LOW ORDER UNIVERSAL PORTFOLIOS GENERATED BY SPECIAL DISTRIBUTIONS

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A project report submitted in partial fulfilment of the requirements for the award of Bachelor of Science (Honours) Actuarial Science

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DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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APPROVAL FOR SUBMISSION

I certify that this project report entitled **"LOW ORDER UNIVERSAL PORTFOLIOS GENERATED BY SPECIAL DISTRIBUTIONS"** was prepared by **LING HOONG CHUAN** has met the required standard for submission in partial fulfilment of the requirements for the award of Bachelor of Science (Honours) Actuarial Science at Universiti Tunku Abdul Rahman.

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ABSTRACT

Universal Portfolio is an investment strategy that enables investors to maximize their wealth by reallocating their invested wealth in the everyday portfolio. There is no stochastic model being assumed for the stock price in the universal portfolio. Five different companies are selected from Kuala Lumpur Stock Exchange (KLSE) to form three portfolios containing three different companies. This project's main objective is to generate a low-order universal portfolio with selected distributions with two or three parameters. As parameters of the distribution will affect the wealth generated by each distribution, therefore parameter sensitivity test was performed in this project to identify the best parameter for each distribution. To study the performance of a universal portfolio in the long term, we collected opening and closing stock prices for 2500 trading days. We generated the wealth for each 500, 1000, 1500, 2000, and 2500 trading days and compared each trading period's performance. Finally, we also study the universal portfolio's performance with a non-diversified and diversified portfolio. The result generated for the best constant rebalanced portfolio (BCRP) and constant rebalanced portfolio (CRP) are used as a benchmark to compare the performance of each distribution. The results obtained will be discussed in this project.

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LIST OF SYMBOLS / ABBREVIATIONS

b	portfolio vector
b_{ni}	proportion of current wealth invested in stocks i on n^{th} trading days
$b_{n+1,k}$	proportion of current wealth invested in k^{th} stock on $n+1^{th}$ trading days
$E(X^k)$	k^{th} moment of distribution
S_n	wealth generated on n^{th} day
X	stock market vector
x_{ni}	ratio of closing price to opening price of stock i on day n
$x_{n+1,k}$	ratio of closing price to opening price k^{th} stock on $n+1^{th}$ trading days

BCRP	best constant rebalance portfolio
CRP	constant rebalance portfolio
KLSE	Kuala Lumpur stock exchange
MAMR	multiperiodical asymmetric mean reversion
MCR	mixture-current run
OEA	online expert aggregation
SCRP	successive constant rebalanced portfolios
VBA	visual basic for application
WAA	weak aggregating algorithm

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CHAPTER 1

INTRODUCTION

1.1 Background

Investing is one of the most challenging courses for most investors as it is not just simply putting all the eggs in one basket; it involves professional financial knowledge and analysis of the market condition. In addition, we must invest in various investment instruments so that diversification can help us reduce the investment risk and generate higher investment returns simultaneously. Therefore, the universal portfolio is being introduced to help investors to generate higher investment returns. It is helpful for the investors who possess limited knowledge of the actual distribution underlying the market.

A portfolio is an investment strategy that helps investors diversify investment risk by allocating the investor's wealth into different stocks. A diversified portfolio consists of different investment instruments from various sectors. The reason is that whenever there is a negative impact on one of the investment instruments, the alternative investment counterbalances it and minimizes the potential loss. The main goal of a portfolio is to maximize the investors' wealth. Cover (1991) wished to outperform the best constant rebalance portfolio. Therefore, he introduced Universal Portfolio, which can update the portfolio every day to achieve maximum wealth.

In a universal portfolio, there is no stochastic model that is being assumed for the stock price. We denote $\boldsymbol{X} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_m})^{\boldsymbol{t}}$, where $\boldsymbol{X} \geq \boldsymbol{0}$, be the stock market vector, and portfolio $\boldsymbol{b} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \dots, \boldsymbol{b_m})^t$ be the proportion of current wealth invested in each of the m stock where $\boldsymbol{b_i} \geq \boldsymbol{0}$, $\sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \boldsymbol{b_i} = \boldsymbol{1}$. Therefore, the increase in wealth by using portfolio $\boldsymbol{b}$ is denoted by $S_n = \boldsymbol{b^t x} = \sum_{i=1}^{m} b_i x_i$. The ratio of closing price to the opening price of stock i on day n is denoted by $\boldsymbol{x_{ni}}$. The wealth of constant rebalance portfolio $\boldsymbol{b}$ is calculated with $S_n(\boldsymbol{b}) = \prod_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{b^t}\, \boldsymbol{x_i}$ where initial wealth $S_0(\boldsymbol{b}) = \boldsymbol{1}$, and $S_n^*(\boldsymbol{b}) = \max S_n(\boldsymbol{b})$ is maximum wealth to be achieved by the universal portfolio. Therefore, the maximum wealth achieves by the investor on day n by using universal portfolio strategy, $\widehat{\boldsymbol{b}}_{\boldsymbol{i}}$ is $\hat{S}_n = \prod_{i=1}^{n} \hat{b}_i\, x_i$.

1.2 Problem Statement

The investors' main interest is to maximize their wealth, and portfolios help them to diversify risk. However, they are having trouble deciding how much they should allocate in each stock to achieve maximum wealth. Therefore, a low-order universal portfolio generated by special distributions is proposed in this project. We studied different distribution's performance and how parameters will affect the universal portfolios' performance. Besides, we also studied the effect of the number of trading days, diversified or non-diversified portfolios, and the sensitivity of the parameters of different distributions on the performance of a universal portfolio.

1.3 Aim and Objectives

This project aims to generate low-order universal portfolios, orders 1, 2, and 3, with special probability distributions. Using a low-order universal portfolio will shorten the implementation time and require lesser computer memory than the moving-order universal portfolio. In addition, we investigated how the performance of the universal portfolio will be affected by using different portfolio stock components, different numbers of trading days, and different parameter values used in running the model.

Three data sets, A, B, and C, are used in this project to compare the result. Each set of data consist of a 3-stocks portfolio. In addition, data set with opening and closing prices for 1500 trading days collected from Yahoo Finance will be used. Five selected distributions were chosen to generate the universal portfolio with the data sets. We compared the wealth generated by each distribution with the best constant rebalance portfolio (BCRP) and constant rebalance portfolio (CRP). First, Visual Basic in Application (VBA) in excel will calculate the wealth of universal portfolios generated by selected distributions. Since the calculation will be more complicated if we consider the transaction costs, we assumed there is no transaction cost in this project. Next, we studied the sensitivity of parameters in generating a wealth of universal portfolios using various parameters for the distribution. Then, we determined the best parameter for distribution to generate the maximum wealth of the portfolio. Moreover, we identified which distribution that able to generate the highest wealth for the low-order universal portfolio.

Finally, a different number of trading days was tested in this project; 500, 1000, 1500, 2000, and 2500 trading days. This was to study the performance of the universal portfolio in the short run and long run. We also compared the wealth generated by the diversified and non-diversified universal portfolio.

1.4 Scope and Limitation of the Study

The universal portfolios generated by Thomas M. Cover are in finite order, which will need an enormous computer memory and longer implementation time to generate the result. Therefore, we used different continuous random variable distributions in this project to generate a low-order universal portfolio. We will only generate universal portfolios with order 1, order 2, and order 3 due to low computer memory and implementation time. To do that, we need to find the random distribution's expected moment. Then we compared results generated by different distributions with few sets of portfolios.

Since the universal portfolio will update the portfolio daily to reallocate the wealth in each stock, there will be transaction costs for each trading. Therefore, we assumed that there is no transaction cost to simplify the derivation of the mathematical model. We also not considering dividends distributed by the company as the adjusted closing price was not used in this project.

CHAPTER 2

LITERATURE REVIEW

Many past researchers studied portfolios and introduced new investment theories. Markowitz (1952) studied portfolio selection by introducing a portfolio selection combining portfolio return's mean and variance. Investors should choose portfolios that increase the mean and variance of the portfolio return. He suggested that investors should avoid high covariances securities and choose efficient portfolios according to their preferences. Diversification across different industries should be considered in a portfolio, as they have different market characteristics, resulting in lower covariances.

Sharpe (1963) continued the study of Markowitz, and he found some problems in his works. He suggested that there might be a situation where the portfolio is efficient but with a high expected return and variance or vice versa. The computation task for portfolio analysis is complicated and can be simplified with a diagonal model that considers the relationship among securities. He studied portfolio investment by considering lending the remainder money to earn interest or borrow money to purchase the portfolio. The diagonal code allows the investor to decide how much interest they wish to receive or pay. He found that some efficient portfolios will become inefficient and not be considered when considering these alternatives. He also concluded that by using a diagonal model, the portfolio analysis' cost is lower.

The theory of rebalance portfolio is introduced by Kelly (1956). She introduced this theory from the point of view of a gamble. She argued that if the gambler repeated the same bet every time with the same amount of money, he would maximize his capital. However, she did not consider reinvestment of the income or investment in other securities. Breiman (1961) continued the study of Kelly, which examines how much time is needed to achieve a fixed wealth and how much wealth will achieve a fixed number of trials.

Cover (1991) researched universal portfolios, updating the portfolio and changing allocation daily to generate maximum profit. The theory of rebalance portfolio is used by Cover in introducing a universal portfolio. Cover and Ordenlich (1996) continue the universal portfolio's research, including side information and the market's past performance. The portfolio will be updated according to the side information, such as whether the stock performed well in previous trading days, allowing investors to allocate their wealth in the best-performing stock every day. However, in this paper, the researchers have not considered the transaction cost or the investment commissions. Moreover, the availability of side information in real-world markets is also a challenging problem that needs to be considered.

Blum and Kalai (1999) study the universal portfolios' performance by considering transaction costs. It is assumed that the transaction cost will only occur when purchasing the stocks only. They concluded that some of the selected stock's set works very well in the presence of transaction costs. The portfolio can perform better with lesser transactions, which means rebalancing the portfolio should not be done too often. Therefore, they perform rebalancing monthly for the sets of stocks they used in the research. They suggested choosing the period length of time that can give the best performance with historical data when deciding how often the portfolio needs to rebalance.

Gaivoronski and Stella (2000) studied universal portfolios by introducing Successive Constant Rebalanced Portfolios (SCRP). They derived SCRP from nonstationary and stochastic optimizations. In addition, they used an algorithmic approach that was able to analyze a large number of stocks.

Goetzmann and Kumar (2008) studied the extent to which portfolio diversification exists among investors. First, they investigate how preferences of portfolio diversification related to investors' personal preferences and characteristics. Next, they examined the relationship between portfolio performance and diversification of portfolio. Diversification is divided into two levels, either diversifying the portfolio by holding multiple securities or reducing the risk by selecting negatively correlated stocks. They found that a highly diversified portfolio can earn a higher return compared to the least diversified portfolio.

Portfolio investment aimed to maximize wealth earned by the investor by reducing the risk through diversification. Abu Bakar and Rosbi (2018) suggest that investors will have a better-diversified portfolio when they hold global securities compare to domestic only. They investigated the performance of the portfolio with modern portfolio theory. Modern portfolio theory suggests that the portfolio model can be optimized by minimizing the risk, which is the variance of the stock price.

Kozat (2011) generated universal portfolios with semi-constant rebalances portfolios. The semi-constant rebalances portfolio will only be rebalanced when the rebalancing benefits do not outweigh the transaction cost. The trading period of the portfolio is divided into a few segments which the portfolio will only rebalance and paying the transaction cost at the start of each segment. He managed to generate the universal semi-constant portfolios and outperform the universal portfolios introduced by Cover and also constant rebalance portfolios.

Due to the long implementation time and ample computer storage needed when computing the maximum wealth from the algorithm, Tan (2013) researched the finite-order universal portfolio that could generate the result by using lesser implementation time. Pang, Liew, and Chang (2019) continue the study by generating a finite-order universal portfolio with Brownian Motion and Ornstein-Uhlenback. With selected parameters for each stochastic process, they can achieve wealth comparable to CRP.

The distribution parameter will influence the performance of universal portfolios generated. An improper selection of parameters will cause the wealth generated by the universal portfolio to be lower. Tan and Lim (2013) estimated the best parameter of universal portfolios generated with Chi-Square Divergence and Helmbold. They studied the Mixture-Current-Run (MCR) universal portfolio, enabling them to combine two or more universal portfolios of the same kind to discover the optimal parameter that corresponds to the best daily wealth. They keep tracking the daily performance of the finite order universal portfolio generated within a range of parameters. The highest wealth achieved was recorded with the best parameter. The result showed that even with the best parameter for the distribution, they could not outperform BCRP for some data sets.

Peng, Xu, and Li (2020) continue studying universal portfolios and adopt the transaction cost model introduced by Blum and Kalai. In addition, they proposed a new method in portfolio selection by using multiperiodical asymmetric mean reversion (MAMR). MAMR can perform online learning techniques in selecting the portfolio and achieve comparable performance than the other portfolio selection techniques in the long run with higher transaction costs.

Moreover, Phoon, Tan, and Pan (2020) study the universal portfolio's performance generated by a special time series: zero mean autoregressive process and zero mean moving average process. Different combination of parameters is used to generate a wealth of universal portfolio that is comparable to BCRP. They conclude that the wealth generated special time series is close to the wealth generated by the best constant portfolio, and the performance will be affected by the parameter used.

He and Yang (2020) suggested that investors to consider various universal portfolio strategies when maximizing their returns. They proposed a new strategy, Online Expert Aggregation (OEA), using Weak Aggregating Algorithm (WAA), which can consider a pool of universal portfolio strategies. The pool universal portfolio strategies are considered expert advice. They showed that OEA had better performance when compared to other portfolio strategies. However, further study is needed to exclude those universal portfolio strategies that are not performing well during the investment period.

CHAPTER 3

METHODOLOGY AND WORK PLAN

3.1 Low Order Universal Portfolio

The proportion of current wealth invested in each of the m stock will be denoted by $\boldsymbol{b} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \dots, \boldsymbol{b_m})^t$, where $\boldsymbol{b_i} \geq 0$, $\sum_{i=1}^{m} \boldsymbol{b_i} = 1$. Then, $\boldsymbol{b_{ni}}$ is the proportion of current wealth invested in the stock on the n^{th} trading day. The stock market vector will be denoted by $\boldsymbol{X} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_m})^{\boldsymbol{t}}$, where $\boldsymbol{X} \geq 0$. The closing price to opening price ratio of stock i on day n is denoted by $\boldsymbol{x_{ni}}$. Therefore, the increase in wealth is denoted by:

$$\boldsymbol{b_j^t x_j} = \sum_{i=1}^{m} b_{ji} x_{ji} \tag{3.1}$$

We will define the wealth generated by portfolio **b** with

$$S_n = \prod_{j=1}^{n} \boldsymbol{b_j^t x_j}. \tag{3.2}$$

where initial wealth $S_0\,(\boldsymbol{b}) = 1$.

In this project, we studied the finite-order universal portfolios generated by special distributions. Let $Y_1, Y_2, \dots, Y_m$ be the identical independent random variable with probability density function $f(y_1), f(y_2), \dots, f(y_m)$. The joint probability density function can be written as

$$f(y_1, y_{2,\dots\dots,} y_m) = f(y_1)\, f(y_2) \dots f(y_m)$$

where $\boldsymbol{y} = (y_1, y_2, \dots, y_m) \in \boldsymbol{D}$, **D** is defined by:

$$\boldsymbol{D} = \{(y_1, y_2, \dots, y_m): f_{Yi}(y_i) > 0, for\ all\ i = 1, 2, \dots, m\} \tag{3.3}$$

Let v be fixed positive integer. Then, the order v universal portfolio that generated by an identical independent random variable, $Y_1, Y_2, \ldots., Y_m$,

$$b_{n+1,k} = \frac{\int_D y_k \,(y^t x_n) \ldots \left(y^t x_{n-(v-1)}\right)\boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}}{\int_D (y_1 + \; y_2 + \cdots + y_m)(\, y^t x_n) \ldots \left(y^t x_{n-(v-1)}\right)\boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}} \tag{3.4}$$

where $k = 1,2, \ldots, m$.

3.1.1 Order 1 Universal Portfolio

The moment $E\left[Y_i^j\right]$ is assumed to be positive for $i = 1,2, \ldots. m$, $j = 1,2, \ldots, v + 1$. Order 1 universal portfolio can be derived from equation (3.4) and become:

$$b_{n+1,k} = \frac{\int_D y_k \,(y^t x_n)\boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}}{\int_D (y_1 + \; y_2 + \cdots + y_m)(\, y^t x_n)\, \boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}}$$

$$= \frac{\int_D \sum_{i=1}^m y_k y_i x_{n,i} f(y_1, y_2, \ldots, y_m) dy_1 dy_2 \ldots dy_m}{\int_D (y_1 + y_2 + \cdots + \; y_m) \sum_{i=1}^m y_i x_{n,i} f(y_1, y_2, \ldots, y_m) dy_1 dy_2 \ldots dy_m}$$

$$= \frac{\sum_{i=1}^m \left[x_{n,i} \int_D y_k y_i f(y_1, y_2, \ldots, y_m) dy_1 \, dy_2 \ldots dy_m\right]}{\sum_{i=1}^m \left[x_{n,i} \int_D (y_1, y_2, \ldots, y_m) y_i f(y_1, y_2, \ldots, y_m) dy_1 \, dy_2 \ldots dy_m\right]}$$

$$= \frac{x_{n,1} E\left[Y_{k,} Y_{1,}\right] + x_{n,2,} E\left[Y_{k,} Y_{2,}\right] + \cdots + x_{n,m,} E\left[Y_{k,} Y_m\right]}{\sum_{k=1}^m \left\{x_{n,1} E\left[Y_{k,} Y_{1,}\right] + x_{n,2,} E\left[Y_{k,} Y_{2,}\right] + \cdots + x_{n,m,} E\left[Y_{k,} Y_m\right]\right\}}$$

$$= \zeta_{n,1} \left(\sum_{i=1}^m x_{n,i} E[Y_k Y_i] \right) \tag{3.5}$$

where $\zeta_{n,1} = \left(\sum_{i=1}^m \left\{x_{n,1} E[Y_k Y_1] + x_{n,2} E[Y_k Y_2] + \cdots + x_{n,m} E[Y_k Y_m]\right\}\right)^{-1}$

$$= \left[\sum_{k=1}^m \left(\sum_{i=1}^m x_{n,i} E[Y_k Y_i]\right)\right]^{-1}$$

Since $Y_1, Y_2, \ldots, Y_m$ are identical independent random variables,

$$E[Y_k Y_i] = \begin{cases} E[Y_k] E[Y_i]\,, & if\ k \neq i \\ E[Y_k^2] \quad\;\, , & if\ k = i \end{cases}$$

3.1.2 Order 2 Universal Portfolio

Order 2 universal portfolio can be derived from equation (3.4) and become:

$$b_{n+1,k} = \frac{\int_D y_k\,(y^t x_n)(y^t x_{n-1})\boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}}{\int_D (y_1 + y_2 + \cdots + y_m)(y^t x_n)\,(y^t x_{n-1})\boldsymbol{f}(\boldsymbol{y})\boldsymbol{dy}}$$

$$= \frac{\int_D y_k(\sum_{i_1}^m y_{i_1} x_{n,i_1})\left(\sum_{i_2}^m y_{i_2} x_{n-1,i_2}\right) f(y_1, y_2, \dots, y_m) dy_1 dy_2 \dots dy_m}{\int_D (y_1 + y_2 + \cdots + y_m)\left(\sum_{i_1}^m y_{i_2} x_{n,i_1}\right)\left(\sum_{i_2}^m y_{i_2} x_{n-1,i_2}\right) f(y_1, y_2, \dots, y_m) dy_1 dy_2 \dots dy_m}$$

$$= \frac{\sum_{i_1}^m \sum_{i_2}^m x_{n,i} x_{n-1,i_2} \int_D y_k y_{i_1} y_{i_2} \times f(y_1, y_2, \dots, y_m) dy_1 dy_2 \dots dy_m}{\sum_{i_1}^m \sum_{i_2}^m x_{n,i} x_{n-1,i_2} \int_D (y_1 + y_2 + \cdots + y_m) y_{i_1} y_{i_2} \times f(y_1, y_2, \dots, y_m) dy_1 dy_2 \dots dy_m}$$

$$= \frac{x_{n,1} x_{n-1,1} E\left[Y_{k,} Y_{1,} Y_{1,}\right] + x_{n,2,} x_{n-1,2} E[Y_k Y_1 Y_2] + \cdots + x_{n,m,} x_{n-1,m} E[Y_k Y_m Y_m]}{\sum_{k=1}^m \left\{x_{n,1} x_{n-1,1} E[Y_k Y_1 Y_1] + x_{n,2,} x_{n-1,2} E[Y_k Y_1 Y_2] + \cdots + x_{n,m,} x_{n-1,m} E[Y_k Y_m Y_m]\right\}}$$

$$= \zeta_{n,2} \left(\sum_{i_1=1}^m \sum_{i_2=1}^m x_{n,i} x_{n-1,i_2} E\left[Y_k Y_{i_1} Y_{i_2}\right] \right) \tag{3.6}$$

where $\zeta_{n,2} = \left[\sum_{k=1}^m \left(\sum_{i_1=1}^m \sum_{i_2=1}^m x_{n,i_1} x_{n-1,i_2} E\left[Y_K Y_{i_1} Y_{i_2}\right]\right)\right]^{-1}$

Since $Y_1, Y_2, \dots, Y_m$ are identical independent random variables,

$$E\left[Y_k Y_{i_1} Y_{i_2}\right] = \begin{cases} E[Y_k]E\left[Y_{i_1}\right]E\left[Y_{i_2}\right], & if\ k \neq i_1 \neq i_2 \\ E[Y_k^2]E\left[Y_{i_2}\right], & if\ k = i_1, k \neq i_2 \\ E[Y_k^2]E\left[Y_{i_1}\right], & if\ k = i_2, k \neq i_1 \\ E\left[Y_{i_2}^2\right]E[Y_k], & if\ k \neq i_1, i_2 = i_1 \\ E[Y_k^3], & if k = i_1 = i_2 \end{cases}$$

3.1.3 Order 3 Universal Portfolio

Order 3 universal portfolio can be derived from equation (3.4) and become:

$$b_{n+1,k} = \frac{\int_D y_k\,(y^t x_n)(y^t x_{n-1})(y^t x_{n-2})\boldsymbol{f(y)dy}}{\int_D (y_1 + y_2 + \cdots + y_m)(\,y^t x_n)\,(y^t x_{n-1})(y^t x_{n-2})\boldsymbol{f(y)dy}}$$

$$= \frac{\int_D y_k(\sum_{i_1}^m y_{i_1}x_{n,i_1})(\sum_{i_2}^m y_{i_2}x_{n-1,i_2})(\sum_{i_3}^m y_{i_2}x_{n-1,i_3})f(y_1,y_2,\dots,y_m)dy_1dy_2\dots dy_m}{\int_D \begin{matrix}(y_1+y_2+\cdots+y_m)(\sum_{i_1}^m y_{i_2}x_{n,i_1})(\sum_{i_2}^m y_{i_2}x_{n-1,i_2})(\sum_{i_2}^m y_{i_2}x_{n-1,i_3})\\ f(y_1,y_2,\dots,y_m)dy_1dy_2\dots dy_m\end{matrix}}$$

$$= \frac{\sum_{i_1}^m\sum_{i_2}^m\sum_{i_3}^m x_{n,i}x_{n-1,}x_{n-2,i_3}\int_D y_k y_{i_1}y_{i_2}y_{i_3}\times f(y_1,y_2,\dots,y_m)dy_1dy_2\dots dy_m}{\sum_{i_1}^m\sum_{i_2}^m\sum_{i_3}^m x_{n,i}x_{n-1,}x_{n-1,i_3}\int_D (y_1+y_2+\cdots+y_m)y_{i_1}y_{i_2}y_{i_3}\times f(y_1,y_2,\dots,y_m)dy_1dy_2\dots dy_m}$$

$$= \zeta_{n,3}\left(\sum_{i_1=1}^m\sum_{i_2=1}^m\sum_{i_3=1}^m x_{n,i}x_{n-1,i_2}x_{n-2,i_3}E\left[Y_kY_{i_1}Y_{i_2}Y_{i_3}\right]\right) \tag{3.7}$$

where $\zeta_{n,3} = \left[\sum_{k=1}^m\left(\sum_{i_1=1}^m\sum_{i_2=1}^m\sum_{i_3=1}^m x_{n,i_1}x_{n-1,i_2}x_{n-2,i_3}E\left[Y_KY_{i_1}Y_{i_2}Y_{i_3}\right]\,\right)\right]^{-1}$

Since $Y_1, Y_2, \dots, Y_m$ are identical independent random variables,

$$E\left[Y_kY_{i_1}Y_{i_2}Y_{i_3}\right]$$

$$= \begin{cases}
E[Y_k]E[Y_{i_1}]E[Y_{i_2}]E[Y_{i_3}], & if\ k \neq i_1 \neq i_2 \neq i_3\\
E[Y_k^2]E[Y_{i_2}]E[Y_{i_3}], & if\ k = i_1, k \neq i_2, k \neq i_3, i_2 \neq i_3\\
E[Y_k^2]E[Y_{i_1}]E[Y_{i_3}], & if\ k = i_2, k \neq i_1, k \neq i_3, i_1 \neq i_3\\
E[Y_k^2]E[Y_{i_1}]E[Y_{i_2}], & if\ k = i_3, k \neq i_1, k \neq i_2, i_1 \neq i_2\\
E[Y_{i_1}^2]E[Y_{i_3}]E[Y_k], & if\ \ i_2 = i_1, k \neq\ i_1, k \neq i_2, k \neq\ i_3\\
E[Y_{i_1}]E[Y_{i_3}^2]E[Y_k], & if\ \ i_2 = i_3, k \neq\ i_1, k \neq i_2, k \neq\ i_3\\
E[Y_{i_2}]E[Y_{i_3}^2]E[Y_k], & if\ \ i_1 = i_3, k \neq\ i_1, k \neq i_2, k \neq\ i_3\\
E[Y_{i_1}^2]E[Y_k^2], & if\ k = i_3, i_1 = i_2, k \neq i_1\\
E[Y_{i_3}^2]E[Y_k^2], & if\ k = i_1, i_2 = i_3, k \neq i_2\\
E[Y_{i_2}^2]E[Y_k^2], & if\ k = i_2, i_1 = i_3, k \neq i_3\\
E[Y_{i_3}]E[Y_k^3], & if\ k = i_1 = i_2, k \neq i_3\\
E[Y_{i_2}]E[Y_k^3], & if\ k = i_1 = i_3, k \neq i_2\\
E[Y_{i_1}]E[Y_k^3], & if\ k = i_2 = i_3, k \neq i_1\\
E[Y_k^4], & ifk =\ i_1 = i_2 = i_3
\end{cases}$$

3.2 Order ν Universal Portfolio's Wealth Function

Applied recursive method to generate wealth function of order ν universal portfolio, $S_{n+1}(x^n)$ as follows:

$$\begin{aligned} S_{n+1} &= \prod_{j=1}^{n+1} \boldsymbol{b_j^t x_j} \\ &= (b_{n+1}^t x_{n+1}) \prod_{j=1}^{n} \boldsymbol{b_j^t x_j} \\ &= (b_{n+1}^t x_{n+1}) S_n \end{aligned} \tag{3.8}$$

where

$$b_{n+1}^t x_{n+1} = \sum_{k=1}^{m} b_{n+1,k} x_{n+1,k} \tag{3.9}$$

From equation (3.4), the universal portfolio's wealth function on $(n+1)^{th}$ day, $b_{n+1}^t x_{n+1}$ for order ν can be evaluated as follows:

$$\begin{aligned} b_{n+1}^t x_{n+1} &= \frac{\sum_{k=1}^{m} x_{n+1,k} \int_D y_k (y^t x_n) \ldots . (y^t x_{n-(\nu-1)}) \boldsymbol{f(y)dy}}{\int_D (y_1 + y_2 + \cdots + y_m)\, (y^t x_n) \ldots \left(y^t x_{n-(\nu-1)}\right) \boldsymbol{f(y)dy}} \\ &= \frac{\int_D (y^t x_{n+1})(y^t x_n) \ldots . (y^t x_{n-(\nu-1)}) \boldsymbol{f(y)dy}}{\int_D (y_1 + y_2 + \cdots + y_m)\, (y^t x_n) \ldots \left(y^t x_{n-(\nu-1)}\right) \boldsymbol{f(y)dy}} \end{aligned} \tag{3.10}$$

3.3 Low Order Universal Portfolio Generated by Five Special Distributions

From equation (3.5), we can generate wealth increase on the $(n+1)^{th}$ day for order 1, order 2, and order 3 Universal Portfolio for three stocks, $b_{n+1}^{t}x_{n+1}$ as follows:

3.3.1 Order 1 Universal Portfolio

$$\sum_{k=1}^{3} b_{n+1,k}x_{n+1,k}$$
$$= \frac{\sum_{k=1}^{3} x_{n+1,k}\{x_{n,1}E[Y_kY_1] + x_{n,2}E[Y_kY_2] + \cdots + x_{n,m}E[Y_kY_m]\}}{\sum_{k=1}^{3}\{x_{n,1}E[Y_kY_1] + x_{n,2}E[Y_kY_2] + \cdots + x_{n,m}E[Y_kY_m]\}}$$
$$= \zeta_{n,1}\left(\sum_{i=1}^{3} x_{n+1,k}\{x_{n,i}E[Y_kY_i]\}\right)$$
$$\begin{aligned} = \ \zeta_{n,1}\{&x_{n+1,1}\left(x_{n,1}E[Y_1^2] + x_{n,2}E[Y_1Y_2] + x_{n,3}E[Y_1Y_3]\right) \\ &+x_{n+1,2}\left(x_{n,1}E[Y_1Y_2] + x_{n,2}E[Y_2^2] + x_{n,3}E[Y_2Y_3]\right) \\ &+x_{n+1,3}\left(x_{n,1}E[Y_1Y_3] + x_{n,2}E[Y_2Y_3] + x_{n,3}E[Y_3^2]\right)\} \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} \zeta_{n,1} = \ \big(&x_{n,1}E[Y_1^2] + x_{n,2}E[Y_1Y_2] + x_{n,3}E[Y_1Y_3] \\ &+x_{n,1}E[Y_1Y_2] + x_{n,2}E[Y_2^2] + x_{n,3}E[Y_2Y_3] \\ &+x_{n,1}E[Y_1Y_3] + x_{n,2}E[Y_2Y_3] + x_{n,3}E[Y_3^2]\big)^{-1} \end{aligned}$$

