater detection accuracy, and, practical computational complexity.

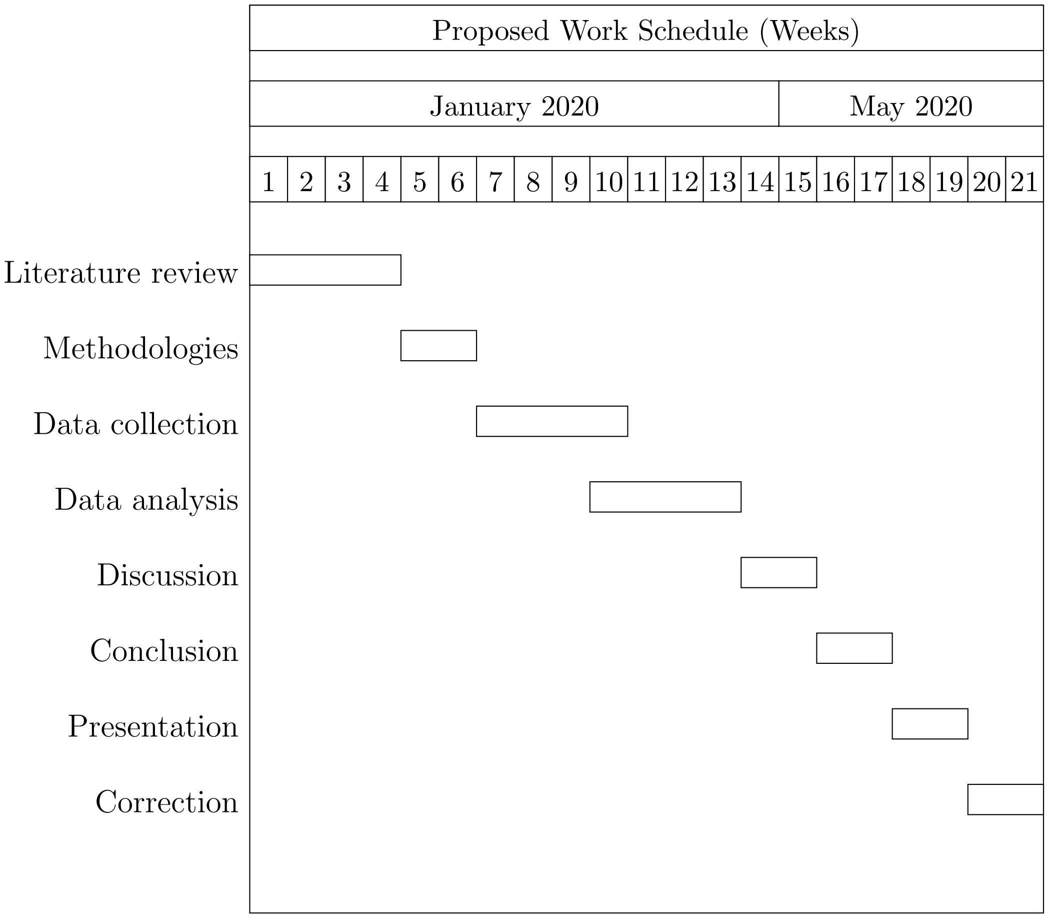
1.4 Research Methodology

This project use the similar technique in double moving average chart developed by Khoo and Wong [11] to construct a double moving range chart. Then, using sample data from Montgomery (2009) [14], an example is illustrated using the proposed chart as well as compared with other studied charts.

1.5 Expected Project Contribution

In the end of this project, it is expected that the proposed control chart can be used as a complement chart together with the existing charts, where it might provides some advantages over the others such as better performance, simpler implementation or higher sensitivity.

1.6 Expected Work Schedule

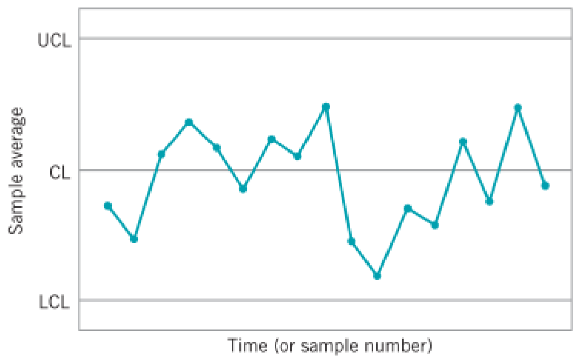


Chapter 2

CONTROL CHARTS

2.1 Basic ideas of control charts

A control chart is one of the primary techniques of statistical quality control (SQC). It contains three lines: centre line (CL), upper control limit (UCL), and lower control limit (LCL). Most of the observations shall fall between the UCL and LCL. Points plotted outside the control limits are considered out of control. This is a signal some unusual sources of variability are present, and one must investigate the sources of variability of the process and then make corrective action to remove these unusualness. A general control chart is also called Shewhart control chart.



