ELECTRO-MODELING OF THE ALUMINIUM-AIR BATTERY

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A project report submitted in partial fulfilment of the requirements for the award of Bachelor of Engineering (Honours) Mechanical Engineering

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DECLARATION

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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APPROVAL FOR SUBMISSION

I certify that this project report entitled **"ELECTRO-MODELING OF THE ALUMINIUM-AIR BATTERY"** was prepared by **KOH YANN YEE** has met the required standard for submission in partial fulfilment of the requirements for the award of Bachelor of Engineering (Honours) Mechanical Engineering at Universiti Tunku Abdul Rahman.

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ABSTRACT

In the modern day, the application of battery as a form of energy storage system has played the important part especially in electric vehicles (EVs). EV is well known as an alternative to internal combustion engine vehicles to reduce carbon emission. Thus, the application of suitable battery types in electric vehicles plays an important role in the long run consideration. The aluminium air battery is considered to be one of the suitable energy storage mediums as it has a high theoretical energy densities value that can provide the EV to have sufficient energy source for long travelling distances. Considering the aluminium-air battery as the alternative energy storage in future, the electro-modeling is applied to study the air battery characteristics. For the application, electro-modeling to determine the aluminium-air battery characteristics is still lacking behind. Thus in this project, electro-modeling such as the RC model can contribute to determining the air battery characteristics without using a smart battery charger. RC battery model generally is a model made of components such as the voltage source, capacitor and resistor to represent the characteristics of a battery. By constructing the equivalent circuit models, the 1 RC and 2 RC models model the discharge characteristics of the battery, the comparison between the simulated and experimental curves is conducted. Through the process of estimation and validation of the parameters, the maximum average error obtained for the 1 RC is 9.54 % while, for the 2 RC is 6.93%.

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LIST OF SYMBOLS / ABBREVIATIONS

RC	resistor-capacitor
SoC	state of charge
SoH	state of health
ECM	equivalent circuit model
PDT	Pulse Discharge Test
MAE	Mean Absolute Error
OCV	open circuit voltage
DKF	dual Kalman filter algorithm

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CHAPTER 1

INTRODUCTION

1.1 General Introduction

With the advancement of technology, the demand for production and the application of a battery as energy storage has continuously increased drastically. There are few types of battery commonly available in the market. For instance, the lithium-ion battery has the characteristics of high energy stored in its mass, low rate of self-discharge, and a significant number of cycle life (Mossali, et al., 2020). With its characteristics, it is widely used in advanced technological devices and as the primary power source of electric vehicles. Nevertheless, it is generally believed that at most 30 % increment of energy density can be achieved in further improvements of lithium-ion batteries (Liu, et al., 2020). There are some existing drawbacks such as high cost, limited energy density and battery degradation issues, lithium ions battery seems not suitable to be applied in large scale applications. With that, extensive research has been done on metal-air battery, a new energy storage system that can be a better replacement.

Metal air battery is mainly made of anode, cathode, separator, and electrolyte. Plus, it is thought of as a hybrid of fuel cells. In the working principle of metal-air battery, the metal anode reacts with electrolyte to form metal hydroxide and release electrons. The electrons will then flow to the cathode and be consumed in the process of formation of oxygen and water (Liu, et al., 2017). While, it utilizes oxygen from the surrounding air at the cathode during the electrochemical process (Liu, et al., 2020). In terms of theoretical energy densities, the metal-air battery has about 3-30 times higher than a lithium-ion battery (Li and Lu, 2017).

Among metal-air batteries, aluminium air has received interest due to its attractive characteristics such as high theoretical energy density, low operating cost, being environmentally friendly, and being recyclable (Park, Choi and Kim, 2017). The electrochemical reaction of electrodes of aluminium air battery can be summarized as below:

$$\text{Anode: } 4Al \rightarrow 4Al^{3+} + 12e^{-} \quad (1.1)$$

$$\text{Cathode: } 3O_2 + 6H_2O + 12e^{-} \rightarrow 12OH^{-} \quad (1.2)$$

$$\text{Overall: } 4Al + 3O_2 + 6H_2O \rightarrow 4Al(OH)_3 \quad (1.3)$$

In battery technology, a model based method is applied to determine the state charge, as well as the state health of the battery that is unable to directly measure from the battery. Through battery modeling, the state charge, and state health of a battery can be estimated accurately and can further be applied in the battery management system (BMS) to prevent the over-discharge plus over-charge phenomenon. Among the model based methods, the electrochemical model (EM) is the model that consists of serials of complex, mutually coupled partial differential equations. It can reflect the potential and the voltage changing process and describe the reaction processes of a battery (Xiong, et al., 2018). Another type of model is the resistor-capacitor (RC) model which is the model with fixed parameters that prevent the model to trace the change in the dynamics behaviour of batteries. Besides, the model accuracy is low, depending on the number of RC networks connecting to the model (Cen, Kubiak and Belharouak, 2016).

In general, each model based method has its strengths and weaknesses. In optimising battery use, electrochemical models are crucial tools that provide a rich description of the battery’s response in the sense of snapshots of its internal electrochemical state. However, their accuracy critically depends on parameter values of the electrochemical (Drummond and Duncan, 2012). While RC model has advantages such as simple built-in, where electrical components are used to study the characteristics of a battery. With the characteristic of clear physical meaning, it is easy to be applied for battery modeling and parameter calibration (Cen, Kubiak and Belharouak, 2016).

1.2 Importance of the Study

Through the study, the findings obtained are important to provide a better understanding of the discharge behaviour of aluminium-air battery by using the RC model. The RC model can reflect the complex static and dynamic habits of a battery through mathematic expressions. Besides, the findings may provide a basis to build a control system for aluminium air batteries. With the study of electro-modeling of aluminium air battery, the approximate SoC value of the battery can be estimated. Thus, an expensive smart battery charger will not require to determine the SoC of an aluminium air battery which is important to ensure the reliable operation and safety of the battery.

1.3 Problem Statement

The research studies of aluminium-air battery have been done for decades where experiment tests were conducted to examine the characteristics of the battery. However, the study of aluminium-air battery discharge characteristics under the RC simulation method system is limited. Electro-modeling of battery is defined as the application of electrical equivalent circuit to represents the behaviour of battery. There are not many research articles regarding the electro-modeling of aluminium air battery that can be obtained. Thus, the study of modeling has to rely on other types of energy storage mediums such as the lithium-ion battery or lead-acid battery-related articles.

1.4 Aim and Objectives

The aim of this study is to build an RC model for an aluminium-air battery. The specific objectives:

- To review different types of battery models available
- To construct the RC model using MATLAB
- To validate the discharge characteristics of the model with the experimental data.

1.5 Scope and Limitation of the Study

Scope:

The study will cover one battery cell only and the type of metal-air battery to be studied will only cover the aluminium air battery.

Limitation:

In this study, the studying of discharge characteristics of aluminium air battery is done at constant temperature (Room temperature). Therefore, the different discharge behaviours of the aluminium air battery under a huge different temperature range will not be covered in the study.

1.6 Contribution of the Study

The study is to contribute to the application of the RC battery model to study the characteristics of the aluminium-air battery. From the study, a set of 1 RC and 2 RC battery model parameter values can be determined. This allows the battery model can be applied to study the discharge characteristic of the aluminium-air battery under different electrolyte concentrations and different current rates. Besides, this study also helps to determine the SoC level of the battery which is crucial for the safety battery operation. The study also contributes to allowing the aluminium-air battery cell operation condition to be evaluated without using a smart battery charger.

1.7 Outline of the Report

In chapter 2, the literature study is to review the working operation of the aluminium-air battery and some existing battery models that are available in the market. The comparison of the equivalent circuit models with other battery models was done. Furthermore, the study of the advantages and disadvantages of the equivalent circuit model was performed.

In chapter 3, the methodology of the project was planned. The 1 RC and 2 RC battery models were constructed in MATLAB to obtain a set of parameter values. The process of comparison of the experimental curve with the simulated curve was discussed.

In chapter 4, comparison results between the experimental data and simulated data were done. The average error of both battery models was calculated to identify the most feasible model to be applied to study the discharge characteristics of the aluminium-air battery. Chapter 5 summarized the result obtained and recommendations were done for future improvement.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Battery models are typically used to describe the interaction between the battery's external electrical characteristics and internal electrical condition mathematically. Using the proven battery models, it is possible to approximate the internal state variables such as state of charge and resistance based on the external variables measured.

An accurate and reliable battery model is important to appropriately analyse the discharge characteristics of a battery. To understand the characteristic of a battery, different simulation models have been established, which generally comprise electrochemical models. The dynamic characteristics of a battery can be determined using the parameters of the electrochemical model. There are numerous model's parameters that need to be identified, the electrochemical model is described to be a complex battery modeling. Therefore, it usually functions to determine the electrochemical process of a battery, and the result estimation from the model is useful for battery design and production.

Meanwhile, another type of simulation model is the equivalent circuit model (ECM). In an ECM, the charging and discharging processes of a battery can be described by using components such as resistances, capacitances, and voltage sources. The model normally is a frequency or time domain built-in.

In this study, the focus is to cover the working principle of aluminium air battery and some studies of literature review for the electrochemical models and the ECMs. Besides, the advantages and disadvantages of the equivalent circuit models and comparison of result estimation between RC models and other battery models will be discussed too. Lastly, the estimation SoC of battery using different methods will also be covered in the study.

2.2 Working principle of the aluminium air battery

An aluminium air battery is mainly made of an air cathode, an aluminium anode with an electrolyte in between. It also consists of a separator that functions to prevent the direct contact between anode and cathode that will result in the short circuit. During the discharging process, the aluminium ions of the anode react with hydroxide ions to form aluminium hydroxide. In the anode oxidation process, the release of electrons transport through an external circuit toward the cathode to involve in the oxygen reduction process. On the surface of the cathode, oxygen molecules combine with the electrons and hydroxide ions are formed through the reduction process of water. The generated hydroxide ions then move across the separator to the anode to form a complete current loop (Liu, et al., 2020).

Liu, et al. (2017) studied the working principle of aluminium air battery by establishing an experimental system to analyse the polarization curve, the performance of constant current discharge and the performance of the monomer battery system under different electrolyte concentrations. The voltage characteristic curve, as seen in Figure 2.1, has clear segmented resistance characteristics. The voltage dropped rapidly at low current density and larger area, with the increase of the current density, while the voltage is linearly decreased in the middle segment; under such conditions, the aluminium air battery will operate under various operating currents according to the power curve. Figure 2.2 shows the battery's output voltage curve in the 80 A under constant current. The battery voltage rises to its maximum value of 1.45 V in the constant current. Since the reaction activation did not exceed the best discharge range at the start of discharge, the activation performance is not totally mirrored, and the output voltage is at its maximum. Liu, et al. (2017) also stated that the corrosion of aluminium electrolyte is greater at low concentrations, and the alkali content is insufficient to meet the needs of the battery's rapid reaction, resulting in an improvement in battery self-discharge and a decrease in storage performance. The viscosity of the electrolyte increases as the electrolyte concentration increases, and the conductivity decreases. The experiment results suggested that the aluminium-air battery has a good discharge performance.

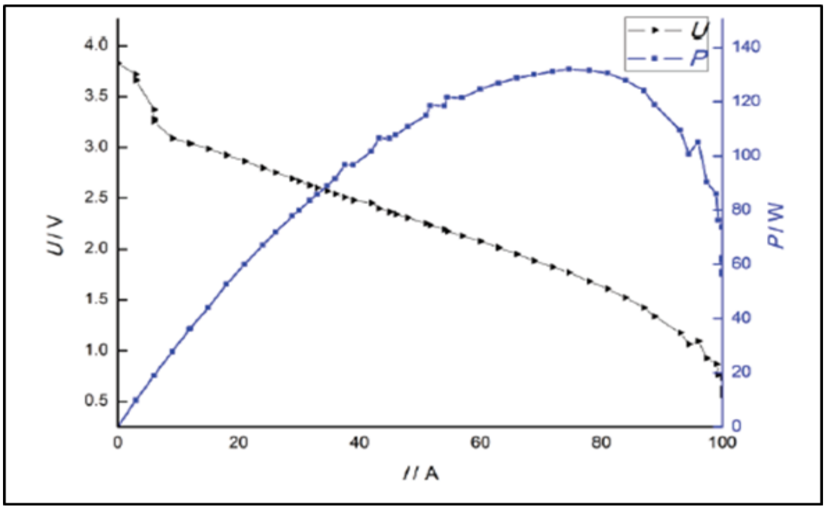

Figure 2.1: Polarization characteristic curve of aluminium air battery (Liu, et al., 2017)

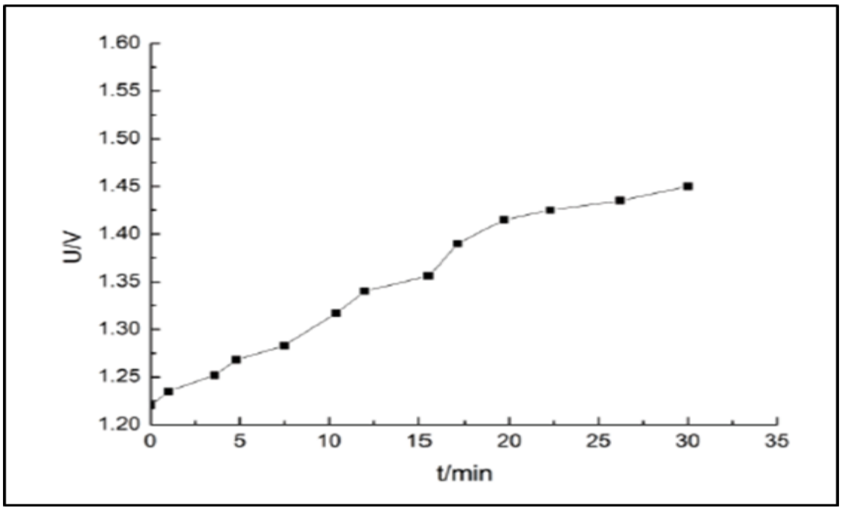

Figure 2.2: Output voltage curve under 80 A constant current discharge (Liu, et al., 2017)

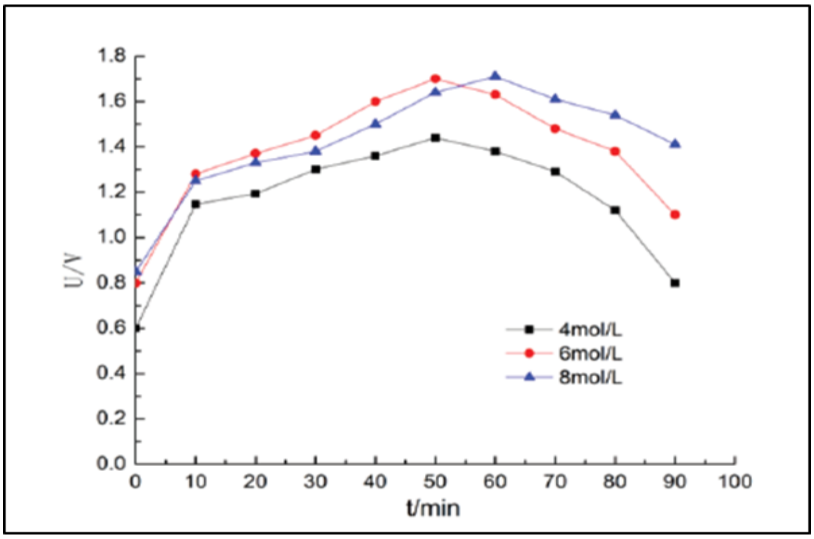

Figure 2.3: Output voltage curve under different electrolyte concentrations (Liu, et al., 2017)

2.3 Electrochemical models

Electrochemical models are a type of battery modeling approach that is useful to identify the electrochemical states and provide accurate information of the battery. These models are normally defined by the number of complex partial differential equations that have to be solved simultaneously and this highly affects the complexity of the model. Due to its capability to explain battery processes in great detail, it has significant computational issues. Besides, there is several parameters that need to be set and identified in the process of setting up these models, leading to over-fitting issues and increase the uncertainty in the output of the model (Nejad, Gladwin and Stone, 2016).

Jongerden and Haverkort (2005) studied and evaluated the types of battery model available and evaluate their ability to fit with the battery. In the study, a type of electrochemical model, Dualfoil was explained able to compute battery properties that varies according to time but there are more than 50 battery related parameters that need to be set. Through an evaluation, Jongerden and Haverkort (2005) stated although the model has high accuracy but the computational complexity is high to solve partial differential equations, which makes the execution duration of the model to be longer. Plus, over detailed information is needed to set all parameters of the battery model, causing the model to be complex.

The electrochemical model is a useful battery model for optimising the usage of the battery. It is able to provide an overall description of the electrochemical state of the battery. However, the parameter values of the electrochemical model highly affect the accuracy of the battery's response. While some electrochemical model parameter values are difficult to predict owing to their relative uncertainty (Drummond and Duncan, 2020). Thus, some simplified forms of electrochemical models were built to reduce the complexity and computational duration. A single particle model (SPM) was studied and determine the parameters of the model (Bizeray, et al., 2017). Bizeray, et al. (2017) have stated that six groups of parameters were actually enough to parametrize the model and some of the models are usually over parameterized with only certain parameters that can be obtained through measurement. Thus, it is important to understand their structural and functional identifiability before estimating a model's parameters. Attempting to identify all model parameters at the same time without understanding their identifiability is likely to result in poor optimization issues.

The parameters of a simplified electrochemical model can be obtained in experimental identification. Speltino, et al. (2009) stated that the electrochemical model can be simplified by neglecting some parameters although some information will be lost. The reduced order model has fewer parameters than the full order model, but there is a number of parameters that are still unknown. In Speltino, et al. (2009) research, some model parameters were found through the parameters identification process which consists of charge and discharge processes under the steady and pulse current profiles to capture the dynamic change of battery behaviour. While some parameters can be retrieved from literature research and from manufacturers' datasheets to minimize the number of parameters to be identified while maintaining high model fitting accuracy.

Li, et al. (2018) formed a simplified version of the electrochemical model based on previous researches' models and some assumptions have been made such as the effect of internal temperature of battery was neglected under room temperature. Under different current excitations, the model parameters were derived from the reaction time of various physicochemical processes. The estimated parameter results were then validated to examine the accuracy of estimation, and the reliability of the estimation method. To test the reliability, the procedure of estimating parameters was performed repeatedly three times to obtain the latest parameters value at different degrading stages. The simulated terminal voltages were used to approximate the parameters to validate the accuracy of the method in estimating the parameter. Besides, the computational efficiency of the model was improved by assuming the battery electrochemical behaviours is independent of temperature change and excluding the thermal related equations. Li, et al. (2018) explained that the temperature of the developed model was considered to be constant to speed up the duration in identifying all parameters.

2.4 Equivalent circuit models

Equivalent circuit models (ECMs) consist of simple components such as capacitors and resistors to illustrate a battery's internal electrochemical process. It is also used to describe the batteries' dynamic characteristics. Equivalent circuit models are easy for implementing battery model parameter identification. There are some of ECMs that commonly use in battery modeling study which is the ECM with a branch of RC networks such as the first-order RC model, the Rint model and the PNGV model (Attanayaka, Karunadasa and Hemapala, 2019).

