# A PRODUCTION INVENTORY MODEL WITH ITEM RECOVERY


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**A project report submitted in partial fulfilment of the requirements for the award of Bachelor of Science (Honours) Applied Mathematics with Computing**

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**DECLARATION**

I hereby declare that this project report is based on my original work except for citations and quotations which have been duly acknowledged. I also declare that it has not been previously and concurrently submitted for any other degree or award at UTAR or other institutions.

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**APPROVAL FOR SUBMISSION**

I certify that this project report entitled **"A PRODUCTION INVENTORY MODEL WITH ITEM RECOVERY"** was prepared by **TIFFANY TEE LIANG JING** has met the required standard for submission in partial fulfilment of the requirements for the award of Bachelor of Science (Honours) Applied Mathematics with Computing at Universiti Tunku Abdul Rahman.

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Date : ____________________

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| | |
|---|---|
| $I(t)$ | inventory level at time, t |
| $D$ | demand rate |
| $P$ | production rate |
| $R$ | repairing rate |
| $\theta$ | deterioration rate |
| $Q$ | ordered size |
| $T$ | the last point of time in a complete cycle |
| $T_P$ | time interval for a production cycle |
| $T_R$ | time interval for a repairing cycle |
| $n$ | number of production cycle |
| | |
| $K$ | fixed ordering cost per order |
| $h$ | unit holding cost per unit time |
| $s$ | unit shortage cost per unit time |
| $d$ | unit deterioration cost per unit time |
| $w_e$ | unit carbon emission cost per unit time for the holding items in warehouse |
| | |
| $K_p$ | production set-up cost |
| $h_p$ | unit holding cost per unit time of service items |
| $c_p$ | unit production cost of service items |
| $Q_p$ | economic production quantity |
| | |
| $K_r$ | repairing set-up cost |
| $h_r$ | unit holding cost per unit time of used items |
| $c_r$ | unit repairing cost of used items |
| $Q_r$ | economic repairing quantity |
| | |
| EOQ | economic ordering quantity |
| EPQ | economic production quantity |
| CE | circular economy |
| TCUT | total cost per unit time |

# CHAPTER 1

# INTRODUCTION

## 1.1 General Introduction

Inventory or stocks refer to the goods and materials that a company holds and plans to sell in the near future (Ahmed and Sultana, 2013). To manage inventory, businesses use a mathematical model known as an inventory model, which determines the optimal level of inventory to maintain in a production system and regulates the supply flow to ensure uninterrupted service to customers. The main objective of inventory management is to maximize the benefits of inventory while minimizing costs. There are different types of inventory models that cater to the specific needs of businesses. The two most commonly used models are the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ).

In a classical logistics system, the flow of the inventory starting with raw materials and ending with the finished goods delivered to the client. This denotes a forward flow. Conversely, the reverse logistics system contains a backward flow that manages the return of recyclable discarded items from customers to manufacturers. Recently, companies are compelled to implement such item recovery systems by environmental conscience. In such manner, natural resources can be preserved for future generations, allowing firms to support initiatives for sustainable development.

The reverse logistics system is strongly related to circular economy (CE). In circular economy, waste being produced in the first place is avoided by extending the life cycle of items. The model emphasises reusing, repairing, and recycling existing and discarded goods to minimise waste (Arruda, et al., 2021).

The objective of this project is to conduct a comprehensive analysis of various conventional inventory models and reverse logistics inventory models. Each model will be evaluated with the aim of developing a function that calculates the total cost per unit time. The ultimate aim is to determine the most cost-effective approach for each model.

## 1.2 Importance of the Study

The implication of this research is the addition of new knowledge to the field of inventory theory. The provision of novel managerial insights on the organisation of closed-loop inventory systems that include reusable goods with a circular economy indicator is what makes this research significant.

## 1.3 Problem Statement

It is widely accepted that human activities, such as extracting resources and generating waste, are causing harm to the environment. To promote economic development while positively impacting the environment, the circular economy has emerged as a promising approach for more efficient resource utilization. It is gaining popularity as an alternative to the current economic model. The reverse logistics inventory model is a key strategy that aligns with the circular economy. Both the circular economy and the reverse logistics model involve redirecting products from their typical endpoint to create value or ensure proper disposal. This has garnered significant attention and interest.

To retain environmental sustainability, many firms are facing challenges to recycle their goods to the greatest extent possible while maximising the profit. Hence, the primary research challenge is: "How to direct inventory management decisions such that the product can be manufactured and remanufactured with a variable level of circularity while profits are maximised, and costs are minimised?".