When designing a control chart, one of the most important decisions to be considered is to decide how wide the control limits should be. The further the control limits from the centre line (a larger gap), the lower the probability of a point falling outside the control limits, which is called the type I error. This means the chart will falsely identify out-of-control signal when there is no such cause present. On the other hand, a wide control limits increase the probability of a point falling between the control limits when the process is actually out of control, which is called the type II error. This means the chart is less sensitive to out-of-control signal.

If the control limits were moved closer to the centre line (a narrower gap), the opposite result is obtained: The probability of type I error increases, while the probability of type II error decreases.

2.1.1 Sample size

When designing a control chart, one must narrow down how large the sample size is and how frequent the sampling take. In general, larger samples will make it easier to detect small shifts in the process. When choosing the sample size, one must often be reminded on the shift magnitude that are intended to detect. If the process shift is comparatively large, then smaller sample sizes is used.

2.1.2 Choosing a suitable control chart

Basically, a control chart deals with one measurement such as sample mean $\bar{x}$, range R and sample standard deviation s or sample variance s^2. These charts analysed the data gathered all at once, without considering the relation between each observations.

For single observation per time period, moving range chart is commonly used. The moving range is defined as the absolute differences of two successive observations. A moving range control chart can also be constructed. Usually, the moving range control chart are used together with individual values chart, they are called *individual and moving range* (X-MR) chart.

The advantage of X-MR chart is that the data does not need to be assumed to follow normal distribution, because it is based on averages of consecutive differences. However, X-MR chart can be easily affected by an outlier (extreme value).

When one are more interested in the detection of small process shifts, the above charts are not powerful to monitor such process. Therefore, other charts are used instead. These charts use information of the entire sequence of points in a process. Two candidates that fulfil the need above are the cumulative sum (CUSUM) control chart, and the exponentially weighted moving average (EWMA) control chart. They are excellent for these situations.

The moving average (MA) chart is also a useful chart for detecting smaller shift of mean . MA chart uses the avrage of the current mean and all the previous means to produce each moving average. Both X-MR and MA charts are useful when only a single response is available at each time point.

2.1.3 Performance Measure of Control Chart

The performance of a control chart can be evaluated using average run length (ARL). Basically, ARL is the average number of points that must be plotted before an out-of-control point is detected. There are other ways to measure the performance of a control chart other than ARL, such as Median Run Length (MRL) and Standard Deviation Run Length (SDRL). The MRL denotes the median number of sample points plotted before the first out-of-control signal is detected. On the other hand, SDRL measures the spread of the run length distribution.

If there is no correlation between the process observations, then the ARL for any Shewhart control chart can be calculated from the following simple formula:

$$ARL = \frac{1}{p} \tag{2.1}$$

where p is the probability that any point falls outside the control limits. This equation can be used to evaluate the performance of the control chart.

2.2 $\overline{X}$ chart

The $\bar{X}$ chart is the simplest and most intuitive control chart, hence it is widely used in the industry to monitor the arithmetic means of successive samples with constant size, n. One of the most important assumption about the sample is that the samples are normally distributed, for the purpose of control limit calculation.

In an $\bar{X}$ chart, the vertical axis represents the sample average, $\bar{x}$. The process is said to be in control if the values of $\bar{x}$ are between UCL and LCL. Otherwise, the process mean is considered out of control. When a process is in statistical control, the mean value for each subgroup is stable over time and the variation within a subgroup is also stable. This chart is effective in detecting large scale shifts in the mean but not so effective in detecting small or moderate shifts in the mean.

The general model of a control chart is as follows: Let x be a sample statistic that measures some quality characteristic of interest and the mean of x is μ_x and the standard deviation is σ_x. The general formula for centre line, upper control limit and lower control limit are calculated using the following equations:

$$UCL = \mu_x + L\sigma_x$$

$$CL = \mu_x$$

$$LCL = \mu_x - L\sigma_x$$

where L controls the "width" of control limits. It is usually set to 3 unless stated otherwise. The following figure shows a $\bar{X}$ chart used to monitor the weekly average bowling scores.

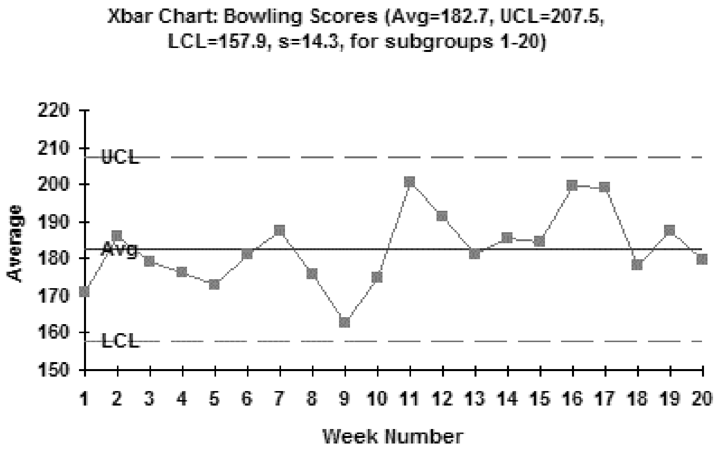


Figure 2.1: $\bar{X}$ chart

If there is point plotted outside the control limits, this might mean there is some variability or that individual point behaves differently compare to others. If a series of points fall outside the control limits, this suggests a shift in the process mean. It is very likely that this is also a clue that the mean is out of control. Therefore, further investigation should be fulfilled. It is most likely