In one of the studies, 3 types of ECM (Shepherd model, Rint model and Thevenin model) were introduced to a battery to compare parameters identification and discharge results. The Thevenin model was parameterized through experiment tests which were quite time-consuming, while others were set up through datasheet information. All models were tested under different operating conditions such as Pulse Discharge Test (PDT), Dynamic Discharge Test and fixed current or voltage recharge. For discharge curve validation, the result of models was then validated with datasheet discharge curves under a different capacity. It was found that for the Shepherd model, the discharge curves were fitted well for the 85 % of discharge process compare with the datasheet curve under lower C rates of 0.2, 1 and 1.67 C as shown in Figure 2.4. Similar to the Rint model and Thevenin model, these models also have better accuracy of comparison at low C rates shown in Figure 2.5 and Figure 2.6. In the paper, Campagna, et al. (2020) summarized that the results of the

Thevenin model were the best under different operating conditions due to its parameters were set to be more accurate through experiment.

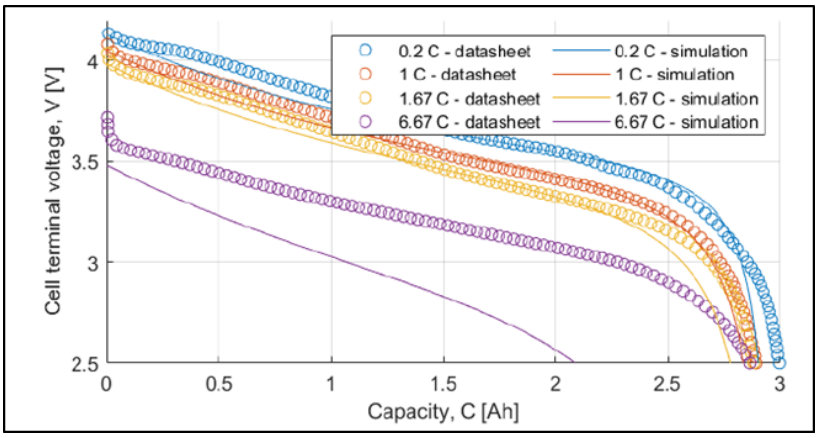

Figure 2.4: Comparison between datasheet and Shepherd model discharge curve (Campagna, et al., 2020)

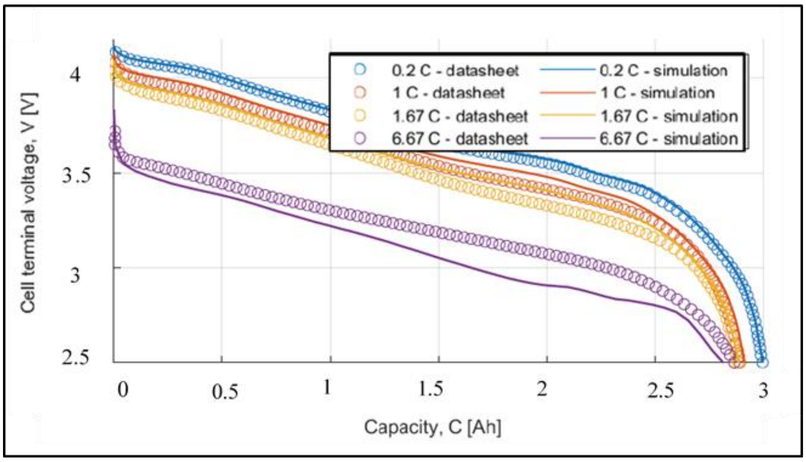

Figure 2.5: Comparison between datasheet and Rint model discharge curve (Campagna, et al., 2020)

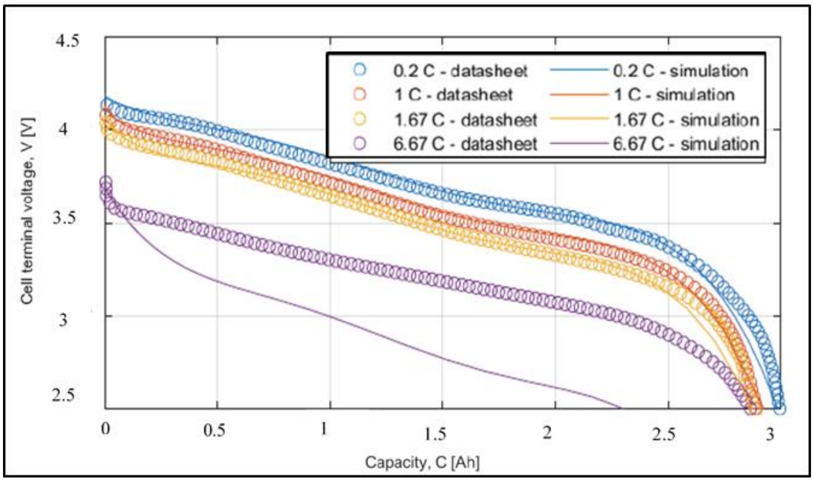

Figure 2.6: Comparison between datasheet and Thevenin model discharge curve (Campagna, et al., 2020)

Liaw, et al. (2004) conducted an experiment whereby the open circuit voltage values was determined from the discharge of a cell at a rate of C/25 to study the SoC and the open circuit voltage correlation. In the ECM shown in Figure 2.4, the R_2 consists sum of two contributions, R'_2, power law and R''_2 an exponential function to approximate the change of resistance with SOC. Liaw et al. (2004) stated that a high degree of accuracy between the simulated results and the actual data was obtained from the ECM simulation of discharge curves at the rate of C/25 and C/1. This suggested that the well-fitted parameters of the model allowed the model to have the ability to obtain a better discharge curve.

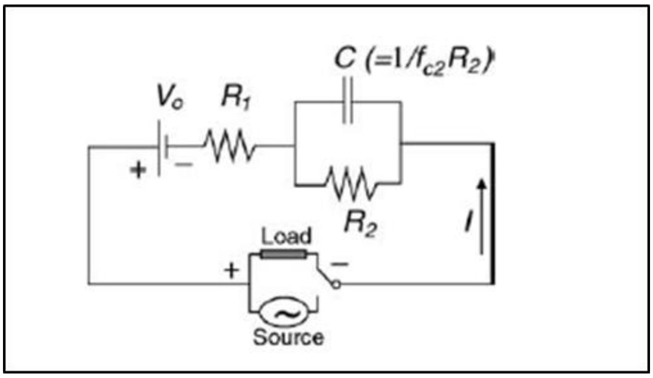

Figure 2.7: Schematic diagram of ECM (Liaw, et al., 2004)

2.4.1 RC model of battery

RC model is a typical ECM that consists of an RC network comprising of capacitor and resistor to estimate a battery's dynamic characteristics. The dynamic characteristics of a battery usually involve the nonlinear voltage, a nonlinear current exhibit by the battery. The RC model is simple and comprises a voltage source, resistor and capacitor components to represent an actual battery where the battery characteristics can be studied through the RC model. In the battery model, the voltage source represents the actual battery's open-circuit voltage. While the series resistor represents the ionic conductivity of the separator of the battery. Besides, the parallel pair of RC networks reflects the diffusional process of battery ions that diffuse through the porous electrodes. These components usually are represented as the function of the SoC of a battery.

Theoretically, the high order of the RC models will have high accuracy of the model. However, this will increase the complexity of the model resulting in the model being difficult to be applied for estimation work. In actual applications, the accuracy of a model depends on the model parameters. Poor accuracy may be happened due to parameter mismatch in a complex RC model. Therefore, when applying ECM, consideration of the accuracy and complexity has to be balanced (Lai, Zheng and Sun, 2018).

Lai, Zheng and Sun (2017) stated increase in the order of the RC model does not imply that the accuracy of the model will be improved but it will cause numerous unknown parameters in the model and an overfitting issue may arise. Based on the study, the models with one RC network and two RC networks were highly reliable and accurate compare to other models tested in the research. An increase in the number of RC networks in the equivalent circuit model will weaken the effects of the SoC result error. Lai, Zheng and Sun (2017) also studied the SoC estimation under the influence of resistor and capacitor by applying equivalent impedance, the SoC result error was found to be small if the equivalent impedance increase when more numbers of RC network is applied.

He, et al. (2012) carried out an evaluation of estimating the terminal voltage accuracy on few battery models experimentally. Among the battery models, there are 3 types of RC equivalent circuit models which are the Rint model, the first order RC model and the DP model. Through the evaluation of the Hybrid Pulse Power Characterization test, the Thevenin and DP models were found to have small voltage errors. In the Dynamic Stress Test, both models showed to have better estimation precision with the existence of small voltage errors. In the Federal Urban Driving Schedule test, the resulting outcome of the Thevenin and DP models were found to be more accurate compared to the remaining models due to both models take the effect of polarization characteristics into consideration. It was concluded that the reason both models seem to be more excel compare to other models in the evaluation of tests was due to both the models took in the voltage relaxation effect of the battery into consideration.

Other than that, the ECM with an RC network and the ECM with two RC networks models as shown in Figure 2.8 have been studied where the parameters of models were obtained through experimental work. The results precision of simulation work were then verified with the actual data.

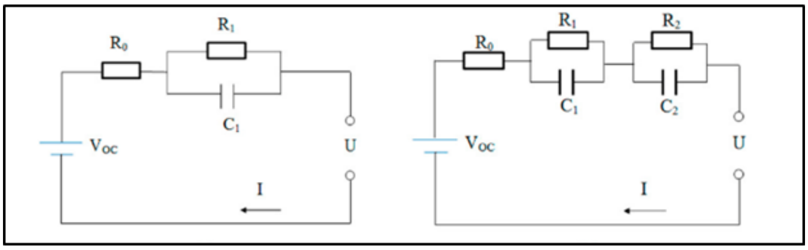

Figure 2.8: The schematic diagram of the ECM with an RC network (left) and the ECM with two RC networks (right) (Zhang, et al., 2017)

The simulation results of the ECM with an RC network and the ECM with two RC networks models in Figure 2.9 and Figure 2.10 showed that the ECM with an RC network has a relatively large error at the beginning and the end of discharge. It was mainly due to the polarization process of batteries are deepening gradually and the contribution of unbalanced electrochemical reactions in batteries.

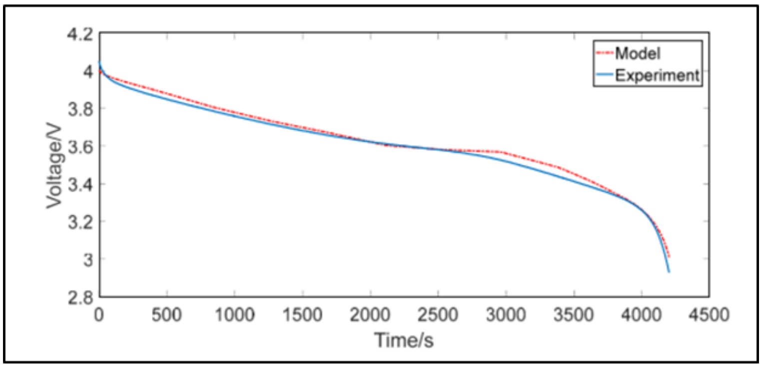

Figure 2.9: The output voltage of the ECM with an RC network under 1C constant current discharge (Zhang, et al., 2017)

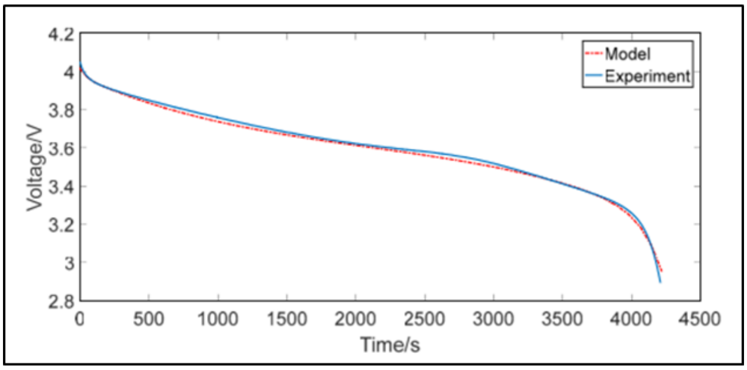

Figure 2.10: The output voltage of the ECM with two RC networks under 1C constant current discharge (Zhang, et al., 2017)

The output error of the models was measured by determining different types of maximum error under the 1C constant current discharging rate. It was obtained that the maximum of the absolute error in terms of voltage for ECM with an RC network is 0.0610 higher compare to ECM with two RC networks with the value of 0.0452. It also determined that the magnitude of maximum relative errors of an RC network model with 1.65 % error higher compare to the two RC networks model with 1.22 %. In terms of root mean squared error the ECM with an RC network has a value of 0.0221 V higher compare to 0.0156 V of the ECM with two RC networks (Zhang, et al., 2017).

Hu, Li and Peng (2011) have performed a study on the comparison of twelve types of ECMs and indicated that using a complex electrical model can improve the model accuracy with the drawbacks of increasing in system complexity and computation cost. In the study, all the models are differentiated with multiple cell datasets obtained under a range of working temperatures. The root mean squared errors of all the models were evaluated using validation datasets under 3 different temperatures. The ECM with an RC network was found to be similar to more complex models under the datasets which suggested that balancing between accuracy and complexity is a wise decision.

Jiang, et al. (2017) used an ECM model with an RC network to fit and compare with the V-I characteristics of a battery with experimental data. The applicability of the model to the battery is analysed according to the simulation results at different current rate tests in the study. The hybrid pulse power characterisation test was used to estimate the model parameters. Jiang et al. (2017) also stated that due to the simple structure of first order RC model, it was not suitable to use to determine the dynamic characteristics of the battery. The experimental data were used to verify the model's accuracy and the error analysis was performed to the SoC range in between 10 percent to 90 percent of SoC in practical applications.

Table 2.1: Comparison between experimental data and simulation under different current rates (Jiang, et al., 2017)

Current rate	Simulation error of charge	Simulation error of discharge
0.1 C	< 1%	< 2%
0.2 C	< 2%	< 3%

The comparison results summarized that the 1 RC model has good applicability for battery in most SoC regions under a relatively lower current rate (Jiang, et al., 2017).

2.5 Advantages and disadvantages of equivalent circuit models (ECMs) in battery simulation modeling

2.5.1 Advantages of ECMs

The ECM composes of basic electric components to reflect a battery behaviour and simulate performance characteristics. It is frequently applied in real-time applications due to its simplicity. Generally, an RC model is equipped with a number of RC networks. The more the number of RC networks in a model, the accuracy of voltage and SoC estimation can be improved with the drawback of larger computational cost. Thus, the balance between accuracy and complexity has to be optimized to produce a suitable battery model (Liu, et al., 2014).

Besides, Zhang, et al. (2017) reported that each component of an ECM has a clear physical meaning, simple mathematical equations are easy to understand, and is easy to handle. The ECM also does not consider the complex electrochemical reaction in a battery. Using ECM for the SoC estimation allow high fidelity and low computational cost results to be determined (Lai, Zheng and Sun, 2017).

The electrical components in the RC model function to describe the electrical behaviour of a battery. In the RC model, the open-circuit voltage of the model represents the voltage source of a battery and a resistor which explains the resistivity of electrolyte and connection of a battery. Plus, the transient response of a battery can represent by parallel RC networks of the ECM battery model (Xiong, et al., 2018). The transient response includes the diffusional process of battery ions across porous electrodes.

2.5.2 Disadvantages of ECMs

The number of order of the RC models is proportional to the degree of accuracy that can be achieved. High order RC model tends to have more parameters that need to be defined, leading to an increase in complexity and not being suitable to be applied in the real application. The increase in parameters also reflects a long computational hour and can lead to poor accuracy due to parameter mismatch (Lai, Zheng and Sun, 2017).

Besides, another drawback of ECMs is that it is unable to describe the electrochemical process in a battery during charging and discharging, leading to poor prediction capabilities. (Liu, et al., 2014). Unlike the electrochemical models which apply coupled complex mathematical equations to describe the actual electrochemical reaction process of battery and are able to capture transient response accurately (Lai, Zheng and Sun, 2017).

2.6 Comparison between equivalent circuit models with other battery models

Meng, et al. (2018) did a comparison of 4 types of battery modeling which one of them are the two RC ECM to understand the performance of modeling methods in terms of accuracy of voltage response and response time. A battery with a nominal capacity of 10 Ah and a nominal voltage of 3.2 V was discharged under 25 Celsius temperature and one second for data acquisition duration. From the result performance, the SVM was found to have the smallest Mean Absolute Error (MAE) of 0.0034 among the models but its execution time has been observed to be longer than others. Compare with two RC ECM, the execution time of 0.0018 seconds longer than ECM which with the duration of 6.3549×10^{-7} seconds. Considering the trade-off, Two RC ECM and combined models are more preferable as long the accuracy is within an acceptable range.

2.7 State of Charge (SoC) of battery

Identifying the SoC is a crucial step to understand the percentage of remaining charge in a battery over the rated capacity of a battery. The SoC percentage can provide manufacturers with informative data regarding the battery performance and also assist in identifying the correct time to recharge the battery. In addition, the SoC estimation results are useful in battery management system for power management purposes. This is because battery power is an expression of impedance that rely on SoC percentage. Therefore, over-charge or over-discharge issues may arise due to inaccurate SoC estimation that may cause permanent damage to the battery cells. A precise SoC estimation can also reflect useful information such as battery performance, remaining life of a battery that ultimately lead to effective management and utilization of the battery power and energy. Furthermore, SoC estimation can be used to extend the battery life cycle and to prevent accelerating ageing and permanent damage to the cell structure of batteries by regulating the overcharge and discharge. Thus, regulate SoC indication is thus important to ensure the battery’s efficiency, safety, and long life cycle.

Tian, et al. (2017) stated the battery's SoC is generally interpreted as the ratio of the amount of capacity that remains in a battery to the nominal capacity of the battery. It was expressed as

$$SoC(t) = SoC(t_o) - \frac{1}{C_n}\int_{t_o}^{t} \eta_c i_L(\tau)d\tau \qquad (2.1)$$

The factors that affected the accuracy of SOC estimation can be identified from the equation which is inaccurate of initial SoC, discharge capacity, measurement of i_L and the Columbic efficiency. These factors cannot be self-correct by the non-model estimation approaches, making the model based methods better options due to the close loop and self-correct characteristics.

2.8 Analyse battery SoC using estimation methods

Attanayaka, Karunadasa, and Hemapala (2019) explained that the precise estimation of battery SoC reflects battery performance, the life span of a battery and regulates the exceed charge and discharge of a battery. While the accurate SoC indication is important to ensure the efficiency of a battery, safety, and long-lasting. Accurate SoC estimation eliminates failures due to thermal runaways and regulates cell balancing. However, it is heavily affected by battery degradation. As batteries degrade, the SoC estimation algorithm may lead to large errors.

In the Battery Management System; the location of the state of charge estimation algorithm programmed in, it regulates the flow of energy in a battery pack with respect to individual cells' voltages, temperature, SoC and SoH. The key role of the BMS is to maintain the battery system in a secure operating condition and to protect it from damage. While battery SoC estimation is a key feature of BMS, due to the non-linear complex electrochemical process in the battery, its accuracy and online estimation are challenging (Attanayaka, Karunadasa and Hemapala, 2019)

Estimation of battery SoC can be done through many methods and the methods are categorise into model based-methods and non-model based as shown in Table 2.2.

Table 2.2: SoC Estimation Methods (Xiong, et al., 2017)

SOC estimation methods	
Model based methods	1. Extended/ Unscented Kalman filter 2. Kalman filter
Non- model based methods	1. Open Circuit Voltage (OCV) method 2. Columb Counting method 3. Neural Network method 4. Support Vector Machines

2.8.1 Model based methods of SoC estimation

Model based methods are able to estimate the conditions of a battery through its current, voltage and temperature by applying a model along with advanced algorithms. The model based methods tend to show higher accuracy compared to other methods.