3.3.2 Order 2 Universal Portfolio

$$\sum_{k=1}^{3} b_{n+1,k}x_{n+1,k}$$
$$= \frac{\sum_{k=1}^{3} x_{n+1,k}\{x_{n,1}x_{n-1,1}E[Y_kY_1Y_1] + x_{n,2}x_{n-1,2}E[Y_kY_1Y_2] + \cdots + x_{n,m}x_{n-1,m}E[Y_kY_mY_m]\}}{\sum_{k=1}^{3}\{x_{n,1}x_{n-1,1}E[Y_kY_1Y_1] + x_{n,2}x_{n-1,2}E[Y_kY_1Y_2] + \cdots + x_{n,m}x_{n-1,m}E[Y_kY_mY_m]\}}$$
$$= \zeta_{n,2}\left(\sum_{i_1=1}^{3}\sum_{i_2=1}^{3} x_{n+1,k}\{x_{n,i_1}x_{n-1,i_2}E\left[Y_kY_{i_1}Y_{i_2}\right]\}\right)$$
$$= \zeta_{n,2}\{x_{n+1,1}\left(x_{n,1}x_{n-1,1}E[Y_1^3] + x_{n,1}x_{n-1,2}E[Y_1^2Y_2] + x_{n,1}x_{n-1,3}E[Y_1^2Y_3]\right.$$

$$+x_{n,2}x_{n-1,1}E[Y_1^2Y_2] + x_{n,2}x_{n-1,2}E[Y_1Y_2^2] + x_{n,2}x_{n-1,3}E[Y_1Y_2Y_3]$$
$$+x_{n,3}x_{n-1,1}E[Y_1^2Y_3] + x_{n,3}x_{n-1,2}E[Y_1Y_2Y_3] + x_{n,3}x_{n-1,3}E[Y_1Y_3^2]\big)$$
$$+x_{n+1,2}\big(x_{n,1}x_{n-1,1}E[Y_1^2Y_2] + x_{n,1}x_{n-1,2}E[Y_1Y_2^2] + x_{n,1}x_{n-1,3}E[Y_1Y_2Y_3]$$
$$+x_{n,2}x_{n-1,1}E[Y_1Y_2^2] + x_{n,2}x_{n-1,2}E[Y_2^3] + x_{n,2}x_{n-1,3}E[Y_2^2Y_3]$$
$$+x_{n,3}x_{n-1,1}E[Y_1Y_2Y_3] + x_{n,3}x_{n-1,2}E[Y_2^2Y_3] + x_{n,3}x_{n-1,3}E[Y_2Y_3^2]\big)$$
$$+x_{n+1,3}\big(x_{n,1}x_{n-1,1}E[Y_1^2Y_3] + x_{n,1}x_{n-1,2}E[Y_1Y_2Y_3] + x_{n,1}x_{n-1,3}E[Y_1Y_3^2]$$
$$+x_{n,2}x_{n-1,1}E[Y_1Y_2Y_3] + x_{n,2}x_{n-1,2}E[Y_2^2Y_3] + x_{n,2}x_{n-1,3}E[Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}E[Y_1Y_3^2] + x_{n,3}x_{n-1,2}E[Y_2Y_3^2] + x_{n,3}x_{n-1,3}E[Y_3^3]\big)\big\} \tag{3.12}$$

where

$$\zeta_{n,2} = \big(x_{n,1}x_{n-1,1}E[Y_1^3] + x_{n,1}x_{n-1,2}E[Y_1^2Y_2] + x_{n,1}x_{n-1,3}E[Y_1^2Y_3]$$
$$+x_{n,2}x_{n-1,1}E[Y_1^2Y_2] + x_{n,2}x_{n-1,2}E[Y_1Y_2^2] + x_{n,2}x_{n-1,3}E[Y_1Y_2Y_3]$$
$$+x_{n,3}x_{n-1,1}E[Y_1^2Y_3] + x_{n,3}x_{n-1,2}E[Y_1Y_2Y_3] + x_{n,3}x_{n-1,3}E[Y_1Y_3^2]$$
$$+x_{n,1}x_{n-1,1}E[Y_1^2Y_2] + x_{n,1}x_{n-1,2}E[Y_1Y_2^2] + x_{n,1}x_{n-1,3}E[Y_1Y_2Y_3]$$
$$+x_{n,2}x_{n-1,1}E[Y_1Y_2^2] + x_{n,2}x_{n-1,2}E[Y_2^3] + x_{n,2}x_{n-1,3}E[Y_2^2Y_3]$$
$$+x_{n,3}x_{n-1,1}E[Y_1Y_2Y_3] + x_{n,3}x_{n-1,2}E[Y_2^2Y_3] + x_{n,3}x_{n-1,3}E[Y_2Y_3^2]$$
$$+x_{n,1}x_{n-1,1}E[Y_1^2Y_3] + x_{n,1}x_{n-1,2}E[Y_1Y_2Y_3] + x_{n,1}x_{n-1,3}E[Y_1Y_3^2]$$
$$+x_{n,2}x_{n-1,1}E[Y_1Y_2Y_3] + x_{n,2}x_{n-1,2}E[Y_2^2Y_3] + x_{n,2}x_{n-1,3}E[Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}E[Y_1Y_3^2] + x_{n,3}x_{n-1,2}E[Y_2Y_3^2] + x_{n,3}x_{n-1,3}E[Y_3^3]\big)^{-1}$$

3.3.3 Order 3 Universal Portfolio

$$\sum_{k=1}^{3} b_{n+1,k}x_{n+1,k}$$
$$= \frac{\sum_{k=1}^{3} x_{n+1,k}\left\{\begin{matrix} x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_kY_1Y_1Y_1] + x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_kY_1Y_1Y_2] + \cdots \\ +x_{n,m}x_{n-1,m}x_{n-2,m}E[Y_kY_mY_mY_m] \end{matrix}\right\}}{\sum_{k=1}^{3}\left\{\begin{matrix} x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_kY_1Y_1Y_1] + x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_kY_1Y_1Y_2] + \cdots \\ +x_{n,m}x_{n-1,m}x_{n-2,m}E[Y_kY_mY_mY_m] \end{matrix}\right\}}$$
$$= \zeta_{n,3}\left(\sum_{i_1=1}^{3}\sum_{i_2=1}^{3}\sum_{i_3=1}^{3} x_{n+1,k}\{x_{n,i}x_{n-1,i}x_{n-2,i}E\big[Y_kY_{i_1}Y_{i_2}Y_{i_3}\big]\}\right)$$

$$
\begin{aligned}
= \zeta_{n,3}\{ & x_{n+1,1}(x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^4] + x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^3Y_2] + x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^3Y_3] \\
& +x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^3Y_2] + x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1^2Y_2^2] \\
& +x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^3Y_3] \\
& +x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1^2Y_3^2] \\
& +x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_2] + x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2^2] \\
& +x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_2Y_3] + x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2^2] \\
& +x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^3] + x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_1Y_2^2Y_3] \\
& +x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3] \\
& +x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_1Y_2Y_3^2] + x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_3] \\
& +x_{n,3}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2Y_3] + x_{n,3}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_3^2] \\
& +x_{n,3}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,3}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^2Y_3] \\
& +x_{n,3}x_{n-1,2}x_{n-2,3}E[Y_1Y_2Y_3^2] + x_{n,3}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_3^2] \\
& +x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_1Y_2Y_3^2] + x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_1Y_3^3]\big) \\
& +x_{n+1,2}(x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_2] + x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2^2] \\
& +x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2^2] \\
& +x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^3] + x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1Y_2^2Y_3] \\
& +x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3] \\
& +x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1Y_2Y_3^2] + x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2^2] \\
& +x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1Y_2^3] + x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1Y_2^2Y_3] \\
& +x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1Y_2^3] + x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_2^4] \\
& +x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_2^3Y_3] + x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1Y_2^2Y_3] \\
& +x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_2^3Y_3] + x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_2^2Y_3^2] \\
& +x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,3}x_{n-1,1}x_{n-2,2}E[Y_1Y_2^2Y_3] \\
& +x_{n,3}x_{n-1,1}x_{n-2,3}E[Y_1Y_2Y_3^2] + x_{n,3}x_{n-1,2}x_{n-2,1}E[Y_1Y_2^2Y_3] \\
& +x_{n,3}x_{n-1,2}x_{n-2,2}E[Y_2^3Y_3] + x_{n,3}x_{n-1,2}x_{n-2,3}E[Y_2^2Y_3^2] \\
& +x_{n,3}x_{n-1,3}x_{n-2,1}E[Y_1Y_2Y_3^2] + x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_2^2Y_3^2] \\
& +x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_2Y_3^3]\big) \\
& +x_{n+1,3}(x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_3] + x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2Y_3] \\
& +x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_3^2] + x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2Y_3] \\
& +x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1Y_2Y_3^2] \\
& +x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3]
\end{aligned}
$$

$$+x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1Y_3^3] + x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1Y_2Y_3^2]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_2^3Y_3]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_2^2Y_3^2] + x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1Y_2Y_3^2]$$
$$+x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_2^2Y_3^2] + x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_3^2] + x_{n,3}x_{n-1,1}x_{n-2,2}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,3}E[Y_1Y_3^3] + x_{n,3}x_{n-1,2}x_{n-2,1}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,2}x_{n-2,2}E[Y_2^2Y_3^2] + x_{n,3}x_{n-1,2}x_{n-2,3}E[Y_2Y_3^3]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,1}E[Y_1Y_3^3] + x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_2Y_3^3]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_3^4]\big\} \qquad (3.13)$$

where

$$\zeta_{n,3} = \big(x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^4] + x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^3Y_2]$$
$$+x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^3Y_3] + x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^3Y_2]$$
$$+x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1^2Y_2^2] + x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^3Y_3] + x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1^2Y_3^2] + x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_2]$$
$$+x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2^2] + x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_2Y_3]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2^2] + x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^3]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_3] + x_{n,3}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2Y_3]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_3^2] + x_{n,3}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,3}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,3}x_{n-1,2}x_{n-2,3}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_3^2] + x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_1Y_3^3] + x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_2]$$
$$+x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2^2] + x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2^2] + x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^3]$$
$$+x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1Y_2^2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1Y_2Y_3^2]$$

$$+x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2^2] + x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1Y_2^3]$$
$$+x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1Y_2^3]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_2^4] + x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_2^3Y_3]$$
$$+x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_2^3Y_3]$$
$$+x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_2^2Y_3^2] + x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_2^2Y_3^2] + x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_2Y_3^3]$$
$$+x_{n,1}x_{n-1,1}x_{n-2,1}E[Y_1^3Y_3] + x_{n,1}x_{n-1,1}x_{n-2,2}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,1}x_{n-2,3}E[Y_1^2Y_3^2] + x_{n,1}x_{n-1,2}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,1}x_{n-1,2}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,1}x_{n-1,2}x_{n-2,3}E[Y_1Y_2Y_3^2]$$
$$+x_{n,1}x_{n-1,3}x_{n-2,1}E[Y_1^2Y_2Y_3] + x_{n,1}x_{n-1,3}x_{n-2,2}E[Y_1Y_2^2Y_3]$$
$$+x_{n,1}x_{n-1,3}x_{n-2,3}E[Y_1Y_3^3] + x_{n,2}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_2Y_3]$$
$$+x_{n,2}x_{n-1,1}x_{n-2,2}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,1}x_{n-2,3}E[Y_1Y_2Y_3^2]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,1}E[Y_1Y_2^2Y_3] + x_{n,2}x_{n-1,2}x_{n-2,2}E[Y_2^3Y_3]$$
$$+x_{n,2}x_{n-1,2}x_{n-2,3}E[Y_2^2Y_3^2] + x_{n,2}x_{n-1,3}x_{n-2,1}E[Y_1Y_2Y_3^2]$$
$$+x_{n,2}x_{n-1,3}x_{n-2,2}E[Y_2^2Y_3^2] + x_{n,2}x_{n-1,3}x_{n-2,3}E[Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,1}E[Y_1^2Y_3^2] + x_{n,3}x_{n-1,1}x_{n-2,2}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,1}x_{n-2,3}E[Y_1Y_3^3] + x_{n,3}x_{n-1,2}x_{n-2,1}E[Y_1Y_2Y_3^2]$$
$$+x_{n,3}x_{n-1,2}x_{n-2,2}E[Y_2^2Y_2^2] + x_{n,3}x_{n-1,2}x_{n-2,3}E[Y_2Y_3^3]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,1}E[Y_1Y_3^3] + x_{n,3}x_{n-1,3}x_{n-2,2}E[Y_2Y_3^3]$$
$$+x_{n,3}x_{n-1,3}x_{n-2,3}E[Y_3^4]\Big)^{-1}$$

3.4 Special Distributions

In this project, five distributions are chosen to generate the moment function required by the low-order universal portfolio function. The distributions and their corresponding k^{th} moment function is as follow:

Table 3.1: k^{th} Moment of Two Parameters Distribution

Two-Parameters Distribution	**k^{th} moment function, $E(X^k)$**
Pareto – α, θ $0 < k < \alpha$	$\frac{\theta^k \Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}$
Loglogistic – γ, θ $-\gamma < k < \gamma$	$\theta^k \Gamma\left(1+\frac{k}{\gamma}\right)\Gamma\left(1-\frac{k}{\gamma}\right)$
Paralogistic – α, θ $-\alpha < k < \alpha^2$	$\frac{\theta^k \Gamma\left(1+\frac{k}{\alpha}\right)\Gamma\left(\alpha-\frac{k}{\alpha}\right)}{\Gamma(\alpha)}$

Table 3.2: k^{th} Moment of Three Parameters Distribution

Three-Parameters Distribution	**k^{th} moment function, $E(X^k)$**
Burr - α, θ, γ $-\gamma < k < \alpha\gamma$	$\frac{\theta^k \Gamma\left(1+\frac{k}{\gamma}\right)\Gamma\left(\alpha-\frac{k}{\gamma}\right)}{\Gamma(\alpha)}$
Transformed Gamma - α, θ, τ $-\alpha\tau < k$	$\frac{\theta^k \Gamma\left(\alpha+\frac{k}{\tau}\right)}{\Gamma(\alpha)}$

Table 3.3: First Four Moment of Two-Parameters Distribution

Two-Parameters Distribution	$\boldsymbol{E(X)}$	$\boldsymbol{E(X^2)}$	$\boldsymbol{E(X^3)}$	$\boldsymbol{E(X^4)}$
Pareto – α, θ	$\frac{\theta\Gamma(2)\Gamma(\alpha-1)}{\Gamma(\alpha)}$	$\frac{\theta^2\Gamma(3)\Gamma(\alpha-2)}{\Gamma(\alpha)}$	$\frac{\theta^3\Gamma(4)\Gamma(\alpha-3)}{\Gamma(\alpha)}$	$\frac{\theta^4\Gamma(5)\Gamma(\alpha-4)}{\Gamma(\alpha)}$
Loglogistic – γ, θ	$\theta\Gamma\left(1+\frac{1}{\gamma}\right)\Gamma\left(1-\frac{1}{\gamma}\right)$	$\theta^2\Gamma(1+\frac{2}{\gamma})\Gamma(1-\frac{2}{\gamma})$	$\theta^3\Gamma(1+\frac{3}{\gamma})\Gamma(1-\frac{3}{\gamma})$	$\theta^4\Gamma\left(1+\frac{4}{\gamma}\right)\Gamma\left(1-\frac{4}{\gamma}\right)$
Paralogistic – α, θ	$\frac{\theta\Gamma\left(1+\frac{1}{\alpha}\right)\Gamma\left(\alpha-\frac{1}{\alpha}\right)}{\Gamma(\alpha)}$	$\frac{\theta^2\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(\alpha-\frac{2}{\alpha}\right)}{\Gamma(\alpha)}$	$\frac{\theta^3\Gamma\left(1+\frac{3}{\alpha}\right)\Gamma\left(\alpha-\frac{3}{\alpha}\right)}{\Gamma(\alpha)}$	$\frac{\theta^4\Gamma\left(1+\frac{4}{\alpha}\right)\Gamma\left(\alpha-\frac{4}{\alpha}\right)}{\Gamma(\alpha)}$

Table 3.4: First Four Moment of Three-Parameters Distribution

Three-Parameters Distribution	$\boldsymbol{E(X)}$	$\boldsymbol{E(X^2)}$	$\boldsymbol{E(X^3)}$	$\boldsymbol{E(X^4)}$
Burr - α, θ, γ	$\frac{\theta\Gamma\left(1+\frac{1}{\gamma}\right)\Gamma\left(\alpha-\frac{1}{\gamma}\right)}{\Gamma(\alpha)}$	$\frac{\theta^2\Gamma\left(1+\frac{2}{\gamma}\right)\Gamma\left(\alpha-\frac{2}{\gamma}\right)}{\Gamma(\alpha)}$	$\frac{\theta^3\Gamma\left(1+\frac{3}{\gamma}\right)\Gamma\left(\alpha-\frac{3}{\gamma}\right)}{\Gamma(\alpha)}$	$\frac{\theta^4\Gamma\left(1+\frac{4}{\gamma}\right)\Gamma\left(\alpha-\frac{4}{\gamma}\right)}{\Gamma(\alpha)}$
Transformed Gamma - α, θ, τ	$\frac{\theta\Gamma\left(\alpha+\frac{1}{\tau}\right)}{\Gamma(\alpha)}$	$\frac{\theta^2\Gamma\left(\alpha+\frac{2}{\tau}\right)}{\Gamma(\alpha)}$	$\frac{\theta^3\Gamma\left(\alpha+\frac{3}{\tau}\right)}{\Gamma(\alpha)}$	$\frac{\theta^4\Gamma\left(\alpha+\frac{4}{\tau}\right)}{\Gamma(\alpha)}$

3.4.1 Order 1 Universal Portfolio Generated by Pareto Distribution

By setting parameters of Pareto distribution as $(\alpha_1, \alpha_2, \alpha_3)$ and $(\theta_1, \theta_2, \theta_3)$, we can generate probability density function, $f(y_1), f(y_2)$ and $f(y_3)$.

To generate the wealth increase of k stock on the $(n+1)^{th}$ day for order 1 Universal Portfolio, $b_{n+1}^{t} x_{n+1}$ with Pareto distribution, we will substitute the respective k^{th} moment function, $E(X^K)$ from Table 3.3 into equation (3.11) and become:

$$
\begin{aligned}
\sum_{k=1}^{3} b_{n+1,k} x_{n+1,k} = \zeta_{n,1} \Bigg\{ & x_{n+1,1} \Bigg(x_{n,1} \left[\frac{{\theta_1}^2 \Gamma(3)\Gamma(\alpha_1 - 2)}{\Gamma(\alpha_1)}\right] \\
& + x_{n,2} \left[\frac{\theta_1 \Gamma(2)\Gamma(\alpha_1 - 1)}{\Gamma(\alpha_1)} \times \frac{\theta_2 \Gamma(2)\Gamma(\alpha_2 - 1)}{\Gamma(\alpha_2)}\right] \\
& + x_{n,3} \left[\frac{\theta_1 \Gamma(2)\Gamma(\alpha_1 - 1)}{\Gamma(\alpha_1)} \times \frac{\theta_3 \Gamma(2)\Gamma(\alpha_3 - 1)}{\Gamma(\alpha_3)}\right] \Bigg) \\
& + x_{n+1,2} \Bigg(x_{n,1} E \left[\frac{\theta_1 \Gamma(2)\Gamma(\alpha_1 - 1)}{\Gamma(\alpha_1)} \times \frac{\theta_2 \Gamma(2)\Gamma(\alpha_2 - 1)}{\Gamma(\alpha_2)}\right] \\
& + x_{n,2} \left[\frac{\theta_2^2 \Gamma(3)\Gamma(\alpha_2 - 2)}{\Gamma(\alpha_2)}\right] \\
& + x_{n,3} \left[\frac{\theta_2 \Gamma(2)\Gamma(\alpha_2 - 1)}{\Gamma(\alpha_2)} \times \frac{\theta_3 \Gamma(2)\Gamma(\alpha_3 - 1)}{\Gamma(\alpha_3)}\right] \Bigg) \\
& + x_{n+1,3} \Bigg(x_{n,1} \left[\frac{\theta_1 \Gamma(2)\Gamma(\alpha_1 - 1)}{\Gamma(\alpha_1)} \times \frac{\theta_3 \Gamma(2)\Gamma(\alpha_3 - 1)}{\Gamma(\alpha_3)}\right] \\
& + x_{n,2} \left[\frac{\theta_2 \Gamma(2)\Gamma(\alpha_2 - 1)}{\Gamma(\alpha_2)} \times \frac{\theta_3 \Gamma(2)\Gamma(\alpha_3 - 1)}{\Gamma(\alpha_3)}\right] \\
& + x_{n,3} \left[\frac{\theta_3^2 \Gamma(3)\Gamma(\alpha_3 - 2)}{\Gamma(\alpha_3)}\right] \Bigg) \Bigg\}
\end{aligned}
\tag{3.14}
$$

3.4.2 Order 1 Universal Portfolio Generated by Loglogistic Distribution

By setting parameters of Loglogistic distribution as $(\gamma_1, \gamma_2, \gamma_3)$ and $(\theta_1, \theta_2, \theta_3)$, we can generate probability density function, $f(y_1), f(y_2)$ and $f(y_3)$.

To generate the wealth increase of k stock on the $(n+1)^{th}$ day for order 1 Universal Portfolio, $b_{n+1}^{t} x_{n+1}$ with Loglogistic distribution, we will substitute the respective k^{th} moment function, $E(X^K)$ from Table 3.3 into equation (3.11) and become:

$$
\begin{aligned}
\sum_{k=1}^{3} b_{n+1,k} x_{n+1,k} = \zeta_{n,1} \Bigg\{ & x_{n+1,1} \Bigg(x_{n,1} \left[{\theta_1}^2 \Gamma(1 + \frac{2}{\gamma_1}) \Gamma(1 - \frac{2}{\gamma_1}) \right] \\
& + x_{n,2} \left[\theta_1 \Gamma\left(1 + \frac{1}{\gamma_1}\right) \Gamma\left(1 - \frac{1}{\gamma_1}\right) \theta_2 \Gamma\left(1 + \frac{1}{\gamma_2}\right) \Gamma\left(1 - \frac{1}{\gamma_2}\right) \right] \\
& + x_{n,3} \left[\theta_1 \Gamma\left(1 + \frac{1}{\gamma_1}\right) \Gamma\left(1 - \frac{1}{\gamma_1}\right) \theta_3 \Gamma\left(1 + \frac{1}{\gamma_3}\right) \Gamma\left(1 - \frac{1}{\gamma_3}\right) \right] \Bigg) \\
& + x_{n+1,2} \Bigg(x_{n,1} \left[\theta_1 \Gamma\left(1 + \frac{1}{\gamma_1}\right) \Gamma\left(1 - \frac{1}{\gamma_1}\right) \theta_2 \Gamma\left(1 + \frac{1}{\gamma_2}\right) \Gamma\left(1 - \frac{1}{\gamma_2}\right) \right] \\
& + x_{n,2} \left[{\theta_2}^2 \Gamma(1 + \frac{2}{\gamma_2}) \Gamma(1 - \frac{2}{\gamma_2}) \right] \\
& + x_{n,3} \left[\theta_2 \Gamma\left(1 + \frac{1}{\gamma_2}\right) \Gamma\left(1 - \frac{1}{\gamma_2}\right) \theta_3 \Gamma\left(1 + \frac{1}{\gamma_3}\right) \Gamma\left(1 - \frac{1}{\gamma_3}\right) \right] \Bigg) \\
& + x_{n+1,3} \Bigg(x_{n,1} \left[\theta_1 \Gamma\left(1 + \frac{1}{\gamma_1}\right) \Gamma\left(1 - \frac{1}{\gamma_1}\right) \theta_3 \Gamma\left(1 + \frac{1}{\gamma_3}\right) \Gamma\left(1 - \frac{1}{\gamma_3}\right) \right] \\
& + x_{n,2} \left[\theta_2 \Gamma\left(1 + \frac{1}{\gamma_2}\right) \Gamma\left(1 - \frac{1}{\gamma_2}\right) \theta_3 \Gamma\left(1 + \frac{1}{\gamma_3}\right) \Gamma\left(1 - \frac{1}{\gamma_3}\right) \right] \\
& + x_{n,3} E \left[\theta_3 \Gamma\left(1 + \frac{1}{\gamma_3}\right) \Gamma\left(1 - \frac{1}{\gamma_3}\right) \right] \Bigg) \Bigg\}
\end{aligned}
\tag{3.15}
$$

3.4.3 Order 1 Universal Portfolio Generated by Paralogistic Distribution

By setting parameters of Paralogistic distribution as $(\alpha_1, \alpha_2, \alpha_3)$ and $(\theta_1, \theta_2, \theta_3)$, we can generate probability density function, $f(y_1), f(y_2)$ and $f(y_3)$.

To generate the wealth increase of k stock on the $(n+1)^{th}$ day for order 1 Universal Portfolio, $b_{n+1}^t x_{n+1}$ with Paralogistic distribution, we will substitute the respective k^{th} moment function, $E(X^K)$ from Table 3.3 into equation (3.11) and become:

$$
\begin{aligned}
\sum_{k=1}^{3} b_{n+1,k}x_{n+1,k} = \zeta_{n,1}\Bigg\{ & x_{n+1,1}\Bigg(x_{n,1}\left[\frac{{\theta_1}^2\Gamma\left(1+\frac{2}{\alpha_1}\right)\Gamma\left(\alpha_1-\frac{2}{\alpha_1}\right)}{\Gamma(\alpha_1)}\right] \\
& +x_{n,2}\left[\frac{\theta_1\Gamma\left(1+\frac{1}{\alpha_1}\right)\Gamma\left(\alpha_1-\frac{1}{\alpha_1}\right)}{\Gamma(\alpha_1)}\frac{\theta_2\Gamma\left(1+\frac{1}{\alpha_2}\right)\Gamma\left(\alpha_2-\frac{1}{\alpha_2}\right)}{\Gamma(\alpha_2)}\right] \\
& +x_{n,3}\left[\frac{\theta_1\Gamma\left(1+\frac{1}{\alpha_1}\right)\Gamma\left(\alpha_1-\frac{1}{\alpha_1}\right)}{\Gamma(\alpha_1)}\frac{\theta_3\Gamma\left(1+\frac{1}{\alpha_3}\right)\Gamma\left(\alpha_2-\frac{1}{\alpha_3}\right)}{\Gamma(\alpha_3)}\right]\Bigg) \\
& +x_{n+1,2}\Bigg(x_{n,1}\left[\frac{\theta_1\Gamma\left(1+\frac{1}{\alpha_1}\right)\Gamma\left(\alpha_1-\frac{1}{\alpha_1}\right)}{\Gamma(\alpha_1)}\frac{\theta_2\Gamma\left(1+\frac{1}{\alpha_2}\right)\Gamma\left(\alpha_2-\frac{1}{\alpha_2}\right)}{\Gamma(\alpha_2)}\right] \\
& +x_{n,2}\left[\frac{{\theta_2}^2\Gamma\left(1+\frac{2}{\alpha_2}\right)\Gamma\left(\alpha_2-\frac{2}{\alpha_2}\right)}{\Gamma(\alpha_2)}\right] \\
& +x_{n,3}\left[\frac{\theta_2\Gamma\left(1+\frac{1}{\alpha_2}\right)\Gamma\left(\alpha_2-\frac{1}{\alpha_2}\right)}{\Gamma(\alpha_2)}\frac{\theta_3\Gamma\left(1+\frac{1}{\alpha_3}\right)\Gamma\left(\alpha_3-\frac{1}{\alpha_3}\right)}{\Gamma(\alpha_3)}\right]\Bigg) \\
& +x_{n+1,3}\Bigg(x_{n,1}\left[\frac{\theta_1\Gamma\left(1+\frac{1}{\alpha_1}\right)\Gamma\left(\alpha_1-\frac{1}{\alpha_1}\right)}{\Gamma(\alpha_1)}\frac{\theta_3\Gamma\left(1+\frac{1}{\alpha_3}\right)\Gamma\left(\alpha_3-\frac{1}{\alpha_3}\right)}{\Gamma(\alpha_3)}\right] \\
& +x_{n,2}\left[\frac{\theta_2\Gamma\left(1+\frac{1}{\alpha_2}\right)\Gamma\left(\alpha_2-\frac{1}{\alpha_2}\right)}{\Gamma(\alpha_2)}\frac{\theta_3\Gamma\left(1+\frac{1}{\alpha_3}\right)\Gamma\left(\alpha_3-\frac{1}{\alpha_3}\right)}{\Gamma(\alpha_3)}\right] \\
& +x_{n,3}\left[\frac{{\theta_3}^2\Gamma\left(1+\frac{2}{\alpha_3}\right)\Gamma\left(\alpha_3-\frac{2}{\alpha_3}\right)}{\Gamma(\alpha_3)}\right]\Bigg)\Bigg\}
\end{aligned}
\tag{3.16}
$$

3.4.4 Order 1 Universal Portfolio Generated by Burr Distribution

By setting parameters of Burr distribution as $(\alpha_1, \alpha_2, \alpha_3)$, $(\theta_1, \theta_2, \theta_3)$ and $(\gamma_1, \gamma_2, \gamma_3)$, we can generate probability density function, $f(y_1), f(y_2)$ and $f(y_3)$.