## 1.4 Aim and Objectives

The goal of the research includes:

1. To research some of the classical inventory models.
2. To research some of the classical reverse logistics inventory models.
3. To extend the (1, 1) reverse logistics inventory model from Objective 2 to the case of incorporating emission costs and a circularity indicator as a decision variable.

## 1.5 Scope and Limitation of the Study

The classical inventory models indicated in the first objective of the project are limited to the basic Economic Order Quantity (EOQ) model, the basic Economic Production Quantity (EPQ) model, and the extension of the EPQ model to the case of fully back-logged shortages and to the case of constant deterioration. The study of these models is limited to the formulation of the total cost per unit time (TCUT) function, the formulation of the method to find the ideal TCUT, and the demonstration of the method to find the optimal TCUT through a numerical example.

The classical reverse logistics inventory models indicated in the second objective of the project are limited to several basic extensions of the EPQ model to incorporate the reuse of used items that are collected at a rate that is proportional to the demand rate. Two models will be considered, which are the (1, 1) model and the (1, n) model, where n stands for n production setups per inventory cycle. The study of these models is also limited to the formulation of the total cost per unit time (TCUT) function, the formulation of the method to find the ideal TCUT, and the demonstration of the method to find the optimal TCUT through a numerical example.

For the final portion of the project, the (1, 1) reverse logistics inventory model from Objective 2 will be extended to include carbon emission costs which are incurred from all stages of keeping the relevant inventories, and a circularity indicator as a second decision variable. The circularity indicator is assumed to affect the demand rate logarithmically and profit rate exponentially (as assumed in Rabta, 2020).

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 First EOQ Model and EPQ Model

Undoubtedly, the Economic Order Quantity (EOQ) model is one of the earliest inventory analysis models documented in literature. The EOQ model, initially introduced by Ford Whitman Harris in 1913, was the first mathematical model to address inventory or production-related challenges in production systems. The model determines the optimal batch size by balancing the hidden inventory costs and the visible ordering costs, under the assumption of a consistently constant demand rate.

The classical Economic Production Quantity (EPQ) model was developed by E.W. Taft in 1918 and incorporated a finite replenishment rate. Unlike the EOQ model, the EPQ model features an inventory level that increases continuously until it reaches the maximum level, after which it begins to decline at a constant demand rate.

Table 2.1: Comparison Between the EOQ Model and EPQ Model.

| EOQ Model | EPQ Model |
|---|---|
| The demand is independent. | The demand is dependent. |
| It computes the order quantity. | It computes the production lot size. |

Nowadays, the simplicity and efficiency of the EOQ and EPQ inventory control models caused them to be widely used by numerous businesses (Andriolo, et al., 2014). However, there are several modifications and extensions of the fundamental EOQ/EPQ models that aim to alter their unsophisticated considerations in order to better reflect the real system.

## 2.2 First Reverse Logistics Inventory Model

Over the past few years, scholars have shown considerable interest in reverse logistics inventory models. Numerous studies and investigations have been conducted in this field. It is not surprising that the Economic Order Quantity (EOQ) technique has been widely used by researchers in the past to explore recovery systems. This is due to the fact that simple EOQ models typically yield closed-form solutions. The first reverse logistics model was studied by Schrady in 1967. He explored the issue of the EOQ model for repairable items, which assumes that production and repair rates are instantaneous and free from disposal costs (Alamri, 2020).

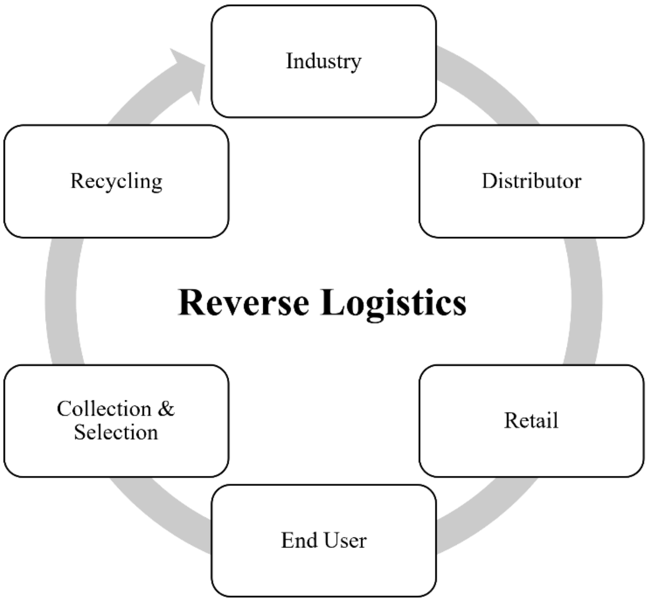


Figure 2.1: Basic Flow of Reverse Logistics.