In the methods, the model with the highest accuracy in representing cell behaviour should be considered. This is because the model accuracy can greatly affect the model based method's performance. In a battery model, the operation of a battery can be represented by several parameters of the battery model. Once, all the model parameters have been identified, the SoC estimation can be further processed and estimated. The model based methods show very accurate estimation result in the battery monitoring approaches. Besides, both extended, and unscented Kalman filters are usually applied with these methods.

To evaluate the electrical properties of batteries at varying ambient temperatures, Xu, et al. (2019) designed a temperature-based second order RC model. Based on the existing model, a dual Kalman filter algorithm (DKF) was proposed to predict the battery SoC, which synthesizes the integration method and algorithms of few SoC estimation methods such as the Coulomb counting the Kalman filter and extended Kalman filter. The SoC estimation results were compared with the experimental SoC value, the SoC estimation errors were found to be maintained within 0.4 percent under various temperature conditions. Furthermore, it is shown that the proposed DKF algorithm is resistant to temperature fluctuations and initial SoC errors.

Baba and Adachi (2012) proposed a combination of two Kalman filters to improve the result estimation of SoC. It is stated that the method was able to solve the issue of battery parameter variation during the process of estimation. In the series of filters, the first filter was used to reduce the disturbance in the battery parameters estimation process, while the second one was for OCV estimation. The effectiveness of the model was determined through the comparison with simulation results under a normal hybrid electric vehicle operating condition. Under the nominal condition, the proposed method showed high accuracy result. Besides, the result under the proposed method is not affected by the presence of the error in the initial SoC.

However, the Kalman filter consists of mathematical expressions that are only able to deal with battery models with linear characteristics. Therefore, the extended Kalman filter will normally be applied to a battery model which consists of non-linear characteristics. It can linearize the battery's characteristics through partial derivatives and Taylor series expansion. An extended Kalman filter has been applied with the two RC models to estimate the SoC of a battery. The impact of noise disturbance can be neglected under the presence of an extended Kalman filter. To verify the extended Kalman filter in removing noise, a comparison of battery current with or without adding the external noise current was performed. It was found that the SoC estimation with extended Kalman filter under the influence of noise has a slightly different outcome of 0.04% compared to the actual measure current value (Chen, Fu and Mi, 2013).

2.8.2 Non model-based methods of SoC estimation

A neural network is a computational tool that comprises a set of neurons that has the ability to process information and acquire knowledge, making it widely used for system modeling. With sufficient processing elements, it can fit the nonlinear expression of the battery and adapt to change by learning and updating its internal structure. He, et al. (2014) stated the neural network requires a large set of data for the training process in order to improve its accuracy.

SoC estimation can also be identified using open circuit voltage which the main idea is to create a stable one to one battery OCV and the SoC correlation. When a battery in the unloaded state, its internal equilibrium will reach a steady state where the battery's open circuit voltage is assumed to be the same as the battery terminal. Then, the OCV of the battery can convert to SoC estimate through the SoC and OCV. The estimation result of the method is very accurate but to measure the OCV, the battery has to come to a complete rest which is not practical. Hence it is difficult to be utilized in real applications (Ramadan, Becherif and Claude, 2017)

Coulomb counting method is another type of SoC estimation approach in which the discharge current of the battery is integrated along with time to determine the remaining energy. The method is a relatively simple SOC estimation method. With the initial SoC value provided, it can identify an SoC easily. The coulomb counting method can determine the change in the battery's coulombs based on the integration of battery current over time. However, the technique is not practical due to all battery cells are assumed to be balanced. In the actual condition, the voltage and temperature values of battery cells vary under different modes of operation, which makes cells individual SoC vary. (Movassagh, Raihan and Balasingam, 2019). Moreover, long integration hours of Coulomb counting method can cause the accumulated errors in the final SOC (Ramadan, Becherif and Claude, 2017)

2.9 Summary

Different battery models have been significantly used to study the behaviour of battery cells. Among the battery models, the equivalent circuit model such as the RC model has certain advantages compared to other models. However, the battery model still facing some drawbacks in terms of balancing the trade-off between the accuracy of model estimation and the computational complexity.

CHAPTER 3

METHODOLOGY AND WORK PLAN

3.1 Introduction

In this chapter, the setup to obtain an aluminium air battery's discharge curve through experimental and simulation will be discussed. The main purpose of these setups is to obtain a set of optimized parameters of the battery model's components which can be used to reflect the discharge characteristic of the aluminium-air battery. A simple battery curve that can accurately fit with the aluminium-air battery discharge characteristics can represent the battery characteristics and the battery model can be applied to design a battery charger.

3.2 Experimental setup

In the experiment, an aluminium-air battery composes an aluminium anode, air cathode, separator made of polypropylene and potassium hydroxide electrolyte. Aluminium-air battery's performance highly depending on the material used to assemble the battery. In this case, the metal anode is made out of aluminium foil. While the electrolyte of the battery usually will be an alkaline solution or sodium chloride solution which functions to transfer the charges between the anode and cathode of the battery. In the experiment, the effect of different discharge rates and electrolyte concentrations on the air battery performance was carried out. These variables are important as they influence the battery's discharge performance. The aluminium-air battery was discharged under different constant currents of 10 mA, 20 mA, and 30 mA at the electrolyte concentration of 1 mole (1 M) to obtain the discharge performance. The discharge process was repeated with different electrolyte concentrations of 2 M and 3 M. These tests were performed at a room temperature of 25 Celsius. The battery's life cycle can be determined by applying current to release the voltage of the battery cell.

The discharge test of the aluminium-air battery was done by using an electrochemical workstation. An electrochemical workstation such as ZIVE SP1 is a useful instrument in battery research. It consists of a potentiostat, a galvanostat, and an electrochemical impedance analyser. It is able to perform standard measurements as well as programmed tests and is used to investigate the air battery's electrochemical impedance spectroscopy.

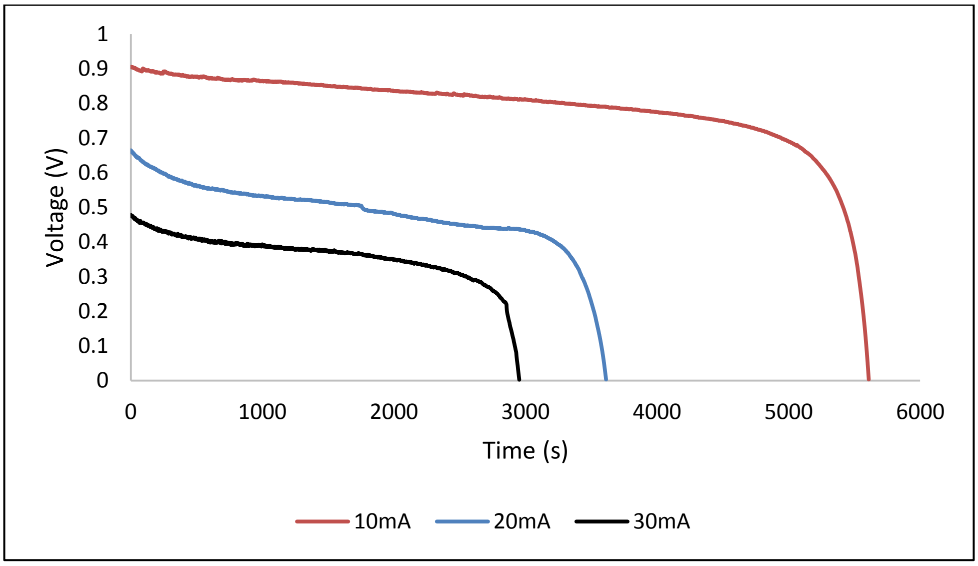

Figure 3.1: Experiment data of aluminium air battery at 1 M electrolyte

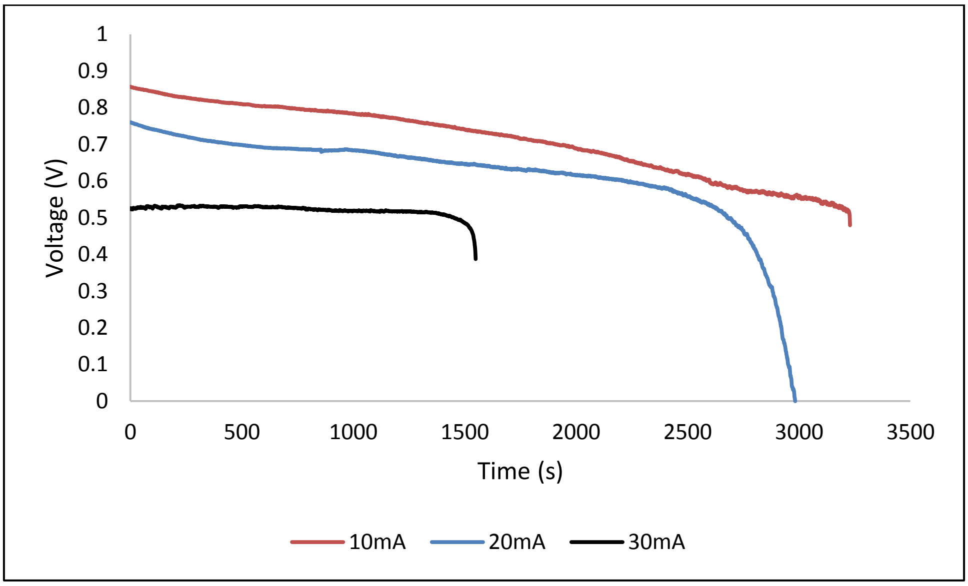

Figure 3.2: Experiment data of aluminium air battery at 2 M electrolyte

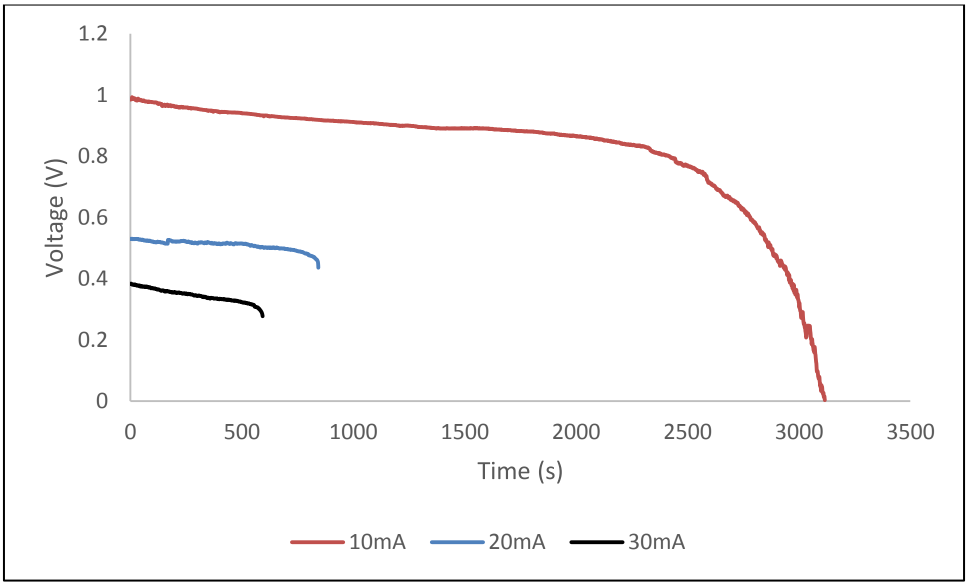

Figure 3.3: Experiment data of aluminium air battery at 3 M electrolyte

3.3 Simulation setup

The main objective of the simulation is to determine the optimized parameter values of the battery model's components which are the voltage source, series resistor and pair of RC network. An accurate battery model parameters can be applied to represent the actual discharge curve for the air battery. In the simulation study, MATLAB software was used to determine the simulation discharge curve of the battery model and compare it with the experimental curve.

A template of an equivalent circuit model (Figure 3.4) with an RC network (Figure 3.5) and two RC networks (Figure 3.6) were modified in MATLAB. To obtain the discharge curve of the aluminium-air battery through simulation, the parameters of the battery model have to be determined initially. The parameters such as voltage source, ohmic resistor, and RC network were pre-determined as an initial guess in form of lookup tables that correspond to the values of SoC.

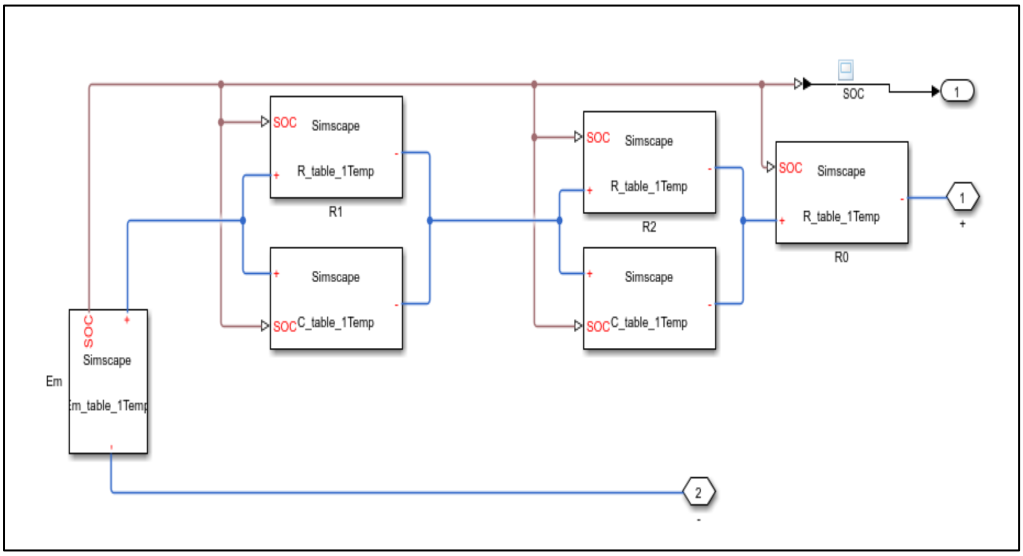

Figure 3.4: Equivalent circuit model template

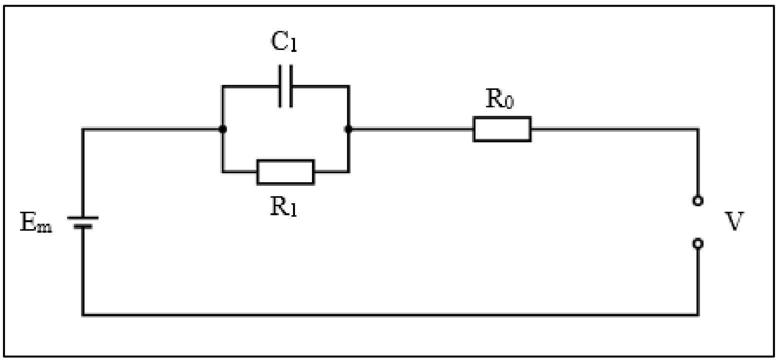

Figure 3.5: Schematic circuit diagram of first-order RC model

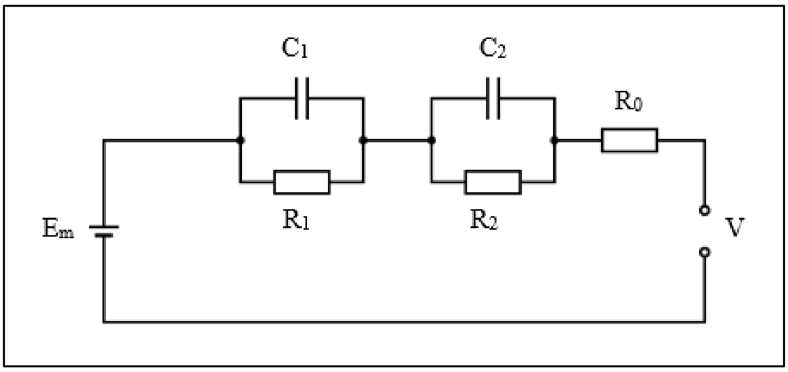

Figure 3.6: Schematic circuit diagram of second-order RC model

The equivalent model was then discharged under constant current throughout the whole period and the output voltage was recorded and tabulated. The battery cell was set to discharge with 10 mA, 20 mA and 30 mA current at a 25 °C temperature. The lookup tables for each circuit component were formed to be based on SoC. and temperature of 25 Celsius. This implied that each component of the model is the two-dimensional lookup table of values. The model is able to estimate the changes in the battery voltage during discharge operation based on different levels of SoC as shown in Figure 3.7 where the values of parameters were corresponding to the breakpoints.

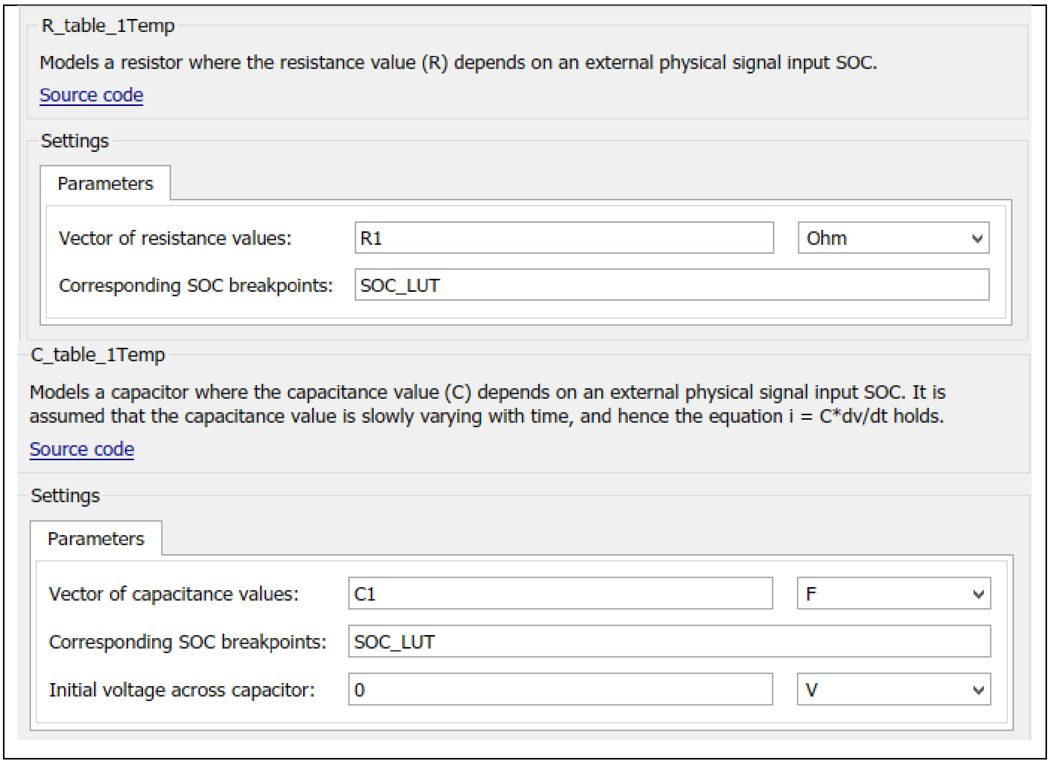

Figure 3.7: The properties of battery model R and C components

3.4 The comparison of the experiment and the simulation data

Validation of the battery model was done to ensure the behaviour of the battery model was able to adequately mimic the characteristic of the actual battery. A set of lithium-ion battery experiment data were used to compare with the modified battery model to obtain the discharge curves of 1 C rated, 2 C rated, and 3 C rated as shown in Figure 3.8 and Figure 3.9. Once the discharge curves have been obtained, they were used to apply with the aluminium-air battery experimental data. Since the discharge curve of the lithium-ion battery is almost similar to the aluminium air battery. The simulation curve can be manually adjusted by changing the parameter values of the equivalent circuit.