2.3 Moving average (MA) chart

The moving average (MA) chart is a time weighted control chart. It is used to monitor the mean of a process based on samples taken at given period, which can be hours, shifts, days, weeks, months and so on. [15] The concept of moving average is that instead of computing all the mean as a whole (which is from the first group up to current group), only the current mean and some recent means are taken into computation, hence the name "moving". A more details definition is as follows.

Suppose that individual observations X_{ij} follows a normal distribution with mean μ and variance σ^2. The observations are separated into i subgroups where each subgroup contains n observations. Thus $i = 1, 2, \ldots$ and $j =$

$1,,2,\ldots,n$. For each subgroup, the subgroup averages are

$$\overline{X}_i = \frac{X_{i1} + X_{i2} + \cdots + X_{in}}{n} \tag{2.2}$$

so there are $\overline{X}_1, \overline{X}_2, \ldots$ subgroup averages as long as the process is undergoing.

Now define w as the span, which is the 'width" or the number of most recent subgroups that is wished to taken into computation. Then the MA statistics of span w at time i computed from the subgroup averages $\overline{X}_1, \overline{X}_{i-1}, \ldots$ is defined as

$$MA_i = \frac{\overline{X}_i + \overline{X}_{i-1} + \cdots + \overline{X}_{i-w+1}}{w} \tag{2.3}$$

Note that the formula is true for $i \geq w$ only. For measurements less than the span, there is not enough subgroup averages to compute the MA statistics. For these periods, the MA is defined by the average of all subgroups up to period i. When the process is in control, the mean and variance (for $i \geq w$) of MA_i are:

$$E(MA_i) = E(\overline{X}) = \mu \tag{2.4}$$

and

$$\begin{aligned} Var(MA_i) &= \frac{1}{w^2} \sum_{j=i-w+1}^{i} Var(\overline{X}_j) \\ &= \frac{1}{w^2} \sum_{j=i-w+1}^{i} \frac{\sigma^2}{n} \\ &= \frac{\sigma^2}{nw} \end{aligned} \tag{2.5}$$

respectively, where μ is the overall mean for all observations as defined above. For $i < w$ the variance is $Var(MA_i) = \frac{\sigma^2}{ni}$. The control limits of the

MA chart are:

$$UCL/LCL = \begin{cases} \mu \pm \frac{3\sigma}{\sqrt{nw}} & i \geq w \\ \mu \pm \frac{3\sigma}{\sqrt{ni}} & i < w \end{cases} \tag{2.6}$$

2.4 Double moving average chart

The constuction of double moving average chart was proposed by Khoo & Wong [11] which is based on the idea of computing the MA of subgroup averages twice. The computation of MA statistic is explained in Eq. (2.3). With similar idea, the DMA statistic of span w at time i is computed by:

$$DMA_i = \frac{MA_i + MA_{i-1} + \cdots + MA_{i-w+1}}{w} \tag{2.7}$$

Note that the formula is also true for $i \geq w$ only. For measurements less than the span, there is not enough MAs to compute the DMA statistics. For these periods, the DMA is defined by the average of MAs up to period i. When the process is in control, the mean of DMA$_i$ are:

$$E(DMA_i) = \frac{1}{w} E\left(\sum_{j=i-w+1}^{i} MA_j\right) = \frac{1}{w}(w\mu) = \mu \tag{2.8}$$

which is similar to Eq. (2.4). The formula for the variance of the DMA$_i$ statistic for $w > 2$ is more complicated:

$$Var(DMA_i) = \begin{cases} \dfrac{\sigma^2}{ni^2} \displaystyle\sum_{j=1}^{i} \frac{1}{j}, & i \leq w \\ \dfrac{\sigma^2}{nw^2} \left[\displaystyle\sum_{j=i-w1}^{w-1} \frac{1}{j} + (i-w+1)\left(\frac{1}{w}\right) \right], & w < i < 2w-1 \\ \frac{\sigma^2}{nw^2}, & i \geq 2w-1 \end{cases} \tag{2.9}$$

Note that for $w = 2$, Var(DMA$_i$) is computed using only the first and third line of Eq. (2.9). The control limits of the DMA chart for $w > 2$

are:

$$UCL/LCL = \begin{cases} \mu \pm \dfrac{L\sigma}{i\sqrt{n}}\sqrt{\displaystyle\sum_{j=1}^{i}\frac{1}{j}}, & i \le w \\ \mu \pm \dfrac{L\sigma}{w\sqrt{n}}\sqrt{\displaystyle\sum_{j=i-w+1}^{w-1}\frac{1}{j} + (i-w+1)\left(\frac{1}{w}\right)}, & w < i < 2w-1 \\ \mu \pm \frac{L\sigma}{w\sqrt{n}}, & i \ge 2w-1 \end{cases} \tag{2.10}$$