## 2.3 EOQ Extensions Incorporate Circular Economy

Although not explicitly mentioning the strategy, some extensions of the Economic Order Quantity (EOQ) model incorporate one or more fundamental concepts of the Circular Economy (CE), such as repair, remanufacturing, and recycling. Richter (1996) presents an EOQ model with repair and waste disposal, including changeable setup numbers, which was further analysed with integer setup numbers in 1999. Dobos and Richter (2000) examine a repair and waste disposal model for integer EOQ. Dobos and Richter (2004) introduce an integrated production/recycling model that incorporates linear costs for waste disposal, recycling, manufacture, and buyback. The study demonstrates that a pure strategy (either production or recycling) is optimal. In Dobos and Richter (2006), the model is extended further to consider quality control. The study demonstrates that outsourcing quality control and only acquiring reusable products is preferable to reduce overall EOQ and non-EOQ related expenses.

## 2.4 Circularity of a Product in Circular Economy

Circularity of a product refers to its ability to be reused, repurposed, or recycled at the end of its life cycle, rather than being disposed of as waste. In other words, a circular product is one that contributes to a circular economy by being part of a closed-loop system, where waste is minimized and resources are conserved. Circularity indicators are metrics used to measure the circularity of products, processes, or systems. These indicators can help companies and organizations track their progress towards more sustainable practices and identify areas for improvement.

A valuable case study of the implementation of circularity indicators in a real-world manufacturing company has been provided by Bracquené, Lindemann and Duflou in 2022. They conclude that the implementation of circularity indicators can help companies identify areas for improvement and guide the transition towards a circular economy. However, they also acknowledge that the implementation of circularity indicators can be challenging and requires a significant investment of time and resources. In the recent paper, Oliveira and Oliveira (2023) argued that many of the indicators proposed in the literature are too simplistic and fail to capture the complexity of circularity. They proposed a set of criteria for evaluating circularity indicators and applied them to a sample of existing indicators. Overall, both articles emphasised the need for more rigorous and comprehensive circularity indicators to support the transition to a circular economy.

## 2.5 Sustainable EOQ Model

Several sustainable versions of the EOQ inventory model have been proposed. Hua, Cheng, and Wang (2011) presented a carbon-emission aware EOQ model that considers carbon emissions from shipping and warehousing operations. The model explores the impact of carbon emissions, overall costs, and the effect of carbon trade, carbon price, and carbon cap on the order decisions. Similarly, Chen, Benjaafar, and Elomri (2013) and Hovelaque and Bironneau (2015) both proposed carbon-constrained EOQ models. These models demonstrate that adjusting order quantities can reduce carbon emissions without affecting the company's profitability. Liao and Deng (2018) demonstrated how modifying the carbon tax would affect the best purchasing decisions in their EOQ model for the carbon cap and trade system. Although EOQ-like models are simple, they can effectively reveal possibilities. Dobos, Pishchulov, and Gössinger (2018) studied the reverse logistics system, where products are returned with deteriorating quality after a period of usage and can be either rebuilt or discarded.

## 2.6 Summary

The authors mentioned above have shown that incorporating circularity measures into inventory decision-making is feasible. This not only provides environmental and social benefits but can also be economically viable and financially justifiable. Adjusting order quantities may allow for a switch to circular products without significant profit loss, and determining the optimal circularity level can lead to substantial earnings growth. Mathematical decision models demonstrate the potential of the circular economy, giving businesses a strong incentive to adopt circular practices and accelerate the transition. Further research is needed to develop quantitative models of circular economy that complement the extensive body of qualitative and empirical research in this and related areas. Expanding the proposed concepts to various inventory, manufacturing, and supply chain models is clearly a goal for future research.

# CHAPTER 3

# METHODOLOGY AND WORK PLAN

## 3.1 Methodology

The following figure illustrated the flow of this research project.