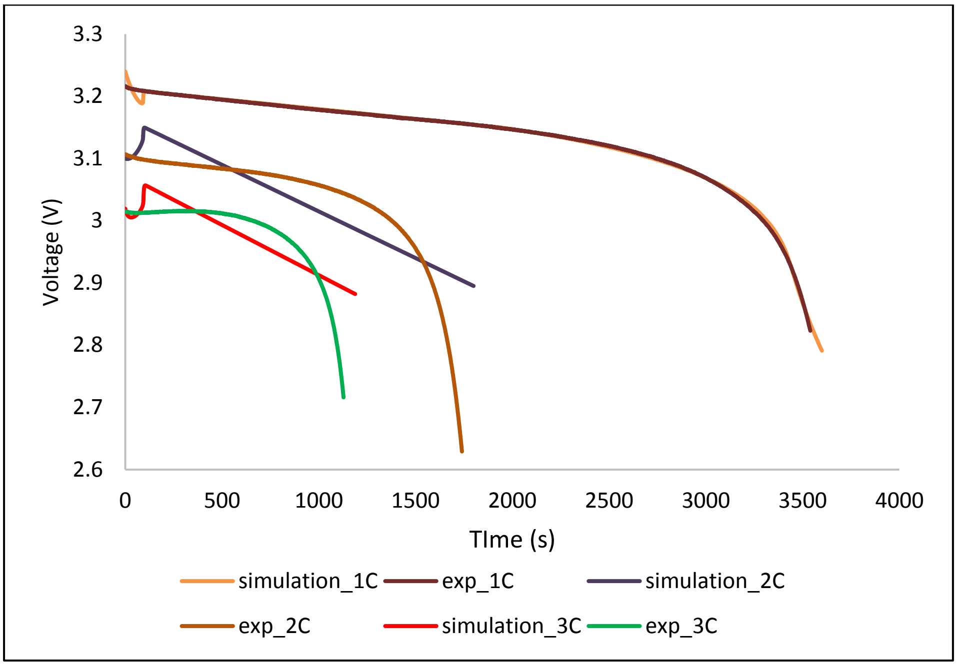

Figure 3.8: Comparison of discharge curve of experiment and simulation of 1 RC model at the different C rate

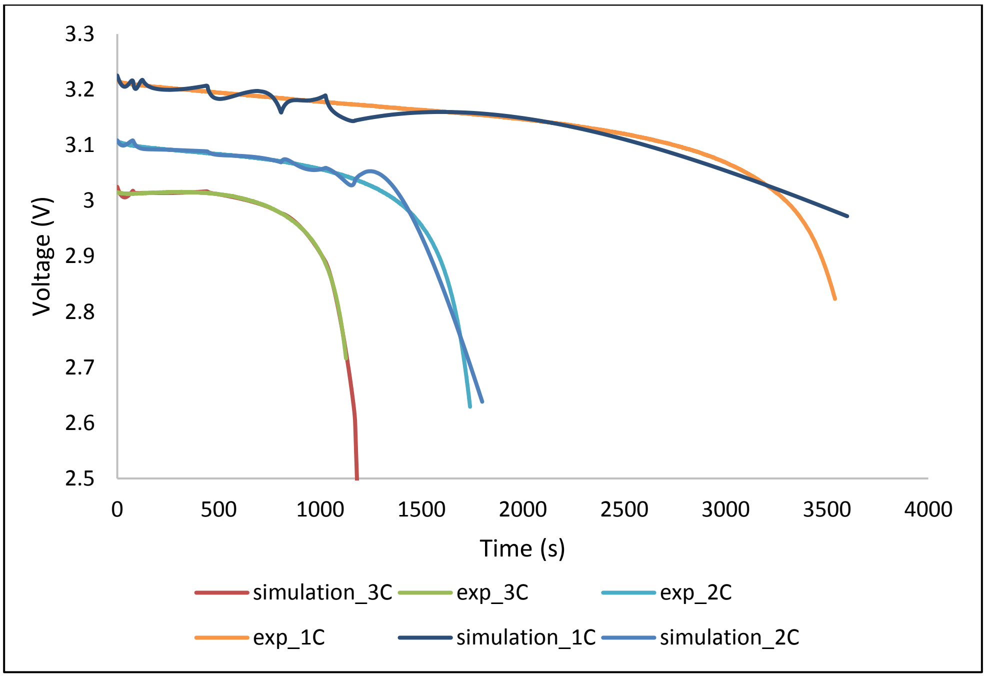

Figure 3.9: Comparison of discharge curve of experiment and simulation of 2 RC models at the different C rate

After obtaining the experimental curve of the aluminium-air battery, the data of the curve was extracted out and inserted into the MATLAB software. In Simulink, the modified battery model was used to fit with the aluminium-air battery experiment data. The experiment data sets were tested with the 1 RC and 2 RC models. The number of RC network branches is proportional to the unknown battery model parameters that need to be defined. After all the parameters of the battery model were pre-defined, the pre-defined discharge curves were produced and compared with the experimental curves. The comparison processes were performed for two different order RC models which are the first-order RC model and second-order RC model to determine the suitability of the model in fitting to obtain the discharge curve.

The parameter estimation method in the Simulink software was applied to adjust and estimate the components' parameters of the battery model until the data was generated by the Simulink model able to satisfy the measured experimental data. Such an optimization process can estimate the parameter to minimize the difference between the battery model and measured data. If the difference between curves is relatively large, the parameters of the model will be re-adjusted until the simulated discharge curve has approximately the same shape compare to the experimental result. With that, the open-circuit voltage (OCV), ohmic resistance, and RC network branch at different SoC were determined during parameter identification.

3.5 The work plan of the project

Figure 3.10 shows the overall process flow of the project that the experimental setup will be performed first and followed by the simulation setup. Once the electro model has been set up, the parameters estimation process will be carried out to determine a set of optimized parameters values that can fit well to the experiment result. The project activities of this trimester mainly focus on understanding the composition of aluminium air battery and its working principle. Besides, an extensive study on battery modeling will be applied to identify the discharge characteristic of the battery.

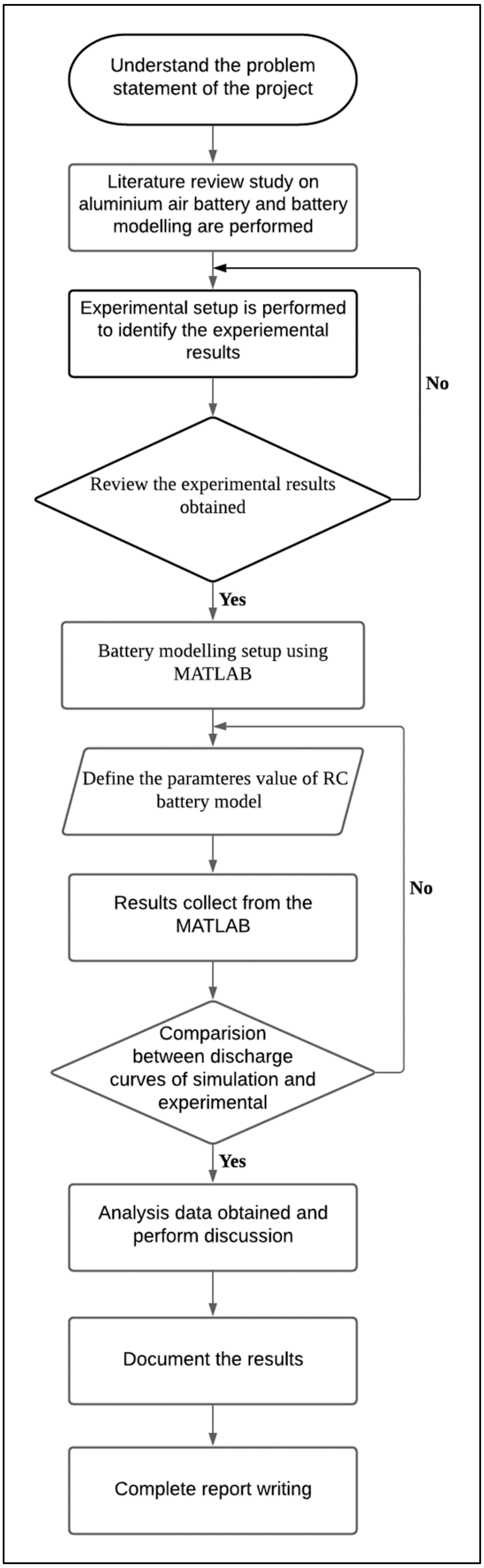

Figure 3.10: Flow chart of the project

3.6 Summary

The overall methodology of the project mainly involves setting up the experiment for an aluminium-air battery to be tested under different test conditions to obtain a set of discharge data. Then, a battery model template is modified in the Simulink and the experimental data will be applied into the battery modeling to determine the value of the battery model's parameters. Validation will be done to ensure the optimized parameters able to fit well with the experiment data. Once the validation of results is done, the documentation of report writing will be carried out until complete.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

In this chapter, the results obtained from the battery model will be discussed. The discussion will be mainly covered identifying the battery components, validate the RC model, and determine the optimized value of model parameters, the SoC breakpoints and selecting a feasible RC battery model. In this project, it will only consider 1 RC and 2 RC battery models to be used will be sufficient practically to conduct the analysis and computation. The general form of all the parameters of the battery model will be SoC dependent at 25 Celsius room temperature.

4.2 Model parameters identification

To build a good quality battery model, the model parameters need to able accurately computed from test data. To identify these parameters, data from particular tests, such as current pulse discharge or constant current discharge tests at different current rates have to be collected.

The parameters of the equivalent circuit model's elements are dependent on the SoC at 25 Celsius. The parameters are obtained through the two-dimensional lookup table of values of each element at different SoC, where the columns of the lookup table represent the SoC values and the temperature row.

In the RC model, the voltage drop in the ohmic resistance and polarization resistance is controlled by the ohmic effect equation of

$$R = V/I \tag{4.1}$$

where the increase of voltage value will result in the linear increase of current. Whereby the instantaneous voltage drop in the model is controlled by the ohmic resistance, which does not change with respect to time. While the transient voltage drop is determined by the polarization resistance and the polarization capacitance that governs by the equation of

$$i = C\frac{dV}{dt} \tag{4.2}$$

The i is the current flow in the parallel RC network that describes the non-linear polarization response of the aluminium-air battery.

To identify the optimized set of parameters, the constant current discharge test at various current rates to obtain data. The lookup tables for each circuit element were set based on 11 different SoC breakpoints of (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1) at 25 Celsius to reflect the change of voltage throughout the discharging process. Defining more breakpoints allows the battery model to accurately represent the discharge curve. However, it will increase the computational complexity as more parameters need to be determined. Since the 1 RC model has only a pair of parallel RC branches, it will be less complex compare to the 2 RC model. While the two pairs of RC allows the second-order RC model to describe the transient effects of a battery more accurate. These transient effects are related to electrochemical and concentration polarization effects that occur in the battery.

4.3 Model parameters validation

To validate the degree of fitting of the simulation result, 9 sets of experimental data of discharge current rates (10 mA, 20 mA, and 30 mA) at 1 mole, 2 mole and 3 mole concentration of electrolyte have been compared with the simulation data to obtain an optimized set of parameters values which are a function of SoC at 25 Celsius. From Figure 4.1, it can be observed that the simulation curves can fit well at the beginning of the voltage drop of the curves. While comparing the difference at the end of voltage drop, there is a significant difference between the accuracy and error of both models compare to experimental data.

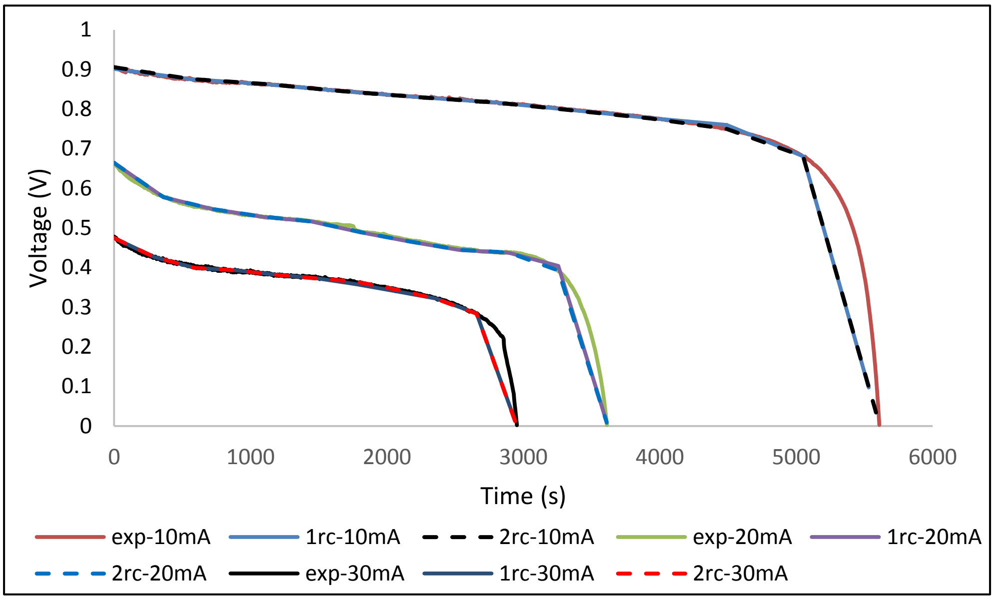

Figure 4.1: Comparison of simulated curves with experiment curve at 1 Mole electrolyte

Figure 4.2, and Figure 4.3 show the comparison of the model results with the measured data at different electrolyte concentrations. Although at different current rates and electrolyte concentrations, the models still match the measured data while the value of the parameter remains the same. The difference between the measured and simulated voltage has a small variation at the beginning of discharge is observed from the simulation result. It can be observed that the simulated curves are mostly unable to fit the curvy region due to not enough SoC breakpoints allocated.

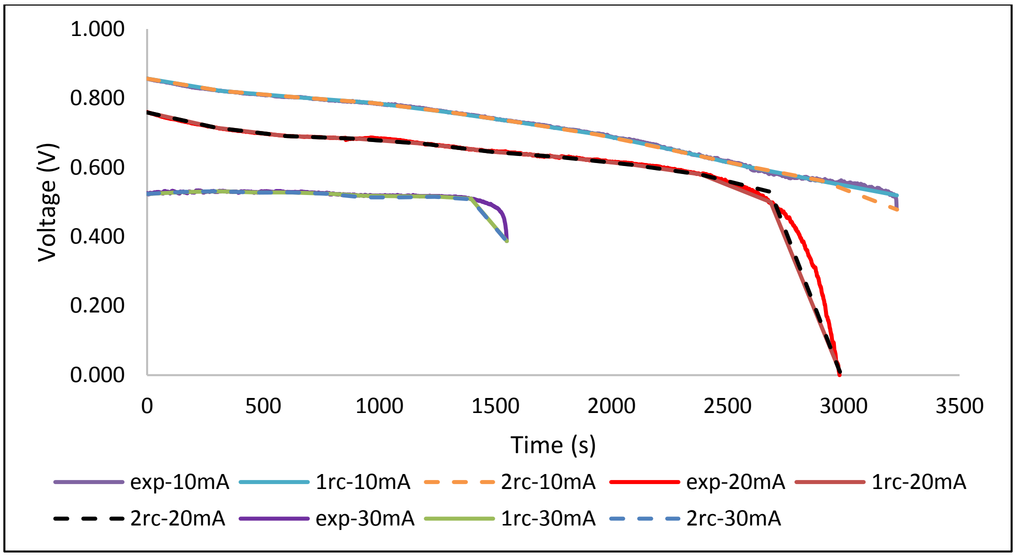

Figure 4.2: Comparison of simulated curves with experiment curve at 2 Mole electrolyte

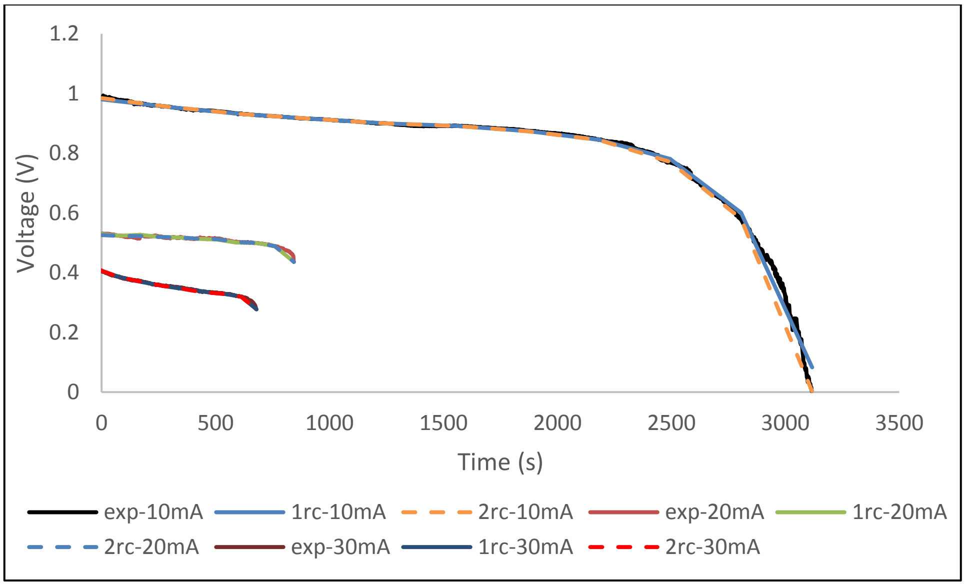

Figure 4.3: Comparison of simulated curves with experiment curve at 3 Mole electrolyte

For the validation, the average error between simulated and experimental data is calculated where the total error for the whole discharge process is divided by the total period of discharge duration. Based on Table 4.1, the 2 RC model has a smaller average error of 2.61 % difference compared to 1 RC model at 1 Mole, 10 mA current rate. However, at certain conditions, the 1 RC error is slightly lower than the 2 RC due to the difference between the experiment voltage value and simulation at the ending of discharge. Plus, some of the tuning processes were carried out manually instead of applying the parameter estimation, resulting in deviation of accuracy occurred. From the overall comparison, the 2 RC still able to fit better with the experiment data. This indicates that the higher order of RC model can adapt more at the transient response region due to the extra RC network branch. As the result, one of the most significant criteria for determining the number of branches is that the curve corresponded closely to the transient part of the experiment data. Otherwise, when applying the battery model in studying the discharge characteristic of aluminium air battery, the optimization will push the entire simulated curve down, resulting in a poor overall fitting.

Table 4.1: The average error at different discharge current rates and mole

		Error (%)	
Mole	Current rate (mA)	1 RC	2 RC
1	10	9.54	6.93
	20	3.97	3.51
	30	7.67	6.54
2	10	0.26	0.53
	20	3.38	2.82
	30	0.90	0.85
3	10	7.38	3.11
	20	0.41	0.48
	30	0.62	0.47

By conducting a simple constant discharge test on the battery cell from 0% and 100% state of charge at a constant current, the voltage of the battery cell is observed. In comparison with the battery model, it can be observed the ability of the model to fit the discharge behaviour, although it still lacks precision at the end of the discharge process. It is quite important to ensure the lower SoC points where the ending of the discharge curve to be accurate, as it will affect the battery life spans. The life spans of the battery are mainly depending on the deep discharge of the battery which might cause permanent damage if the battery model unable to represent the battery data well.