To generate the wealth increase of k stock on the $(n+1)^{th}$ day for order 1 Universal Portfolio, $b_{n+1}^t x_{n+1}$ with Burr distribution, we will substitute the respective k^{th} moment function, $E(X^K)$ from Table 3.4 into equation (3.11) and become:

$$\sum_{k=1}^{3} b_{n+1,k} x_{n+1,k} = \zeta_{n,1} \left\{ x_{n+1,1} \left(x_{n,1} \left[\frac{{\theta_1}^2 \Gamma\left(1+\frac{2}{\gamma_1}\right) \Gamma\left(\alpha_1 - \frac{2}{\gamma_1}\right)}{\Gamma(\alpha_1)} \right] \right. \right.$$
$$+ x_{n,2} \left[\frac{\theta_1 \Gamma\left(1+\frac{1}{\gamma_1}\right) \Gamma\left(\alpha_1 - \frac{1}{\gamma_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_2 \Gamma\left(1+\frac{1}{\gamma_2}\right) \Gamma\left(\alpha_2 - \frac{1}{\gamma_2}\right)}{\Gamma(\alpha_2)} \right]$$
$$\left. + x_{n,3} \left[\frac{\theta_1 \Gamma\left(1+\frac{1}{\gamma_1}\right) \Gamma\left(\alpha_1 - \frac{1}{\gamma_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_3 \Gamma\left(1+\frac{1}{\gamma_3}\right) \Gamma\left(\alpha_3 - \frac{1}{\gamma_3}\right)}{\Gamma(\alpha_3)} \right] \right)$$
$$+ x_{n+1,2} \left(x_{n,1} \left[\frac{\theta_1 \Gamma\left(1+\frac{1}{\gamma_1}\right) \Gamma\left(\alpha_1 - \frac{1}{\gamma_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_2 \Gamma\left(1+\frac{1}{\gamma_2}\right) \Gamma\left(\alpha_2 - \frac{1}{\gamma_2}\right)}{\Gamma(\alpha_2)} \right] \right.$$
$$+ x_{n,2} \left[\frac{{\theta_2}^2 \Gamma\left(1+\frac{2}{\gamma_2}\right) \Gamma\left(\alpha_2 - \frac{2}{\gamma_2}\right)}{\Gamma(\alpha_2)} \right]$$
$$\left. + x_{n,3} \left[\frac{\theta_2 \Gamma\left(1+\frac{1}{\gamma_2}\right) \Gamma\left(\alpha_2 - \frac{1}{\gamma_2}\right)}{\Gamma(\alpha_2)} \frac{\theta_3 \Gamma\left(1+\frac{1}{\gamma_3}\right) \Gamma\left(\alpha_3 - \frac{1}{\gamma_3}\right)}{\Gamma(\alpha_3)} \right] \right)$$
$$+ x_{n+1,3} \left(x_{n,1} \left[\frac{\theta_1 \Gamma\left(1+\frac{1}{\gamma_1}\right) \Gamma\left(\alpha_1 - \frac{1}{\gamma_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_3 \Gamma\left(1+\frac{1}{\gamma_3}\right) \Gamma\left(\alpha_3 - \frac{1}{\gamma_3}\right)}{\Gamma(\alpha_3)} \right] \right.$$
$$+ x_{n,2} \left[\frac{\theta_2 \Gamma\left(1+\frac{1}{\gamma_2}\right) \Gamma\left(\alpha_2 - \frac{1}{\gamma_2}\right)}{\Gamma(\alpha_2)} \frac{\theta_3 \Gamma\left(1+\frac{1}{\gamma_3}\right) \Gamma\left(\alpha_3 - \frac{1}{\gamma_3}\right)}{\Gamma(\alpha_3)} \right]$$
$$\left. \left. + x_{n,3} \left[\frac{{\theta_3}^2 \Gamma\left(1+\frac{2}{\gamma_3}\right) \Gamma\left(\alpha_3 - \frac{2}{\gamma_3}\right)}{\Gamma(\alpha_3)} \right] \right) \right\} \tag{3.17}$$

3.4.5 Order 1 Universal Portfolio Generated by Transformed Gamma Distribution

By setting parameters of Transformed Gamma distribution as $(\alpha_1, \alpha_2, \alpha_3)$, $(\theta_1, \theta_2, \theta_3)$ and (τ_1, τ_2, τ_3), we can generate probability density function, $f(y_1), f(y_2)$ and $f(y_3)$.

To generate the wealth increase of k stock on the $(n+1)^{th}$ day for order 1 Universal Portfolio, $b_{n+1}^t x_{n+1}$ with Transformed Gamma distribution, we will substitute the respective k^{th} moment function, $E(X^K)$ from Table 3.4 into equation (3.11) and become:

$$\sum_{k=1}^{3} b_{n+1,k} x_{n+1,k} = \zeta_{n,1} \Bigg\{ x_{n+1,1} \Bigg(x_{n,1} \left[\frac{{\theta_1}^2 \Gamma\left(\alpha_1 + \frac{3}{\tau_1}\right)}{\Gamma(\alpha_1)} \right]$$

$$+ x_{n,2} \left[\frac{\theta_1 \Gamma\left(\alpha_1 + \frac{1}{\tau_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_2 \Gamma\left(\alpha_2 + \frac{1}{\tau_2}\right)}{\Gamma(\alpha_2)} \right] + x_{n,3} \left[\frac{\theta_1 \Gamma\left(\alpha_1 + \frac{1}{\tau_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_3 \Gamma\left(\alpha_3 + \frac{1}{\tau_3}\right)}{\Gamma(\alpha_3)} \right] \Bigg)$$

$$+ x_{n+1,2} \Bigg(x_{n,1} \left[\frac{\theta_1 \Gamma\left(\alpha_1 + \frac{1}{\tau_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_2 \Gamma\left(\alpha_2 + \frac{1}{\tau_2}\right)}{\Gamma(\alpha_2)} \right] + x_{n,2} \left[\frac{{\theta_2}^2 \Gamma\left(\alpha_2 + \frac{3}{\tau_2}\right)}{\Gamma(\alpha_2)} \right]$$

$$+ x_{n,3} \left[\frac{\theta_2 \Gamma\left(\alpha_2 + \frac{1}{\tau_2}\right)}{\Gamma(\alpha_2)} \frac{\theta_3 \Gamma\left(\alpha_3 + \frac{1}{\tau_3}\right)}{\Gamma(\alpha_3)} \right] \Bigg)$$

$$+ x_{n+1,3} \Bigg(x_{n,1} \left[\frac{\theta_1 \Gamma\left(\alpha_1 + \frac{1}{\tau_1}\right)}{\Gamma(\alpha_1)} \frac{\theta_3 \Gamma\left(\alpha_3 + \frac{1}{\tau_3}\right)}{\Gamma(\alpha_3)} \right]$$

$$+ x_{n,2} \left[\frac{\theta_2 \Gamma\left(\alpha_2 + \frac{1}{\tau_2}\right)}{\Gamma(\alpha_2)} \frac{\theta_3 \Gamma\left(\alpha_3 + \frac{1}{\tau_3}\right)}{\Gamma(\alpha_3)} \right] + x_{n,3} \left[\frac{{\theta_3}^2 \Gamma\left(\alpha_3 + \frac{3}{\tau_3}\right)}{\Gamma(\alpha_3)} \right] \Bigg) \Bigg\} \quad (3.18)$$

3.4.6 Wealth Function of Order 1 Universal Portfolio

From equations (3.14), (3.15), (3.16), (3.17), and (3.18), the wealth function of order 1 universal portfolio on $(n+1)^{th}$ day can be generated by assuming initial wealth $S_0 = 1$, where

$$S_{n+1} = \left(\sum_{k=1}^{3} b_{n+1,k}x_{n+1,k}\right)S_n$$

3.5 Expected Work Schedule

Table 3.5: Gantt Chart of Project 1

	Week												
ACTIVITIES	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Identifying topic of research													
Studying on Literature review													
Writing proposal													
Mock presentation													
Writing interim report													
Hand In Interim Report													
Preparation for presentation													
Biweekly Report													

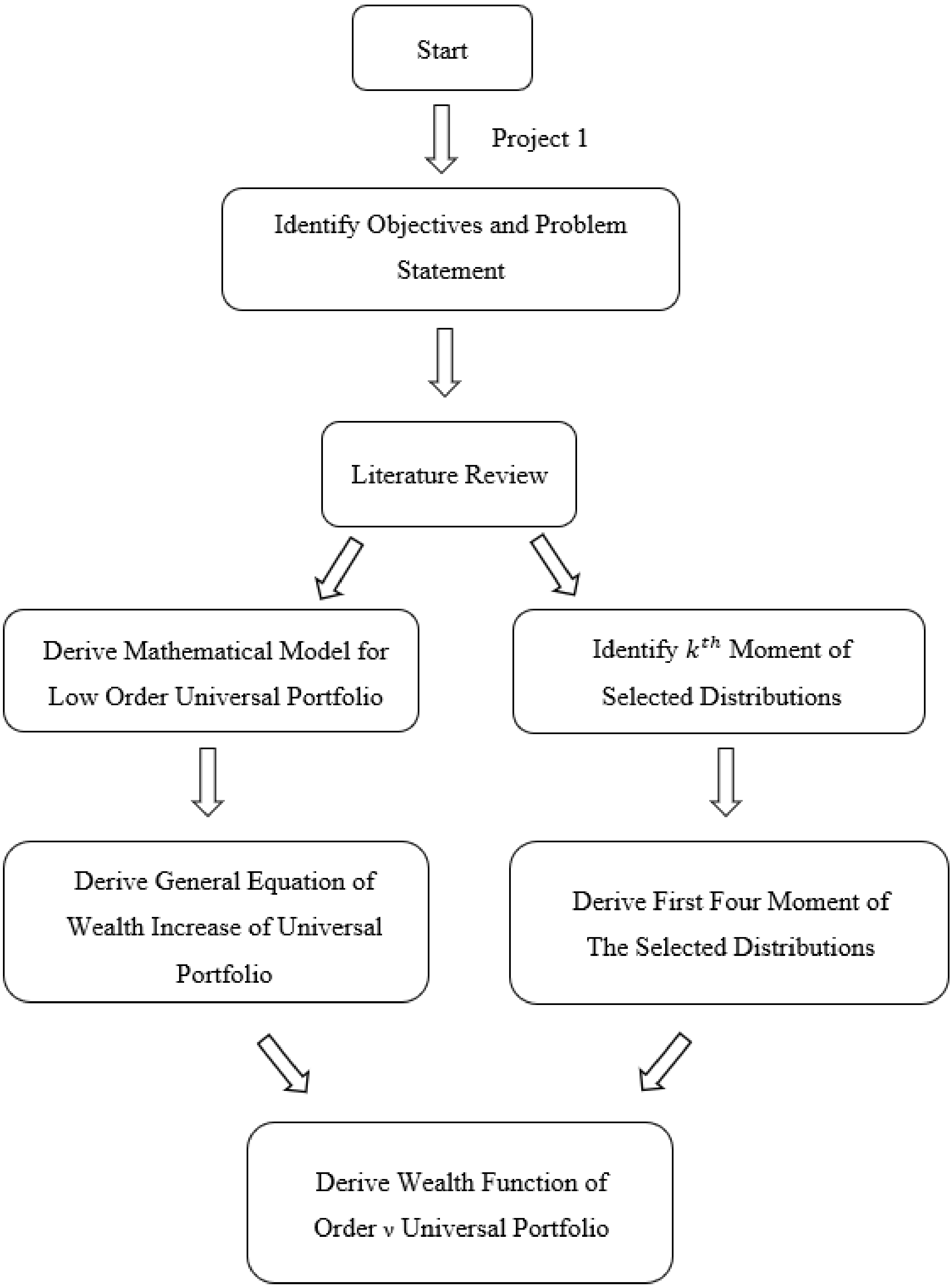
Table 3.6: Gantt Chart for Project 2

	Week												
ACTIVITIES	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Derive Mathematical Model													
Collecting Data													
Build up the Model and Writing VBA Code													
Writing Final Report													
Preparing FYP Poster													
Submission of Final Report Draft													
Submission of FYP Poster													
Submission of Final Report to Turnitin.com													
Submission of Final Report													
Oral Presentation													

3.6 Flow Chart

To conduct this project, first, we need to identify the objectives and problem statements. Then, we researched the journal articles written by other researchers about universal portfolios. From the literature review, we obtained more ideas for conducting this project. After reviewing the articles, we derived the mathematical model for low order universal portfolio. We defined $\boldsymbol{b_{ni}}$ as the proportion of current wealth invested in the stock on the n^{th} trading day and $\boldsymbol{x_{ni}}$ as the ratio of the closing price to the opening price of stock i on day n. The general equation to identify the wealth increase of low order universal portfolio on the $(n+1)^{th}$ day was derived in research methodology. In this project, we only consider order 1, order 2, and order 3 universal portfolios. We identified some special distributions to generate the low-order universal portfolio. Then, we find the k^{th} moment function for the chosen distributions. After that, we derived the first four moments of the distributions and substitute them into respective general equations for order 1, order 2, and order 3 universal portfolio.

Next, we build three different data sets. Each of the data sets has three different companies. First, the opening and closing prices were collected from Yahoo Finance for 1500 days. Then, we decided on the range of the distributions' parameters. With the derived mathematical model and data collected, we generated low order universal portfolio generated by special distributions with VBA in excel. From the result generated, we compared the performance of low order universal portfolio. Then, we identified which set of parameters can generate the highest wealth for each distribution. Next, we compared the low order universal portfolio performance generated with 500, 1000, 1500, 2000, and 2500 trading days. Finally, we identified whether the performance of a universal portfolio would be affected by diversified and non-diversified portfolios.


Start
Project 1
Identify Objectives and Problem Statement
Literature Review
Derive Mathematical Model for Low Order Universal Portfolio
Identify k^{th} Moment of Selected Distributions
Derive General Equation of Wealth Increase of Universal Portfolio
Derive First Four Moment of The Selected Distributions
Derive Wealth Function of Order v Universal Portfolio

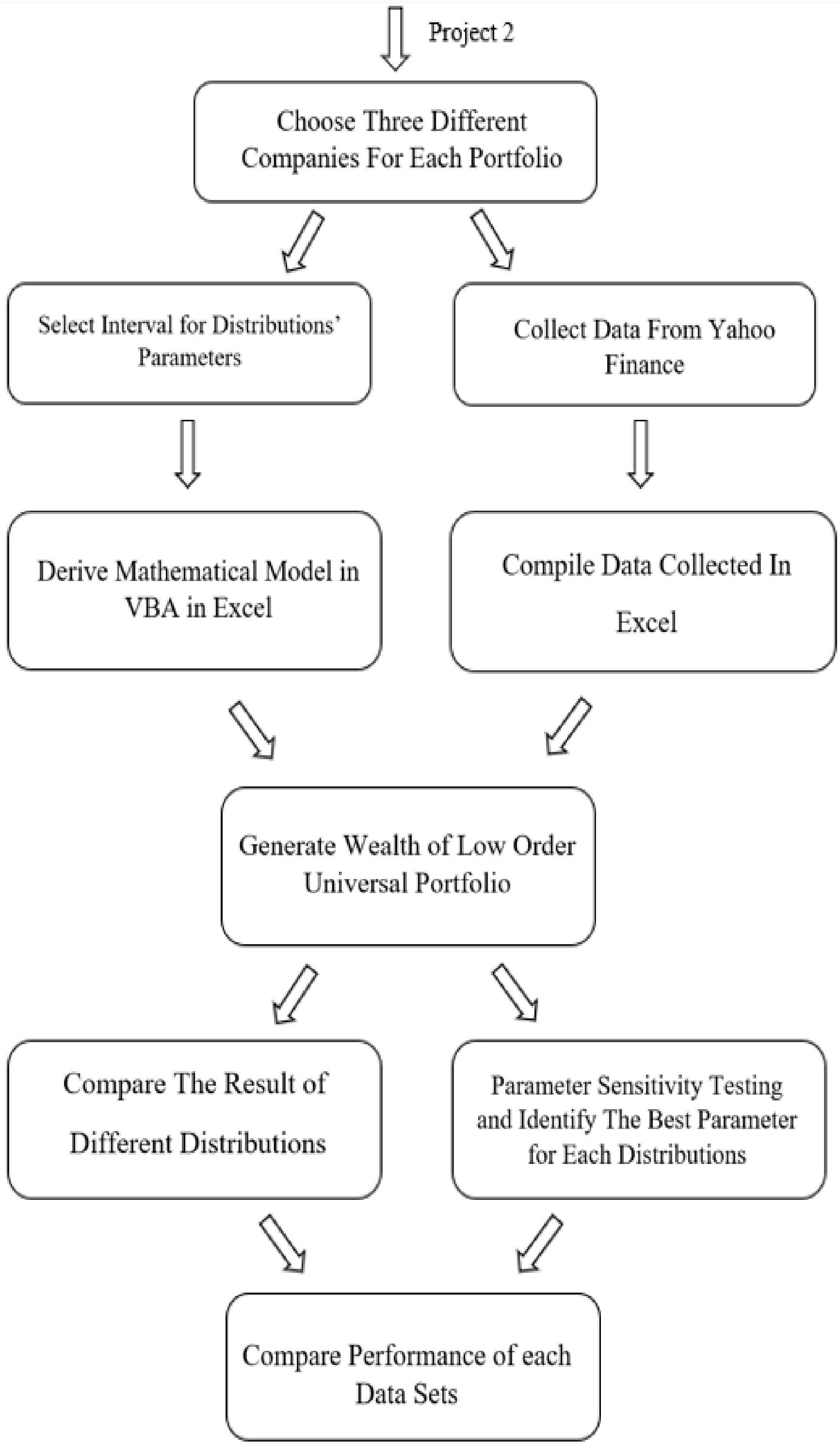


Figure 3.1: The Process of Generating Low Order Universal Portfolio With Special Distributions

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Low Order Universal Portfolio Generated by Selected Distributions

Five special distributions were used to generate a universal portfolio with three sets of data. Each data set contained 1500 days of opening and closing's price for three different companies' stock from Kuala Lumpur Stock Exchange (KLSE). The opening and closing price of the stock was taken from 5th January 2015 to 27th January 2021.

Table 4.1: Portfolio consists of three companies selected from KLSE.

Portfolio	Companies
A	(Hong Leong Bank Berhad, Malayan Banking Berhad, Public Bank berhad)
B	(Top Glove Corporation Berhad, Malayan Banking Berhad, Fraser & Neave Holding Berhad)
C	(Hong Leong Bank Berhad, Malayan Banking Berhad, Top Glove Corporation Berhad)

Assume initial wealth, $S_0 = 1$, and $b_0 = (0.3333, 0.3333, 0.3334)$, the low order universal portfolio is generated with Pareto, Loglogistic, Paralogistic, Burr, and Transformed Gamma distribution. The parameters of the distribution are randomly generated with a selected range of values. Each portfolio is generated for 500 trials, and the highest wealth achieved by each distribution is collected. Order 1, order 2, and order 3 universal portfolio are generated, and the terminal wealth obtained after 1500 trading days is compared with the best constant rebalances portfolio (BCRP) and constant rebalance portfolio (CRP). Tables 4.2 and 4.3 show the computation result for BCRP and CRP.

For the Pareto distribution, its parameters were randomly generated within $[4,103]$ for α and $[1,100]$ for θ to generated order 1 universal portfolio. The higher-order universal portfolio will be generated by increasing the range of α by 1 while θ remain unchanged. The highest wealth of Portfolio A, which is 2.7637, can be generated within order 1 when its parameter, $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3)$ is $(3, 54, 18, 63, 20, 3)$. The highest wealth achievable with Pareto distribution in portfolio B is 6.7674 within order 2, with parameters $(94, 67, 4, 17, 13, 40)$. While for portfolio C, the highest wealth for Pareto distribution is 3.2732, which generates within order 2 with parameters $(6, 34, 27, 5, 1, 40)$.

For the Loglogistic distribution, its parameters were randomly generated within $[4,103]$ for γ and $[1,100]$ for θ to generated order 1 universal portfolio. The higher-order universal portfolio will be generated by increasing the range of γ by 1 while θ remains unchanged. The highest wealth of Portfolio A, which is 2.5998, can be generated within order 3 when its parameter, $(\gamma_1, \gamma_2, \gamma_3, \theta_1, \theta_2, \theta_3)$ is $(50, 72, 67, 86, 11, 1)$. The highest wealth achieved with Loglogistic distribution in portfolio B is 6.3041 within order 3, with parameters $(7, 35, 28, 5, 1, 40)$. While for portfolio C, the highest wealth for Loglogistic distribution is 3.2456, which generates within order 3 with parameters $(37, 15, 34, 55, 2, 89)$.

For the Paralogistic distribution, its parameters were randomly generated within $[2,101]$ for α and $[1,100]$ for θ to generated order 1 universal portfolio. The higher-order universal portfolio will be generated by increasing the range of α by 1 while θ remain unchanged. The highest wealth of Portfolio A, which is 2.6389, can be generated within order 1 when its parameter, $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3)$ is $(93, 96, 100, 85, 6, 2)$. The highest wealth achieved with Paralogistic distribution in portfolio B is 6.3754 within order 1, with parameters $(4, 32, 25, 5, 1, 40)$. While for portfolio C, the highest wealth for Paralogistic distribution is 3.2533, which generates within order 1 with parameters $(94, 43, 61, 54, 1, 64)$.

For the Burr distribution, its parameters were randomly generated within $[1,100]$ for α, $[1,100]$ for θ and $[4, 103]$ for γ to generated order 1 universal portfolio. The higher-order universal portfolio is generated by increasing the range of γ by 1 while α and θ remain unchanged. The highest wealth of Portfolio A, which is 2.5799, can be generated with order 1 when its parameter, $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3, \gamma_1, \gamma_2, \gamma_3)$ is $(99, 65, 34, 94, 7, 5, 50, 74, 37)$. The highest wealth achieved with Burr distribution in portfolio B is 6.2614 within order 1, with parameters $(59, 12, 68, 18, 1, 76, 31, 7, 66)$. While for portfolio C, the highest wealth for Burr distribution is 3.2548, which generates within order 1 with parameters $(61, 91, 95, 66, 2, 67, 99, 3, 98)$.

For the Transformed Gamma distribution, its parameters are randomly generated within [1,100] for α, [1,100] for θ and $[1, 100]$ for τ to generated order 1 universal portfolio. The higher-order universal portfolio is generated without changing the range of α, θ, and τ. The highest wealth of Portfolio A, which is 2.7466, can be generated with order 1 when its parameter, $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3, \tau_1, \tau_2, \tau_3)$ is $(72, 54, 33, 41, 25, 17, 1, 83, 84)$. In portfolio B, the highest wealth achieved with Transformed Gamma distribution is 4.7336 within order 2, with parameters $(70, 53, 76, 13, 28, 73, 57, 66, 1)$. While for portfolio C, the highest wealth for Transformed Gamma distribution is 3.2570, which generate within order 3 with parameters $(64, 20, 81, 36, 1, 83, 97, 95, 25)$.

Based on the result obtained, the lower-order universal portfolio needs lesser time to generate wealth. Besides, the result showed that the universal portfolio generated by the Pareto distribution could outperform the universal portfolio generated by the other distribution and CRP. However, it can not outperform the BCRP. Since the parameters used to generate the wealth of a low order universal portfolio are randomly generated, 500 trials may not be sufficient to determine the maximum wealth generated by each distribution. Therefore, testing the parameter's sensitivity for all the distributions was studied to determine the best parameters.

Table 4.2: Best Constant Rebalance Portfolio (BCRP) for portfolios A, B, and C with 1500 trading days.

Portfolio	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	**Terminal Wealth, S_{1500}**
A	1.0000	0	0	2.7740
B	0	0	1.0000	6.7750
C	0.4000	0	0.6000	3.2750

Table 4.3: Constant Rebalance Portfolio (CRP) for portfolios A, B, and C with 1500 trading days.

Portfolio	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	0.8000	0.1000	0.1000	2.4399
B	0.1000	0.1000	0.8000	5.6782
C	0.3000	0.1000	0.6000	3.0889

Table 4.4: The wealth achieved for order 1 universal portfolio generated by selected distributions with random parameters for portfolio A and their respective implementation times.

Distribution	Order 1		
	Parameter	Terminal Wealth, S_{1500}	Timer (second)
Pareto	(3, 54, 18, 63, 20, 3)	2.7637	1.05
Loglogistic	(10, 32, 61, 81, 5, 6)	2.5669	1.04
Paralogistic	(93, 96, 100, 85, 6, 2)	2.6389	1.09
Burr	(99, 65, 34, 94, 7, 5, 50, 74, 37)	2.5799	1.45
Transformed Gamma	(72, 54, 33, 41, 25, 17, 1, 83, 84)	2.7466	1.46

Table 4.5: The wealth achieved for order 2 universal portfolio generated by selected distributions with random parameters for portfolio A and their respective implementation times.

Distribution	Order 2		
	Parameter	Terminal Wealth, S_{1500}	Timer (second)
Pareto	(5, 80, 85, 72, 6, 42)	2.7593	2.62
Loglogistic	(4, 39, 27, 23, 7, 1)	2.5427	2.79
Paralogistic	(22, 13, 61, 98, 6, 8)	2.5611	2.73
Burr	(81, 71, 54, 91, 5, 6, 23, 97, 22)	2.5689	4.76
Transformed Gamma	(77, 45, 14, 83, 68, 82, 1, 58, 62)	2.7284	3.73

Table 4.6: The wealth achieved for order 3 universal portfolio generated by selected distributions with random parameters for portfolio A and their respective implementation times.

Distribution	**Order 3**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (Second)**
Pareto	(6, 81, 86, 71, 5, 42)	2.7624	11.95
Loglogistic	(50, 72, 67, 86, 11, 1)	2.5998	11.06
Paralogistic	(10, 32, 61, 81, 5, 6)	2.5323	10.79
Burr	(8, 30, 59, 81, 5, 6, 61, 36, 46)	2.2735	12.45
Transformed Gamma	(94, 64, 76, 69, 23, 3, 2, 85, 20)	2.3977	11.55

Table 4.7: The wealth achieved for order 1 universal portfolio generated by selected distribution with random parameters for Portfolio B and their respective implementation times.

Distribution	**Order 1**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (second)**
Pareto	(85, 89, 3, 24, 28, 91)	6.7664	1.84
Loglogistic	(17, 79, 29, 14, 5, 70)	5.9005	0.73
Paralogistic	(4, 32, 25, 5, 1, 40)	6.3754	1.67
Burr	(59, 12, 68, 18, 1, 76, 31, 7, 66)	6.2614	1.19
Transformed Gamma	(59, 12, 68, 18, 1, 76, 29, 5, 64)	6.1120	1.77

Table 4.8: The wealth achieved for order 2 universal portfolio generated by selected distribution with random parameters for Portfolio B and their respective implementation times.

Distribution	**Order 2**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (second)**
Pareto	(94, 67, 4, 17, 13, 40)	6.7674	4.59
Loglogistic	(11, 14, 37, 14, 1, 54)	6.0990	4.94
Paralogistic	(18, 81, 101, 6, 9, 83)	5.9083	4.35
Burr	(17, 80, 100, 6, 9, 83, 81, 37, 68)	5.8918	4.59
Transformed Gamma	(70, 53, 76, 13, 28, 73, 57, 66, 1)	6.7336	4.77

Table 4.9: The wealth achieved for order 3 universal portfolio generated by selected distribution with random parameters for Portfolio B and their respective implementation times.

Distribution	**Order 3**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (Second)**
Pareto	(63, 77, 19, 26, 3, 83)	6.7232	11.57
Loglogistic	(7, 35, 28, 5, 1, 40)	6.3041	11.47
Paralogistic	(10, 13, 36, 13, 1, 54)	6.2115	11.85
Burr	(15, 77, 27, 14, 5, 70, 89, 56, 85)	5.2505	12.46
Transformed Gamma	(71, 46, 65, 92, 1, 76, 36, 23, 1)	6.0343	11.43

Table 4.10: The wealth achieved for order 1 universal portfolio generated by selected distribution with random parameters for Portfolio C and their respective implementation times.

Distribution	**Order 1**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (second)**
Pareto	(95, 44, 62, 54, 1, 64)	3.2530	1.48
Loglogistic	(88, 11, 24, 76, 2, 56)	3.2002	0.58
Paralogistic	(94, 43, 61, 54, 1, 64)	3.2533	1.22
Burr	(61, 91, 95, 66, 2, 67, 99, 3, 98)	3.2548	1.33
Transformed Gamma	(55, 61, 61, 47, 2, 96, 58, 79, 56)	3.2418	0.72

Table.4.11: The wealth achieved for order 2 universal portfolio generated by selected distribution with random parameters for Portfolio C and their respective implementation times.

Distribution	**Order 2**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (second)**
Pareto	(6, 34, 27, 5, 1, 40)	3.2732	2.62
Loglogistic	(69, 5, 70, 97, 1, 96)	3.2411	3.17
Paralogistic	(11, 67, 79, 46, 1, 54)	3.2489	2.64
Burr	(64, 20, 81, 36, 1, 83, 100, 98, 28)	3.2440	2.67
Transformed Gamma	(50, 94, 48, 98, 2, 46, 94, 84)	3.2525	2.69

Table 4.12: The wealth achieved for order 3 universal portfolio generated by selected distribution with random parameters for Portfolio C and their respective implementation times.

Distribution	**Order 3**		
	Parameter	**Terminal Wealth, S_{1500}**	**Timer (Second)**
Pareto	(28, 53, 30, 35, 5, 49)	3.2407	11.95
Loglogistic	(37, 15, 34, 55, 2, 89)	3.2456	12.46
Paralogistic	(68, 4, 69, 97, 1, 96)	3.2379	12.47
Burr	(61, 91, 95, 66, 2, 67, 101, 5, 100)	3.2351	12.04
Transformed Gamma	(64, 20, 81, 36, 1, 83, 97, 95, 25)	3.2570	11.09

Figure 4.1: The order 1 universal portfolio performance generated by selected distributions with random parameters for portfolio A.

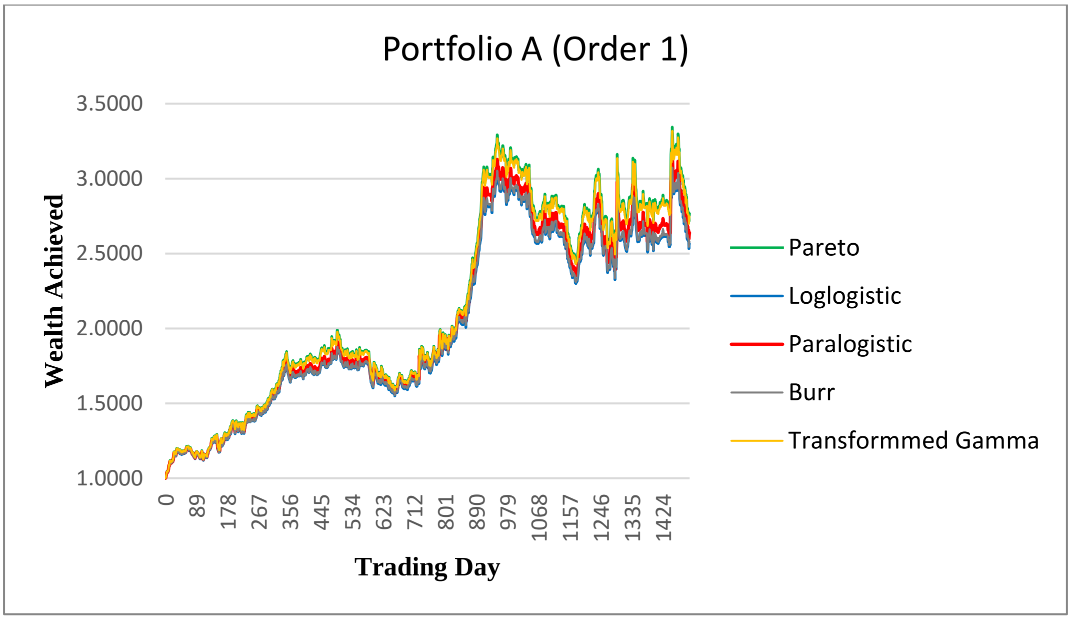


Figure 4.2: The order 2 universal portfolio performance generated by selected distributions with random parameters for portfolio A.