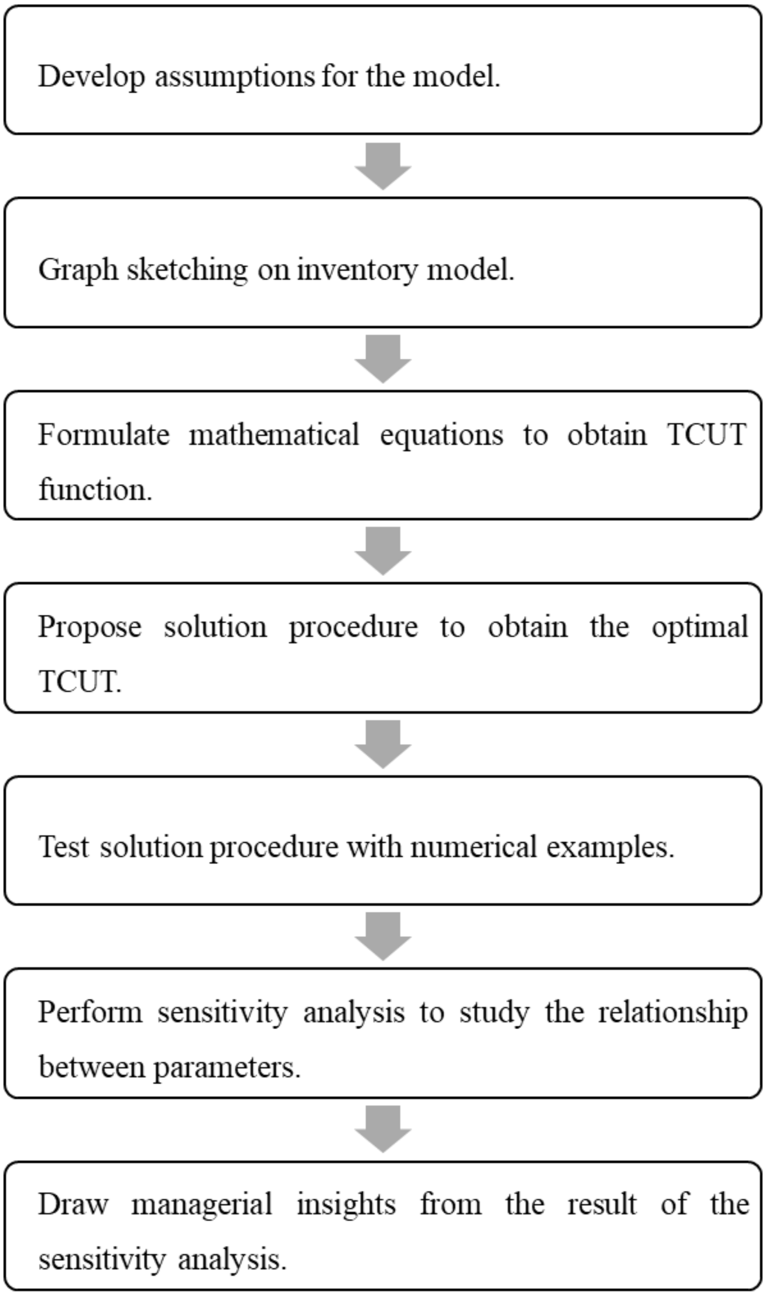


Figure 3.1: Basic Flow of the Project.

After conducting a preliminary literature review, the inventory model will be mathematically formulated based on several assumptions. Before presenting the mathematical formulation, a graphical representation of the inventory status will be provided. The relevant cost components will be calculated, and the equations governing the inventory models will be stated. Additionally, the total inventory costs will be defined.

Subsequently, a function for the total inventory cost will be established, and solution procedures to determine the optimal total cost will be proposed. The solution procedures will be tested by substituting the values of all the parameters into the total cost function using Python's data science libraries. To gain managerial insights, sensitivity analysis will be performed to examine the relationship between the parameters. Finally, the results of the sensitivity analysis will be used to provide managerial insights.

## 3.2 Work Plan

The following tables are the proposed work plan for Project I and Project II.