The equivalent circuit model such as the RC model is one of the simple alternative approaches to replace the other complex model in studying the characteristics of the battery. Thus, it is clearly shown that the parameter values of an equivalent circuit model can apply to compare with the physical data of a battery cell. However, there is a limitation in terms of accuracy where the model is not able to completely fit well with the nonlinear electrochemical phenomenon. The complex electrochemical phenomenon in a battery can be expressed by a more complex model. However, as the consideration to reduce the difficulty of simulation work, slightly less reliability has to be sacrificed in order to maintain the balance between the computational effort and reliability to avoid the duration of the lengthy calculation of the electrochemical process.

4.4 Effect of SoC breakpoints

Defining the location of the SoC breakpoint is also an important aspect in electro-modeling as it generally affects the battery model output result. At the nonlinear part of the discharge curve, more breakpoints of SOC with smaller equally difference breakpoint values should be placed, to obtain better mutation parameters in the circuit. Based on Figure 4.4, the SoC breakpoints of (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1) can be seen that both models (1 RC and 2 RC) curves can fit well for the beginning part of the experiment curve. However, both models are unable to fit well at the transient region of the experimental curve. This is because the number of SoC breakpoint at the low SoC is less and the breakpoints are far apart causing the electrical parameters at low SOC levels to be inaccurate. Compare with the beginning of the SoC level where the aluminium air battery is full charged, the end of SoC experience fast response discharge while the beginning of SoC experiences slower discharge response which can be seen the part almost looks like a linear curve. Thus, the fewer breakpoints at the end are unable to catch all the data values to allow the simulation curve to fit well with the experiment curve.

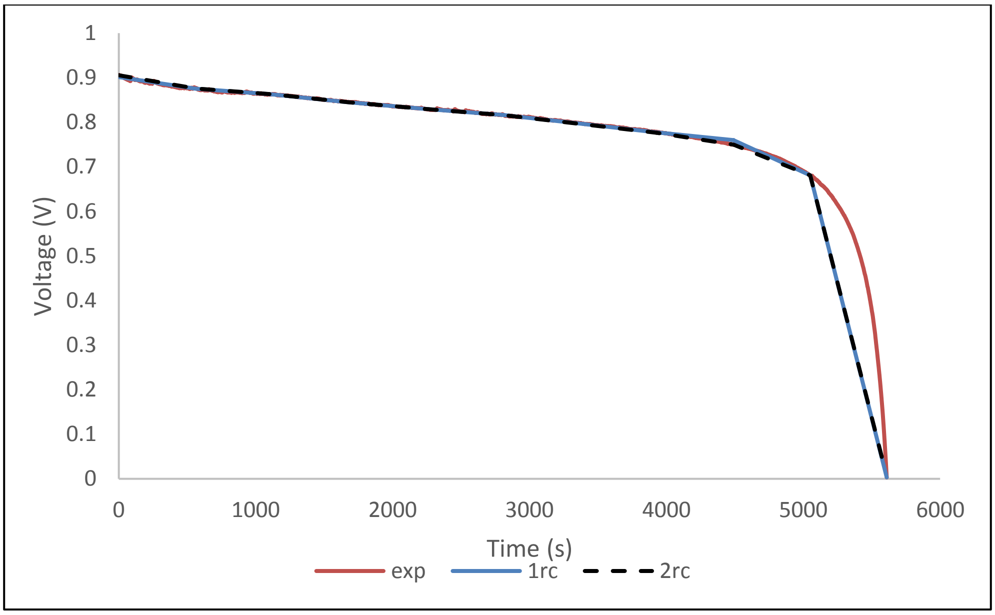

Figure 4.4: Comparison between the experiment data and battery models with the SoC breakpoints of (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1)

Figure 4.5 shows a similar comparison of experimental data as Figure 4.4 but with the increasing of numbers SoC breakpoints at the transient part. Increasing the number of breakpoints is better to represent the model curve rather than neglect the other region of the curve by setting fewer breakpoints in that region. However, as the number of breakpoints increase, the number of unknown parameters will increase, leading to the CPU time and memory usage increasing significantly as well. A reliable and practicable electro model depends on accurate SOC estimation.

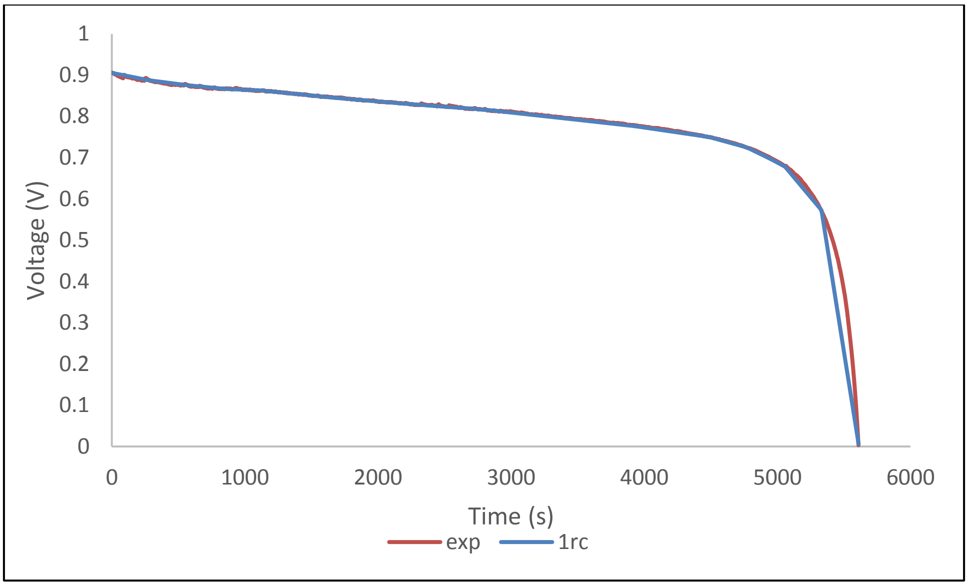

Figure 4.5: Comparison between the experiment data and battery models with the SoC breakpoints of (0; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.85; 0.9; 0.95; 1)

In general, a good battery model has two major characteristics which are it can accurately represent the dynamic and static states, and it has a minimal complexity. When it comes to achieving excellent precision in the model, the correct adjustment of the parameters involved is critical, which is why locations of the SoC breakpoint is crucial in the formation of the lookup table. Figure 4.6 shows the most fit 1 RC battery model curve with total number of 16 SoC breakpoints (0; 0.025; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.85; 0.9; 0.95; 1) where one more extra point is located at the end of discharge. The battery model seems to fit well with the experiment curve with the minimum deviation. Compare with the battery model in Figure 4.5 and Figure 4.6, more nearer SOC breakpoints are located with a bias toward the beginning and end of the discharge to reflect the change of the cell voltage at low and high SOC more accurately. Since the model parameters are depending on the battery SoC, the SoC has to be set equally and near to each so that model can catch more experiment data from the experiment curve. Besides, for this battery model with a large number of SoC breakpoints, the battery model to be tested is only 1 RC model because using the 2 RC models will increase the number of parameters that need to be defined. Plus, as the 1 RC model is able to fit well the experiment curve, it should suffice to represent the discharge characteristics of the aluminium air battery in this case to avoid a large number of unknown parameters.

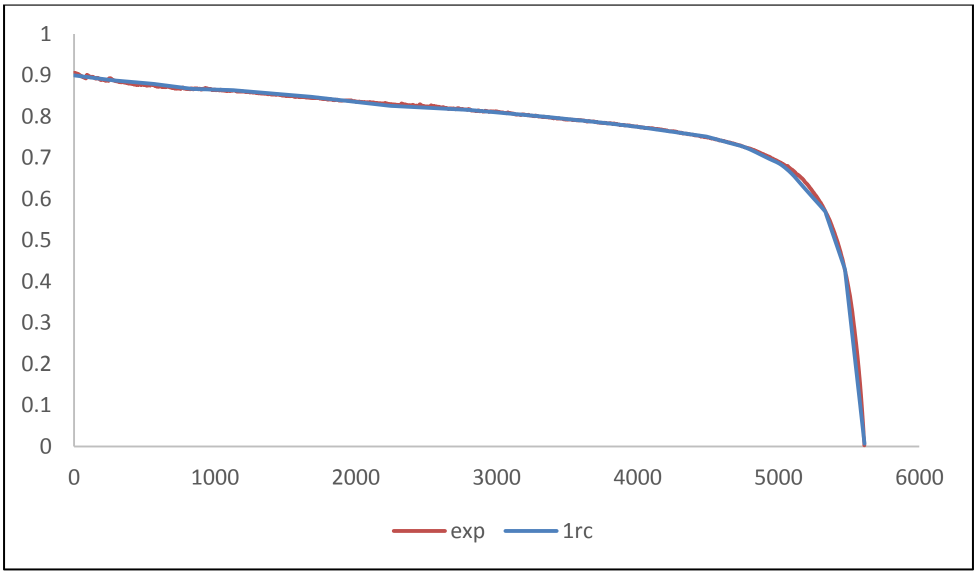

Figure 4.6: Comparison between the experiment data and 1 RC model with the SoC breakpoints of (0; 0.025; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.85; 0.9; 0.95; 1)

In Figure 4.7, the overall comparison between the experiment data and 1 RC model with the different SoC breakpoints can be observed. Therefore, it is also important to do SoC estimation to identify the optimized breakpoints location to produce a well-fit model. Nevertheless in this model, the estimation of SoC is not conducted as it tends to fix the SoC breakpoints for different discharge current rates at different electrolyte concentrations.

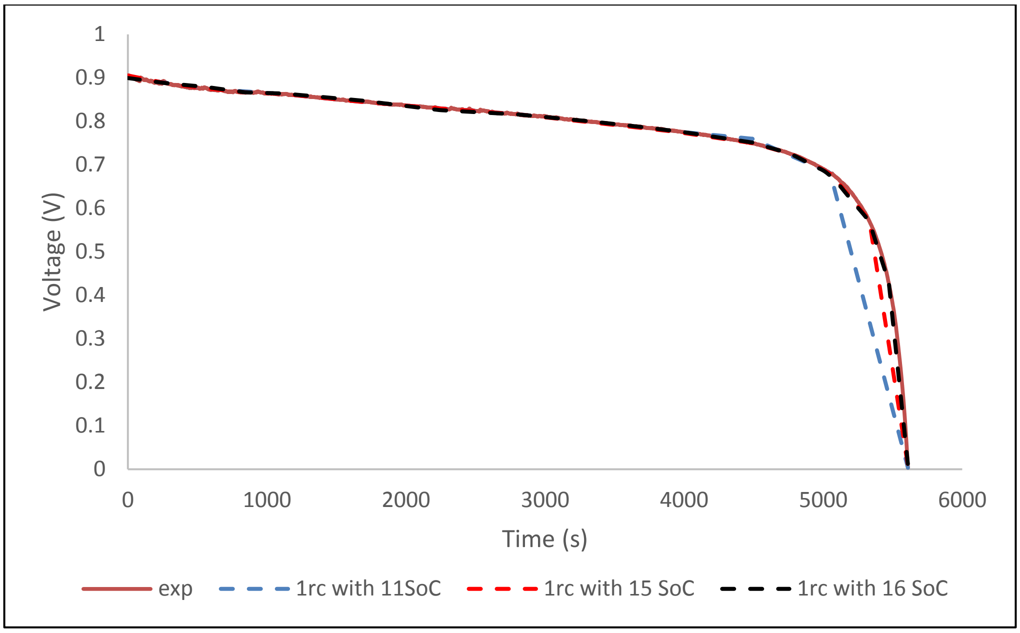

Figure 4.7: Comparison between the experiment data and 1 RC model with the different SoC breakpoints

Table 4.2 shows that the comparison of error among the 1 RC model with the different SoC breakpoints, the battery model with the most points has a smaller error deviation. While the battery model with 11 points tends to have a large error deviation.

Table 4.2: Average error of 1 RC model with different SoC breakpoints

No. of SoC point	Average error (%)
11	9.54
15	4.00
16	2.87

The finding of the terminal voltage at a certain SoC level for the discharge process can be determined by knowing the value of the SoC level. In the project, the SoC is defined by using the equation of

$$SoC = 1 - \frac{t}{t_{Total}} \tag{4.3}$$

where the t is the duration of extracted charge in discharging process and t_{Total} is the total charge of the battery cell in the discharge period. By identifying the voltage value at a certain SoC level, the voltage parameter of the battery model can adjust well through only manual processes instead of using parameter estimation in Simulink. This will allow the user to obtain the discharge curve of the battery easily.

By changing the discharge curve to be in form of voltage vs. SoC as shown in Figure 4.8, it is able to see clearly that both battery models starting to show a different variation with the experiment data around the SoC value of 0.1. Increase the number of SOC points can create diminishing advantages to reflect the cell voltage. However, the parameter estimation process will delay due to the number of parameter values to be defined. When the battery SOC level is high, the modeling result matches fairly well with the measured battery terminal voltage. At low SOC values, the discrepancy between experimental and modeling results becomes significant.

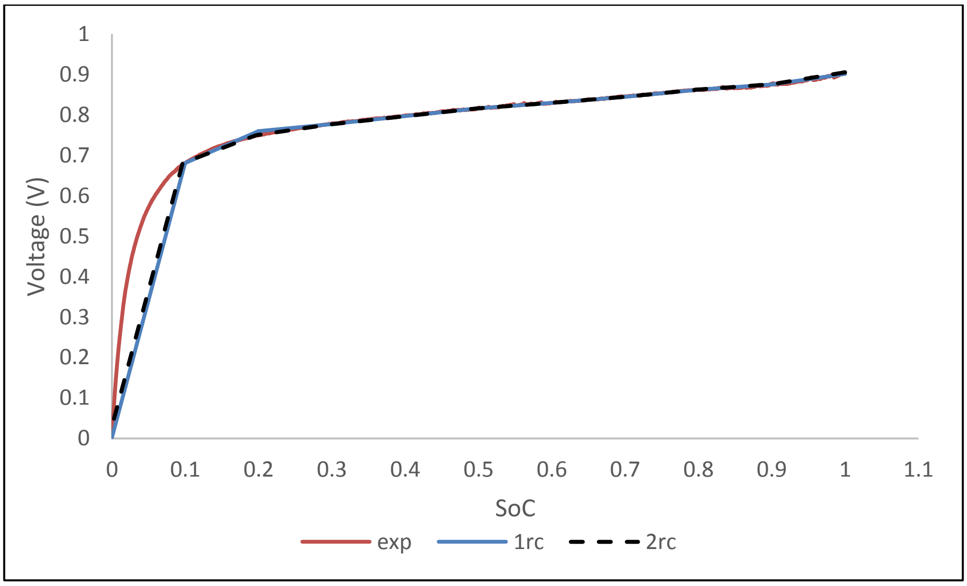

Figure 4.8: The discharge curves to be voltage vs. SoC at 1 M electrolyte and 10 mA with 11 SoC breakpoints

Compare with Figure 4.9, it can be seen that defining the smaller interval SoC breakpoints at the transient region of the curve allows the model to generate a better curve fitting. The addition of SoC points is only performed on the1 RC battery model to compare with the experiment data. This is because, with only the 1 RC model, there are 15 undefined parameters for each element of the battery model corresponding to the SoC points. Using the 2 RC models will result in a total of 90 undefined parameters (6 lookups tables * 15 SoC breakpoints) that need to be determined, causing the battery model to be much complicated.

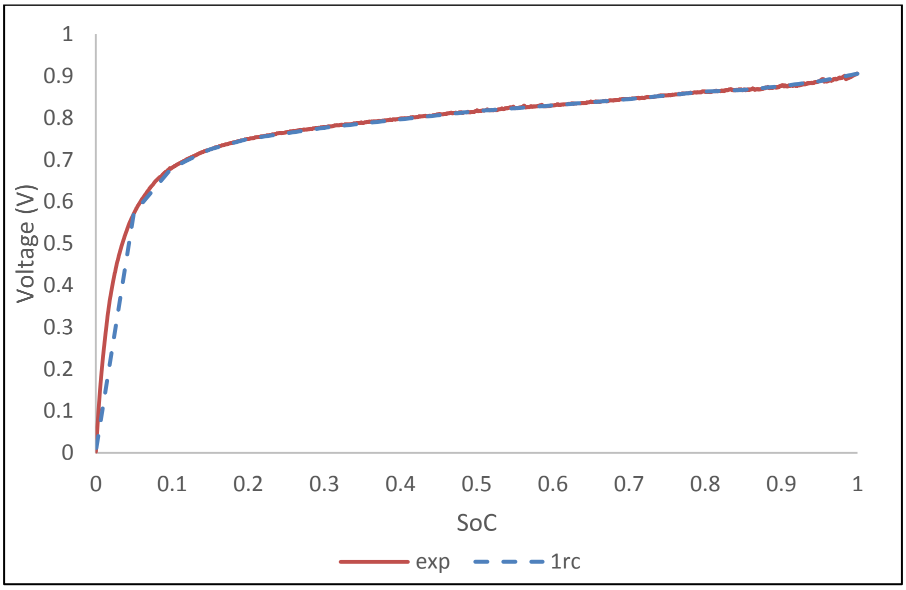

Figure 4.9: The discharge curves to be voltage vs. SoC at 1 M electrolyte and 10 mA with 15 SoC breakpoints

Regardless, this model obtained still has some room for improvement. Rather than defining more points towards the beginning and end of SoC, the SoC estimation should be carried out whereby some SoC estimation methods such as the Kalman filter method can be applied to the estimation of SoC for the model but the complexity of the model might as well increase.

4.5 The value of model parameters

The following shows the optimized values of the model parameter of 1 RC model and 2 RC model with SoC breakpoints (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1) at 1 mole concentration of electrolyte with 10 mA current rates. The SOC values range from to 0 to 1, indicating that the aluminium air battery in full charge (1) and fully discharged (0).

Since the equivalent circuit model parameters is a function of SOC. Parameter estimation results of various SOC levels are listed in Table 4.3 for the 1 RC model and Table 4.4 for 2 RC model.

Table 4.3: Parameters value of 1 RC model at different SoC (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1)

SoC	E_m (V)	R_0 (Ω)	R_1 (Ω)	C_1 (F)
0	0.003	4.9194e-06	0.80321	2303.3
0.1	0.681	9.5376e-06	0.011021	36350
0.2	0.750	9.5376e-06	0.019521	25641
0.3	0.778	0.15377	5.52e-05	3.2538e+05
0.4	0.798	0.15376	7.36e-05	72875
0.5	0.817	0.078698	9.17e-17	15376
0.6	0.830	0.011845	0.008017	17196
0.7	0.845	0.01184	0.009366	17953
0.8	0.863	0.011604	0.00739	24370
0.9	0.876	0.011911	0.004003	36379
1	0.906	0.047591	0.004375	24893

Table 4.4: Parameters value of 2 RC model at different SoC (0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1)

SoC	E_m (V)	R_0 (Ω)	R_1 (Ω)	C_1 (F)	R_2 (Ω)	C_2 (F)
0	0.003	0.000146	0.695160	3944.1	3.797900	396250
0.1	0.681	0.000145	0.007960	1600200	0.241680	6238700
0.2	0.750	0.000150	0.000102	243130	0.104270	9467700
0.3	0.778	0.063866	0.000134	4036100	0.000136	4332.7
0.4	0.798	0.080500	0.000123	67651	0.003908	9850.3
0.5	0.817	0.043354	0.000150	14355	0.011674	1803.2
0.6	0.830	0.006104	0.003812	16439	0.004668	8024.4
0.7	0.845	0.006156	0.004734	16181	0.007840	11254
0.8	0.863	0.018124	0.009357	57662	0.030641	25821
0.9	0.876	0.052874	0.009162	29065	0.034552	37690
1	0.906	0.453740	0.006802	81586	51.341000	419570

For all curves at different concentrations of electrolyte and different discharge current rates, one set of optimized model parameters has been obtained. The values of ohmic resistance, RC network branches at different SoC levels are fixed for different current rates at different electrolyte concentrations. While the value of voltage and the capacity of the battery will vary as the voltage level and total capacity of all curves are different. By doing so, the model study of aluminium air battery can be simplified as the user is only required to adjust the voltage and capacity values when using the MATLAB model to study the discharge characteristic of the air battery in future.