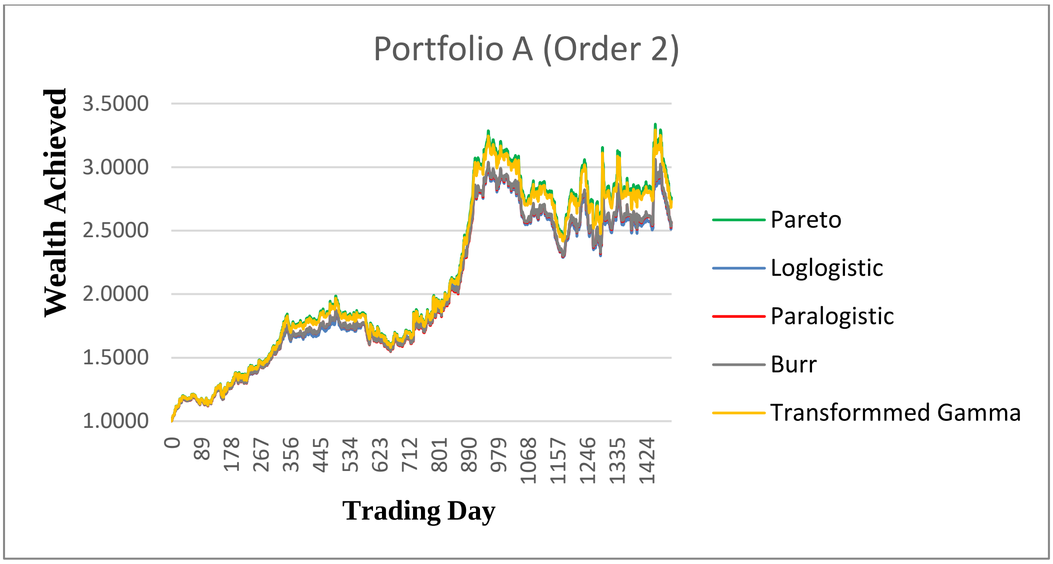


Figure 4.3: The order 3 universal portfolio performance generated by selected distributions with random parameters for portfolio A.

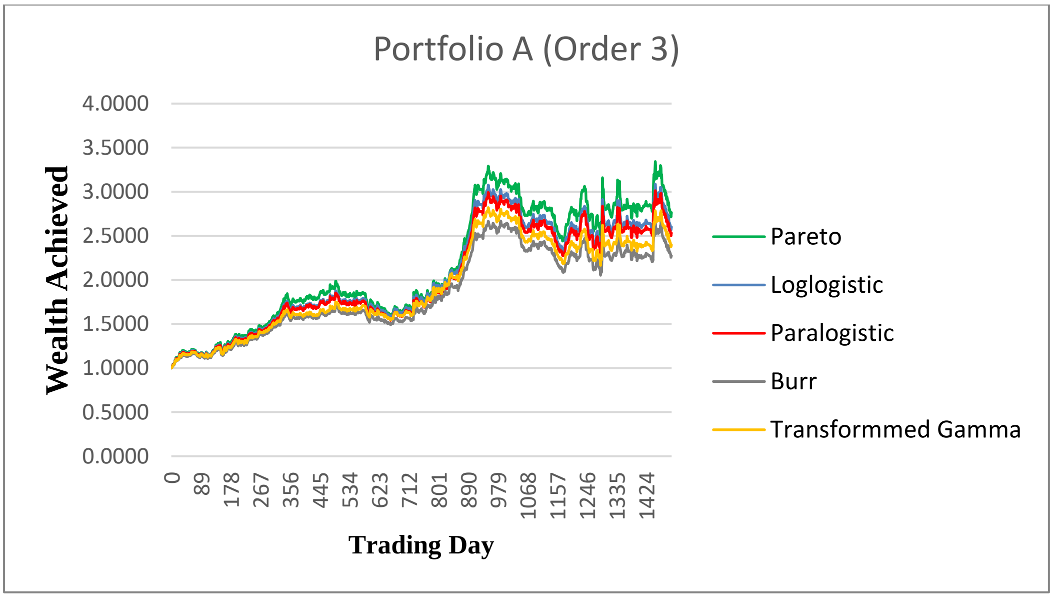


Figure 4.4: The order 1 universal portfolio performance generated by selected distributions with random parameters for portfolio B.

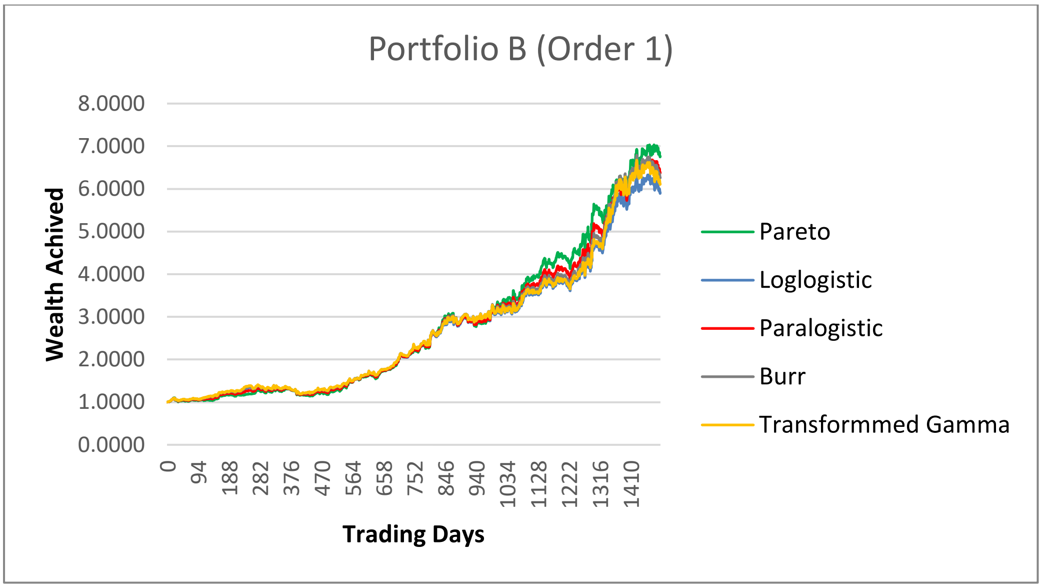


Figure 4.5: The order 2 universal portfolio performance generated by selected distributions with random parameters for portfolio B.

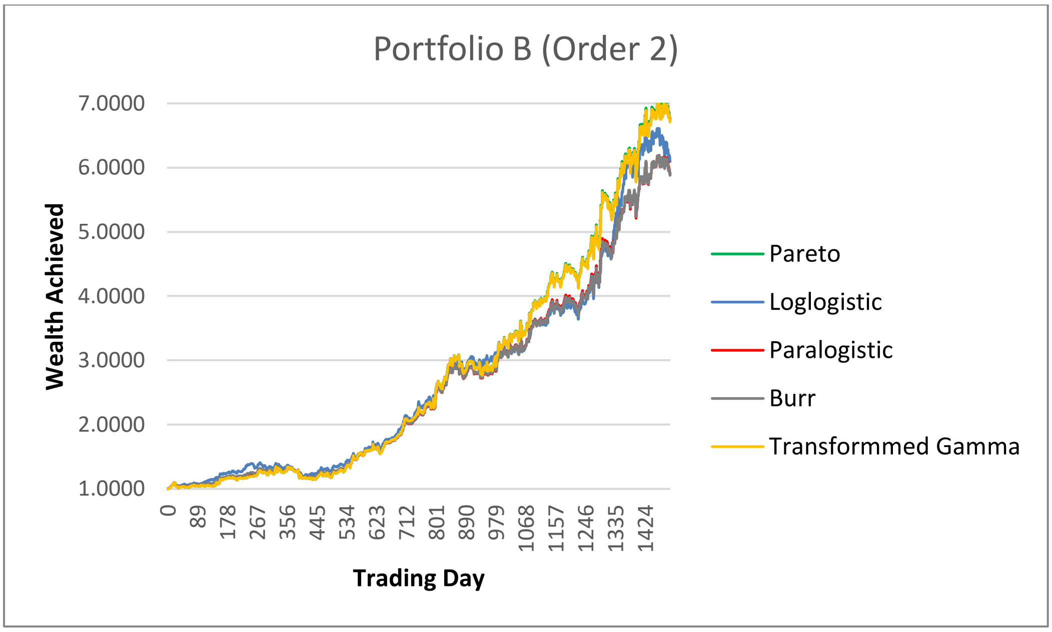


Figure 4.6: The order 3 universal portfolio performance generated by selected distributions with random parameters for portfolio B.

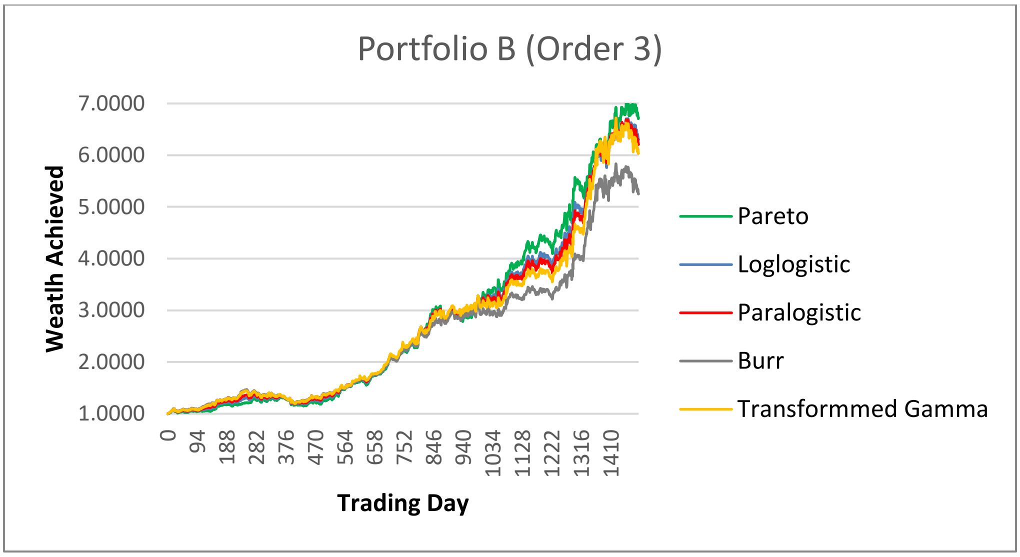


Figure 4.7: The order 1 universal portfolio performance generated by selected distributions with random parameters for portfolio C.

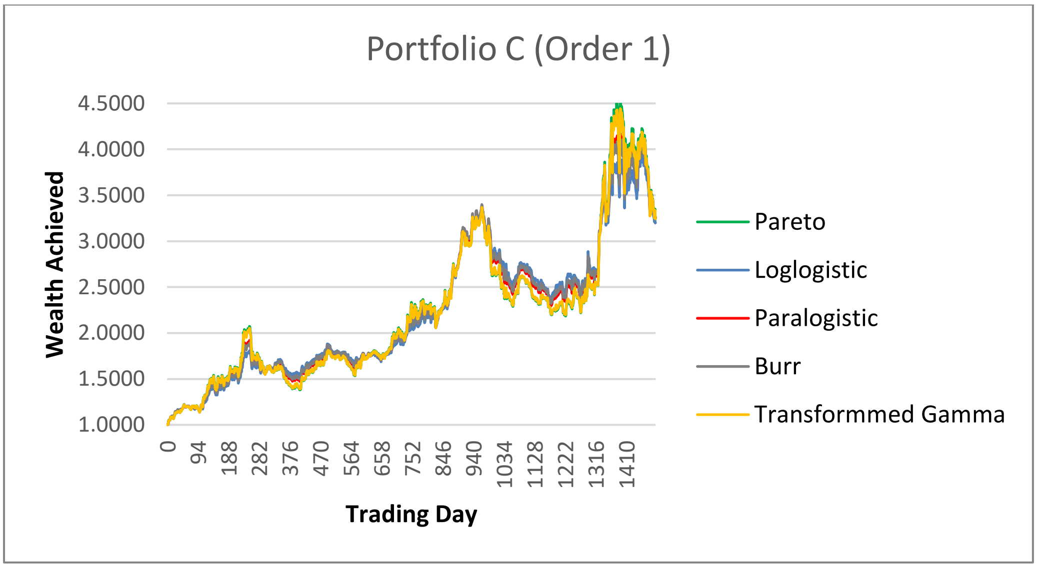


Figure 4.8: The order 2 universal portfolio performance generated by selected distributions with random parameters for portfolio C.

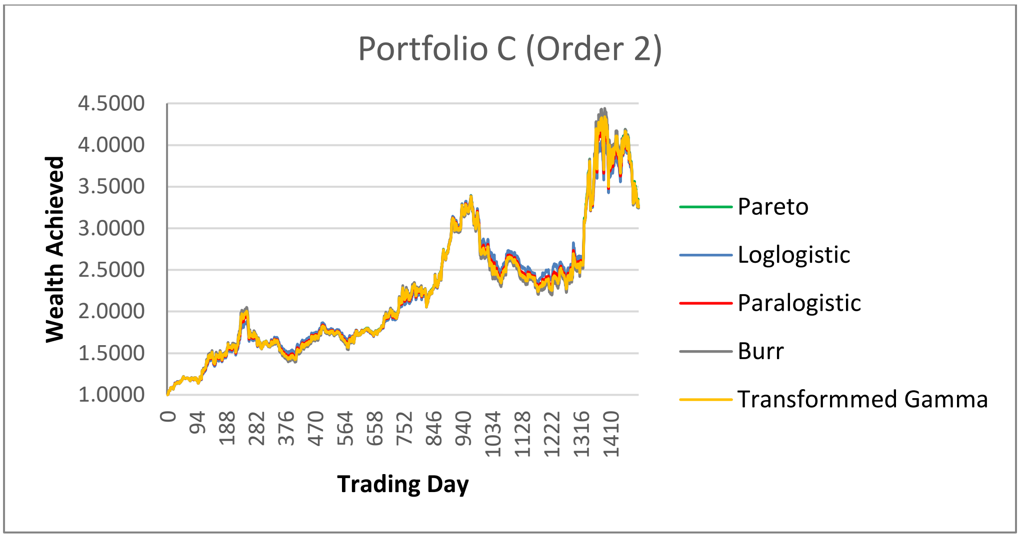
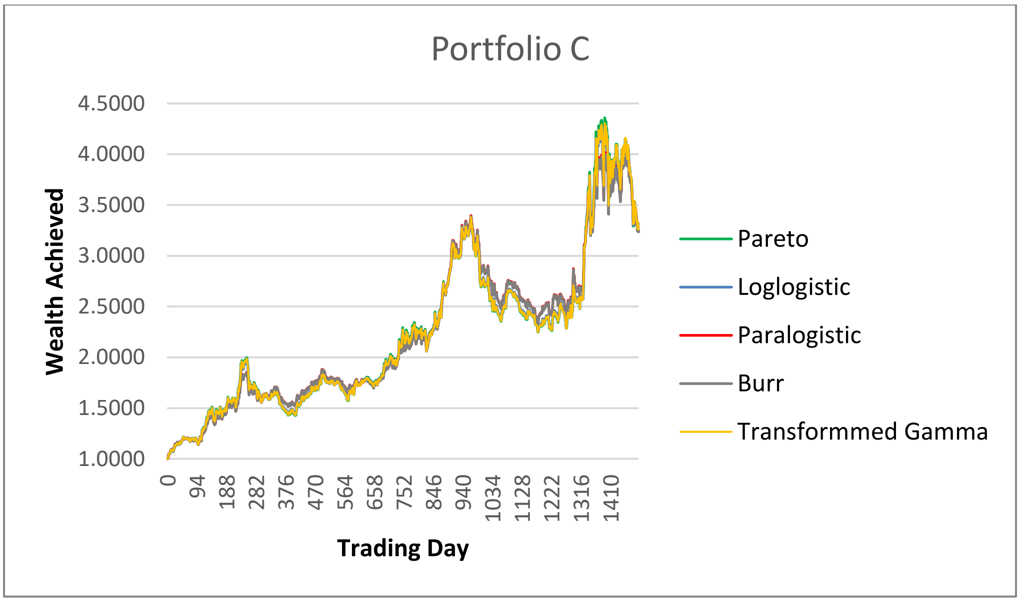


Figure 4.9: The order 3 universal portfolio performance generated by selected distributions with random parameters for portfolio C.



4.2 Parameter's Sensitivity Test

Sensitivity testing on the distributions' parameters was carried out to determine the most significant parameter contributing to generating the maximum wealth. For each portfolio, there was a different parameter that influenced the terminal wealth. In this section, an order 1 universal portfolio with 1500 trading days was generated to observe changes in the parameters on the wealth generated by the portfolio. Each distribution's parameter value was tested by selecting a value from 1 to 3000. Tables 4.68, 4.69, and 4.70 show the highest wealth achieved by each distribution with the selected parameter value.

4.2.1 Pareto Distribution $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3)$

Three parameters in Pareto distribution affect the wealth of Portfolio A. As the value of α_2, α_3 and θ_1 increases, the wealth of Portfolio A generated will increase. While for α_1 , θ_2 and θ_3, they are set to be 50 as they are either need to be as small as possible or have not much effect on the wealth generated by the portfolio. From Figure 4.10, we observe that when α_2, α_3 and $\theta_1$increases together, and there will be a drastic change in the terminal wealth. The highest wealth achieved by Pareto distribution by changing these parameters is 2.3398, which we set the parameter to be $(50, 100, 100, 100, 50, 50)$. From Table 4.15, when θ_1 increases to 100, generating a wealth of 2.0308, the highest wealth compared to the other two parameters. Based on the empirical result, we found that θ_2 and θ_3 need to be small and α_2 has not much effect on the wealth generated by Portfolio A. Therefore, to obtain the highest wealth of the portfolio, we set θ_2 and θ_3 to be 1, and α_2 to be 5 which following the parameter condition of the Pareto distribution, α needs to be more than 4 to generate order 3 universal portfolio. The highest wealth generated by Pareto distribution is 2.7708 within order 2 and order 3 with parameters $(5, 170, 170, 170, 1, 1)$.

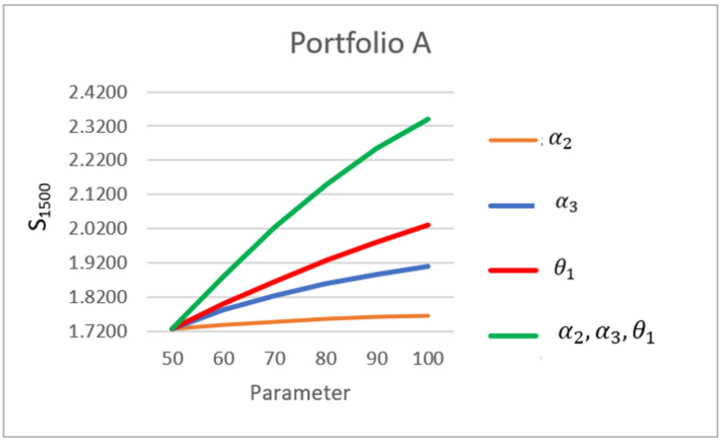


Figure 4.10: The Performance of the universal portfolio generated by the Pareto distribution by changing i) α_2, ii) α_3, iii) θ_1, and iv) $\alpha_2, \alpha_3, \theta_1$ for Portfolio A.

For Portfolio B, two parameters will affect the wealth of Portfolio B. As the value of α_2 and θ_3 increases, the wealth of Portfolio B generated will increase. While for α_1 , α_3, θ_1 and θ_2, they are set to be 50 as they are either need to be as small as possible or have not much effect on the wealth generated by the portfolio. From Figure 4.11, we observe that when α_2 and θ_3 increases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Pareto distribution by changing these parameters is 5.1782, which we set the parameter to be $(50, 100, 50, 50, 50, 100)$. Both α_2 and θ_3 have almost the same influence on the wealth generated. Based on the result, we found that α_3, θ_1 and θ_2 need to be small and α_1 has not much effect on the wealth generated by Portfolio B. Therefore, to obtain the highest wealth of the portfolio, we set α_1 and α_3 to be 100 and 5 respectively, θ_1 and θ_2 to be 1. The highest wealth achieved by Pareto distribution for Portfolio B is 6.7857 with parameters $(100, 100, 5, 1, 1, 100)$ for order 3 universal portfolio.

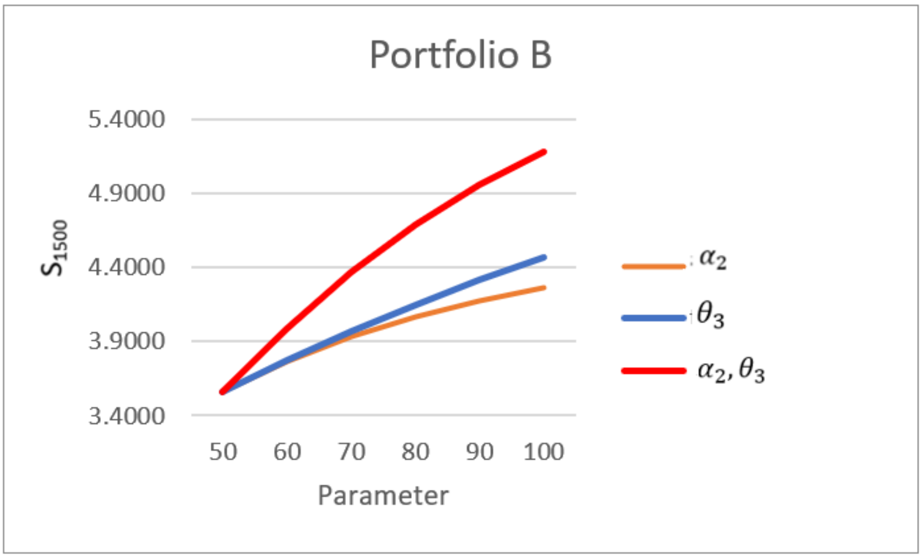


Figure 4.11: The Performance of the universal portfolio generated by the Pareto distribution by changing i) α_2, ii) θ_3And iii) α_2, θ_3 for Portfolio B.

For Portfolio C, two parameters will affect the wealth of Portfolio C. As the value of α_2 and θ_3 increase, the wealth of Portfolio C generated will increase. While for α_1 , α_3, θ_1 and θ_2, they are set to be 50 as they are either need to be as small as possible or have not much effect on the wealth generated by the portfolio. From Figure 4.12, we observe that when α_2 and θ_3 increases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Pareto distribution by changing these parameters is 3.0705, which we set the parameter to be $(50, 100, 50, 50, 50, 100)$. Both α_2 and θ_3 have almost the same influence on the wealth generated. According to the analysis, we found that α_1, θ_1 and θ_2 need to be small and α_3 has not much effect on the wealth generated by Portfolio C. Therefore, to obtain the highest wealth of the portfolio, we set α_1 and α_3 to be 5 and 170 respectively, θ_1 and θ_2 to be 1. The highest wealth that we can generate for Portfolio C is within order 3 with $(5, 170, 170, 1, 1, 100)$ as the parameter of Pareto distribution, which is 4.5595.

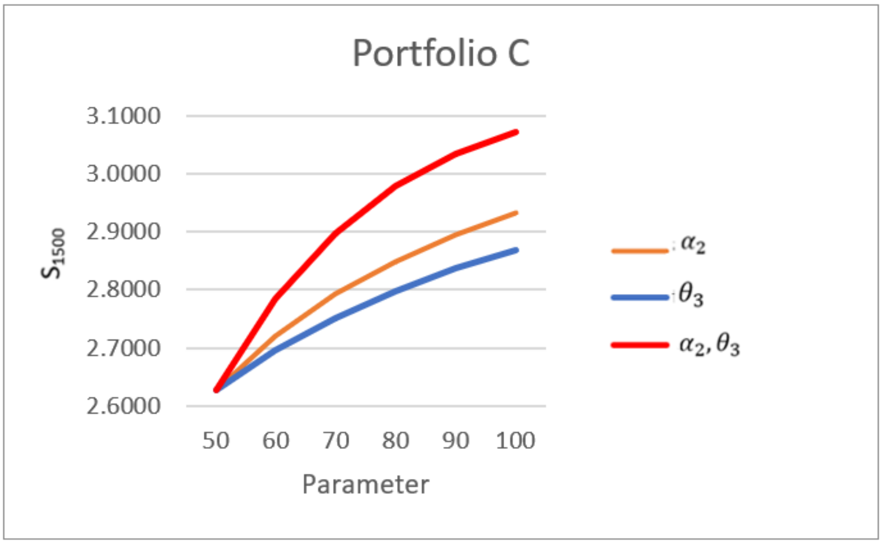


Figure 4.12: The Performance of the universal portfolio generated by the Pareto distribution by changing i) α_2, ii) θ_3, and iii) α_2, θ_3 for Portfolio C.

Table 4.13: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2 value for Portfolio A.

Portfolio	α_2	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	0.3345	0.3320	0.3334	1.7284
	60	0.3594	0.2824	0.3582	1.7406
	70	0.3783	0.2447	0.3770	1.7496
	80	0.3930	0.2153	0.3916	1.7564
	90	0.4048	0.1919	0.4033	1.7617
	100	0.4143	0.1729	0.4128	1.7659

Table 4.14: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_3 value for Portfolio A.

Portfolio	$\boldsymbol{\alpha_3}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	0.3345	0.3320	0.3334	1.7284
	60	0.3596	0.3568	0.2836	1.7834
	70	0.3787	0.3756	0.2457	1.8260
	80	0.3935	0.3903	0.2162	1.8597
	90	0.4054	0.4019	0.1927	1.8868
	100	0.4150	0.4114	0.1736	1.9091

Table 4.15: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected θ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_1}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	0.3345	0.3320	0.3334	1.7284
	60	0.3876	0.3056	0.3068	1.8006
	70	0.4350	0.2819	0.2830	1.8667
	80	0.4773	0.2609	0.2619	1.9268
	90	0.5148	0.2422	0.2430	1.9814
	100	0.5482	0.2255	0.2263	2.0308

Table 4.16: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2, α_3 and θ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\alpha_2}$	$\boldsymbol{\alpha_3}$	$\boldsymbol{\theta_1}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	50	50	0.3345	0.3320	0.3334	1.7284
	60	60	60	0.4453	0.2768	0.2779	1.8812
	70	70	70	0.5443	0.2275	0.2282	2.0250
	80	80	80	0.6271	0.1862	0.1868	2.1504
	90	90	90	0.6938	0.1529	0.1533	2.2549
	100	100	100	0.7466	0.1265	0.1268	2.3398

Table 4.17: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2 value for Portfolio B.

Portfolio	$\boldsymbol{\alpha_2}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
B	50	0.3313	0.3338	0.3348	3.5574
	60	0.3561	0.2839	0.3600	3.7674
	70	0.3749	0.2460	0.3791	3.9330
	80	0.3895	0.2165	0.3940	4.0657
	90	0.4012	0.1929	0.4059	4.1738
	100	0.4107	0.1737	0.4156	4.2634

Table 4.18: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected θ_3 value for Portfolio B.

Porforlio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3313	0.3338	0.3348	3.5574
	60	0.3049	0.3071	0.3879	3.7704
	70	0.2814	0.2833	0.4354	3.9676
	80	0.2604	0.2621	0.4776	4.1486
	90	0.2417	0.2432	0.5151	4.3140
	100	0.2250	0.2264	0.5486	4.4646

Table 4.19: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2 and θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\alpha_2}$	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	50	0.3313	0.3338	0.3348	3.5574
	60	60	0.3253	0.2599	0.4148	3.9897
	70	70	0.3106	0.2055	0.4839	4.3660
	80	80	0.2924	0.1652	0.5424	4.6857
	90	90	0.2733	0.1349	0.5918	4.9541
	100	100	0.2547	0.1119	0.6334	5.1782

Table 4.20: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2 value for Portfolio C.

Portfolio	$\boldsymbol{\alpha_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3359	0.3333	0.3308	2.6276
	60	0.3611	0.2835	0.3554	2.7211
	70	0.3802	0.2456	0.3741	2.7929
	80	0.3952	0.2162	0.3887	2.8492
	90	0.4071	0.1926	0.4003	2.8944
	100	0.4168	0.1735	0.4097	2.9313

Table 4.21: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3359	0.3333	0.3308	2.6276
	60	0.3094	0.3071	0.3835	2.6953
	70	0.2856	0.2837	0.4307	2.7515
	80	0.2645	0.2627	0.4728	2.7980
	90	0.2456	0.2441	0.5103	2.8364
	100	0.2288	0.2274	0.5438	2.8681

Table 4.22: The performance of Order 1 universal portfolio, which generated by the Pareto distribution with selected α_2 and θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\alpha_2}$	$\boldsymbol{\theta_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, $\mathbf{S}_{1500}$
C	50	50	0.3359	0.3333	0.3308	2.6276
	60	60	0.3302	0.2599	0.4099	2.7844
	70	70	0.3157	0.2059	0.4784	2.8977
	80	80	0.2975	0.1657	0.5368	2.9775
	90	90	0.2783	0.1355	0.5862	3.0327
	100	100	0.2596	0.1124	0.6280	3.0705

4.2.2 Loglogistic Distribution $(\boldsymbol{\gamma_1, \gamma_2, \gamma_3, \theta_1, \theta_2, \theta_3})$

Four parameters will affect the wealth of Portfolio A. As the value of θ_1 increases and the value of γ_1, θ_2 and θ_3 decreases, the wealth of Portfolio A generated will increase. While for γ_2 and γ_3, they are set to be 50 as they are either need to be as small as possible or have not much effect on the wealth generated by the portfolio. From Figure 4.13, we observe that when γ_1, θ_2 and, θ_3 decreases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Loglogistic distribution by changing these parameters is 2.4939, which we set the parameter to be $(10, 50, 50, 100, 10, 10)$. From Table 4.23, when θ_1 increases to 100; it can generate a wealth of 1.9617, the highest wealth compared to changing the other three parameters. Therefore, it indicates that θ_1 has more influence on the wealth generated by Portfolio A. Based on the result obtained, we found that γ_2 and γ_3 has not much effect on the wealth generated by Portfolio A. Therefore, to obtain the highest wealth of the portfolio, we set γ_2 and γ_3 to be 100. The highest wealth generated by Loglogistic distribution is 2.7544 within order 3 with parameters $(5, 100, 100, 100, 1, 1)$.