2.5 Moving range chart

Another way to monitor the process is by looking at the range between observations. Again, suppose that each observation X_{ij} follows a normal distribution with mean μ and variance σ^2, where $i = 1, 2, \ldots,$ represents ith subgroup of the samples; and $j = 1, 2, \ldots, n$ represents jth item within ith subgroup. The range between two observations is defined as:

$$R_{ij} = |X_{ij} - X_{i,j+1}| \tag{2.11}$$

Therefore for each subgroup with n observations, there are only $n-1$ range samples. It is also assumed that R_{ij} follows a normal distribution with mean R_0 and variance σ_R^2. The subgroup range average is

$$\overline{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i,n-1}}{n-1} \tag{2.12}$$

for ith subgroup with n subgroup size.

The moving range (MR) is the average of current and the most recent few mean ranges. Given a span w which is the number of most recent subgroup range averages $\overline{R}_i, \overline{R}_{i-1}, \ldots,$ the moving range is defined as

$$MR = \frac{\overline{R}_i + \overline{R}_{i-1} + \overline{R}_{i-w+1}}{w} \tag{2.13}$$

Note that this is only true for $i \geq w$. There is not enough subgroup averages to compute a MR when $i < w$. Thus, the MR for these periods are defined by the average of all subgroups $\overline{R}$ up to period i. When the process is in-control, the mean of MR_i are:

$$E(MR_i) = E(\overline{R}) = R_0 \tag{2.14}$$

and variance (for $i \geq w$) is

$$\begin{aligned} Var(MR_i) &= \frac{1}{w^2} \sum_{j=i-w+1}^{i} Var(\overline{R}_j) \\ &= \frac{1}{w^2} \sum_{j=i-w+1}^{i} \frac{\sigma_R^2}{n-1} \\ &= \frac{\sigma_R^2}{(n-1)w} \end{aligned} \tag{2.15}$$

respectively. For $i < w$ the variance is defined as

$$Var(MR_i) = \frac{\sigma_R^2}{(n-1)i} \tag{2.16}$$

instead. The control limits of the MR chart are:

$$UCL/LCL = \begin{cases} \mu_0 \pm \frac{3\sigma_R}{\sqrt{(n-1)w}} & i \geq w \\ \mu_0 \pm \frac{3\sigma_R}{\sqrt{(n-1)i}} & i < w \end{cases} \tag{2.17}$$

Chapter 3

DOUBLE MOVING RANGE CHART

3.1 Construction of double moving range chart

Using the same concept on double moving average chart, the construction of double moving range chart is similar. The computation of MR statistic is explained in Eq. (3.2). With similar technique, the DMR statistic of span w at time i is computed using the following formula:

$$DMR_i = \frac{MR_i + MR_{i-1} + \cdots + MR_{i-w+1}}{w} \tag{3.1}$$

for $i \geq w$. If $i < w$, the DMR statistic is only computed as the average of all MRs up to period i.

From Eq. (2.14), the mean of DMR_i which assumed to be an in-control process is:

$$\begin{aligned} E(DMR_i) &= \frac{1}{w} E\left(\sum_{j=i-w+1}^{i} MR_j \right) \\ &= \frac{1}{w}(wR_0) \\ &= R_0 \end{aligned} \tag{3.2}$$

for all periods of i. By modifying Equation (2.9) , the variance of DMR_i statistic for $w > 2$ is as follows:

$$Var(DMR_i) = \begin{cases} \frac{\sigma_R^2}{(n-1)i^2} \sum_{j=1}^{i} \frac{1}{j}, & i \le w \\ \frac{\sigma_R^2}{(n-1)w^2} \left[\sum_{j=i-w+1}^{w-1} \frac{1}{j} + (i-w+1)\left(\frac{1}{w}\right) \right], & w < i < 2w-1 \\ \frac{\sigma_R^2}{(n-1)w^2}, & i \ge 2w-1 \end{cases} \tag{3.3}$$

whereas for $w = 2$, $Var(\text{DMR})_i$ is computed using only the first and third line of Eq. (3.3). Thus the control limits of the DMR chart for $w > 2$ are:

$$UCL/LCL = \begin{cases} \mu_0 \pm \frac{L\sigma_R}{i\sqrt{n-1}} \sqrt{\sum_{j=1}^{i} \frac{1}{j}}, & i \le w \\ \mu_0 \pm \frac{L\sigma_R}{w\sqrt{n-1}} \sqrt{\sum_{j=i-w+1}^{w-1} \frac{1}{j} + (i-w+1)\left(\frac{1}{w}\right)}, & w < i < 2w-1 \\ \mu_0 \pm \frac{L\sigma_R}{w\sqrt{n-1}}, & i \ge 2w-1 \end{cases} \tag{3.4}$$

where L is usually set to be 3.