Table 3.1: Work Plan of Project I

| Project Task | Week | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Collecting and reading research materials | | | ■ | ■ | ■ | ■ | | ■ | ■ | ■ | | | | |
| Building classical inventory models | | | | ■ | ■ | ■ | | ■ | ■ | ■ | | | | |
| Work on proposal | | | | | ■ | ■ | | | | | | | | |
| Mock presentation for proposal | | | | | | | ■ | | | | | | | |
| Submission of proposal | | | | | | | ■ | | | | | | | |
| Work on interin report | | | | | | | | ■ | ■ | ■ | ■ | | | |
| Mock presentation for interin report | | | | | | | | | | | ■ | | | |
| Submission of interin report and turnItIn report | | | | | | | | | | | | ■ | | |
| Oral presentation for Project I | | | | | | | | | | | | | ■ | |

Table 3.2: Work Plan of Project II

| Project Task | Week | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| Collecting and reading research materials | | | | | | | | | | | | | | |
| Building (1,1) reverse logistics inventory model | | | | | | | | | | | | | | |
| Numerical process on the built model | | | | | | | | | | | | | | |
| Sensitivity analysis on the built model | | | | | | | | | | | | | | |
| Preparation of final report | | | | | | | | | | | | | | |
| Submission of final report draft | | | | | | | | | | | | | | |
| Preparation of project poster | | | | | | | | | | | | | | |
| Submission of project poster | | | | | | | | | | | | | | |
| Submission of final report and turnItIn report | | | | | | | | | | | | | | |
| Oral presentation for Project II | | | | | | | | | | | | | | |

# CHAPTER 4

# CLASSICAL INVENTORY MODELS

## 4.1 Formulation of Basic EOQ Model

Assumptions and Notations:

1. $I(t)$: Inventory level at time $t$.
2. $D$: Constant demand rate.
3. $Q$: Ordered size.
4. $K$: Fixed ordering cost per order.
5. $h$: Unit holding cost per unit time.
6. Shortages are not allowed.
7. The planning horizon is infinite in length.
8. The inventory policy is cyclic.

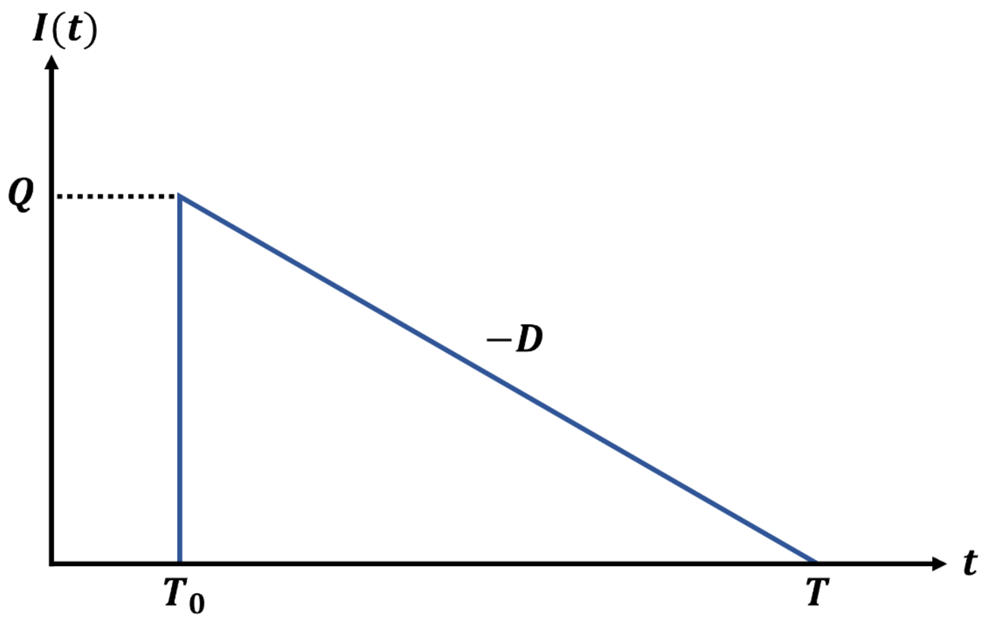


Figure 4.1: Overview of Inventory Status of Basic EOQ Model Over Time.

The total cost for the inventory cycle is the sum of fixed ordering cost and the unit holding cost per time from $T_0$ to $T$. Hence, the total cost per unit time

$(TCUT)$ is the total inventory cost divided by $T$ and it is governed by the following equation:

$$TCUT = \frac{1}{T}\left\{K + h\int_{T_0}^{T} I(t)\,dt\right\} \tag{4.1}$$

From the graph above, we notice that the area under graph is an area of triangle. Without loss of generality, set $T_0 = 0$. Then, we obtain:

$$TCUT = \frac{1}{T}\left\{K + h\frac{QT}{2}\right\} = \frac{K}{T} + \frac{hQ}{2} \tag{4.2}$$