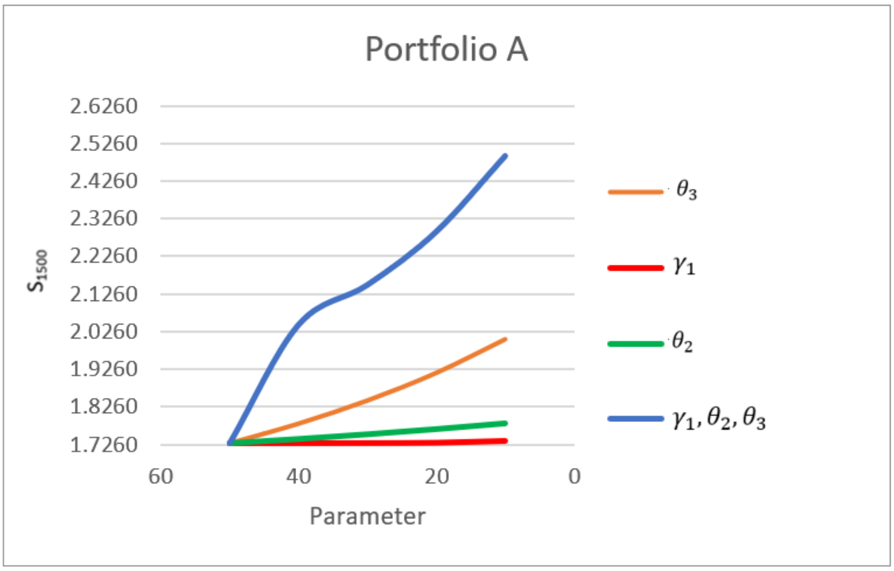


Figure 4.13: The Performance of the universal portfolio generated by the Loglogistic distribution by changing i) θ_3, ii) γ_1, iii) θ_2And iv) $\gamma_1, \theta_2, \theta_3$ for Portfolio A.

Those are four parameters that will affect the wealth of Portfolio B. As the value of γ_2 and θ_3 increases and the value of γ_3 and θ_2 decreases, the wealth of Portfolio B generated will increase. While for γ_1 and θ_1, they are set to be 50 as they are either need to be as small as possible or have not much effect on the wealth generated by the portfolio. From Figure 4.14, we observe that when γ_3 and θ_2 decreases together; there will be a drastic change in the terminal wealth. The highest wealth achieved by Loglogistic distribution by changing these parameters is 5.3998, which we set the parameter to be $(50, 100, 10, 50, 10, 100)$. From Table 4.26, when θ_3 decreases to 10; it can generate a wealth of 2.0070, the highest wealth generated by changing the other three parameters. Therefore, it indicates that θ_3 has more influence on the wealth generated by Portfolio B. According to the analysis, we found that γ_1 and θ_1 has not much effect on the wealth generated by Portfolio B. Therefore, to obtain the highest wealth of the portfolio, we set γ_1 and θ_1 to be 5 and 1, respectively. The highest wealth generated by Loglogistic distribution is 6.7328 within order 3 with parameters $(5, 100, 5, 1, 1, 100)$.

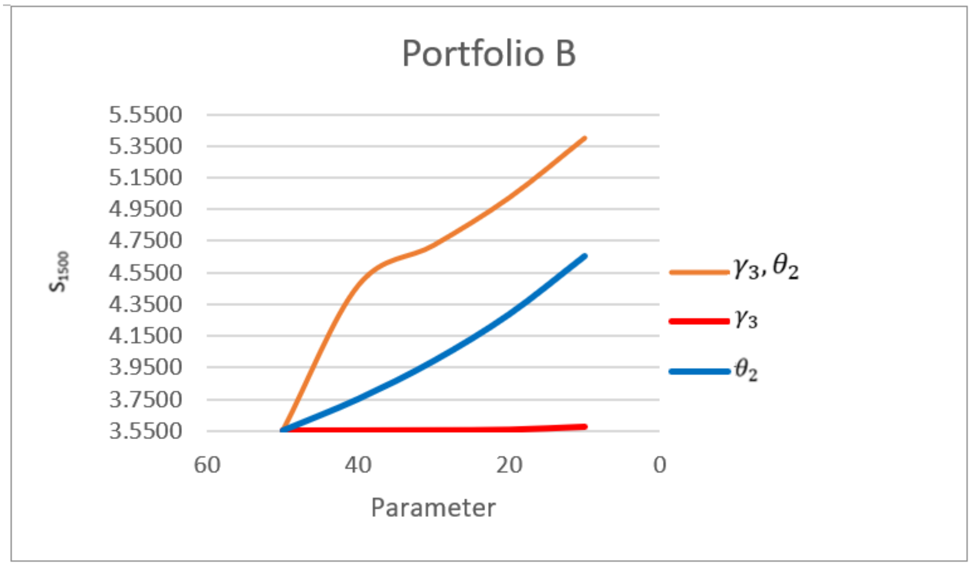


Figure 4.14: The Performance of the universal portfolio generated by the Loglogistic distribution by changing i) γ_3, ii) θ_2And iii) γ_3, θ_2 for Portfolio B.

Four parameters will affect the wealth of Portfolio C. As the value of θ_1 and θ_3 increases and the value of γ_3 and θ_2 decreases, the wealth of Portfolio C generated will increase. While for γ_1 and γ_2, they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Figure 4.15, we observe that when γ_3 and θ_2 decreases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Loglogistic distribution by changing these parameters is 3.1713, which we set the parameter to be $(50, 50, 10, 100, 10, 100)$. From Table 4.31, when θ_2 decrease to 10; it can generate a wealth of 3.0831, the highest wealth generated by changing the other three parameters. Therefore, it indicates that θ_2 has more influence on the wealth generated by the Portfolio C. Hence, the highest wealth can be achieved by setting γ_1 and γ_2 to be 100 respectively for portfolio C, which is 3.2619 within order 3 with parameters $(100, 100, 5, 100, 1, 100)$.

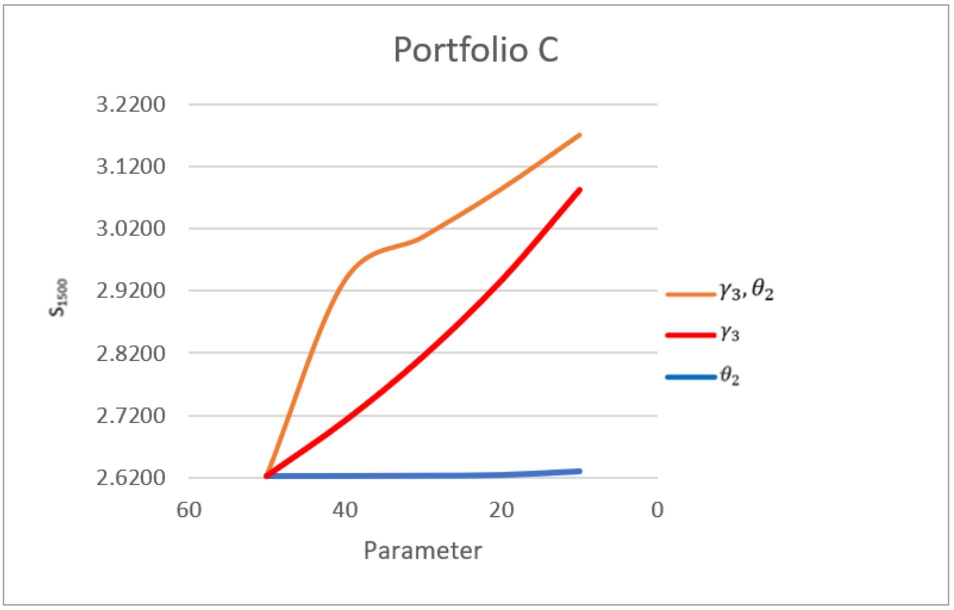


Figure 4.15: The Performance of the universal portfolio generated by the Loglogistic distribution by changing i) γ_3, ii) θ_2, and iii) γ_3, θ_2 for Portfolio C.

Table 4.23: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_1}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	**Terminal Wealth, $\mathbf{S_{1500}}$**
A	50	0.3333	0.3333	0.3333	1.7281
	60	0.3750	0.3125	0.3125	1.7847
	70	0.4118	0.2941	0.2941	1.8357
	80	0.4445	0.2777	0.2778	1.8817
	90	0.4738	0.2631	0.2631	1.9236
	100	0.5001	0.2500	0.2500	1.9617

Table 4.24: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\gamma_1}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	**Terminal Wealth, $\mathbf{S_{1500}}$**
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.3335	0.3333	0.3333	1.7283
	30	0.3338	0.3331	0.3331	1.7287
	20	0.3346	0.3327	0.3327	1.7299
	10	0.3394	0.3303	0.3303	1.7362

Table 4.25: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_2 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.3572	0.2857	0.3572	1.7398
	30	0.3846	0.2307	0.3846	1.7528
	20	0.4167	0.1666	0.4167	1.7671
	10	0.4546	0.0909	0.4546	1.7828

Table 4.26: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.3572	0.3571	0.2857	1.7807
	30	0.3846	0.3846	0.2307	1.8428
	20	0.4167	0.4167	0.1666	1.9169
	10	0.4546	0.4546	0.0909	2.0070

Table 4.27: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_1, θ_2 and θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\gamma_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	50	50	0.3333	1.7281	1.7281	1.7281
	40	40	40	0.5559	0.2221	0.2221	2.0440
	30	30	30	0.6258	0.1871	0.1871	2.1501
	20	20	20	0.7162	0.1419	0.1419	2.2921
	10	10	10	0.8394	0.0803	0.0803	2.4939

Table 4.28: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\gamma_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3334	0.3333	0.3334	3.5518
	70	0.3334	0.3332	0.3334	3.5520
	80	0.3334	0.3332	0.3334	3.5521
	90	0.3334	0.3332	0.3334	3.5522
	100	0.3334	0.3332	0.3334	3.5522

Table 4.29: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3125	0.3125	0.3750	3.7187
	70	0.2941	0.2941	0.4118	3.8705
	80	0.2777	0.2778	0.4445	4.0086
	90	0.2631	0.2631	0.4738	4.1348
	100	0.2500	0.2500	0.5001	4.2505

Table 4.30: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\gamma_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.3333	0.3333	0.3335	3.5520
	30	0.3331	0.3331	0.3338	3.5532
	20	0.3327	0.3327	0.3346	3.5566
	10	0.3303	0.3303	0.3394	3.5752

Table 4.31 The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.3571	0.2857	0.3572	3.7514
	30	0.3846	0.2307	0.3846	3.9923
	20	0.4167	0.1666	0.4167	4.2873
	10	0.4546	0.0909	0.4546	4.6558

Table 4.32: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_3 and θ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\gamma_3}$	$\boldsymbol{\theta_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	50	0.3333	0.3333	0.3333	3.5515
	40	40	0.2630	0.2103	0.5267	4.4691
	30	30	0.2774	0.1663	0.5563	4.7227
	20	20	0.2928	0.1170	0.5902	5.0230
	10	10	0.3053	0.0610	0.6337	5.3998

Table 4.33: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_1 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_1}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3750	0.3125	0.3125	2.6439
	70	0.4118	0.2941	0.2941	2.6621
	80	0.4445	0.2777	0.2777	2.6772
	90	0.4738	0.2631	0.2631	2.6899
	100	0.5001	0.2500	0.2500	2.7007

Table 4.34: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3125	0.3125	0.3750	2.6748
	70	0.2941	0.2941	0.4118	2.7192
	80	0.2778	0.2778	0.4445	2.7567
	90	0.2631	0.2631	0.4737	2.7885
	100	0.2500	0.2500	0.5001	2.8157

Table 4.35: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\gamma_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.3333	0.3333	0.3335	2.6219
	30	0.3331	0.3331	0.3338	2.6223
	20	0.3327	0.3327	0.3346	2.6235
	10	0.3304	0.3304	0.3393	2.6297

Table 4.36: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_2}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.3572	0.2857	0.3571	2.7106
	30	0.3846	0.2307	0.3846	2.8144
	20	0.4167	0.1666	0.4167	2.9368
	10	0.4546	0.0909	0.4546	3.0831

Table 4.37: The performance of Order 1 universal portfolio, which generated by the Loglogistic distribution with selected γ_3 and θ_2 value for Portfolio C.

Portfolio	γ_3	θ_2	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	50	0.3333	0.3333	0.3333	2.6217
	40	40	0.4166	0.1666	0.4168	2.9370
	30	30	0.4344	0.1303	0.4353	3.0070
	20	20	0.4532	0.0906	0.4562	3.0842
	10	10	0.4692	0.0469	0.4840	3.1713

4.2.3 Paralogistic distribution $(\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3)$

Three parameters will affect the wealth of Portfolio A. As the value of θ_1 increases and the value of θ_2 and θ_3 decreases, the wealth of Portfolio A generated will increase. While for α_1, α_2 and α_3, they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Figure 4.16, we observe that when θ_2 and, θ_3 decreases together; there will be a drastic change in the terminal wealth. The highest wealth achieved by Paralogistic distribution by changing these parameters is 2.4839, which we set the parameter to be $(50, 50, 50, 100, 10, 10)$. From Table 4.40, when θ_3 increases to 10; it can generate a wealth of 2.2361, which is the highest wealth compared to changing the other two parameters. Therefore, it indicates that θ_3 has more influence on the wealth generated by Portfolio A. Therefore, to obtain the highest wealth of the portfolio, we set α_1, α_2 and α_3 to be 4. The highest wealth that can generate by Paralogistic distribution is 2.7702 within order 2 with parameters $(4, 4, 4, 3500, 1, 1)$.

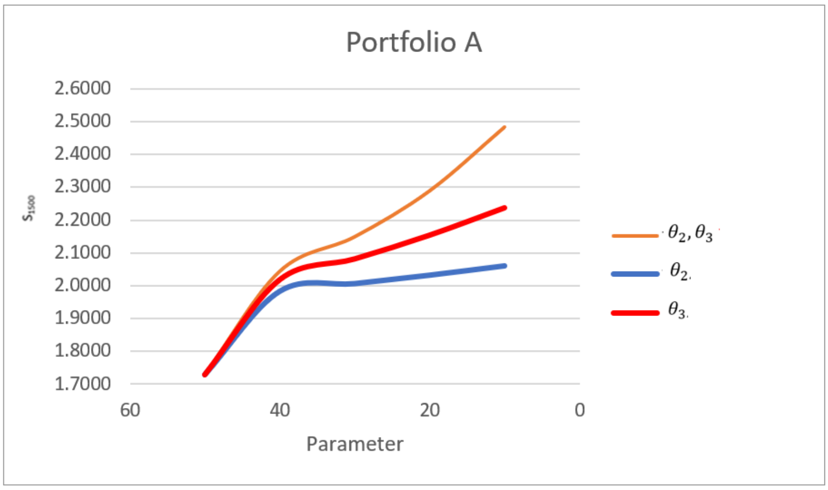


Figure 4.16: The Performance of the universal portfolio generated by the Paralogistic distribution by changing i) θ_2, ii) θ_3 and iii) θ_2, θ_3 for Portfolio A.

Three parameters will affect the wealth of Portfolio B. As the value of θ_3 increases and the value of θ_1 and θ_2 decreases, the wealth of Portfolio B generated will increase. While for α_1, α_2 and α_3, they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Figure 4.17, we observe that when θ_1 and θ_2 decreases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Paralogistic distribution by changing these parameters is 2.4839, which we set the parameter to be $(50, 50, 50, 10, 10, 100)$. From Table 4.43, we noticed that when θ_1 decreases to 10 by itself; the terminal wealth will decrease. Therefore, it indicates that θ_1 and θ_2 need to decrease together to increase the wealth generated by the Portfolio B. Therefore, to obtain the highest wealth of the portfolio, we set α_1, α_2 and α_3 to be 4. The highest wealth generated by Paralogistic distribution is 6.7710 within order 2 with parameters $(4, 4, 4, 1, 1, 500)$.

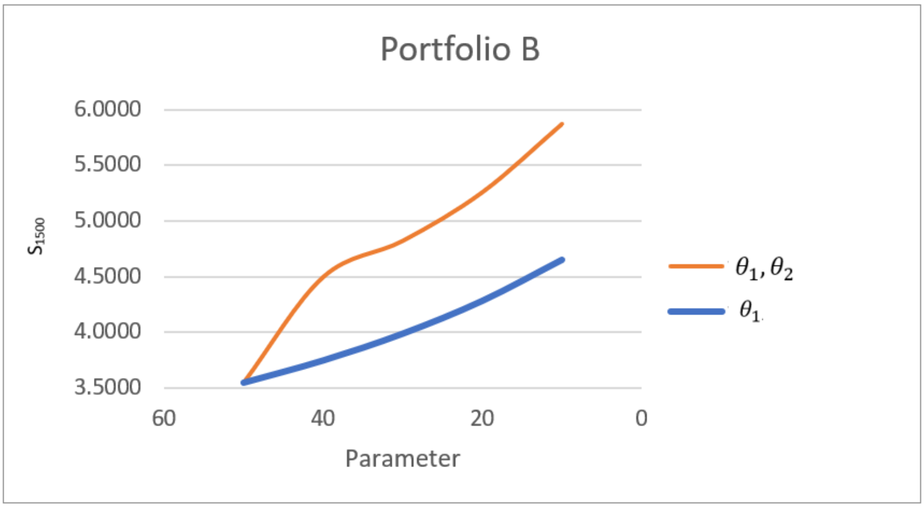


Figure 4.17: The Performance of the universal portfolio generated by the Paralogistic distribution by changing i) θ_1 and ii) θ_1, θ_2 for Portfolio B.

Three parameters will affect the wealth of Portfolio C. As the value of θ_3 increases and the value of α_2 and θ_2 decreases, the wealth of Portfolio C generated will increase. While for α_1, α_3 and θ_1, they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Figure 4.18, we observe that when α_2 and θ_2 decreases simultaneously; there will be a drastic change in the terminal wealth. The highest wealth achieved by Loglogistic distribution by changing these parameters is 3.1737, which we set the parameter to be $(50, 10, 50, 50, 10, 100)$. From Table 4.47, when θ_2 decreases to 10, it can generate a wealth of 3.1559, which is comparable to the wealth generated by decreasing both α_2 and θ_2 together. Therefore, it indicates that θ_2 has more influence on the wealth generated by the Portfolio C. To obtain the highest wealth of the portfolio, we set α_1, α_3 and θ_1 to be 100 respectively. The highest wealth generated by Paralogistic distribution is 3.2598 within order 2 with parameters $(5, 5, 150, 100, 1, 100)$.

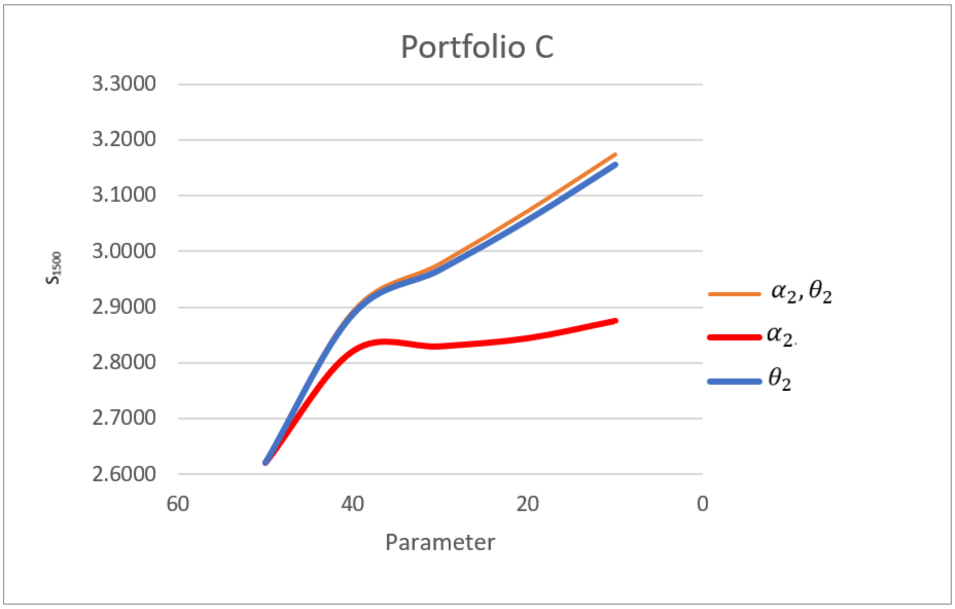


Figure 4.18: The Performance of the universal portfolio generated by the Paralogistic distribution by changing i) α_2, ii) θ_2 and iii) α_2, θ_2 for Portfolio C.

Table 4.38: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_1 value for Portfolio A.

Portfolio	θ_1	**b**$_{1500,1}$	**b**$_{1500,2}$	**b**$_{1500,3}$	**Terminal Wealth,** S_{1500}
A	50	0.3333	0.3333	0.3333	1.7281
	60	0.3750	0.3125	0.3125	1.7847
	70	0.4118	0.2941	0.2941	1.8357
	80	0.4445	0.2778	0.2778	1.8817
	90	0.4737	0.2631	0.2631	1.9235
	100	0.5000	0.2500	0.2500	1.9616

Table 4.39: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_2 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5264	0.2105	0.2631	1.9830
	30	0.5556	0.1666	0.2778	2.0065
	20	0.5883	0.1176	0.2941	2.0323
	10	0.6251	0.0625	0.3125	2.0608

Table 4.40: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5264	0.2631	0.2105	2.0174
	30	0.5556	0.2778	0.1666	2.0806
	20	0.5883	0.2941	0.1176	2.1528
	10	0.6251	0.3125	0.0625	2.2361

Table 4.41: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_2 and θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
A	50	50	0.3333	0.3333	1.7281	1.7281
	40	40	0.5556	0.2222	0.2222	2.0437
	30	30	0.6251	0.1875	0.1875	2.1491
	20	20	0.7144	0.1428	0.1428	2.2892
	10	10	0.8334	0.0833	0.0833	2.4839

Table 4.42: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3125	0.3125	0.3750	3.7187
	70	0.2941	0.2941	0.4118	3.8704
	80	0.2778	0.2778	0.4445	4.0085
	90	0.2631	0.2631	0.4737	4.1347
	100	0.2500	0.2500	0.5000	4.2503

Table 4.43: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_1 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_1}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.2857	0.3571	0.3571	3.5405
	30	0.2308	0.3846	0.3846	3.5194
	20	0.1666	0.4167	0.4167	3.4836
	10	0.0909	0.4546	0.4546	3.4262

Table 4.44: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_2}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.3571	0.2857	0.3571	3.7514
	30	0.3846	0.2308	0.3846	3.9922
	20	0.4167	0.1666	0.4167	4.2872
	10	0.4546	0.0909	0.4546	4.6557

Table 4.45: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_1 and θ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
B	50	50	0.3333	0.3333	0.3333	3.5515
	40	40	0.2222	0.2222	0.5556	4.5007
	30	30	0.1875	0.1875	0.6251	4.8257
	20	20	0.1428	0.1428	0.7144	5.2623
	10	10	0.0833	0.0833	0.8334	5.8753

Table 4.46: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_3}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3125	0.3125	0.3750	2.6748
	70	0.2941	0.2941	0.4118	2.7192
	80	0.2778	0.2778	0.4445	2.7566
	90	0.2631	0.2631	0.4737	2.7884
	100	0.2500	0.2500	0.5000	2.8157

Table 4.47: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_2}$	**b$_{1500,1}$**	**b$_{1500,2}$**	**b$_{1500,3}$**	**Terminal Wealth, S$_{1500}$**
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.2631	0.2105	0.5264	2.8879
	30	0.2778	0.1666	0.5556	2.9679
	20	0.2941	0.1176	0.5883	3.0568
	10	0.3125	0.0625	0.6251	3.1559

Table 4.48: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected α_2 value for Portfolio C.

Portfolio	$\boldsymbol{\alpha_2}$	**b$_{1500,1}$**	**b$_{1500,2}$**	**b$_{1500,3}$**	**Terminal Wealth, S$_{1500}$**
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.2510	0.2469	0.5021	2.8213
	30	0.2526	0.2423	0.5052	2.8298
	20	0.2552	0.2343	0.5105	2.8443
	10	0.2608	0.2174	0.5217	2.8753

Table 4.49: The performance of Order 1 universal portfolio, which generated by the Paralogistic distribution with selected α_2 and θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\alpha_2}$	$\boldsymbol{\theta_2}$	**b$_{1500,1}$**	**b$_{1500,2}$**	**b$_{1500,3}$**	**Terminal Wealth, S$_{1500}$**
C	50	50	0.3333	0.3333	0.3333	2.6217
	40	40	0.2640	0.2078	0.5282	2.8929
	30	30	0.2797	0.1609	0.5594	2.9783
	20	20	0.2970	0.1090	0.5940	3.0723
	10	10	0.3158	0.0525	0.6317	3.1737

4.2.4 Burr Distribution $(\boldsymbol{\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3, \gamma_1, \gamma_2, \gamma_3})$

Three parameters will affect the wealth of Portfolio A. As the value of θ_1 increases and the value of θ_2 and θ_3 decreases, the wealth of Portfolio A generated will increase. While for $\alpha_1, \alpha_2, \alpha_3, \gamma_1, \gamma_2$ and γ_3 , they are set to be 50 as they have not much effect on the wealth generated by the portfolio. The highest wealth achieved by Burr distribution by decreasing θ_2 and θ_3 together is 2.4839, which we set the parameter to be $(50, 50, 50, 100, 10, 10, 50, 50, 50)$. We also observed that when θ_3 decreases to 10, generating a wealth of 2.2361, which is the highest wealth compared to changing the other two parameters. Therefore, it indicates that θ_3 has more influence on the wealth generated by Portfolio A. Therefore, to obtain the highest wealth of the portfolio, we set

α_1, α_2, α_3 and γ_1 to be 1 while γ_2 and γ_3 to be 100. The highest wealth generated by Burr distribution is 2.7475 within order 2 with parameters $(1, 1, 1, 100, 1, 1, 5, 100, 100)$.

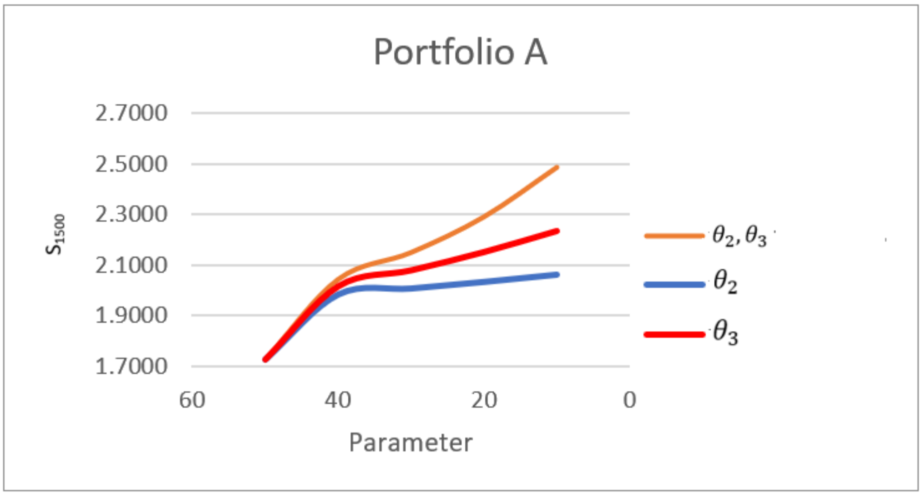


Figure 4.19: The Performance of the universal portfolio generated by the Burr distribution by changing i) θ_2, ii) θ_3 and iii) θ_2, θ_3 for Portfolio A.

Four parameters will affect the wealth of Portfolio B. As the value of θ_3 and γ_3 increases and the value of θ_1 and θ_2 decreases, the wealth of Portfolio B generated will increase. While for $\alpha_1, \alpha_2, \alpha_3, \gamma_1$ and γ_2 they are set to be 50 as they do not affect the wealth generated by the portfolio. From Figure 4.20, we observe that when θ_3 and γ_3 increases together, the highest wealth achieved by Burr distribution is 4.2995, which we set the parameter to be $(50, 50, 50, 50, 50, 100, 50, 50, 100)$. From Figure 4.21, we observe that when θ_1 and θ_2 decrease together, the highest wealth achieved by Burr distribution is 5.9073, which we set the parameter to be $(50, 50, 50, 10, 10, 100, 50, 50, 100)$. From Table 4.58, we noticed that when θ_2 decreases to 10 by itself; it will generate a wealth of 5.4067, which is the highest compare to by changing the other parameters. Therefore, it indicates that θ_2 is sensitive in affecting the wealth of Portfolio B. Therefore, to obtain the highest wealth of the portfolio, we set α_1, α_2 and α_3 to be 1 while γ_1 and γ_2 to be 5. The highest wealth that able to generate by Burr distribution is 6.7621 within order 1 with parameters $(1, 1, 1, 1, 1, 1, 500, 5, 5, 100)$.

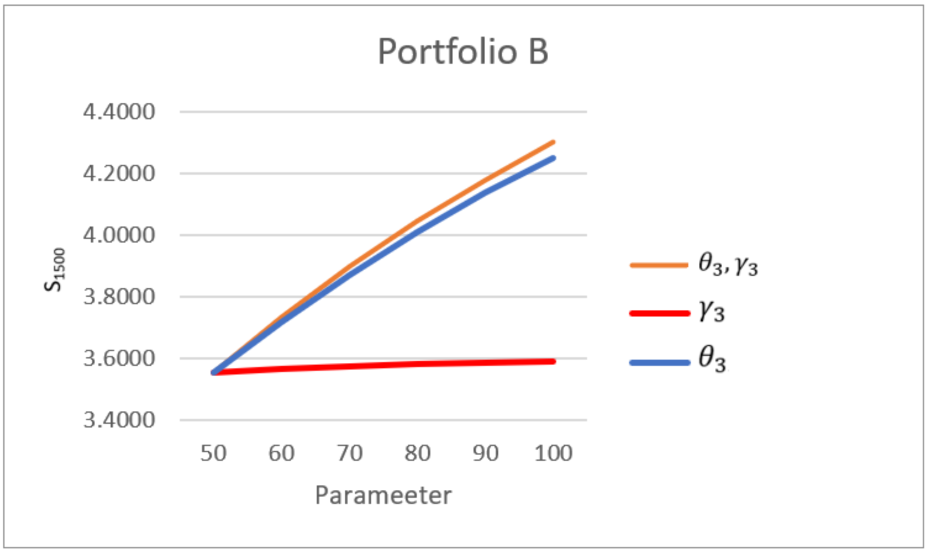


Figure 4.20: The Performance of the universal portfolio generated by the Burr distribution by changing i) θ_3, ii) γ_3 and iii) θ_3, γ_3 for Portfolio B.