3.2 An example with MA, DMA, MR and DMR charts

A sample data is taken from Montgomery (2009) [14], it is listed below. The data consists of 45 subgroups, each with 5 individual observations, results in four range measurements per subgroup. The data is assumed to follows normal distribution.

Subgroup	x_1	x_2	x_3	x_4	x_5
1	132.35	141.28	167.44	145.73	169.14
2	143.14	135.92	160.75	146.66	161.09

3	142.84	148.71	149.32	143.24	156.74
4	150.28	163.52	138.41	128.31	155.07
5	156.04	127.35	152.65	143.63	164.41
6	159.55	154.51	135.74	132.81	141.98
7	162.74	150.64	183.66	141.77	151.44
8	141.9	143.03	166.37	160.67	155.19
9	138.84	172.77	153.55	151.76	136.88
10	140.39	166.97	150.89	146.27	152.2
11	141.58	176.67	142.78	159.28	141.81
12	158.21	133.55	157.77	139.08	175.59
13	128.56	141.06	144.47	163.98	119.28
14	149.51	140.36	158.93	164.58	149.69
15	135.89	128.63	159.96	124.97	154.71
16	157.47	153.01	151.71	118.39	186.62
17	136.8	172.69	139.57	150.14	144.49
18	141.63	138.64	130.57	162.1	155.73
19	157.96	141.85	165.41	151.16	172.47
20	171.06	144.12	123.61	138.2	176.01
21	143.71	150.51	134.85	156.7	148.8
22	147.38	159.36	165.83	149.73	147.2
23	159.17	143.33	155.51	152.95	168.66
24	163.99	152.43	157.05	155.63	155.3
25	157.97	136.63	162.4	137.32	168.87
26	144.83	154.58	145.38	143.03	162.06
27	154.35	168.99	158.3	133.58	141.87
28	151.75	134.46	147.23	166.57	166.61
29	154.54	109.31	140.72	150.39	152.64
30	144.18	150.59	151.24	146.2	162.63
31	143.01	127.25	159.45	153.97	152.52

32	149.81	145.06	161.74	158.37	149.62
33	130.09	150.6	162.31	158.31	164.54
34	141.32	146.03	158.08	171.11	173.13
35	138.17	131.35	149.53	148.94	145.96
36	157.65	170.14	140.26	127.73	145.41
37	149.36	143.73	151.39	148.08	152.93
38	157.29	167.38	150.48	156.51	174.73
39	180.89	155.13	182.5	143.89	165.58
40	162.36	153.93	167.38	186.98	150.36
41	141.2	179.31	173.45	163.91	177.91
42	173.72	156.63	149.1	178.09	155.04
43	159.71	173.94	168.32	166.77	179.74
44	142.95	165.36	191.34	172.72	143.7
45	162.17	182.2	179.15	167.44	194.04

Table 3.1: Sample data with 45 subgroups, each with 5 data

3.2.1 MA chart with $w = 5$

A MA control chart is built using span $w = 5$. The value of w is inversely proportional to the magnitude of shifts detected. Larger w means it smaller shifts are less easily detected.

Measurement	LCL	UCL
1	133.9259627	172.4420373
2	139.5665112	166.8014888
3	142.065367	164.302633
4	143.5549814	162.8130186
≥ 5	144.5715439	161.7964561

Table 3.2: UCLs and LCLs for MA chart with $w = 5$

The table shows that the first four UCLs and LCLs are all different, gradually closer to each other. After 5th measurement, the UCLs and LCLs become the same. This is reflected on the graph.

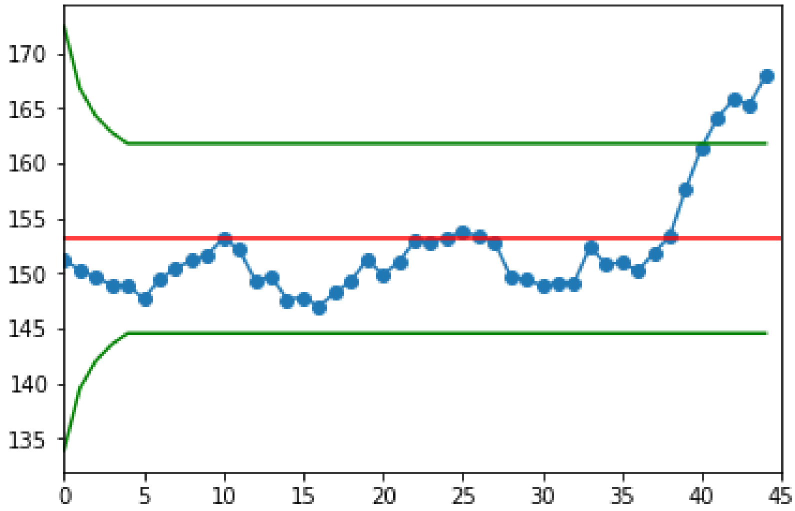


Figure 3.1: MA chart with $w = 5$

From the graph, it can be seen that the last few measurements are out of control. This is because $\bar{X}_{38}, \ldots \bar{X}_{45}$ are all larger than 160. This is reflected on the graph which shows a increase value for the last few measurements. Initially the gap between control limits are large, but quickly stabilised after 5th measurement.