In Table 4.5 shows the optimized set of parameter of the 1 RC model with SoC breakpoints (0; 0.025; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.85; 0.9; 0.95; 1) at 1 mole concentration of electrolyte with 10 mA current rates..

Table 4.5: Parameters value of 1 RC model at different SoC (0; 0.025; 0.05; 0.1; 0.15; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.85; 0.9; 0.95; 1)

SoC	E_m (V)	R_0 (Ω)	R_1 (Ω)	C_1 (F)
0	0.003	0.00015	0.75979	15740
0.025	0.573	0.00015	0.00582	327350
0.05	0.681	0.00015	0.00437	203850
0.1	0.725	0.00015	0.00670	272820
0.15	0.750	0.00015	0.01084	217070
0.2	0.778	0.36543	0.00014	1525400
0.3	0.798	0.15326	0.00016	72687
0.4	0.817	0.13512	0.00015	14705
0.5	0.830	0.00553	0.00451	18554
0.6	0.845	0.00457	0.00538	55481
0.7	0.863	0.11339	0.01231	35525
0.8	0.867	0.09181	0.02383	34428

0.85	0.876	0.01172	0.00856	8487
0.9	0.888	0.47334	0.01106	26732
0.95	0.894	0.13668	0.01037	29045
1	0.906	0.03320	0.00462	25765

Figure 4.10 is the discharge curves generated by using the fixed ohmic resistance values and RC branch value from Table 4.5. This set of parameter values has a max average error is less than 10% as shown in Table 4.6.

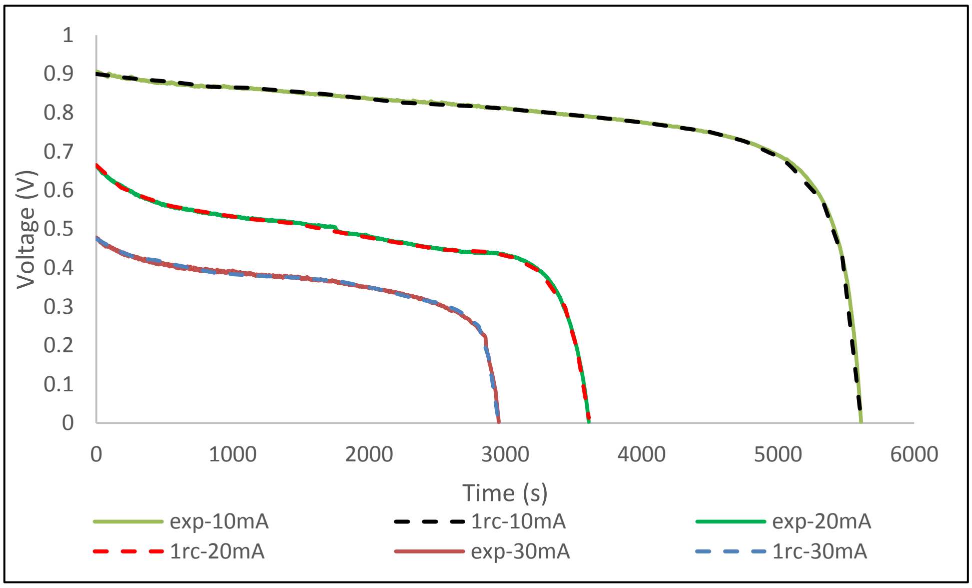

Figure 4.10: Comparison between the experiment data and 1 RC model at different current rates and 1 Mole with 16 SoC points

Table 4.6: Average error of 1 RC model at different current rate and 1 Mole with 16 SoC points

Current rate (mA)	Average error (%)
10	2.87
20	1.51
30	3.62

4.6 Selecting RC model with feasible the number of RC branches

It is important to choose wisely the number of RC branches to be used to balance between reliability and complexity. The number of RC pairs is proportional to the number of time constants that describe battery transients. The experimental data sets can be well matched by a very complicated high order equivalent circuit, but owing to the long computational duration, it is unsuitable for embedded control applications with limited computational resources.

To determine the number of RC branches to use, the decision lies on the ability of the RC branch to mimic the transient response of experiment data. The transient response is controlled by the R_0 and the parallel RC branches of the equivalent circuit. However, in this project, although one or more exponential RC branches have been fitted to the data using parameter estimation. The results show that the difference between the 1 RC and 2RC battery models have no large difference in terms of output.

Figure 4.11 shows the faster part of the transient, it shows more differentiation for the higher number of RC exponentials. However, the insufficient RC branches pull the overall simulated curve down. This result shows that only one or two exponential time constant terms is not enough to produce a satisfactory fitting to the data as the shape of the transient response seems to require extra time constant to represent the measured results. While three RC branches may be considered as a compromise between accuracy and complexity.

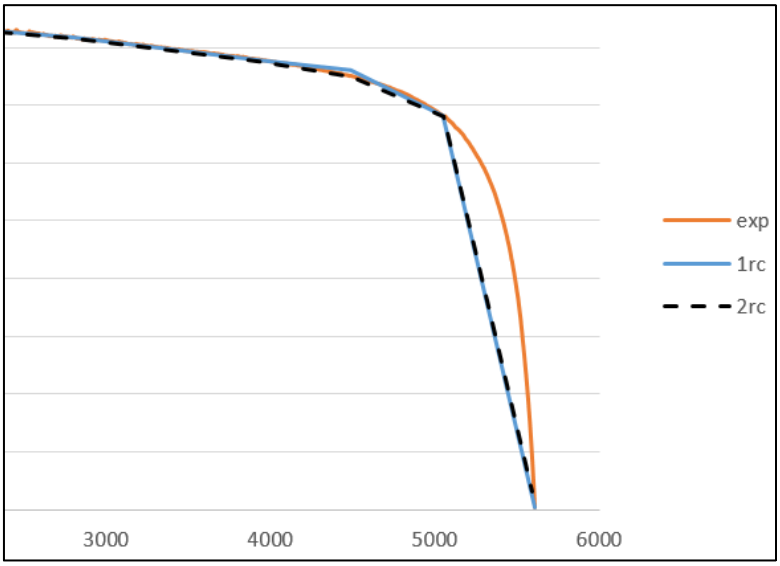

Figure 4.11: Zooming in of transient part of discharge curves

The results of Figure 4.10 show a good alignment between simulation results and experimental data. Thus the model with the optimized value of parameters shown in Table 4.5 can be applied to determine the battery performance under different current rates conditions.

Increasing the battery model order can improve the estimation accuracy because the battery performs different dynamic characteristics during different discharge periods. However, the high order battery requires more computation resources.

4.7 Summary

Overall, the 1 RC and 2 RC battery models with fewer SoC breakpoints were unable to mimic the experiment discharge curve well. However, with the increase in the number of SoC breakpoints, the results have shown that the 1 RC battery can work quite well in representing the actual battery discharge characteristic. Thus, it can be concluded that more SoC breakpoints should be allocated at the beginning and ending region of the discharge curve and an increase in the number of RC branches can help to improve the accuracy. While comparing the battery model, the 2 RC model was able to provide more accurate results compared to 1 RC. Nevertheless, the balance between the accuracy of the model and complexity has to be optimized. Plus, the SoC points estimation should be conducted to produce a good quality battery model.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this project, different types of battery models which are the electrochemical model and equivalent circuit model are reviewed. The modeling of discharge characteristics of aluminium-air battery using an equivalent circuit model is done using MATLAB and Simulink. The simulation for the battery model is performed using the equivalent circuit model: 1 RC and 2 RC. Under the constant discharge current, the 1 RC battery model has a maximum average error of 9.54 %, while the 2 RC model has a maximum error of 6.93 % compared to the experimental data.

The 1 RC equivalent circuit model with smaller SoC interval points at the beginning and end of the discharge curve is sufficient to capture the discharge characteristic of the aluminium air battery cell compare to the 2 RC model. The simulation model can apply to other aluminium-air battery experimental data by using parameter estimation to identify the parameter of the higher complexity battery model. The battery models for the aluminium-air battery is useful in practical aluminium-air battery management systems of electric vehicles or use as a battery charger in future.

Although the battery model is independent of temperature change. The thermal model, heat generation in the battery cell and heat loss to the surrounding should be included in the model. If a battery cell pack is used to study, the temperature of the cell pack has to be considered as it is different from the temperature of the individual cell where the heat generation between cells will occur during battery operation. This consideration is not included in this study, but it can be an improvement in future studies. Plus, the focus of the investigation on the condition of different working temperatures of aluminium-air battery discharge using an equivalent circuit model should be done in future work.

5.2 Recommendations for future work

Modeling the same battery models with including the effect of various operating temperatures into consideration by carrying out different experiments where the aluminium-air battery cell is tested under different temperatures. The surrounding temperature of battery is one of the parameters that affect battery performance. The excessive temperature difference causes the voltage, and internal resistance to be varied. Plus, under huge different working temperatures, there will be a significant effect on the battery discharge capacity and life span. Thus, creating an optimal battery model that can work under several temperature conditions is important.

Furthermore, include the battery properties such as the mass, battery dimension, and battery capacity was changed accordingly to be the same as the battery properties into battery model is important. Such properties are important to set up the thermal model that exists in the equivalent circuit. Battery performance and efficiency are generally affected by the surrounding temperature and discharging process. Thus, the thermal model of the battery is generally used to estimate the cell temperature.

Besides, the simulation study should cover more battery cells. The combination of battery cells provides different SoC values as the SoC for each battery cell is different. Plus, stacking of the battery in series or parallel will cause heat generation between the cells. Thus, the application of a cooling system into the battery model has to be done in order to dissipate the heat generated through the convection process and allow the battery model to operate under the optimum temperature.

Another recommendation is using the parallel computing function in MATLAB that require using a multicore computer to run the estimation of the battery model in parallel mode. This allows MATLAB to distribute the load to several computers to speed up the computation period. For one battery cell electro-modeling study, one set of equivalent circuit parameters are defined during each estimation process. It is still relatively can be determined with less computational complexity. However, if the study of pairs of battery cells is done, the parallel mode might require as each cell will have its own set of equivalent parameters causing more parameters required to be identified.

The model improvement can also be done by adding more RC branches into the equivalent circuit model to improve the reliability. An equivalent circuit model with 3 RC branches (Figure 5.1) can be tested to check the improvement in terms of the accuracy and whether the improvement is significant to be done while ignoring the computational complexity. Plus, different types of experiment tests should be conducted to validate the parameters. Test such as pulse discharge test is one of the suitable tests, where the battery is discharged and follow by a resting period to allow the battery back to the equilibrium state. Through the observation of the relaxation period in Figure 5.2, the proper number of RC branches can be determined as the number of time constants corresponding to the RC branch that affect the shape of the response curve of the voltage.

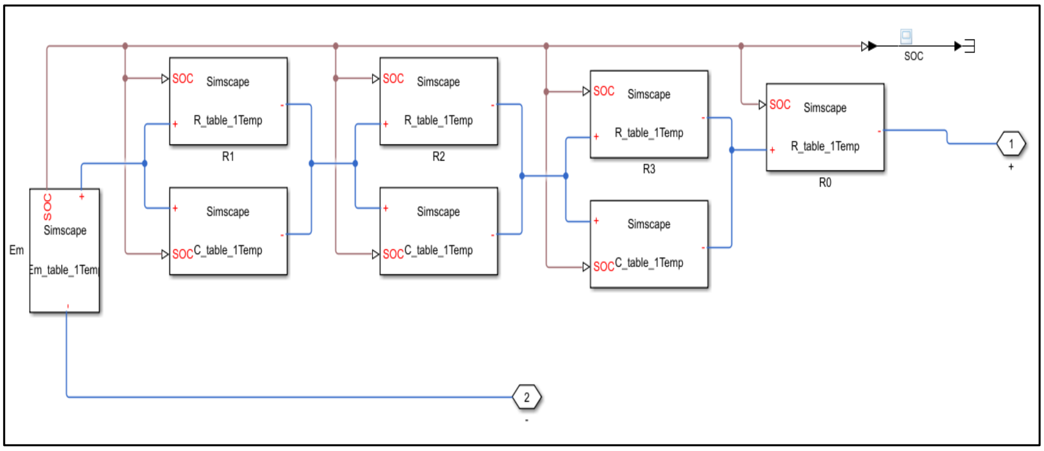

Figure 5.1: 3 RC battery model block diagram

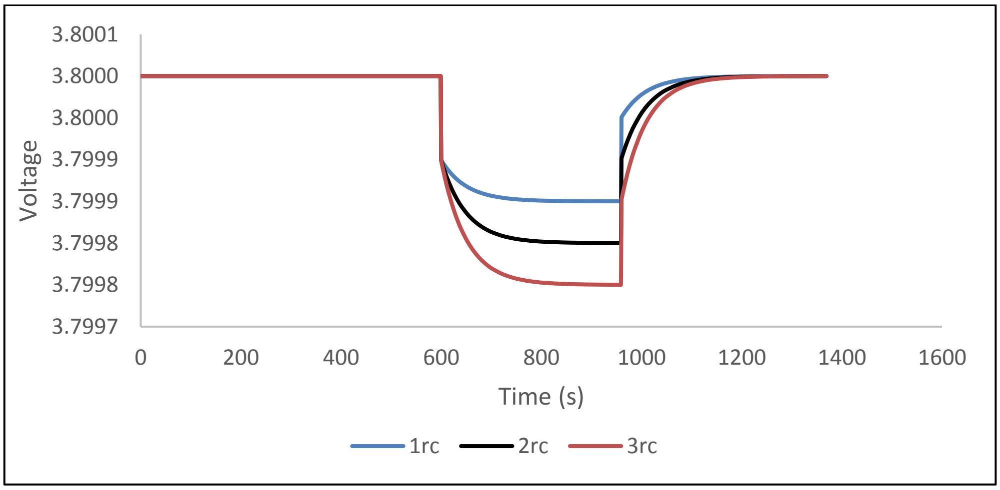

Figure 5.2: Pulse constant current test on different RC branch battery model

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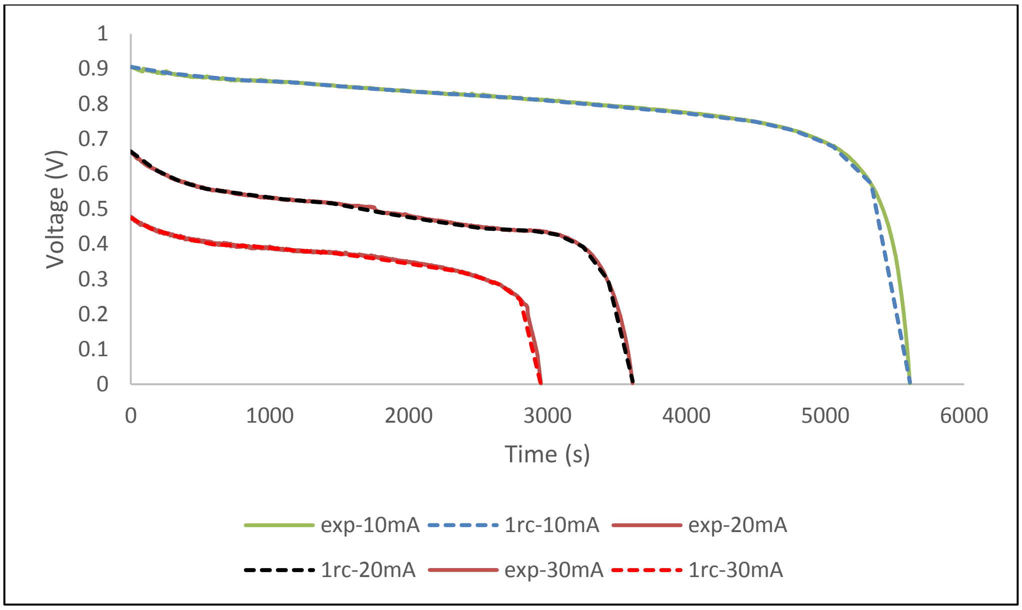
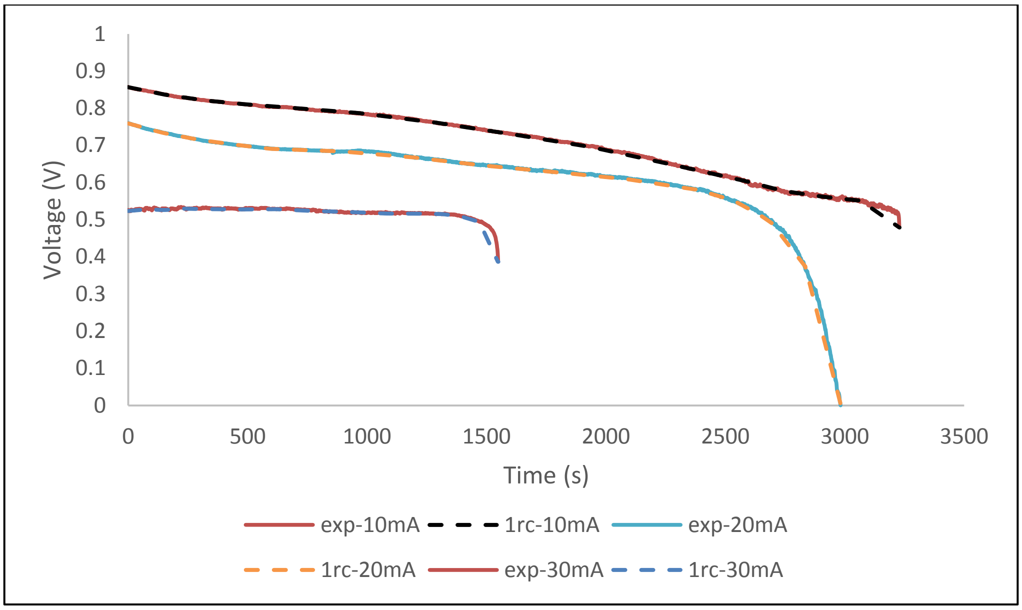
APPENDICES

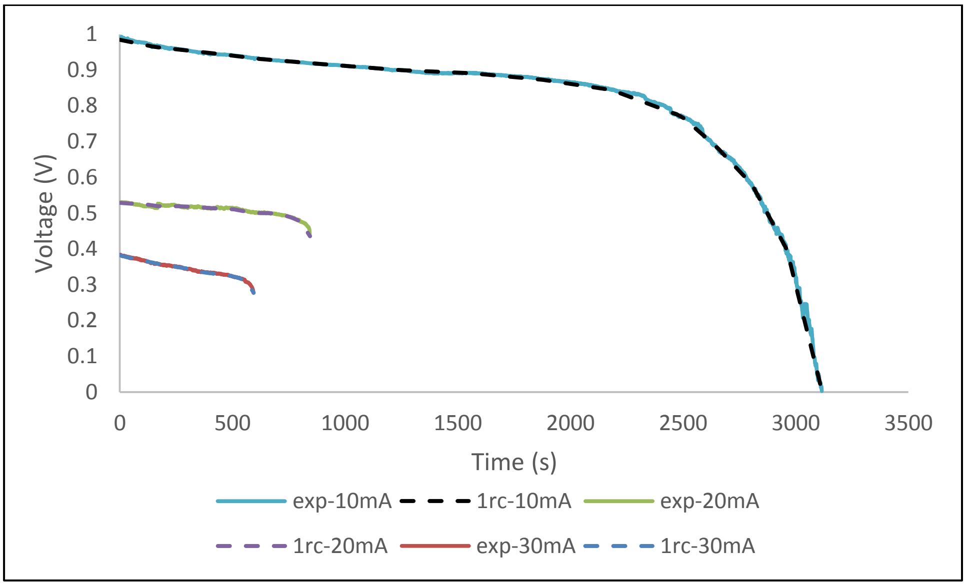
Appendix A: Figure



Comparison of simulated curves with experiment curve at 1 Mole electrolyte with 15 SoC breakpoints