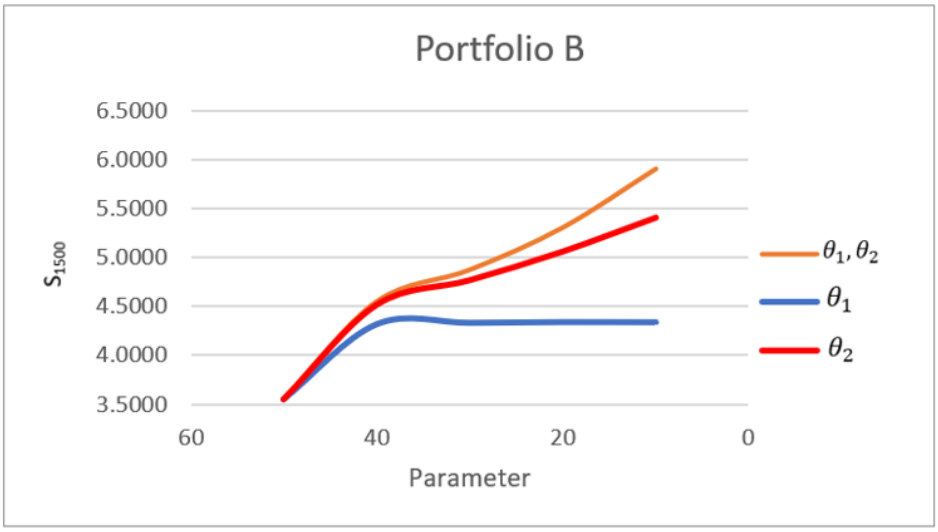


Figure 4.21: The Performance of the universal portfolio generated by the Burr distribution by changing i) θ_1, ii) θ_2 and iii) θ_1, θ_2 for Portfolio B.

Four parameters will affect the wealth of Portfolio C. As the value of θ_1 and θ_3 increases and the value of θ_2 and γ_2 decreases, the wealth of Portfolio C generated will increase. While for $\alpha_1, \alpha_2, \alpha_3$, γ_1 and γ_3 , they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Table 4.62, we observe that when increasing both θ_1 and θ_3, the highest wealth achieved by Burr distribution is 2.8729, which we set the parameter to be (50, 50, 50, 100, 50, 100, 50,50, 50). From Table 4.56, we observe that when θ_2 and γ_2 decreases together, the highest wealth achieved by Burr distribution is 3.1937, which we set the parameter to be (50, 50, 50, 100, 10, 100, 50, 10, 50). From Table 4.63, when θ_2 decreases to 10; it can generate a wealth of 3.1672, which is higher than changing the other parameters. Therefore, it indicates that θ_2 has more influence on the wealth generated by the Portfolio C. To obtain the highest wealth of the portfolio, we set α_1, α_2 and α_3 to be 150 while γ_1 and γ_3 to be 100. The highest wealth generated by Burr distribution is 3.2584 within order 1 with parameters (150, 150, 150, 200, 1, 200, 100, 5, 100).

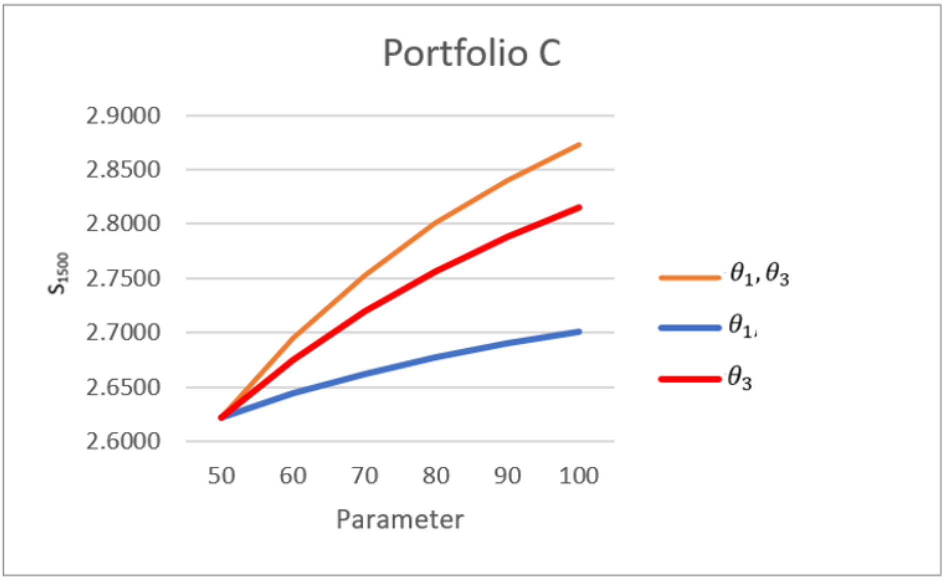


Figure 4.22: The Performance of the universal portfolio generated by the Burr distribution by changing i) θ_1, ii) θ_3 and iii) θ_1, θ_3 for Portfolio C.

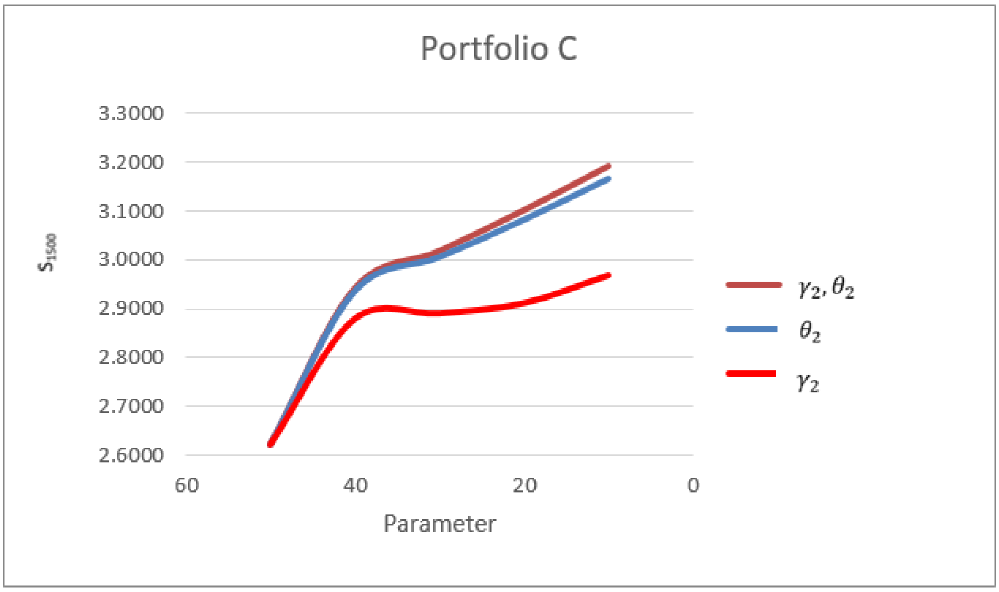


Figure 4.23: The Performance of the universal portfolio generated by the Burr distribution by changing i) γ_2, ii) θ_2 and iii) γ_2, θ_2 for Portfolio C.

Table 4.50: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_1}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	0.3333	0.3333	0.3333	1.7281
	60	0.3750	0.3125	0.3125	1.7847
	70	0.4118	0.2941	0.2941	1.8357
	80	0.4445	0.2778	0.2778	1.8817
	90	0.4737	0.2631	0.2631	1.9235
	100	0.5000	0.2500	0.2500	1.9616

Table 4.51: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_2 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	**$b_{1500,1}$**	**$b_{1500,2}$**	**$b_{1500,3}$**	**Terminal Wealth, S_{1500}**
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5264	0.2105	0.2631	1.9830
	30	0.5556	0.1666	0.2778	2.0065
	20	0.5883	0.1176	0.2941	2.0323
	10	0.6251	0.0625	0.3125	2.0608

Table 4.52: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_3}$	$\mathbf{b_{1500,\,1}}$	$\mathbf{b_{1500,\,2}}$	$\mathbf{b_{1500,\,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5264	0.2631	0.2105	2.0174
	30	0.5556	0.2778	0.1666	2.0806
	20	0.5883	0.2941	0.1176	2.1528
	10	0.6251	0.3125	0.0625	2.2361

Table 4.53: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_2 and θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\mathbf{b_{1500,\,1}}$	$\mathbf{b_{1500,\,2}}$	$\mathbf{b_{1500,\,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
A	50	50	0.3333	0.3333	1.7281	1.7281
	40	40	0.5556	0.2222	0.2222	2.0437
	30	30	0.6251	0.1875	0.1875	2.1491
	20	20	0.7144	0.1428	0.1428	2.2892
	10	10	0.8334	0.0833	0.0833	2.4839

Table 4.54: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_3}$	$\mathbf{b_{1500,\,1}}$	$\mathbf{b_{1500,\,2}}$	$\mathbf{b_{1500,\,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3125	0.3125	0.3750	3.7187
	70	0.2941	0.2941	0.4118	3.8704
	80	0.2778	0.2778	0.4445	4.0085
	90	0.2631	0.2631	0.4737	4.1347
	100	0.2500	0.2500	0.5000	4.2503

Table 4.55: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\gamma_3}$	$\mathbf{b_{1500,\,1}}$	$\mathbf{b_{1500,\,2}}$	$\mathbf{b_{1500,\,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3317	0.3317	0.3366	3.5645
	70	0.3305	0.3305	0.3390	3.5739
	80	0.3296	0.3296	0.3408	3.5809
	90	0.3289	0.3289	0.3422	3.5865
	100	0.3284	0.3284	0.3433	3.5909

Table 4.56: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_3 and γ_3 value for Portfolio B.

Portfolio	θ_3	γ_3	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	50	0.3333	0.3333	3.5515	3.5515
	60	60	0.3108	0.3108	0.3785	3.7328
	70	70	0.2910	0.2910	0.4179	3.8961
	80	80	0.2737	0.2737	0.4527	4.0437
	90	90	0.2582	0.2582	0.4835	4.1776
	100	100	0.2444	0.2444	0.5111	4.2995

Table 4.57: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_1 value for Portfolio B.

Portfolio	θ_1	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.2056	0.2570	0.5374	4.3186
	30	0.1626	0.2709	0.5665	4.3331
	20	0.1146	0.2865	0.5990	4.3409
	10	0.0608	0.3039	0.6354	4.3392

Table 4.58: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_2 value for Portfolio B.

Portfolio	θ_2	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.2570	0.2056	0.5374	4.5164
	30	0.2709	0.1626	0.5665	4.7673
	20	0.2865	0.1146	0.5990	5.0605
	10	0.3039	0.0608	0.6354	5.4067

Table 4.59: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_1 and θ_2 value for Portfolio B.

Portfolio	θ_1	θ_2	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
B	50	50	0.3333	0.3333	0.3333	3.5515
	40	40	0.2167	0.2167	0.5665	4.5508
	30	30	0.1823	0.1823	0.6354	4.8750
	20	20	0.1384	0.1384	0.7233	5.3071
	10	10	0.0803	0.0803	0.8394	5.9073

Table 4.60: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_1 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_1}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3750	0.3125	0.3125	2.6439
	70	0.4118	0.2941	0.2941	2.6621
	80	0.4445	0.2778	0.2778	2.6772
	90	0.4737	0.2631	0.2631	2.6899
	100	0.5000	0.2500	0.2500	2.7006

Table 4.61: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3125	0.3125	0.3750	2.6748
	70	0.2941	0.2941	0.4118	2.7192
	80	0.2778	0.2778	0.4445	2.7566
	90	0.2631	0.2631	0.4737	2.7884
	100	0.2500	0.2500	0.5000	2.8157

Table 4.62: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_1 and θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_3}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	50	0.3333	0.3333	0.3333	2.6217
	60	60	0.3529	0.2941	0.3529	2.6949
	70	70	0.3684	0.2631	0.3684	2.7531
	80	80	0.3810	0.2381	0.3810	2.8005
	90	90	0.3913	0.2174	0.3913	2.8398
	100	100	0.4000	0.2000	0.4000	2.8729

Table 4.63: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_2}$	$b_{1500,1}$	$b_{1500,2}$	$b_{1500,3}$	Terminal Wealth, S_{1500}
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.4167	0.1666	0.4167	2.9368
	30	0.4348	0.1304	0.4348	3.0065
	20	0.4546	0.0909	0.4546	3.0830
	10	0.4762	0.0476	0.4762	3.1672

Table 4.64: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected γ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\gamma_2}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.4018	0.1965	0.4018	2.8797
	30	0.4046	0.1907	0.4046	2.8906
	20	0.4102	0.1797	0.4102	2.9118
	10	0.4250	0.1499	0.4250	2.9689

Table 4.65: The performance of Order 1 universal portfolio, which generated by the Burr distribution with selected γ_2 and θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\gamma_2}$	$\boldsymbol{\theta_2}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	Terminal Wealth, $\mathbf{S_{1500}}$
C	50	50	0.3333	0.3333	0.3333	2.6217
	40	40	0.4182	0.1636	0.4182	2.9426
	30	30	0.4381	0.1239	0.4381	3.0192
	20	20	0.4597	0.0805	0.4597	3.1032
	10	10	0.4830	0.0340	0.4830	3.1937

4.2.5 Transformed Gamma Distribution $(\boldsymbol{\alpha_1, \alpha_2, \alpha_3, \theta_1, \theta_2, \theta_3, \tau_1, \tau_2, \tau_3})$

Three parameters will affect the wealth of Portfolio A. As the value of θ_1 increases and the value of θ_2 and θ_3 decreases, the wealth of Portfolio A generated will increase. While for $\alpha_1, \alpha_2, \alpha_3, \tau_1, \tau_2$ and τ_3 , they are set to be 50 as they have not much effect on the wealth generated by the portfolio. The highest wealth was achieved by Transformed Gamma distribution by decreasing both θ_2 and θ_3 together is 2.4838, which we set the parameter to be $(50, 50, 50, 100, 10, 10, 50, 50, 50)$. We also observed that when θ_3 increases to 10; it can generate a wealth of 2.2360, the highest wealth compared to changing the other two parameters. Therefore, it indicates that θ_3 has more influence on the wealth generated by Portfolio A. Therefore, to obtain the highest wealth of the portfolio, we set α_1 and α_2 to be 150, α_3 and τ_1 to be 1 while τ_2 and τ_3 to be 100. The highest wealth that able to generate by Transformed Gamma distribution is 2.7708 within order 1 and order 2 with parameters $(150, 150, 1, 500, 1, 1, 1, 100, 100)$.

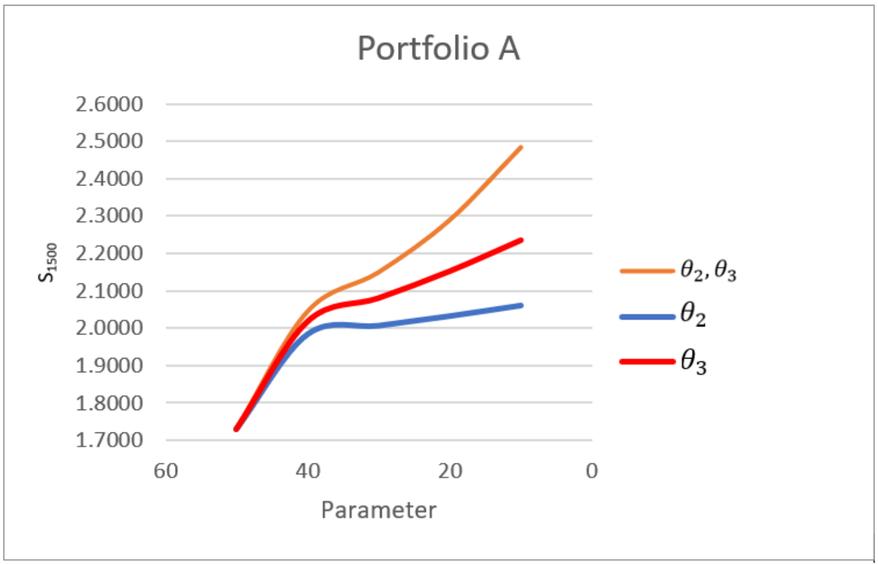


Figure 4.24: The Performance of the universal portfolio generated by the Transformed Gamma distribution by changing i) θ_2, ii) θ_3 and iii) θ_2, θ_3 for Portfolio A.

Three parameters will affect the wealth of Portfolio B. As the value of θ_3 increases and the value of θ_2 and τ_3 decreases, the wealth of Portfolio B generated will increase. While for $\alpha_1, \alpha_2, \alpha_3, \theta_1, \tau_1$ and τ_2 they are set to be 50 as they do not affect the wealth generated by the portfolio. From Figure 4.25, we observe that when θ_2 and τ_3 decreases together, the highest wealth achieved by Transformed Gamma distribution is 5.6494, which we set the parameter to be (50, 50, 50, 50, 10, 100, 50, 50, 10). From Table 4.72, we noticed that when θ_2 decreases to 10 by itself; it will generate a wealth of 4.9804, which is the highest compare to by changing the other parameters. Therefore, it indicates that θ_2 has more influence in affecting the wealth of Portfolio B. Therefore, to obtain the highest wealth of the portfolio, we set α_1, α_2, θ_1, τ_1 and τ_2 to be 1, while α_3 to be 150. The highest wealth generated by Transformed Gamma distribution is 6.7859 within order 1 and order 2 with parameters (1, 1, 150, 1, 1, 1000, 1, 1, 1).

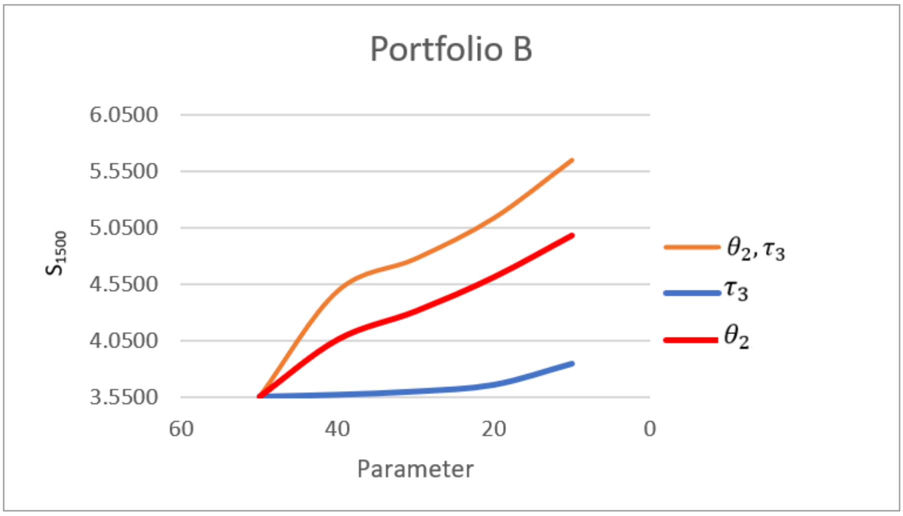


Figure 4.25: The Performance of the universal portfolio generated by the Transformed Gamma distribution by changing i) θ_2, ii) τ_3 and iii) θ_2, τ_3 for Portfolio B.

Three parameters will affect the wealth of Portfolio C. As the value of θ_3 increases and the value of θ_2 and τ_3 decreases, the wealth of Portfolio C generated will increase. While for $\alpha_1, \alpha_2, \alpha_3, \theta_1, \gamma_1$ and γ_2 , they are set to be 50 as they have not much effect on the wealth generated by the portfolio. From Table 4.77, we observe that when decreasing both θ_2 and τ_3, the highest wealth achieved by Transformed Gamma distribution is 3.1626, which we set the parameter to be $(50, 50, 50, 50, 10, 100, 50, 50, 10)$. From Table 4.75, when θ_2 decreases to 10; it can generate a wealth of 3.083, higher than changing the other parameters. Therefore, it indicates that θ_2 has more influence on the wealth generated by the Portfolio C. To obtain the highest wealth of the portfolio, we set α_2, α_3, θ_1 and γ_2 to be 1, while α_1 and γ_1 to be 100. The highest wealth that able to generate by Transformed Gamma distribution is 3.2721 within order 1 with parameters $(100, 1, 1, 1, 1, 100, 1, 100, 1)$.

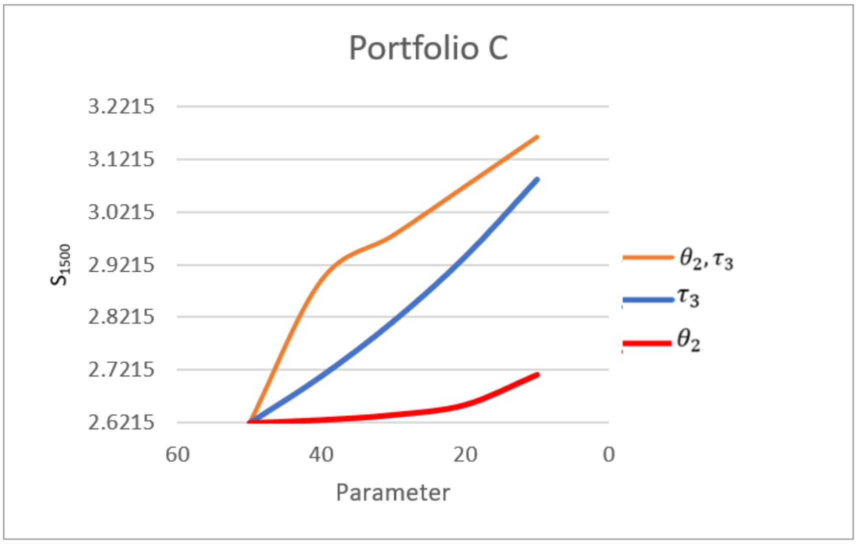


Figure 4.26: The Performance of the universal portfolio generated by the Transformed Gamma distribution by changing i) θ_2, ii) τ_3 and iii) θ_2, τ_3 for Portfolio C.

Table 4.66: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_1 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_1}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	**Terminal Wealth, $\mathbf{S_{1500}}$**
A	50	0.3333	0.3333	0.3333	1.7281
	60	0.3750	0.3125	0.3125	1.7847
	70	0.4118	0.2941	0.2941	1.8356
	80	0.4444	0.2778	0.2778	1.8817
	90	0.4737	0.2632	0.2632	1.9235
	100	0.5000	0.2500	0.2500	1.9616

Table 4.67: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$\mathbf{b_{1500,1}}$	$\mathbf{b_{1500,2}}$	$\mathbf{b_{1500,3}}$	**Terminal Wealth, $\mathbf{S_{1500}}$**
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5263	0.2105	0.2632	1.9830
	30	0.5556	0.1667	0.2778	2.0064
	20	0.5882	0.1176	0.2941	2.0322
	10	0.6250	0.0625	0.3125	2.0607

Table 4.68: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_3}$	**b1500, 1**	**b1500, 2**	**b1500, 3**	**Terminal Wealth, S1500**
A	50	0.3333	0.3333	0.3333	1.7281
	40	0.5263	0.2632	0.2105	2.0173
	30	0.5556	0.2778	0.1667	2.0805
	20	0.5882	0.2941	0.1176	2.1527
	10	0.6250	0.3125	0.0625	2.2360

Table 4.69: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 and θ_3 value for Portfolio A.

Portfolio	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	**b1500, 1**	**b1500, 2**	**b1500, 3**	**Terminal Wealth, S1500**
A	50	50	0.3333	0.3333	1.7281	1.7281
	40	40	0.5556	0.2222	0.2222	2.0436
	30	30	0.6250	0.1875	0.1875	2.1490
	20	20	0.7143	0.1429	0.1429	2.2891
	10	10	0.8333	0.0833	0.0833	2.4838

Table 4.70: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_3}$	**b1500, 1**	**b1500, 2**	**b1500, 3**	**Terminal Wealth, S1500**
B	50	0.3333	0.3333	0.3333	3.5515
	60	0.3125	0.3125	0.3750	3.7187
	70	0.2941	0.2941	0.4118	3.8703
	80	0.2778	0.2778	0.4444	4.0084
	90	0.2632	0.2632	0.4737	4.1345
	100	0.2500	0.2500	0.5000	4.2501

Table 4.71: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected τ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\tau_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	**Terminal Wealth, $\mathbf{S}_{1500}$**
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.3312	0.3312	0.3377	3.5687
	30	0.3275	0.3275	0.3450	3.5978
	20	0.3201	0.3201	0.3598	3.6573
	10	0.2970	0.2970	0.4059	3.8460

Table 4.72: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_2}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	**Terminal Wealth, $\mathbf{S}_{1500}$**
B	50	0.3333	0.3333	0.3333	3.5515
	40	0.3158	0.2526	0.4316	4.0575
	30	0.3371	0.2022	0.4607	4.3086
	20	0.3614	0.1446	0.4940	4.6108
	10	0.3896	0.0779	0.5325	4.9804

Table 4.73; The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 and τ_3 value for Portfolio B.

Portfolio	$\boldsymbol{\theta_2}$	$\boldsymbol{\tau_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	**Terminal Wealth, $\mathbf{S}_{1500}$**
B	50	50	0.3333	0.3333	0.3333	3.5515
	40	40	0.2580	0.2064	0.5355	4.4887
	30	30	0.2662	0.1597	0.5741	4.7755
	20	20	0.2682	0.1073	0.6245	5.1354
	10	10	0.2421	0.0484	0.7095	5.6494

Table 4.74: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	**Terminal Wealth, $\mathbf{S}_{1500}$**
C	50	0.3333	0.3333	0.3333	2.6217
	60	0.3125	0.3125	0.3750	2.6748
	70	0.2941	0.2941	0.4118	2.7191
	80	0.2778	0.2778	0.4444	2.7566
	90	0.2632	0.2632	0.4737	2.7884
	100	0.2500	0.2500	0.5000	2.8156

Table 4.75: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_2}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, $\mathbf{S}_{1500}$
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.3571	0.2857	0.3571	2.7106
	30	0.3846	0.2308	0.3846	2.8143
	20	0.4167	0.1667	0.4167	2.9367
	10	0.4545	0.0909	0.4545	3.0830

Table 4.76: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected τ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\tau_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, $\mathbf{S}_{1500}$
C	50	0.3333	0.3333	0.3333	2.6217
	40	0.3312	0.3312	0.3377	2.6274
	30	0.3275	0.3275	0.3450	2.6369
	20	0.3201	0.3201	0.3598	2.6558
	10	0.2970	0.2970	0.4059	2.7123

Table 4.77: The performance of Order 1 universal portfolio, which generated by the Transformed Gamma distribution with selected θ_2 and τ_3 value for Portfolio C.

Portfolio	$\boldsymbol{\theta_2}$	$\boldsymbol{\tau_3}$	$\mathbf{b}_{1500,1}$	$\mathbf{b}_{1500,2}$	$\mathbf{b}_{1500,3}$	Terminal Wealth, $\mathbf{S}_{1500}$
C	50	50	0.3333	0.3333	0.3333	2.6217
	40	40	0.2632	0.2076	0.5292	2.8921
	30	30	0.2725	0.1613	0.5662	2.9767
	20	20	0.2769	0.1092	0.6139	3.0694
	10	10	0.2569	0.0507	0.6924	3.1626

Table 4.78: The performance of low order universal portfolio generated by selected distribution with selected parameter value for portfolio A.

Distribution	Parameter	Order 1	Order 2	Order 3
Pareto	(5, 170, 170, 170, 1, 1)	2.7707	2.7708	2.7708
Loglogistic	(5, 100, 100, 100, 1, 1)	2.7427	2.7475	2.7544
Paralogistic	(4, 4, 4, 3500, 1, 1)	2.7699	2.7702	2.7701
Burr	(1, 1, 1, 100, 1, 1, 5, 100, 100)	2.7427	2.7475	2.3840
Transformed Gamma	(150, 150, 1, 500, 1, 1, 1, 100, 100)	2.7708	2.7708	2.3709

Table 4.79: The performance of low order universal portfolio generated by selected distribution with selected parameter value for portfolio B.

Distribution	Parameter	Order 1	Order 2	Order 3
Pareto	(100, 100, 5, 1, 1, 100)	6.7843	6.7852	6.7857
Loglogistic	(5, 100, 5, 1, 1, 100)	6.6952	6.7107	6.7328
Paralogistic	(4, 4, 4, 1, 1, 500)	6.7656	6.7710	6.7684
Burr	(1, 1, 1, 1, 1, 500, 5, 5, 100)	6.7621	6.7620	5.4227
Transformed Gamma	(1, 1, 150, 1, 1, 1000, 1, 1, 1)	6.7859	6.7859	5.4423

Table 4.80: The performance of low order universal portfolio generated by selected distribution with selected parameter value for portfolio C.

Distribution	Parameter	Order 1	Order 2	Order 3
Pareto	(5, 170, 170, 1, 1, 100)	3.2486	3.2470	4.5595
Loglogistic	(100, 100, 5, 100, 1, 100)	3.2603	3.2577	3.2619
Paralogistic	(5, 5, 150, 100, 1, 100)	3.2666	3.2598	3.2591
Burr	(150, 150, 150, 200, 1, 200, 100, 5, 100)	3.2584	3.2508	3.2462
Transformed Gamma	(100, 1, 1, 1, 1, 100, 1, 100, 1)	3.2721	3.2553	3.2694

Based on the empirical result, most of the distribution cannot outperform the wealth generated by BCRP, 2.7740 and 6.7750 for Portfolio A and Portfolio B, respectively. However, we noticed that Pareto, Paralogistic, and Transformed Gamma could generate wealth comparable to BCRP. While for Portfolio C, it is evident that only Pareto distribution can outperform BCRP for more than 1.2845. So, this may indicate that we could use Pareto distribution to generate the maximum wealth of a universal portfolio. However, further investigation is needed to determine whether the range of trading days will affect the performance of the universal portfolio generated by the selected distribution. Figure 4.27, Figure 4.28, and Figure 4.29 show the performance comparison for each portfolio generated by selected distributions with selected parameter values and the performance of CRP and BCRP.

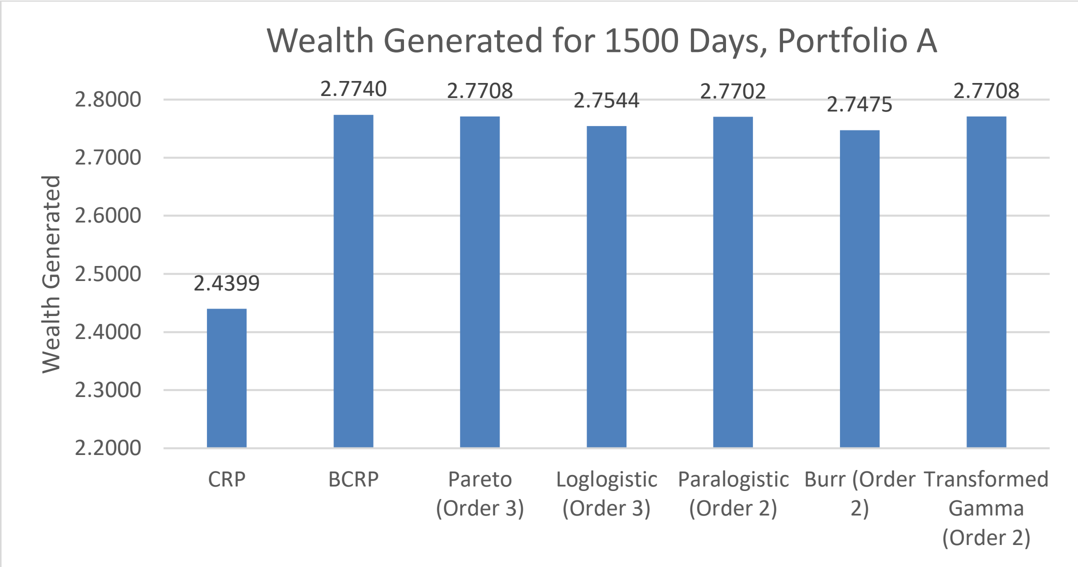


Figure 4.27: The best performance of universal portfolio generated by selected distribution with selected parameter value for portfolio A.