3.2.2 DMA chart with $w = 5$

The LCLs and UCLs table for DMA chart are similar to MA chart. The first LCL/UCL pair is the same as MA chart. Similarly, the difference between control limits slowly decreases. However, it only stabilised after 9th measurement. This is due to the variance formula derived in Khoo & Wong [11]. Notice that the 5th and 6th control limits are the same even though computed using different line from Equation (2.10). The coincidence is suspected to have relationship with the span w, choosing another span may not result this phenomenon.

Measurement	LCL	UCL
1	133.9259627	172.4420373
2	141.3909088	164.9770912
3	144.4921648	161.8758352
4	146.2348544	160.1331456
5	147.3639505	159.0040495
6	147.3639505	159.0040495
7	148.0405055	158.3274945
8	149.2372768	157.1307232
≥ 9	149.3323925	157.0356075

Table 3.3: UCLs and LCLs for the DMA chart with $w = 5$

In the graph below, the control limits can be viewed to have three ”sections”, caused by three lines from Equation (2.10). Compare to MA chart, the final UCLs and LCLs are significantly narrower, result in more out of control signals. The centre line and plots pattern does not differ much, although a smoother plot is seen. The last few measurements shows an increasing trend to a large degree, implying increasing value of observations.

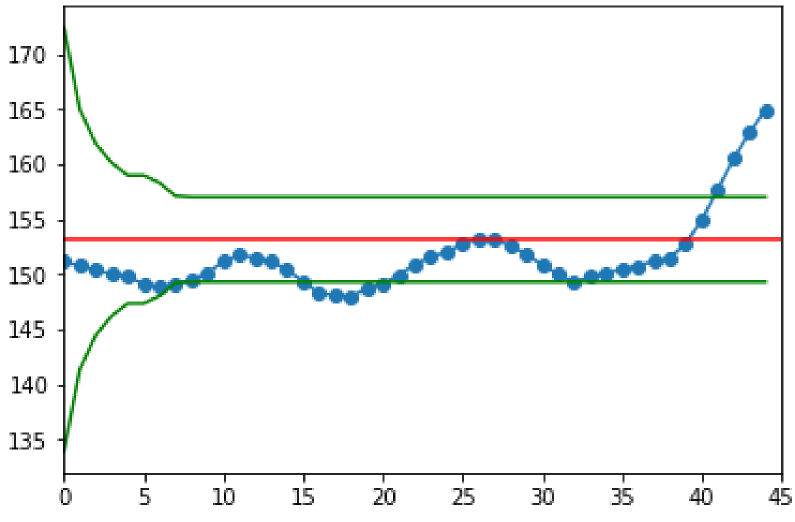


Figure 3.2: DMA chart with $w = 5$

3.2.3 MR chart with $w = 5$

The table shows that the control limits are getting closer to each other in the first four measurements. After measurement 5, the UCLs and LCLs become the same. The first LCL obtained is actually negative by computation, but then set to zero since range is defined to be positive.

Measurement	LCL	UCL
1	0	32.93758232
2	3.82324777	27.94235223
3	6.036216836	25.72938316
4	7.35540884	24.41019116
≥ 5	8.255669478	23.50993052

Table 3.4: UCLs and LCLs for MR chart with $w = 5$

The MR chart with span $w = 5$ is shown as below. Compare to the MA chart, all measurements are inside the control limits, although some are fluctuating. It does not detect the trend of last few measurements shown in the MA chart. However, one can say that the last few measurements does not differ much. Overall, the process is in control.