Comparison of simulated curves with experiment curve at 2 Mole electrolyte with 15 SoC breakpoints



Comparison of simulated curves with experiment curve at 3 Mole electrolyte with 15 SoC breakpoints

Appendix B: Tables

Calculation of battery model 1 RC under 10 mA, and 1 Mole condition with 16 SoC breakpoints

SoC	Time (s)	Simulated voltage (V)	Experimental voltage (V)	Absolute error (%)
1.00	0.0000	0.900	0.906	0.677
1.00	0.0025	0.900	0.906	0.677
1.00	0.0051	0.900	0.906	0.677
1.00	0.0076	0.900	0.906	0.677
1.00	0.0329	0.900	0.906	0.678
1.00	0.0582	0.900	0.906	0.678
1.00	0.0835	0.900	0.906	0.678
1.00	0.3365	0.900	0.906	0.679
1.00	0.5894	0.900	0.906	0.680
1.00	0.8424	0.900	0.906	0.681
1.00	3.3723	0.900	0.905	0.660
1.00	5.9021	0.899	0.906	0.706
1.00	8.4319	0.899	0.905	0.616
1.00	10.9617	0.899	0.904	0.527
1.00	26.2920	0.899	0.902	0.363
0.99	41.6224	0.898	0.899	0.165
0.99	56.9527	0.897	0.897	0.033
0.99	72.2830	0.897	0.895	0.163
0.98	87.6133	0.896	0.894	0.192
0.98	108.4994	0.895	0.897	0.181
0.98	129.3854	0.894	0.895	0.077
0.97	150.2714	0.893	0.893	0.061
0.97	171.1575	0.892	0.893	0.108
0.97	192.0435	0.891	0.889	0.270
0.96	228.8035	0.890	0.887	0.367
0.95	265.5635	0.888	0.891	0.291
0.95	290.5738	0.887	0.886	0.118
0.94	315.5840	0.887	0.885	0.206
0.94	324.4402	0.886	0.884	0.279
0.94	333.2965	0.886	0.883	0.352
0.94	342.1527	0.886	0.884	0.219
0.94	351.0090	0.885	0.883	0.258
0.94	359.8652	0.885	0.883	0.228
0.93	393.6956	0.884	0.883	0.183
0.92	427.5259	0.883	0.878	0.553
0.92	461.3562	0.882	0.878	0.473
0.91	495.1865	0.881	0.876	0.532

0.90	565.5897	0.879	0.876	0.356
0.90	582.8746	0.878	0.874	0.443
0.89	595.5055	0.878	0.873	0.484
0.89	608.1364	0.877	0.873	0.456
0.89	620.7672	0.877	0.872	0.497
0.89	633.3981	0.876	0.873	0.364
0.88	646.0290	0.875	0.872	0.440
0.88	699.9306	0.873	0.870	0.378
0.87	753.8322	0.871	0.868	0.316
0.86	807.7338	0.868	0.867	0.149
0.85	833.4792	0.867	0.867	0.017
0.85	840.0304	0.867	0.867	0.015
0.85	844.8700	0.867	0.866	0.042
0.85	848.5493	0.867	0.867	0.003
0.85	852.2286	0.867	0.867	0.072
0.85	855.9079	0.867	0.868	0.111
0.85	859.5872	0.867	0.868	0.116
0.85	863.2665	0.867	0.867	0.085
0.84	878.8162	0.866	0.868	0.138
0.84	894.3660	0.866	0.867	0.121
0.84	909.9158	0.866	0.866	0.033
0.84	925.4656	0.866	0.866	0.051
0.83	941.0154	0.866	0.868	0.280
0.83	974.1932	0.865	0.866	0.037
0.82	1007.3710	0.865	0.864	0.136
0.81	1040.5488	0.865	0.865	0.008
0.81	1073.7266	0.864	0.862	0.236
0.80	1106.9044	0.864	0.863	0.162
0.80	1131.7022	0.864	0.863	0.077
0.79	1154.6776	0.863	0.862	0.177
0.79	1171.6984	0.863	0.861	0.155
0.79	1188.7192	0.862	0.861	0.168
0.79	1205.7399	0.862	0.860	0.147
0.78	1222.7607	0.861	0.860	0.125
0.78	1239.7815	0.861	0.860	0.103
0.77	1314.0105	0.858	0.858	0.066
0.75	1388.2395	0.856	0.855	0.135
0.74	1462.4685	0.854	0.853	0.134
0.73	1536.6975	0.852	0.849	0.313
0.70	1683.2180	0.848	0.845	0.283
0.70	1701.4736	0.847	0.845	0.200
0.69	1715.6851	0.846	0.845	0.134
0.69	1729.8967	0.846	0.845	0.105
0.69	1744.1082	0.845	0.844	0.148

0.69	1758.3197	0.845	0.843	0.191
0.68	1772.5313	0.844	0.843	0.162
0.67	1841.5464	0.842	0.840	0.205
0.66	1910.5616	0.839	0.839	0.042
0.65	1979.5768	0.836	0.837	0.072
0.63	2048.5919	0.834	0.835	0.138
0.61	2173.9605	0.829	0.832	0.433
0.60	2235.7113	0.826	0.830	0.430
0.60	2251.2379	0.826	0.829	0.413
0.60	2266.7645	0.825	0.829	0.373
0.59	2278.5562	0.825	0.828	0.361
0.59	2290.3479	0.825	0.828	0.350
0.59	2302.1396	0.825	0.828	0.375
0.59	2313.9313	0.825	0.827	0.326
0.59	2325.7230	0.824	0.832	0.870
0.58	2361.0880	0.824	0.828	0.502
0.57	2396.4531	0.823	0.826	0.394
0.57	2431.8181	0.823	0.826	0.396
0.56	2467.1832	0.822	0.826	0.547
0.52	2681.7616	0.818	0.819	0.083
0.51	2746.1351	0.817	0.818	0.111
0.50	2810.5087	0.816	0.816	0.071
0.50	2818.7071	0.816	0.815	0.047
0.50	2826.9056	0.815	0.815	0.091
0.49	2835.1041	0.815	0.814	0.098
0.49	2843.3026	0.815	0.814	0.104
0.48	2901.6293	0.813	0.812	0.072
0.47	2959.9560	0.811	0.813	0.187
0.46	3018.2827	0.809	0.811	0.145
0.45	3076.6094	0.808	0.809	0.140
0.43	3171.3317	0.805	0.804	0.142
0.42	3266.0540	0.802	0.801	0.122
0.40	3360.7764	0.799	0.798	0.064
0.40	3366.9798	0.799	0.798	0.040
0.40	3367.9104	0.799	0.798	0.036
0.40	3368.8410	0.799	0.798	0.032
0.40	3369.7717	0.799	0.798	0.066
0.40	3370.7023	0.799	0.798	0.062
0.40	3371.6329	0.798	0.798	0.058
0.40	3374.2340	0.798	0.798	0.008
0.40	3376.8351	0.798	0.798	0.034
0.40	3379.4362	0.798	0.798	0.061
0.40	3382.0373	0.798	0.797	0.087
0.40	3384.6384	0.798	0.797	0.114

0.40	3391.4049	0.798	0.797	0.122
0.39	3398.1715	0.798	0.797	0.130
0.39	3404.9380	0.797	0.796	0.176
0.39	3411.7046	0.797	0.796	0.107
0.39	3449.8301	0.796	0.795	0.050
0.38	3487.9556	0.794	0.794	0.069
0.37	3526.0810	0.793	0.793	0.026
0.33	3776.0810	0.784	0.784	0.033
0.28	4026.0810	0.774	0.774	0.022
0.27	4101.0810	0.770	0.771	0.175
0.26	4176.0810	0.767	0.768	0.171
0.24	4251.0810	0.763	0.764	0.167
0.20	4463.2081	0.753	0.751	0.172
0.20	4482.2996	0.752	0.750	0.252
0.20	4488.0270	0.751	0.750	0.175
0.20	4489.7452	0.751	0.750	0.164
0.20	4489.9448	0.751	0.750	0.162
0.20	4490.0037	0.751	0.750	0.162
0.20	4490.0625	0.751	0.750	0.161
0.20	4490.1214	0.751	0.750	0.160
0.20	4490.7101	0.751	0.750	0.153
0.20	4491.2987	0.751	0.750	0.186
0.20	4491.8874	0.751	0.750	0.179
0.20	4492.4760	0.751	0.750	0.171
0.20	4493.3599	0.751	0.750	0.201
0.20	4494.2437	0.751	0.750	0.190
0.20	4495.1275	0.751	0.750	0.179
0.20	4496.0114	0.751	0.750	0.168
0.20	4497.5932	0.751	0.750	0.107
0.20	4499.1750	0.750	0.750	0.128
0.20	4500.7568	0.750	0.750	0.109
0.20	4502.3386	0.750	0.749	0.130
0.20	4505.2348	0.750	0.749	0.134
0.20	4508.1310	0.750	0.749	0.098
0.20	4511.0272	0.749	0.749	0.102
0.20	4513.9234	0.749	0.748	0.107
0.19	4519.5445	0.749	0.748	0.118
0.19	4525.1656	0.748	0.747	0.129
0.19	4530.7866	0.748	0.747	0.099
0.19	4536.4077	0.747	0.746	0.070
0.19	4547.9766	0.746	0.746	0.047
0.19	4559.5455	0.745	0.745	0.017
0.19	4571.1143	0.744	0.743	0.042
0.18	4582.6832	0.743	0.742	0.019

0.18	4608.1634	0.740	0.741	0.057
0.17	4633.6436	0.738	0.738	0.050
0.17	4659.1238	0.735	0.736	0.085
0.17	4684.6040	0.733	0.734	0.121
0.15	4747.8273	0.727	0.728	0.057
0.14	4811.0506	0.718	0.720	0.280
0.13	4874.2740	0.708	0.711	0.487
0.12	4937.4973	0.697	0.702	0.656
0.11	5015.9126	0.684	0.688	0.626
0.09	5094.3279	0.661	0.671	1.515
0.08	5172.7432	0.631	0.648	2.743
0.06	5251.1584	0.601	0.615	2.317
0.05	5329.5737	0.571	0.573	0.295
0.05	5339.2679	0.563	0.566	0.585
0.05	5348.9621	0.553	0.560	1.296
0.04	5358.6563	0.543	0.553	1.752
0.04	5368.3505	0.533	0.545	2.167
0.04	5378.0447	0.524	0.536	2.364
0.03	5415.0913	0.487	0.501	2.865
0.03	5452.1380	0.450	0.457	1.684
0.03	5463.2520	0.439	0.442	0.707
0.03	5466.5862	0.435	0.437	0.285
0.03	5469.9204	0.432	0.432	0.143
0.02	5470.9207	0.431	0.431	0.125
0.02	5471.9209	0.430	0.430	0.035
0.02	5472.1571	0.430	0.429	0.232
0.02	5472.2279	0.430	0.429	0.182
0.02	5472.2988	0.429	0.429	0.132
0.02	5472.3696	0.429	0.429	0.082
0.02	5472.4405	0.429	0.429	0.031
0.02	5472.5785	0.428	0.429	0.067
0.02	5472.7166	0.428	0.429	0.165
0.02	5472.8546	0.428	0.429	0.263
0.02	5472.9927	0.427	0.429	0.362
0.02	5473.1308	0.427	0.427	0.032
0.02	5473.6113	0.425	0.427	0.376
0.02	5474.0919	0.424	0.425	0.290
0.02	5474.5724	0.422	0.425	0.637
0.02	5475.0530	0.421	0.424	0.624
0.02	5475.5335	0.419	0.424	0.975
0.02	5476.5648	0.416	0.422	1.370
0.02	5477.5960	0.413	0.420	1.697
0.02	5478.6273	0.410	0.418	2.028
0.02	5479.6585	0.407	0.417	2.440

0.02	5480.6897	0.404	0.415	2.859
0.02	5483.3942	0.396	0.410	3.764
0.02	5486.0987	0.387	0.405	4.630
0.02	5488.8032	0.379	0.402	5.935
0.02	5491.5077	0.371	0.396	6.887
0.02	5494.2122	0.363	0.391	7.799
0.02	5500.8689	0.342	0.380	10.879
0.02	5507.5256	0.322	0.367	13.968
0.02	5514.1823	0.302	0.351	16.359
0.02	5520.8390	0.282	0.337	19.745
0.01	5527.4957	0.261	0.319	22.137
0.01	5534.1800	0.241	0.300	24.342
0.01	5540.8643	0.221	0.284	28.475
0.01	5547.5487	0.200	0.263	31.162
0.01	5554.2330	0.180	0.242	34.287
0.01	5560.9173	0.160	0.223	39.547
0.01	5570.5155	0.130	0.188	44.360
0.01	5580.1138	0.101	0.150	48.333
0.00	5589.7120	0.072	0.111	53.835
0.00	5599.3102	0.043	0.062	45.458
0.00	5602.8172	0.032	0.047	45.338
0.00	5606.3241	0.021	0.025	18.071
0.00	5609.8310	0.011	0.009	20.726
0.00	5611.0000	0.007	0.003	57.738
			Average error (%)	2.865

Calculation of battery model 1 RC under 20 mA, and 1 Mole condition with 16 SoC breakpoints

SoC	Time (s)	Simulated voltage (V)	Experimental voltage (V)	Absolute error (%)
1.00	0.000	0.664	0.665	0.091
1.00	0.001	0.664	0.665	0.091
1.00	0.003	0.664	0.665	0.091
1.00	0.004	0.664	0.665	0.091
1.00	0.016	0.664	0.665	0.092
1.00	0.029	0.664	0.665	0.092
1.00	0.042	0.664	0.665	0.093
1.00	0.168	0.664	0.665	0.099
1.00	0.295	0.664	0.665	0.105
1.00	0.421	0.664	0.665	0.111
1.00	1.686	0.664	0.664	0.079
1.00	2.951	0.663	0.664	0.047
1.00	4.216	0.663	0.662	0.078

1.00	7.226	0.662	0.661	0.212
1.00	10.237	0.661	0.661	0.068
1.00	13.247	0.660	0.658	0.343
1.00	16.258	0.659	0.658	0.152
0.99	25.950	0.656	0.654	0.294
0.99	35.642	0.653	0.650	0.530
0.99	45.334	0.650	0.646	0.675
0.98	55.027	0.647	0.643	0.537
0.98	64.719	0.644	0.642	0.254
0.98	81.376	0.639	0.635	0.633
0.97	98.032	0.633	0.630	0.485
0.97	114.689	0.628	0.626	0.286
0.96	131.346	0.623	0.622	0.084
0.96	148.003	0.618	0.618	0.023
0.95	175.483	0.609	0.614	0.844
0.95	183.885	0.607	0.611	0.747
0.95	192.288	0.605	0.609	0.549
0.94	200.690	0.604	0.608	0.604
0.94	209.092	0.603	0.605	0.405
0.94	217.494	0.602	0.603	0.204
0.93	243.072	0.598	0.599	0.224
0.93	268.651	0.594	0.595	0.089
0.92	294.229	0.590	0.589	0.256
0.91	319.808	0.587	0.585	0.240
0.90	364.410	0.580	0.579	0.177
0.90	370.837	0.579	0.578	0.208
0.90	377.264	0.579	0.577	0.345
0.89	383.691	0.578	0.576	0.324
0.89	390.118	0.577	0.576	0.143
0.89	396.545	0.576	0.574	0.388
0.88	416.713	0.574	0.573	0.252
0.88	436.880	0.572	0.569	0.438
0.87	457.048	0.569	0.568	0.302
0.87	477.215	0.567	0.564	0.489
0.86	518.852	0.562	0.561	0.243
0.85	532.276	0.561	0.560	0.187
0.85	540.147	0.560	0.558	0.300
0.85	542.121	0.560	0.558	0.260
0.85	544.095	0.560	0.558	0.341
0.85	546.069	0.559	0.557	0.374
0.85	548.043	0.559	0.557	0.406
0.85	550.640	0.559	0.558	0.267
0.85	553.237	0.559	0.559	0.072
0.85	555.833	0.559	0.558	0.207

0.84	561.793	0.558	0.557	0.248
0.84	567.753	0.558	0.556	0.345
0.84	573.713	0.558	0.555	0.496
0.84	579.673	0.557	0.556	0.262
0.84	592.406	0.556	0.555	0.170
0.83	605.139	0.556	0.554	0.299
0.83	617.872	0.555	0.554	0.151
0.83	630.605	0.554	0.553	0.169
0.82	652.158	0.553	0.552	0.029
0.81	673.710	0.551	0.550	0.167
0.81	695.263	0.550	0.550	0.086
0.80	716.815	0.548	0.548	0.052
0.80	724.493	0.548	0.547	0.187
0.80	730.632	0.548	0.546	0.348
0.80	736.771	0.547	0.547	0.116
0.79	742.910	0.547	0.545	0.333
0.79	749.048	0.546	0.546	0.157
0.78	783.835	0.545	0.543	0.246
0.77	818.621	0.543	0.542	0.109
0.76	853.408	0.541	0.539	0.199
0.75	888.195	0.539	0.538	0.061
0.73	986.357	0.533	0.532	0.113
0.70	1084.519	0.528	0.529	0.238
0.70	1096.846	0.527	0.528	0.092
0.69	1106.768	0.527	0.527	0.042
0.69	1116.691	0.526	0.528	0.284
0.69	1126.613	0.526	0.526	0.001
0.69	1136.535	0.526	0.526	0.126
0.68	1146.458	0.525	0.528	0.426
0.67	1208.111	0.523	0.525	0.434
0.65	1269.765	0.521	0.522	0.205
0.63	1331.419	0.519	0.521	0.448
0.61	1393.072	0.517	0.519	0.454
0.60	1436.670	0.515	0.517	0.395
0.60	1447.863	0.515	0.517	0.476
0.60	1452.034	0.514	0.516	0.413
0.60	1455.484	0.514	0.516	0.459
0.60	1458.934	0.514	0.516	0.446
0.60	1462.384	0.514	0.516	0.433
0.59	1465.834	0.513	0.516	0.539
0.59	1469.284	0.513	0.516	0.526
0.59	1480.798	0.512	0.515	0.502
0.59	1492.312	0.512	0.515	0.717
0.58	1503.826	0.511	0.515	0.754