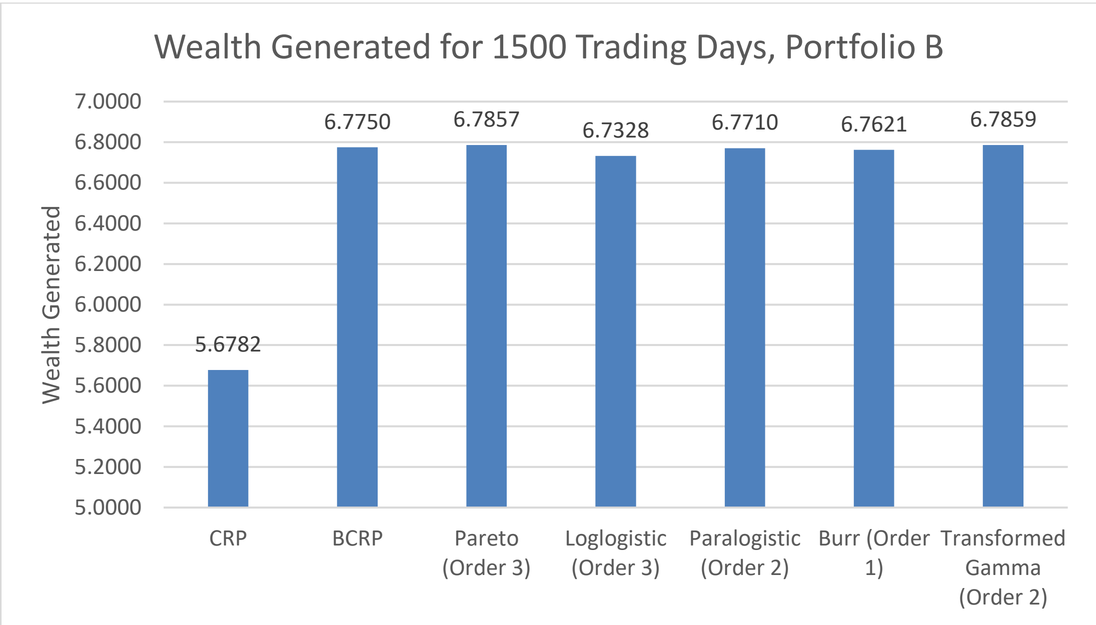


Figure 4.28: The best performance of universal portfolio generated by selected distribution with selected parameter value for portfolio B.

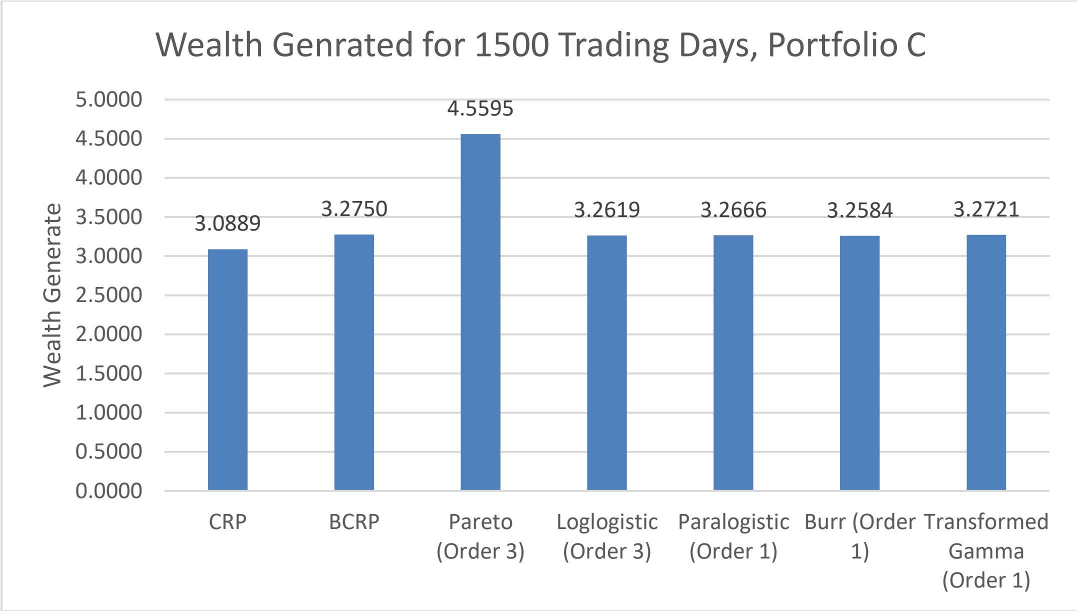


Figure 4.29: The best performance of universal portfolio generated by selected distribution with selected parameter value for portfolio C.

4.3 Performance of Universal Portfolio in The Short and Long Run

To identify how stocks' trading days in a portfolio affect the wealth generated, we used the same parameter for each distribution and order of each portfolio in section 4.2 to generate the highest wealth for 1500 trading days. Opening and closing stock prices are collected from 9th December 2010 to 27th January 2021. Tables 4.81, 4.83, and 4.85 show the result generated by each distribution along with BCRP and CRP. We noticed that, as the range of trading day increases, the terminal wealth would also increase. However, for Portfolio B, we noticed a decrease in wealth for 2500 trading days, and it may be caused by stock that did not perform well during the beginning of 500 days. Therefore, the distribution generated for Portfolio A and B is comparable mainly to wealth generated by BCRP and CRP.

On the other hand, for Portfolio C, the distributions could not generate wealth that outperformed BCRP and CRP for 2500 trading days. Furthermore, the Pareto distribution still shows the highest wealth most of the time than the other distributions. Next, Burr and Transformed Gamma cannot generate wealth for Order 3 Portfolio A and B comparable to the other distributions, BCRP and CRP. So again, it may be because the parameter value selected is not suitable for Burr and Transformed Gamma.

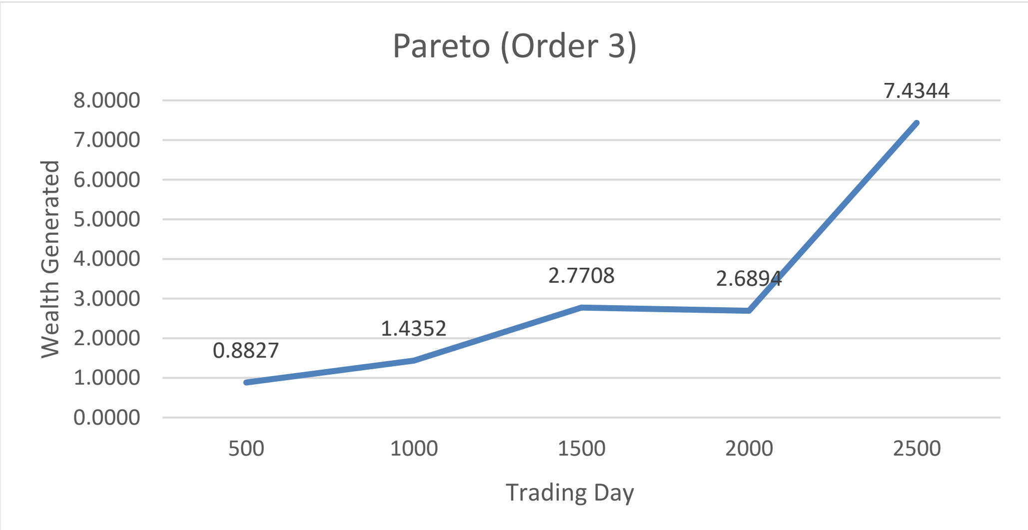


Figure 4.30: The performance of Portfolio A's Order 3 universal portfolio, which was generated by the Pareto distribution for 500, 1000, 1500, 2000, and 2500 trading days.

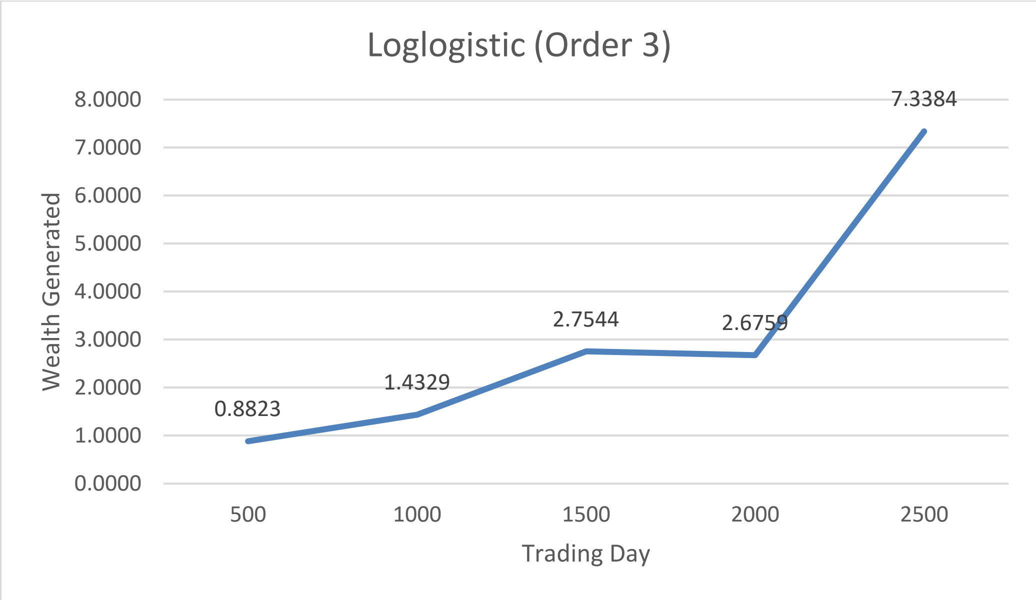


Figure 4.31: The performance of Portfolio A's Order 3 universal portfolio, which was generated by the Loglogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

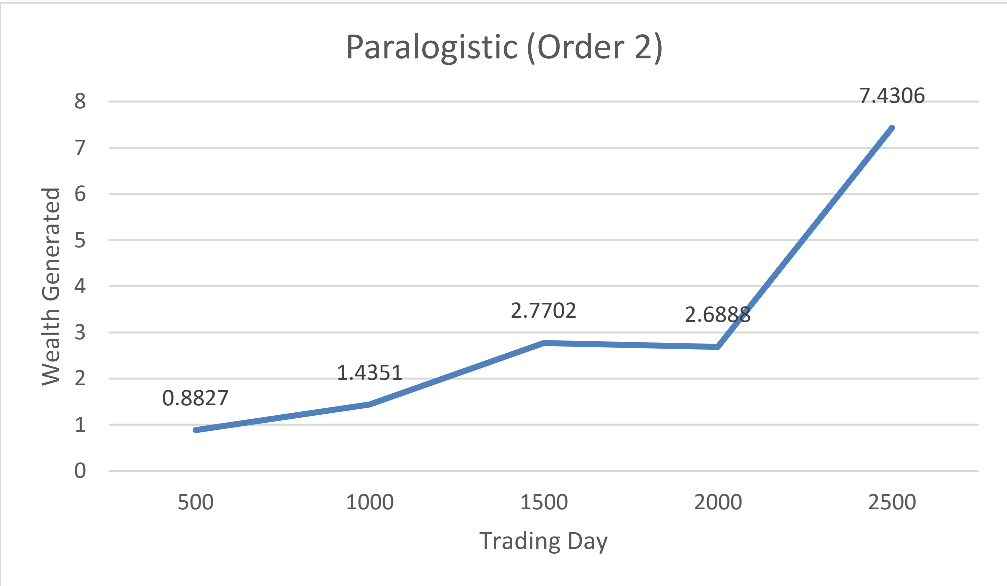


Figure 4.32: The performance of Portfolio A's Order 2 universal portfolio, which was generated by the Paralogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

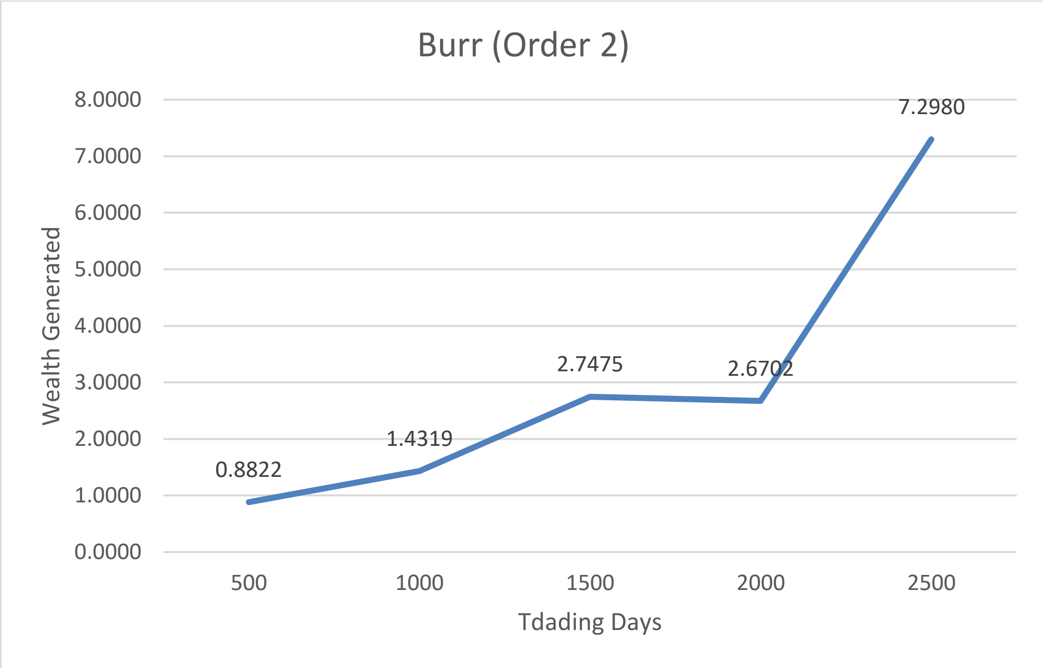


Figure 4.33: The performance of Portfolio A's Order 2 universal portfolio, which was generated by the Burr distribution for 500, 1000, 1500, 2000, and 2500 trading days.

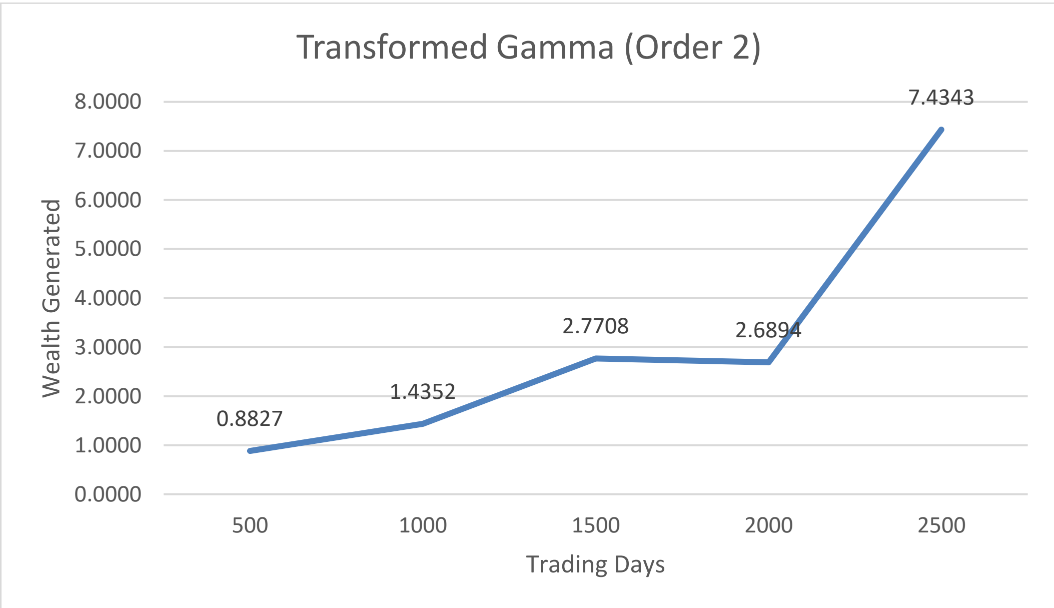


Figure 4.34: The performance of Portfolio A's Order 2 universal portfolio, which was generated by the Transformed Gamma distribution for 500, 1000, 1500, 2000, and 2500 trading days.

Table 4.81: The performance of BCRP, CRP, and universal portfolio generated by selected distributions on Portfolio A for 500, 1000, 1500, 2000, and 2500 trading days.

Distribution	Trading Days				
	500	1000	1500	2000	2500
Pareto (Order 3)	0.8827	1.4352	2.7708	2.6894	7.4344
Loglogistic (Order 3)	0.8823	1.4329	2.7544	2.6759	7.3384
Paralogistic (Order 2)	0.8827	1.4351	2.7702	2.6888	7.4306
Burr (Order 2)	0.8822	1.4319	2.7475	2.6702	7.2980
Transformed Gamma (Order 2)	0.8827	1.4352	2.7708	2.6894	7.4343
CRP	0.8684	1.3810	2.4399	2.3970	5.5591
BCRP	0.8800	1.4370	2.7740	2.6810	7.5070

Table 4.82: The performance of Order 3 universal portfolio generated by Burr and Transformed Gamma distributions on Portfolio A for 500, 1000, 1500, 2000, and 2500 trading days.

Distribution	Trading Days				
	500	1000	1500	2000	2500
Burr (Order 3)	0.8716	1.3745	2.3840	2.3635	5.3610
Transformed Gamma (Order 3)	0.8719	1.3765	2.3937	2.3709	5.3946

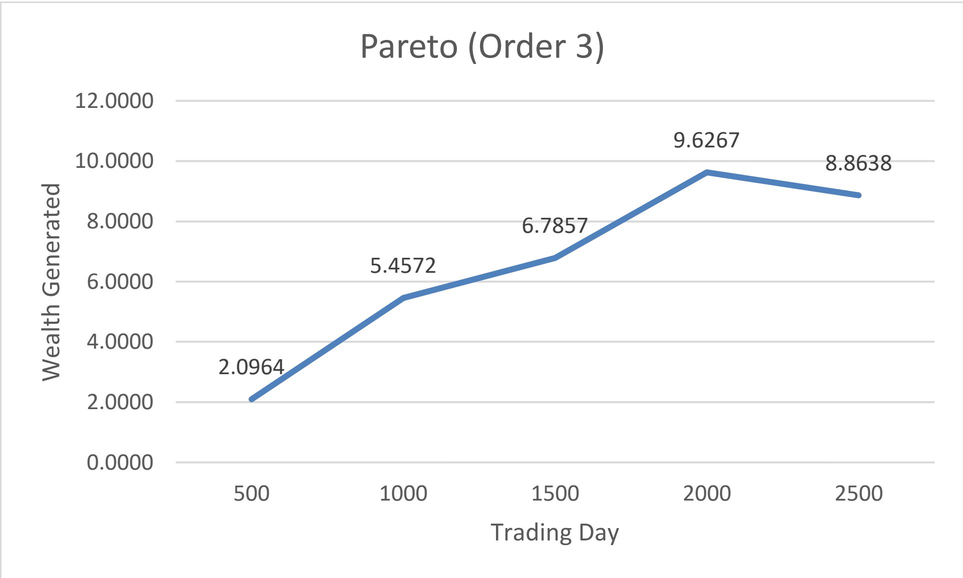


Figure 4.35: The performance of Portfolio B's Order 3 universal portfolio, which was generated by the Pareto distribution for 500, 1000, 1500, 2000, and 2500 trading days.

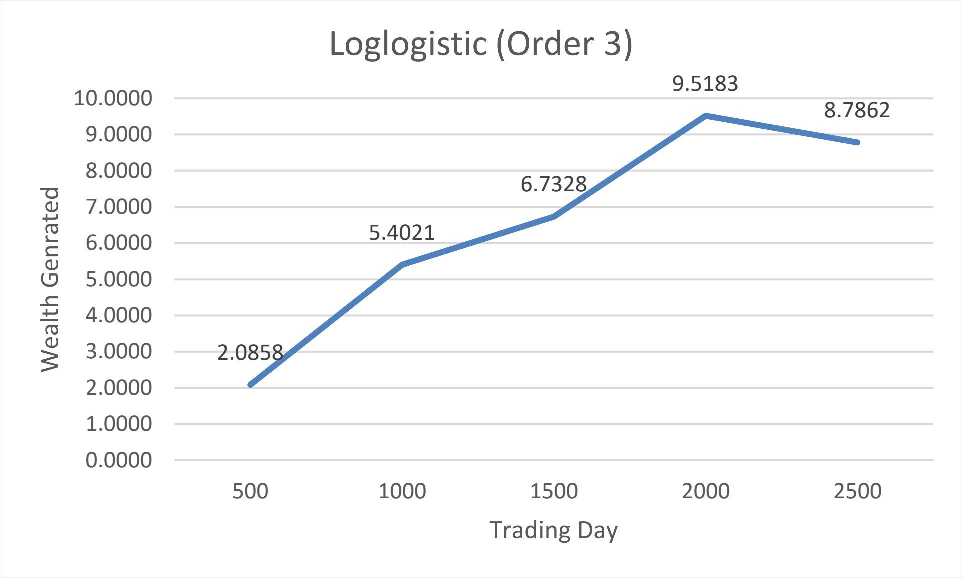


Figure 4.36: The performance of Portfolio B's Order 3 universal portfolio, which was generated by the Loglogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

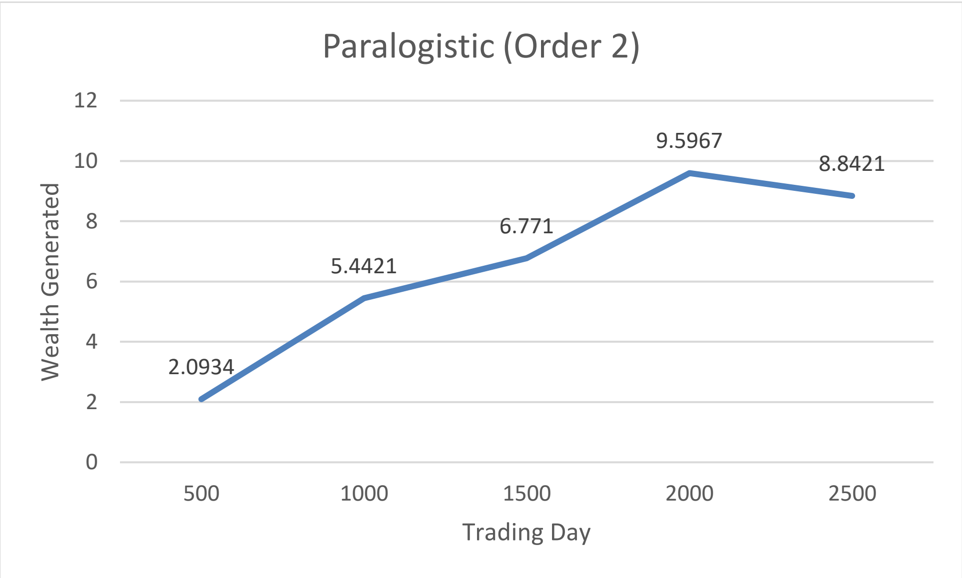


Figure 4.37: The performance of Portfolio B's Order 2 universal portfolio, which was generated by the Paralogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

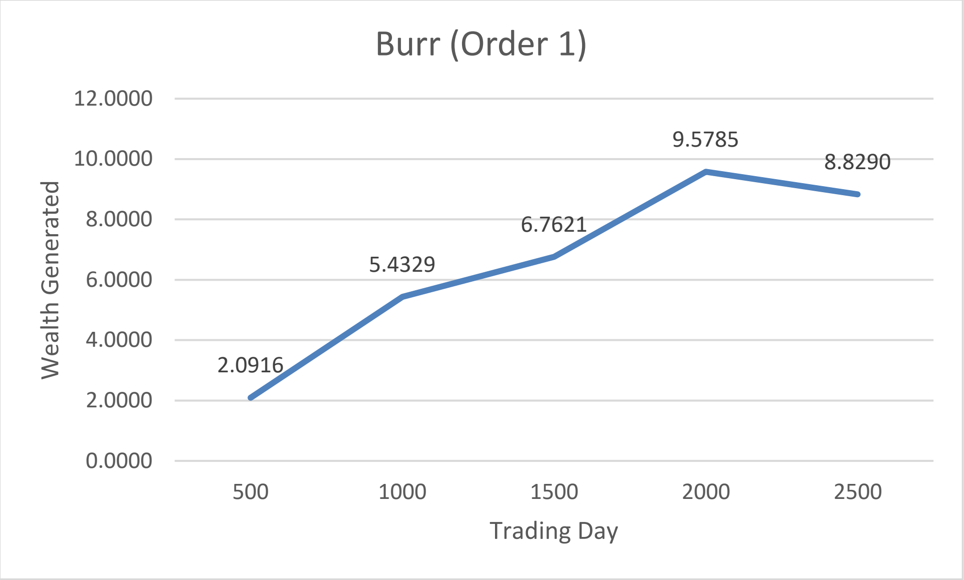


Figure 4.38: The performance of Portfolio B's Order 1 universal portfolio, which was generated by the Burr distribution for 500, 1000, 1500, 2000, and 2500 trading days.

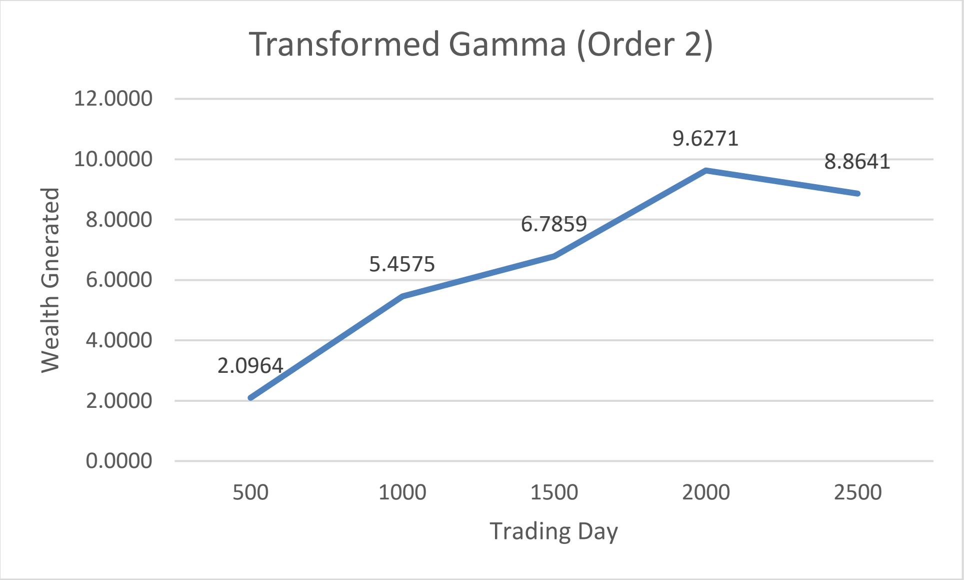


Figure 4.39: The performance of Portfolio B's Order 2 universal portfolio, which was generated by the Transformed Gamma distribution for 500, 1000, 1500, 2000, and 2500 trading days.

Table 4.83: The performance of BCRP, CRP, and universal portfolio generated by selected distributions on Portfolio B for 500, 1000, 1500, 2000, and 2500 trading days.

Distribution	Trading Days				
	500	1000	1500	2000	2500
Pareto (Order 3)	2.0964	5.4572	6.7857	9.6267	8.8638
Loglogistic (Order 3)	2.0858	5.4021	6.7328	9.5183	8.7862
Paralogistic (Order 2)	2.0934	5.4421	6.7710	9.5967	8.8421
Burr (Order 1)	2.0916	5.4329	6.7621	9.5785	8.8290
Transformed Gamma (Order 2)	2.0964	5.4575	6.7859	9.6271	8.8641
CRP	1.8835	4.3670	5.6782	7.5641	7.2975
BCRP	2.1120	5.4190	6.7750	9.7270	8.9110

Table 4.84: The performance of Order 3 universal portfolio generated by Burr and Transformed Gamma distributions on Portfolio B for 500, 1000, 1500, 2000, and 2500 trading days.

Distribution	Trading Days				
	500	1000	1500	2000	2500
Burr (Order 3)	1.8202	4.1245	5.4227	7.0043	6.8629
Transformed Gamma (Order 3)	1.8243	4.1426	5.4423	7.0397	6.8914

Figure 4.40: The performance of Portfolio C's Order 3 universal portfolio, which was generated by the Pareto distribution for 500, 1000, 1500, 2000, and 2500 trading days.