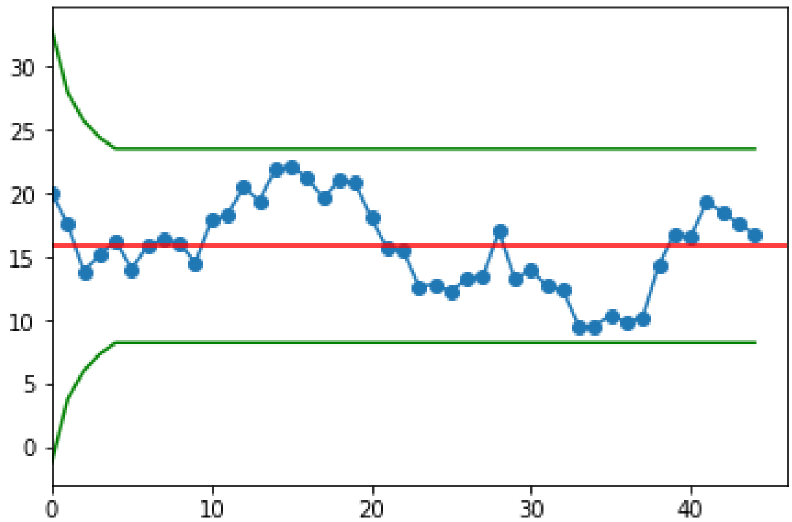


Figure 3.3: MR chart with $w = 5$

3.2.4 DMR chart wiht $w = 5$

By looking at the table of UCLs and LCLs, the values of UCL and LCL stabilized after $i \geq 9$. Note that the UCL and LCL values for 5th and 6th measurement are the same, this is also reflected on the graph, where the control limits show a "neck" between $i = 5$ and $i = 6$.

Measurement	LCL	UCL
1	0	32.93758232
2	5.43892141	26.32667859
3	8.185371858	23.58022814
4	9.728685523	22.03691448
5	10.72860513	21.03699487
6	10.72860513	21.03699487
7	11.32775743	20.43784257
8	12.3876097	19.3779903
≥ 9	12.47184354	19.29375646

Table 3.5: UCLs and LCLs for the DMR chart with $w = 5$

Compare to MR chart of the same span, the plots of DMR chart together form a ”smoother” and less fluctuated pattern. Although the centre line remain unchanged, the UCL and LCL is much narrower. As a result, many of the points are said to be out of control.

Notice that MR_{30} differs much from its neighbouring points, but this does not happen in DMR_{30}.

Compare to DMA chart wit the same span, it detect more out of control signals, although it does not show the trend occurs in the last few measurements.

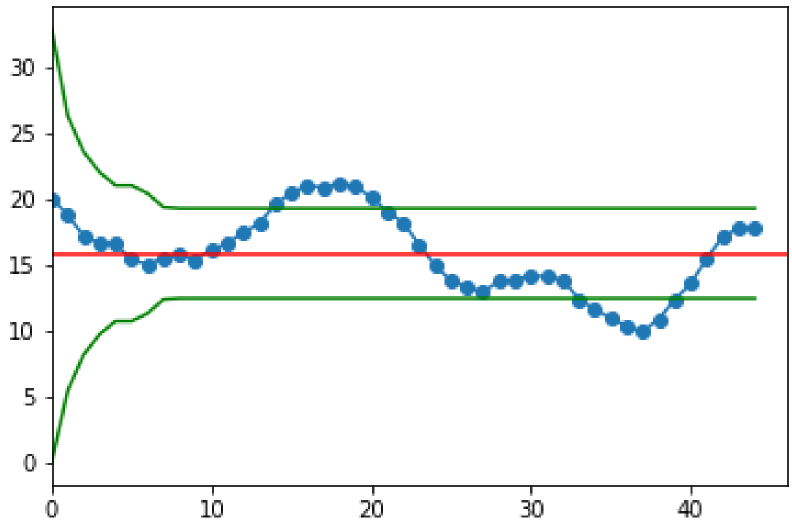


Figure 3.4: DMR chart with $w = 5$

3.2.5 DMR chart with $w = 2$

The same data is applied using DMR with $w = 2$ to compare its difference between spans. The table below shows the UCL and LCL for DMR chart with $w = 2$

Measurement	1	2	≥ 3
UCL	32.93758232	26.32667859	24.41019116
LCL	0	5.43892141	7.35540884

Table 3.6: UCLs and LCLs for the DMR chart with $w = 2$

For $w = 2$ the UCLs and LCLs quickly stabilized after $i \geq 3$. Note that the first and second pair of UCL and LCL are same with those of $w = 5$, this is because the same variance formula is applied. The graph also has larger gap between upper and lower control limits, and thus detect less shift in mean ranges. Two plots are observed to be out of control to a small degree, they are DMR_{13} and DMR_{17}. Other than that, the process is in control. In short, DMR chart with small span is able to detect large shift but not small shift.