0.58	1515.339	0.510	0.513	0.609
0.55	1630.477	0.502	0.509	1.404
0.52	1723.021	0.496	0.506	2.151
0.51	1786.753	0.491	0.492	0.068
0.50	1816.054	0.489	0.490	0.095
0.49	1825.862	0.489	0.489	0.095
0.49	1835.670	0.488	0.489	0.220
0.49	1845.478	0.488	0.489	0.282
0.49	1855.287	0.487	0.489	0.345
0.47	1921.338	0.483	0.485	0.500
0.45	1987.389	0.479	0.483	0.913
0.43	2053.440	0.475	0.477	0.429
0.41	2119.491	0.470	0.472	0.392
0.41	2143.689	0.469	0.470	0.256
0.40	2167.887	0.467	0.469	0.316
0.40	2170.026	0.467	0.469	0.345
0.40	2170.802	0.467	0.469	0.355
0.40	2171.442	0.467	0.469	0.297
0.40	2171.669	0.467	0.469	0.300
0.40	2171.896	0.467	0.469	0.303
0.40	2172.124	0.467	0.469	0.305
0.40	2172.351	0.467	0.469	0.308
0.40	2172.578	0.467	0.469	0.311
0.40	2173.566	0.467	0.469	0.389
0.40	2174.554	0.467	0.469	0.466
0.40	2175.542	0.467	0.469	0.347
0.40	2176.530	0.467	0.469	0.359
0.40	2177.518	0.467	0.469	0.371
0.40	2179.755	0.467	0.468	0.332
0.40	2181.992	0.467	0.468	0.359
0.40	2184.229	0.467	0.468	0.255
0.39	2186.467	0.466	0.467	0.150
0.39	2188.704	0.466	0.467	0.243
0.39	2196.405	0.466	0.467	0.270
0.39	2204.106	0.465	0.467	0.364
0.39	2211.807	0.465	0.467	0.523
0.39	2219.508	0.465	0.467	0.485
0.37	2264.069	0.462	0.464	0.433
0.36	2308.631	0.460	0.462	0.447
0.35	2353.192	0.457	0.457	0.059
0.33	2428.192	0.453	0.453	0.043
0.31	2503.192	0.449	0.450	0.370
0.30	2525.692	0.447	0.449	0.379
0.30	2532.442	0.447	0.448	0.328

0.30	2539.192	0.447	0.447	0.149
0.30	2545.942	0.447	0.447	0.107
0.29	2555.023	0.447	0.447	0.142
0.29	2564.104	0.446	0.448	0.314
0.29	2573.185	0.446	0.447	0.143
0.28	2597.299	0.446	0.447	0.168
0.27	2621.413	0.445	0.445	0.014
0.27	2645.527	0.445	0.444	0.265
0.25	2717.868	0.444	0.440	0.745
0.23	2790.210	0.442	0.439	0.742
0.21	2862.552	0.441	0.438	0.739
0.20	2885.052	0.441	0.439	0.513
0.20	2891.802	0.441	0.438	0.556
0.20	2893.827	0.441	0.438	0.618
0.20	2894.434	0.441	0.438	0.615
0.20	2894.481	0.441	0.438	0.614
0.20	2894.528	0.441	0.438	0.613
0.20	2894.575	0.441	0.438	0.612
0.20	2894.845	0.441	0.438	0.607
0.20	2895.116	0.441	0.438	0.532
0.20	2895.387	0.441	0.438	0.527
0.20	2895.700	0.441	0.438	0.520
0.20	2896.013	0.441	0.438	0.514
0.20	2896.327	0.441	0.438	0.508
0.20	2896.778	0.440	0.438	0.499
0.20	2897.230	0.440	0.438	0.560
0.20	2897.682	0.440	0.438	0.551
0.20	2898.335	0.440	0.438	0.469
0.20	2898.989	0.440	0.438	0.456
0.20	2899.642	0.440	0.439	0.373
0.20	2900.610	0.440	0.438	0.424
0.20	2901.579	0.440	0.438	0.405
0.20	2902.547	0.440	0.439	0.316
0.20	2904.061	0.440	0.439	0.287
0.20	2905.575	0.440	0.438	0.327
0.20	2907.089	0.440	0.439	0.227
0.19	2909.530	0.439	0.438	0.249
0.19	2911.970	0.439	0.439	0.131
0.19	2914.411	0.439	0.438	0.222
0.19	2918.574	0.439	0.437	0.280
0.19	2922.736	0.438	0.438	0.128
0.19	2926.899	0.438	0.438	0.045
0.19	2934.452	0.437	0.438	0.175
0.19	2942.005	0.437	0.437	0.114

0.18	2949.557	0.436	0.437	0.194
0.18	2964.322	0.435	0.436	0.349
0.18	2979.087	0.433	0.436	0.574
0.17	2993.852	0.432	0.435	0.660
0.17	3003.341	0.431	0.433	0.425
0.17	3012.830	0.430	0.432	0.404
0.16	3022.318	0.430	0.431	0.239
0.15	3077.694	0.425	0.427	0.568
0.13	3133.070	0.414	0.419	1.171
0.12	3188.446	0.403	0.409	1.426
0.10	3243.822	0.392	0.395	0.679
0.10	3266.678	0.385	0.389	1.076
0.09	3289.535	0.373	0.382	2.266
0.08	3312.391	0.362	0.372	2.852
0.08	3335.248	0.351	0.362	3.301
0.06	3405.909	0.315	0.321	1.690
0.05	3438.032	0.299	0.293	2.086
0.05	3446.145	0.289	0.286	0.972
0.04	3454.258	0.279	0.278	0.218
0.04	3462.371	0.269	0.271	0.591
0.04	3470.484	0.259	0.263	1.343
0.03	3493.924	0.231	0.236	2.166
0.03	3517.364	0.202	0.205	1.099
0.02	3524.396	0.194	0.195	0.813
0.02	3526.506	0.191	0.192	0.562
0.02	3527.138	0.190	0.191	0.163
0.02	3527.328	0.190	0.191	0.285
0.02	3527.518	0.190	0.191	0.406
0.02	3527.575	0.190	0.191	0.454
0.02	3527.632	0.190	0.191	0.517
0.02	3527.689	0.190	0.191	0.579
0.02	3527.843	0.189	0.191	0.749
0.02	3527.998	0.189	0.191	0.918
0.02	3528.152	0.189	0.189	0.276
0.02	3528.400	0.188	0.189	0.550
0.02	3528.649	0.188	0.189	0.825
0.02	3528.897	0.187	0.189	1.102
0.02	3529.366	0.186	0.187	0.641
0.02	3529.835	0.185	0.187	1.167
0.02	3530.305	0.184	0.187	1.367
0.02	3531.237	0.182	0.185	1.261
0.02	3532.169	0.180	0.183	1.663
0.02	3533.101	0.178	0.182	1.729
0.02	3533.721	0.177	0.182	2.465

0.02	3534.341	0.176	0.180	2.340
0.02	3534.961	0.175	0.180	3.091
0.02	3535.995	0.172	0.178	3.125
0.02	3537.030	0.170	0.175	2.440
0.02	3538.065	0.168	0.172	2.466
0.02	3539.722	0.165	0.171	3.480
0.02	3541.380	0.161	0.168	4.157
0.02	3543.037	0.158	0.164	4.086
0.02	3545.930	0.152	0.161	5.963
0.02	3548.823	0.146	0.156	6.733
0.02	3551.716	0.140	0.151	8.007
0.02	3557.224	0.129	0.141	9.463
0.01	3562.733	0.117	0.132	12.248
0.01	3568.241	0.106	0.120	13.025
0.01	3577.363	0.087	0.101	15.671
0.01	3586.486	0.068	0.080	17.983
0.00	3595.608	0.049	0.058	17.085
0.00	3600.826	0.038	0.043	11.650
0.00	3606.043	0.028	0.025	10.216
0.00	3611.261	0.017	0.009	45.493
0.00	3613.000	0.013	0.003	76.913
			Average error (%)	1.510

Calculation of battery model 1 RC under 30 mA, and 1 Mole condition with 16 SoC breakpoints

SoC	Time (s)	Simulated voltage (V)	Experimental voltage (V)	Absolute error (%)
1.00	0.000	0.475	0.476	0.226
1.00	0.001	0.475	0.476	0.226
1.00	0.002	0.475	0.476	0.226
1.00	0.003	0.475	0.476	0.226
1.00	0.011	0.475	0.476	0.226
1.00	0.019	0.475	0.476	0.226
1.00	0.028	0.475	0.476	0.227
1.00	0.112	0.475	0.476	0.230
1.00	0.196	0.475	0.476	0.234
1.00	0.281	0.475	0.476	0.237
1.00	1.124	0.475	0.478	0.723
1.00	1.967	0.475	0.478	0.758
1.00	2.811	0.474	0.477	0.601
1.00	5.216	0.474	0.475	0.251
1.00	7.622	0.473	0.474	0.094
1.00	10.028	0.473	0.475	0.389

1.00	12.433	0.473	0.474	0.232
0.99	24.857	0.470	0.468	0.412
0.99	37.281	0.468	0.467	0.214
0.98	49.704	0.465	0.462	0.736
0.98	62.128	0.463	0.459	0.868
0.97	74.551	0.460	0.460	0.073
0.97	90.535	0.457	0.455	0.383
0.96	106.518	0.454	0.453	0.226
0.96	122.502	0.451	0.451	0.067
0.95	138.486	0.448	0.447	0.112
0.95	151.105	0.445	0.446	0.126
0.94	163.724	0.444	0.443	0.172
0.94	168.004	0.443	0.443	0.040
0.94	172.285	0.442	0.443	0.023
0.94	176.565	0.442	0.440	0.466
0.94	180.846	0.441	0.438	0.749
0.94	185.126	0.441	0.437	0.894
0.93	205.898	0.438	0.434	0.879
0.92	226.669	0.435	0.436	0.259
0.92	247.440	0.432	0.431	0.212
0.91	268.211	0.429	0.428	0.263
0.90	286.019	0.427	0.427	0.052
0.90	300.809	0.425	0.426	0.227
0.89	315.600	0.425	0.425	0.015
0.89	327.130	0.424	0.424	0.057
0.89	338.659	0.424	0.421	0.548
0.88	350.188	0.423	0.421	0.651
0.88	361.718	0.423	0.422	0.176
0.87	373.247	0.422	0.421	0.422
0.86	421.030	0.420	0.416	0.964
0.85	435.364	0.420	0.415	1.186
0.85	443.492	0.419	0.412	1.754
0.85	444.716	0.419	0.411	1.875
0.85	445.941	0.419	0.412	1.705
0.85	447.165	0.419	0.412	1.608
0.85	448.390	0.419	0.411	1.948
0.85	449.614	0.419	0.414	1.050
0.85	456.665	0.418	0.414	1.055
0.84	463.715	0.418	0.412	1.352
0.84	470.766	0.417	0.411	1.358
0.84	477.816	0.416	0.414	0.703
0.84	484.867	0.416	0.410	1.368
0.83	502.874	0.414	0.410	1.005
0.82	520.880	0.413	0.410	0.640

0.82	538.887	0.411	0.404	1.756
0.81	556.894	0.410	0.404	1.465
0.81	574.901	0.408	0.405	0.799
0.80	594.450	0.407	0.403	1.010
0.80	602.240	0.406	0.399	1.854
0.79	608.243	0.406	0.399	1.526
0.79	614.246	0.405	0.403	0.595
0.79	620.249	0.405	0.402	0.717
0.79	626.252	0.404	0.401	0.764
0.79	632.255	0.404	0.401	0.660
0.77	664.543	0.402	0.402	0.023
0.76	696.830	0.399	0.399	0.066
0.75	729.117	0.397	0.398	0.120
0.74	761.404	0.395	0.394	0.232
0.72	828.077	0.390	0.392	0.571
0.70	894.749	0.386	0.390	1.121
0.69	904.275	0.386	0.392	1.573
0.69	913.800	0.386	0.393	1.947
0.69	923.326	0.385	0.391	1.370
0.68	932.851	0.385	0.388	0.873
0.68	942.377	0.385	0.389	1.088
0.67	970.147	0.384	0.389	1.333
0.66	997.917	0.384	0.391	1.976
0.65	1025.688	0.383	0.389	1.505
0.64	1053.458	0.382	0.387	1.193
0.63	1105.796	0.381	0.384	0.785
0.61	1158.135	0.380	0.382	0.533
0.60	1184.421	0.379	0.380	0.201
0.60	1188.254	0.379	0.381	0.459
0.60	1192.087	0.379	0.380	0.073
0.60	1195.920	0.379	0.381	0.331
0.59	1199.753	0.379	0.382	0.750
0.59	1203.585	0.379	0.380	0.123
0.59	1219.183	0.379	0.382	0.755
0.58	1234.782	0.379	0.381	0.743
0.58	1250.380	0.378	0.379	0.167
0.57	1265.978	0.378	0.381	0.638
0.54	1362.666	0.377	0.376	0.069
0.53	1391.673	0.376	0.379	0.789
0.52	1420.679	0.376	0.377	0.268
0.51	1449.685	0.375	0.375	0.010
0.50	1478.692	0.375	0.377	0.616
0.50	1486.414	0.374	0.375	0.270
0.49	1494.137	0.374	0.375	0.332

0.49	1501.859	0.374	0.375	0.393
0.49	1509.581	0.374	0.373	0.280
0.48	1549.629	0.372	0.375	0.774
0.46	1589.677	0.371	0.370	0.220
0.45	1629.724	0.370	0.369	0.231
0.43	1669.772	0.369	0.367	0.407
0.41	1739.203	0.367	0.366	0.184
0.40	1760.032	0.366	0.365	0.350
0.40	1766.280	0.366	0.362	1.050
0.40	1770.152	0.366	0.363	0.685
0.40	1771.023	0.366	0.363	0.845
0.40	1771.314	0.366	0.363	0.841
0.40	1771.605	0.366	0.363	0.836
0.40	1771.896	0.366	0.363	0.830
0.40	1772.187	0.366	0.363	0.825
0.40	1773.770	0.366	0.362	0.962
0.40	1775.057	0.366	0.361	1.272
0.40	1776.345	0.366	0.361	1.164
0.40	1777.633	0.365	0.363	0.722
0.40	1778.921	0.365	0.363	0.782
0.40	1781.018	0.365	0.362	0.909
0.40	1783.116	0.365	0.363	0.702
0.40	1785.213	0.365	0.361	1.081
0.39	1787.311	0.365	0.363	0.623
0.39	1793.664	0.364	0.362	0.753
0.39	1800.018	0.364	0.363	0.297
0.39	1806.371	0.363	0.363	0.260
0.37	1869.905	0.359	0.356	0.902
0.35	1933.438	0.355	0.352	0.613
0.32	1996.972	0.350	0.349	0.316
0.30	2071.972	0.345	0.344	0.369
0.30	2078.722	0.344	0.344	0.017
0.29	2085.472	0.344	0.344	0.019
0.29	2092.222	0.343	0.342	0.377
0.29	2103.815	0.342	0.343	0.281
0.28	2115.408	0.341	0.342	0.407
0.28	2127.001	0.340	0.342	0.533
0.27	2154.759	0.338	0.338	0.005
0.26	2182.516	0.335	0.337	0.654
0.25	2210.273	0.333	0.334	0.579
0.23	2285.273	0.326	0.328	0.773
0.20	2360.273	0.319	0.323	1.071
0.20	2360.881	0.319	0.323	1.089
0.20	2361.488	0.319	0.322	1.010

0.20	2361.616	0.319	0.322	1.014
0.20	2361.654	0.319	0.322	1.014
0.20	2361.692	0.319	0.322	1.015
0.20	2361.730	0.319	0.322	1.016
0.20	2362.110	0.319	0.322	0.928
0.20	2362.490	0.319	0.322	0.935
0.20	2362.870	0.319	0.322	0.942
0.20	2363.250	0.319	0.322	0.854
0.20	2363.796	0.319	0.322	0.865
0.20	2364.342	0.319	0.321	0.684
0.20	2364.888	0.319	0.321	0.694
0.20	2365.434	0.319	0.321	0.609
0.20	2366.459	0.319	0.320	0.533
0.20	2367.485	0.319	0.320	0.553
0.20	2368.510	0.319	0.320	0.381
0.20	2369.535	0.319	0.320	0.593
0.20	2371.515	0.318	0.320	0.344
0.20	2373.495	0.318	0.320	0.382
0.20	2375.475	0.318	0.319	0.228
0.19	2377.455	0.318	0.322	1.130
0.19	2381.536	0.318	0.321	1.018
0.19	2385.617	0.318	0.322	1.385
0.19	2389.698	0.317	0.321	1.081
0.19	2393.778	0.317	0.320	0.968
0.19	2402.687	0.317	0.319	0.756
0.18	2411.595	0.316	0.317	0.447
0.18	2420.503	0.315	0.317	0.525
0.18	2429.411	0.315	0.315	0.117
0.17	2449.188	0.314	0.314	0.018
0.16	2468.966	0.312	0.311	0.473
0.16	2488.744	0.311	0.310	0.478
0.15	2508.522	0.310	0.305	1.466
0.14	2549.963	0.304	0.301	1.167
0.12	2591.404	0.299	0.292	2.270
0.11	2632.845	0.293	0.288	1.646
0.09	2674.286	0.285	0.280	1.994
0.08	2715.727	0.274	0.270	1.317
0.07	2758.575	0.262	0.258	1.716
0.05	2801.423	0.251	0.244	2.518
0.05	2810.669	0.243	0.240	1.478
0.05	2819.916	0.233	0.236	1.318
0.04	2829.162	0.223	0.231	3.681
0.04	2838.408	0.213	0.228	6.983
0.03	2850.704	0.200	0.224	12.289

0.03	2862.999	0.186	0.194	4.112
0.03	2875.295	0.173	0.172	0.565
0.03	2876.402	0.172	0.171	0.759
0.03	2877.508	0.171	0.169	0.955
0.03	2877.840	0.170	0.169	0.746
0.03	2878.172	0.170	0.168	1.433
0.03	2878.272	0.170	0.168	1.321
0.03	2878.372	0.170	0.168	1.189
0.03	2878.471	0.169	0.168	1.057
0.03	2878.571	0.169	0.168	0.924
0.03	2878.803	0.169	0.168	0.614
0.03	2879.034	0.168	0.166	1.028
0.02	2879.266	0.168	0.166	0.716
0.02	2879.498	0.167	0.166	0.403
0.02	2879.658	0.167	0.166	0.185
0.02	2879.818	0.166	0.166	0.034
0.02	2879.979	0.166	0.166	0.254
0.02	2880.139	0.166	0.164	1.184
0.02	2880.417	0.165	0.164	0.804
0.02	2880.696	0.164	0.164	0.422
0.02	2880.974	0.164	0.164	0.036
0.02	2881.252	0.163	0.162	0.771
0.02	2881.768	0.162	0.162	0.051
0.02	2882.283	0.161	0.160	0.461
0.02	2882.799	0.159	0.160	0.271
0.02	2883.314	0.158	0.158	0.142
0.02	2884.048	0.157	0.156	0.246
0.02	2884.781	0.155	0.156	0.829
0.02	2885.515	0.153	0.155	0.932
0.02	2886.248	0.152	0.154	1.238
0.02	2888.449	0.147	0.151	2.824
0.02	2890.649	0.142	0.148	4.522
0.02	2892.850	0.137	0.145	6.345
0.02	2895.051	0.132	0.141	6.915
0.02	2898.212	0.124	0.136	9.175
0.02	2901.372	0.117	0.131	11.451
0.02	2904.533	0.110	0.126	14.581
0.02	2907.694	0.103	0.121	18.148
0.01	2913.062	0.091	0.111	22.294
0.01	2918.429	0.078	0.102	30.459
0.01	2923.796	0.066	0.093	41.179
0.01	2929.635	0.053	0.083	56.479
0.01	2935.474	0.040	0.060	52.749
0.00	2941.313	0.026	0.043	62.676

0.00	2944.498	0.019	0.032	68.558
0.00	2947.684	0.012	0.022	86.898
0.00	2950.869	0.005	0.012	164.406
0.00	2953.000	0.002	0.002	26.892
			Average error (%)	3.615