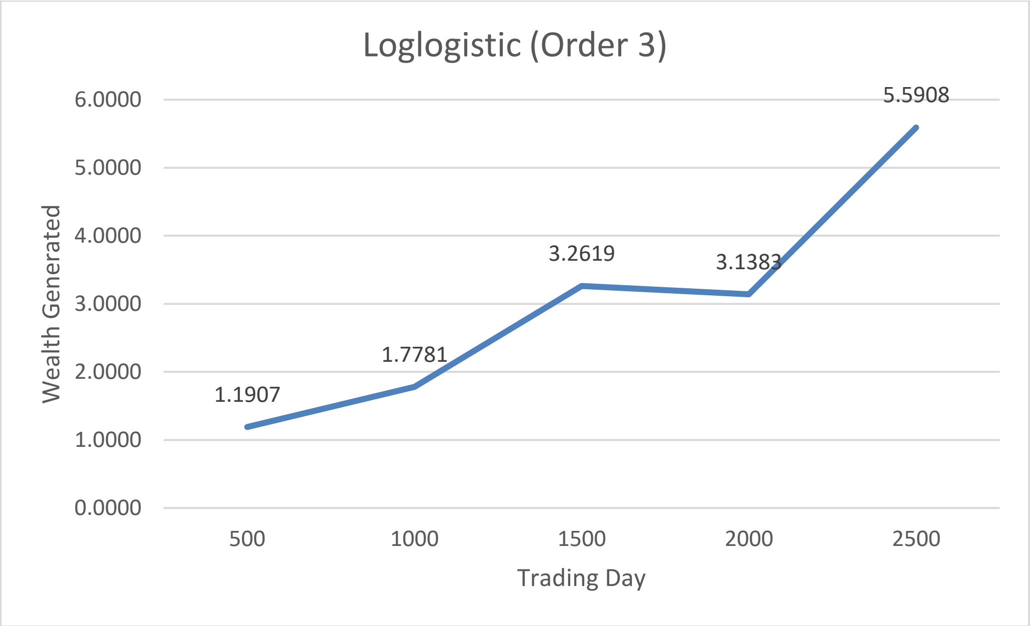


Figure 4.41: The performance of Portfolio C's Order 3 universal portfolio, which was generated by the Loglogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

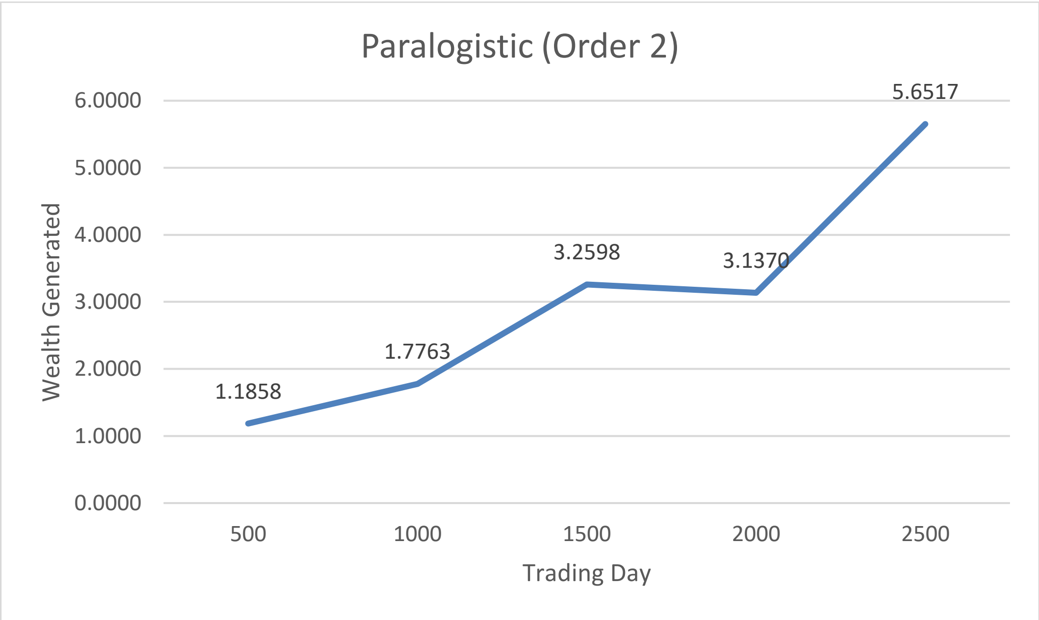


Figure 4.42: The performance of Portfolio C's Order 2 universal portfolio, which was generated by the Paralogistic distribution for 500, 1000, 1500, 2000, and 2500 trading days.

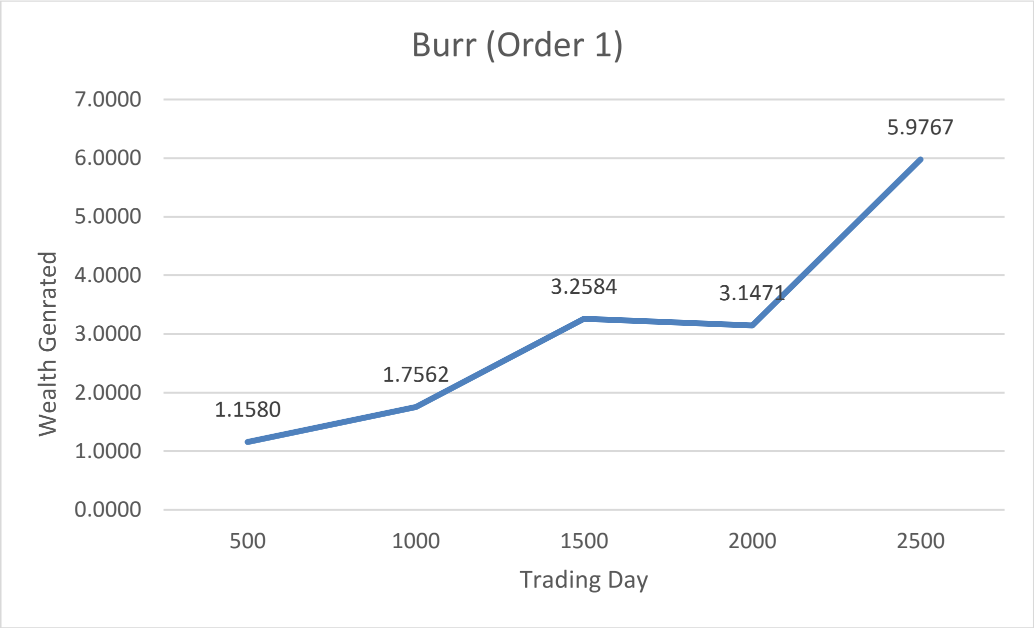


Figure 4.43: The performance of Portfolio C's Order 1 universal portfolio, which was generated by the Burr distribution for 500, 1000, 1500, 2000, and 2500 trading days.

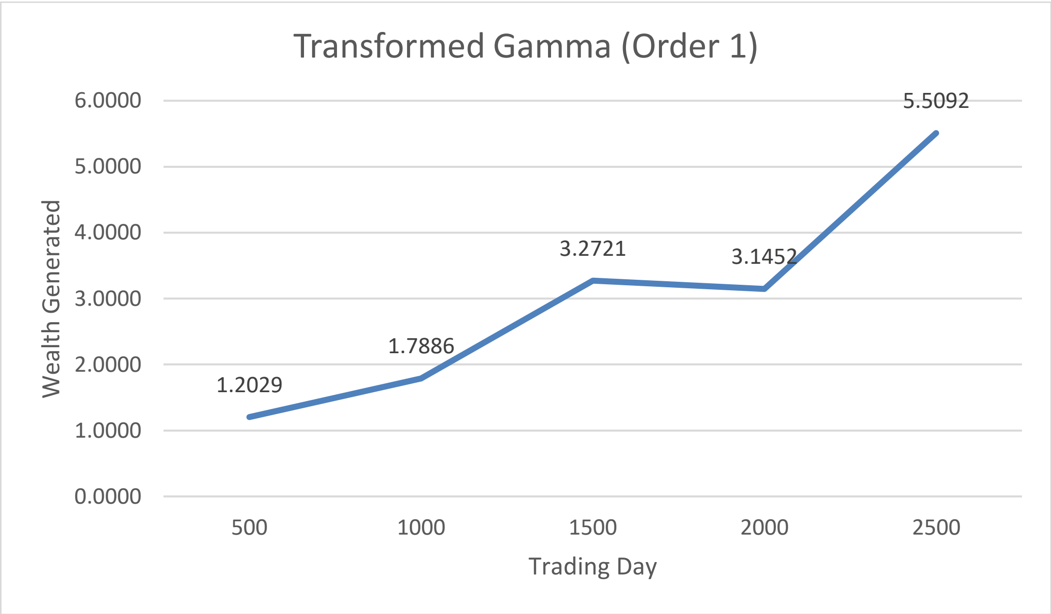


Figure 4.44: The performance of Portfolio C's Order 1 universal portfolio, which was generated by the Transformed Gamma distribution for 500, 1000, 1500, 2000, and 2500 trading days.

Table 4.85: The performance of BCRP, CRP, and universal portfolio generated by selected distributions on Portfolio C for 500, 1000, 1500, 2000, and 2500 trading days.

Distribution	Trading Days				
	500	1000	1500	2000	2500
Pareto (Order 3)	1.3727	2.2478	4.5595	4.8771	5.1274
Loglogistic (Order 3)	1.1907	1.7781	3.2619	3.1383	5.5908
Paralogistic (Order 2)	1.1858	1.7763	3.2598	3.1370	5.6517
Burr (Order 1)	1.1580	1.7562	3.2584	3.1471	5.9767
Transformed Gamma (Order 1)	1.2029	1.7886	3.2721	3.1452	5.5092
CRP	1.2697	1.8185	3.0889	2.9746	6.2685
BCRP	1.3300	1.8450	3.2750	3.1480	7.4140

4.4 Comparison of Performance of Universal Portfolio Between Diversified and Non-Diversified Portfolio

Since Pareto distribution can generate wealth that outperformed most of the other distributions, we compared the wealth generated by Pareto distribution with the selected parameter for each portfolio. The companies that we selected for portfolio A are all from the financing services sector. These companies' stocks are blue-chip stocks where the companies are with a long history of stable earnings and with lesser potential for higher growth. The companies are selected from three different sectors for portfolio B: health care, financing services, and consumer products and services sector. Portfolio B consists of blue-chip stock and non-cyclical stocks where these kinds of stocks' performance will not follow the overall economic growth, and the earning will not be affected if the economy is in recession. While for portfolio C, we have selected two companies from the financing services sector and one from the health care sector; two blue-chip stocks and one non-cyclical stock. From Figure 4.45, we observed that portfolio B could outperform portfolio A and C most of the time. The wealth also generated steadily increasing throughout the trading period. While when comparing portfolios A and C, we noticed that their performance is almost similar as the companies are primarily from the same sector, which may not diversify the risk when the financing services sector is not performing well. Therefore, this indicates that the performance of the universal portfolio will be affected by the company selected and that companies in different industries are preferable in a portfolio.

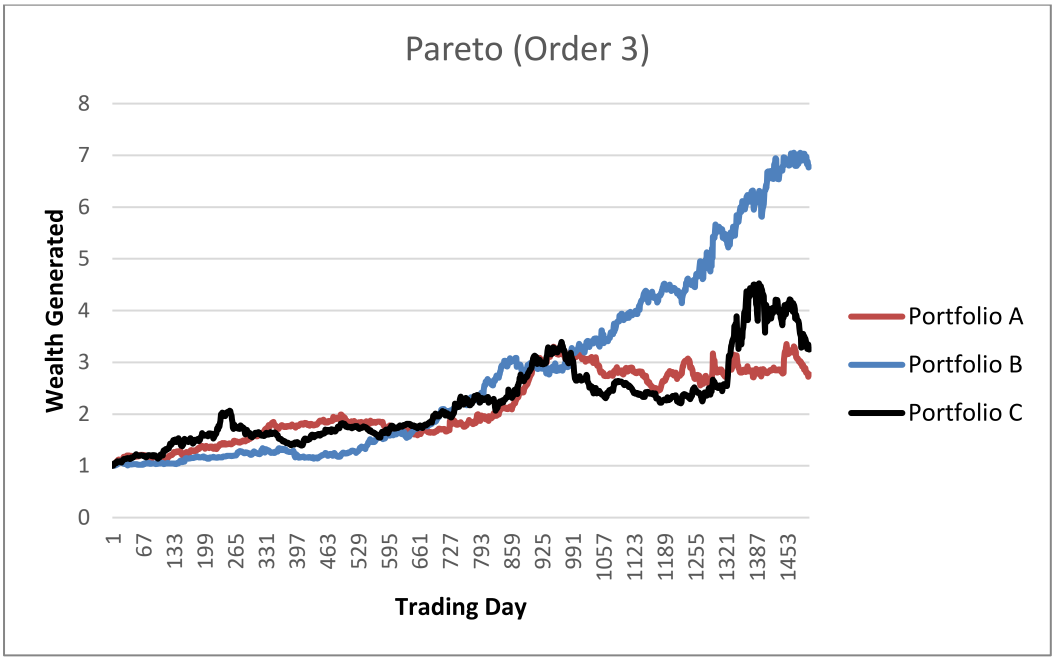


Figure 4.45: The overall performance of the selected portfolios A, B, and C, generated by the Pareto distribution within order 3 Universal Portfolio.

Table 4.86: The wealth achieved in 1500 trading days of selected portfolios A, B, and C, generated by the Pareto distribution within order 3 Universal Portfolio.

Portfolio	Wealth Achieved in 1500 Trading Days
A	2.7708
B	6.7857
C	3.2487

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This project aimed to generate a low-order universal portfolio with five selected distributions: Pareto, Loglogistic, Paralogistic, Burr, and Transformed Gamma distribution. For instance, three data sets each consist of the stocks price of three different companies from KLSE and are collected from Yahoo Finance. The wealth generated with BCRP and CRP is used as benchmarks to study the distribution-generated universal portfolio performance.

After collecting the data, a mathematical model for the universal portfolio is generated with VBA in Excel. The parameters of the selected distributions are randomly generated according to their parameter's property. Each portfolio with 1500 trading days is generated for 500 trials. According to the result obtained, as the order of the universal portfolio increases, the time taken to generate the terminal wealth increases. Moreover, it is found that the Pareto distribution was able to generate the highest wealth among all the other selected distributions. However, most of the distributions are not able to outperform BCRP but can outperform CRP. This indicates that 500 trials may not be sufficient to determine the best parameter for each distribution to generate maximum wealth for the universal portfolio.

Next, a parameter sensitivity test is carried out to determine which distribution parameter will significantly influence the wealth generated. Each parameter is tested with the selected value from the range $[1, 300]$. According to the result obtained, each portfolio will be affected by different parameters of the selected distributions. Next, each distribution parameter is assigned with a selected parameter value to generate the highest possible wealth of the universal portfolio. The wealth generated by each distribution is now comparable to BCRP and CRP, and Pareto distribution can provide the highest wealth within order 3 among the distribution.

Furthermore, to identify whether a universal portfolio will have better performance in the long run, each distribution with selected parameters from the previous section is used to generate a universal portfolio with 2500 trading days. From the result, as the range of trading days of the portfolio increases, the terminal wealth will increase. However, for portfolio B, the terminal wealth for 2000 trading days decreases as it proceeds to 2500 trading days. Moreover, it is found that Burr and Transformed Gamma distribution cannot generate wealth for Order 3 portfolios A and B that are comparable to other distributions, BCRP and CRP. So, again it indicates that the parameter value selected may not be suitable for Burr and Transformed Gamma distribution.

Finally, the final objective of this project is to identify the performance of the diversified and non-diversified universal portfolio. According to the wealth generated by each portfolio with Pareto distribution, diversified universal portfolios can outperform undiversified universal portfolios.

In conclusion, Pareto distribution can generate wealth that is comparable to BCRP and CRP. It was also able to outperform the universal portfolio generated by other distributions. Parameters of the distribution are essential as they are the key to generating the universal portfolio's highest wealth, and the universal portfolio will have better performance in the long run. Lastly, a universal portfolio will have better performance with a diversified portfolio.

5.2 Limitation and Recommendation

In this project, the distribution parameter was either randomly generated or selected from a range of values. This method may exclude the parameter combination that will allow selected distribution to generate the highest wealth of the universal portfolio. This was also why Burr and Transform Gamma distribution could not generate wealth comparable to the other distribution. To obtain the best parameter combination for the distribution, we should loop through a range of values and record the highest wealth obtained. However, VBA in excel cannot loop through an extensive range of values; therefore, we cannot go through all the possible parameter combinations in this project. Therefore, Mixture-Current-Run (MCR) universal portfolio introduced by Tan and Lim (2013) enables them to combine two or more universal portfolios of the same kind to discover the optimal parameter that corresponds to the best daily wealth can be implemented for further study of this project. Moreover, when selecting stocks to form a portfolio, negatively correlated stocks should be selected to get better performance of the universal portfolio. Markowitz (1952) suggested that diversification across different industries should be considered in a portfolio, as they have different market characteristics, resulting in lower covariances.

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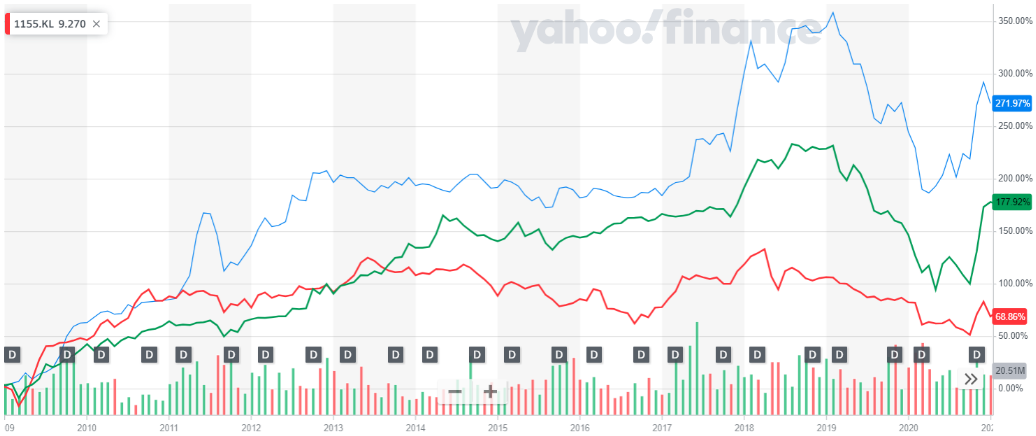
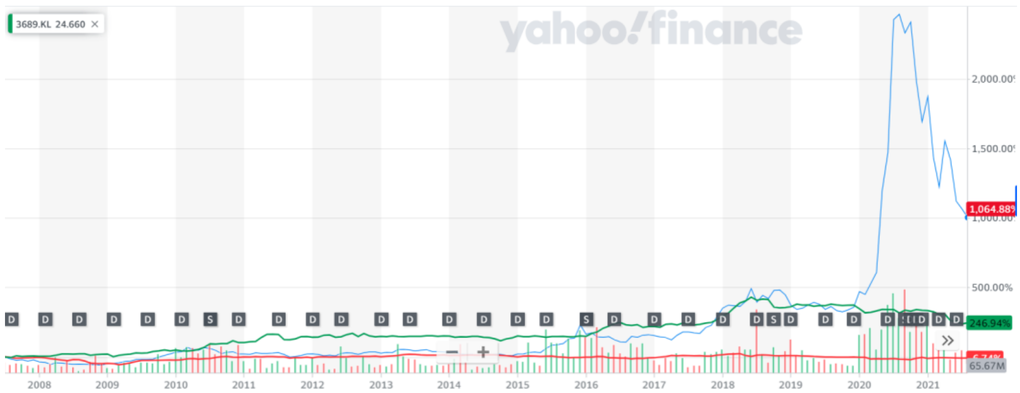
APPENDICES

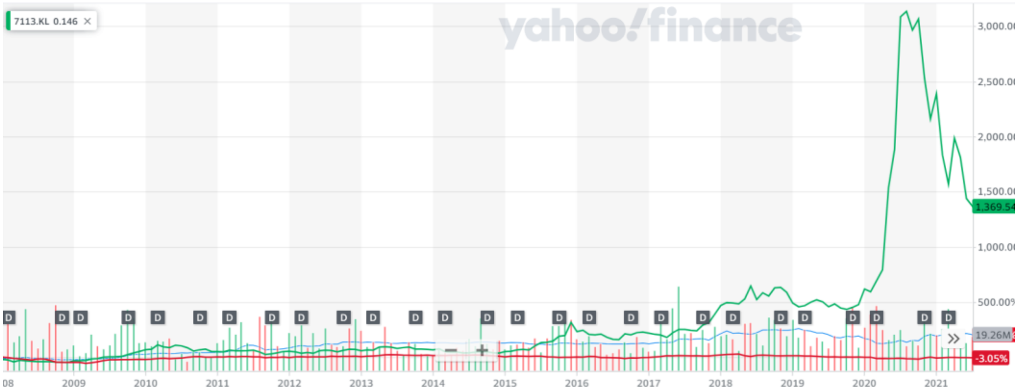
APPENDIX A: Graphs



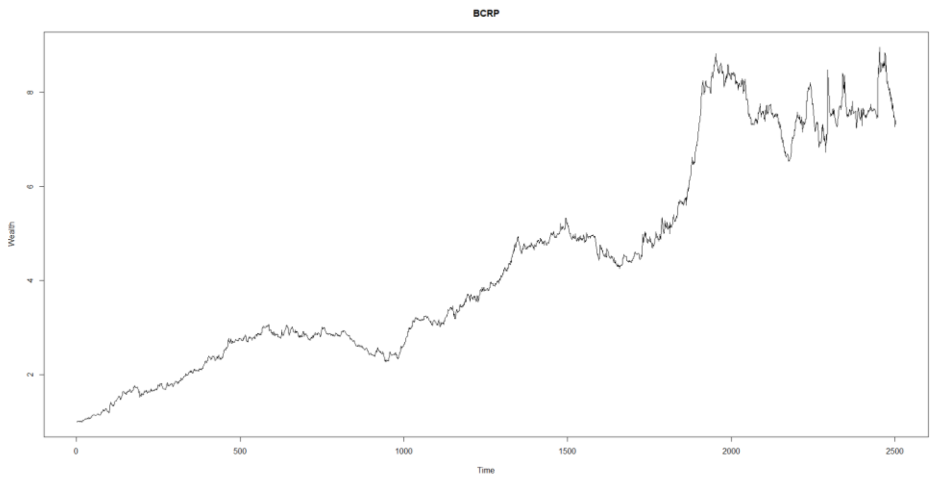
GraphA-1: Performance of Stocks For Hong Leong Bank Berhad (Blue Line), Public Banking Berhad (Green Line) And Malayan Banking Berhad (Red Line)



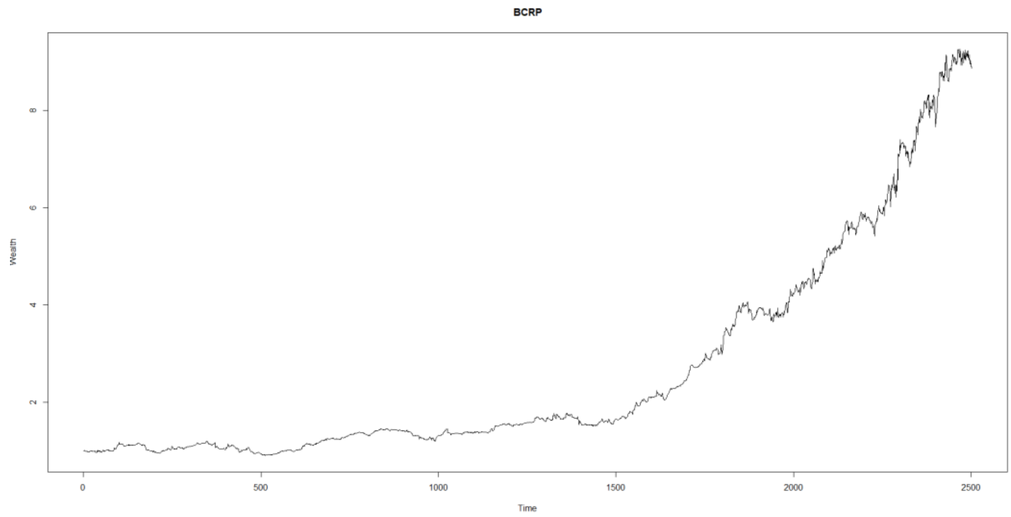
GraphA-2: Performance of Stocks For Top Glove Corporation Berhad (Blue Line), Fraser & Neave Holdings Berhad (Green Line) And Malayan Banking Berhad (Red Line)



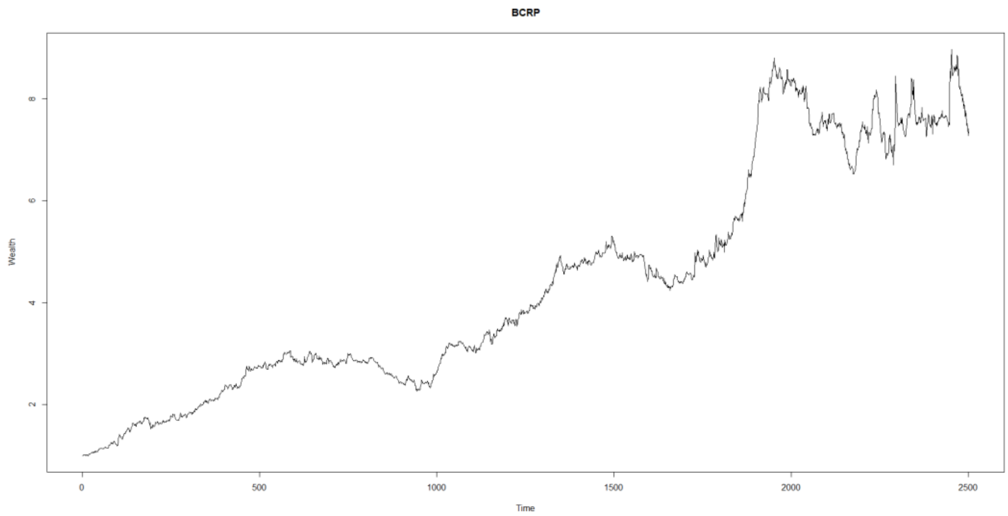
GraphA-3: Performance of Stocks For Top Glove Corporation Berhad (Green Line), Malayan Banking Berhad (Red Line) And Hong Leong Banking Berhad (Blue Line)



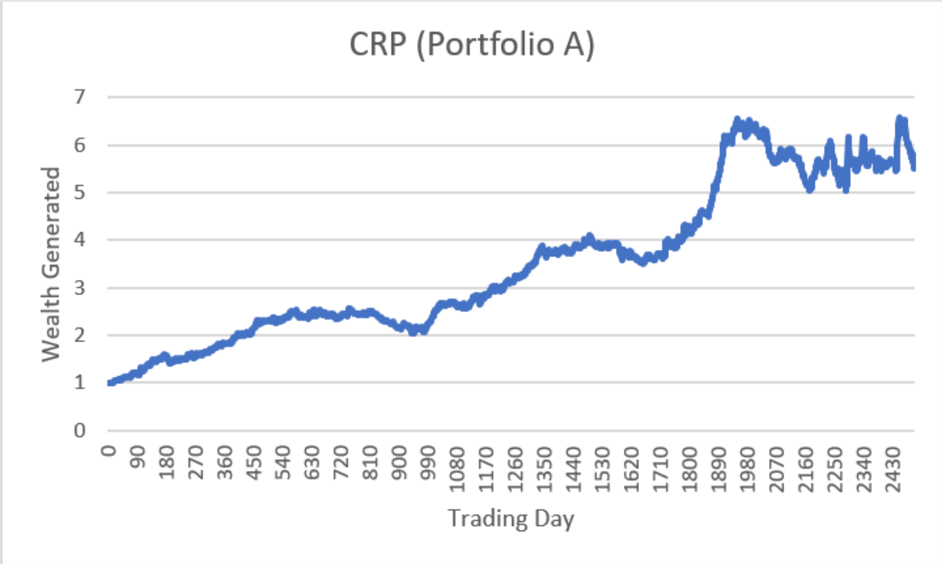
GraphA-4: Wealth of Portfolio A Generated With BCRP For 2500 Trading Days



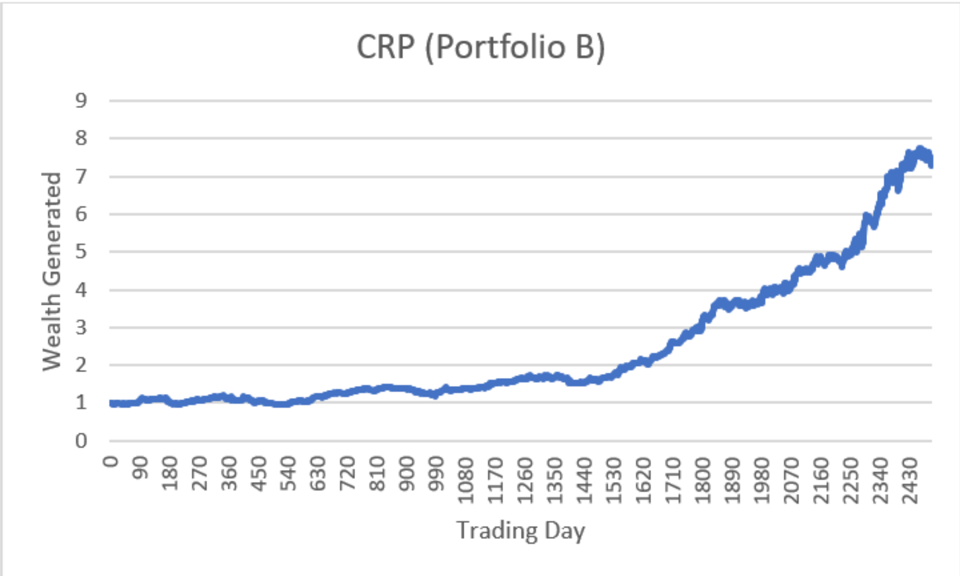
GraphA-5: Wealth of Portfolio B Generated With BCRP For 2500 Trading Days



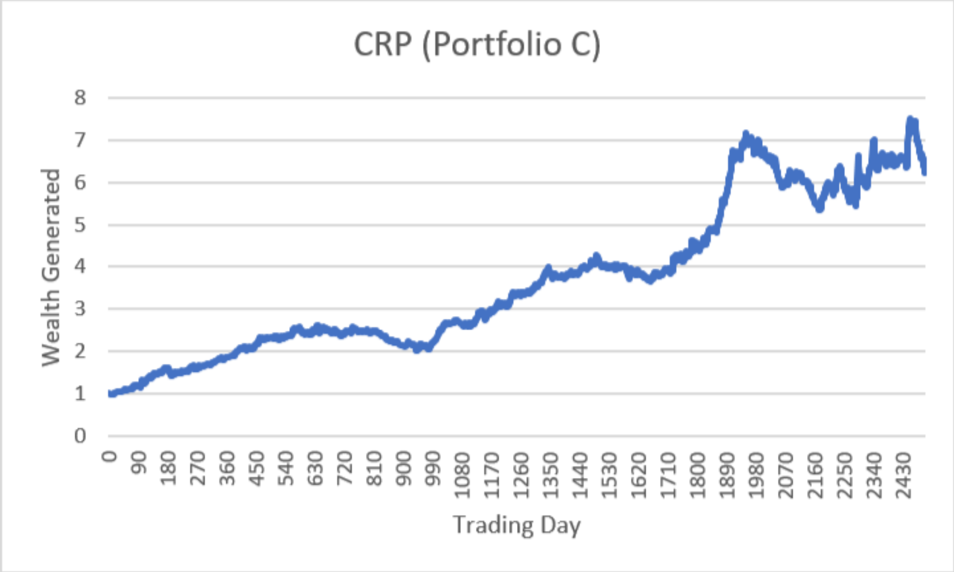
GraphA-6: Wealth of Portfolio C Generated With BCRP For 2500 Trading Days



GraphA-7: Wealth of Portfolio A Generated With CRP For 2500 Trading Days



GraphA-8: Wealth of Portfolio B Generated With CRP For 2500 Trading Days



GraphA-9: Wealth of Portfolio C Generated With CRP For 2500 Trading Days