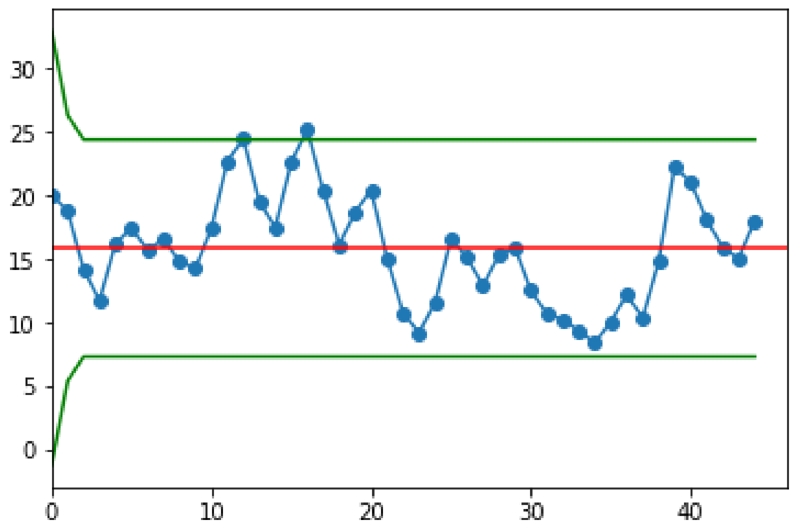


Figure 3.5: DMR chart with $w = 2$

3.2.6 DMR chart with $w = 10$

In this subsection, a large span is chosen to be applied, and its result is discussed. The table of UCLs and LCLs of DMR chart with $w = 10$ are listed below.

Measurement	UCL	LCL
1	0	32.93758232
2	5.43892141	26.32667859
3	8.185371858	23.58022814
4	9.728685523	22.03691448
5	10.72860513	21.03699487
6	11.43364053	20.33195947
7	11.95962722	19.80597278
8	12.36825178	19.39734822
9	12.69553842	19.07006158
10	12.96400554	18.80159446
11	12.7246852	19.0409148
12	12.7246852	19.0409148
13	12.74007286	19.02552714
14	12.81027083	18.95532917
15	12.95571294	18.80988706
16	13.19756893	18.56803107
17	13.5718228	18.1937772
18	14.16787306	17.59772694
≥ 19	14.17732177	17.58827823

Table 3.7: UCLs and LCLs for the DMR chart with $w = 10$

The UCL and LCL stabilized after $i \geq 19$. Note that the tenth measurement has smaller range of control limits than its neighbouring measurements. This is because different formula is applied at 10th measurement. The similar scenario happens at DMR chart with $w = 5$, where UCL/LCL value for the 5th and 6th measurements are the same. It is suspect that when $i = w$, the value of UCL and LCL (which is determined by variance) is equal or smaller than the next measurement.

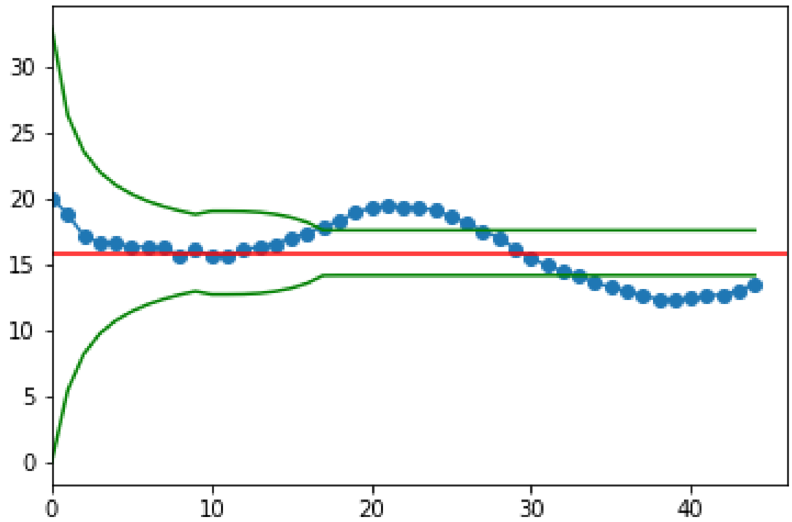


Figure 3.6: DMR chart with $w = 10$

Compare to DMR chart with $w = 5$, the plot are "flatter" or closer to the centre line. However, there are more out of control observations, since the control limits are also much narrower. There are less fluctuation but the trend are carried for longer period. In summary, the chart detect many small shifts but not large shift

Chapter 4

CONCLUSION

Using a span of 5, the MA and DMA chart shows that the process is out of control at the end of the measurements. The DMA chart also detects small shift at the middle of the measurements. In general, DMA chart performs better in detect smaller shifts.

Comparing MA and MR charts with the same span, the graph shows different results. The MR chart does not detects process shift at the end of the measurements like MA chart does. Although the process is considered in control by the MR chart, it can signal some shift within the span.

The DMR control chart detects more shift than the MR control chart of the same span. The DMR chart gives smoother graph than MR control chart. The DMA and DMR chart also give different results.

To study the effect of span, three values ($w = 2, 5, 10$) are selected to plot DMR control charts. It is observed that larger span tends to make the plot closer to the centre line, gives narrower UCLs and LCLs gap, and stabilizes after more measurements. DMR control chart with large span detects more shift in mean ranges.

The demonstrated example use a subgroups of four observations. It is not known whether there is a relationship between the span width and number of observations within subgroup. In general, the value of span is inversely proportional to the magnitude of shifts detected. Hence a control chart with large span can detect small shift more effectively compared to chart with smaller span.

In summary, the DMR contorl chart can be used supplementary with DMA chart to see the if there is a undergoing trend followed by the measurements, where individual outliers can be determined using the MR chart.

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