Since the ordered size must be equal to the demand of the cycle. We have:

$$Q = DT \rightarrow T = \frac{Q}{D} \tag{4.3}$$

$$TCUT = \frac{KD}{Q} + \frac{hQ}{2} \tag{4.4}$$

Our goal is to determine $Q$ that minimizes $TCUT$. Thus, we differentiate $TCUT$ with respect to $Q$ and let the equation equals to 0:

$$\frac{d(TCUT)}{dQ} = -\frac{KD}{Q^2} + \frac{h}{2} = 0 \tag{4.5}$$

This gives us the EOQ formula:

$$Q^* = \sqrt{\frac{2KD}{h}} \tag{4.6}$$

The square root formula for $T$ that minimizes $TCUT$ is easily found as

$$T^* = \frac{Q^*}{D} = \sqrt{\frac{2K}{hD}} \tag{4.7}$$

Also, if there are $N$ orders per unit time, we have

$$1 = NT \rightarrow N = \frac{1}{T^*} \tag{4.8}$$

### 4.1.1 Numerical Example

Let $N = 1$, $D = 100$, $K = 100$, and $h = 10$. Next, we verify the optimality of the TCUT function by demonstrating that it can be optimized with respect to the variable Q. We use Python to perform the calculations and plot the TCUT function, as defined in equation (4.4), over the range of Q values between 40 and 50.

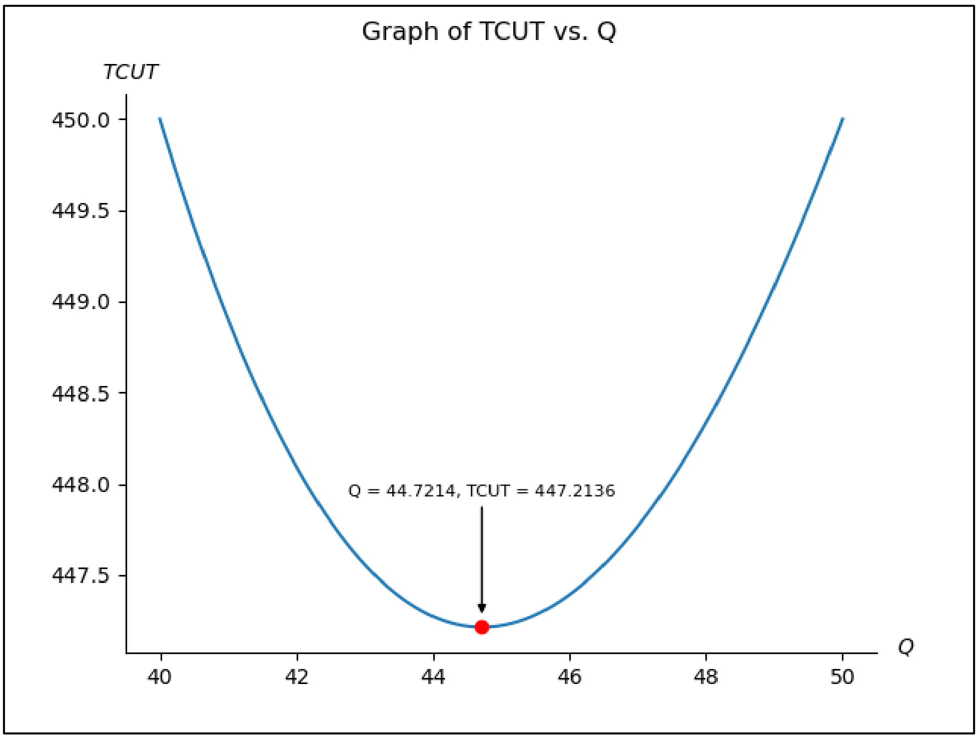


Figure 4.2: Graph of TCUT vs. Q for Basic EOQ Model.

The plot in Figure 4.2 displays a smooth, U-shaped curve with a minimum point. This minimum point confirms that the TCUT function formulated can be optimized by the variable Q. The results indicate that the minimum TCUT for this model is 447.2136, which corresponds to an order size of Q = 44.7214.

## 4.2 Formulation of Basic EPQ Model

Assumptions and notations:

1. $I(t)$: Inventory level at time $t$.
2. $D$: Constant demand rate.
3. $P$: Finite replenishment rate, where $P > D$.
4. $K$: Fixed ordering cost per order.
5. $h$: Unit holding cost per unit time.
6. Shortages are not allowed.
7. The planning horizon is infinite in length.
8. The inventory policy is cyclic.

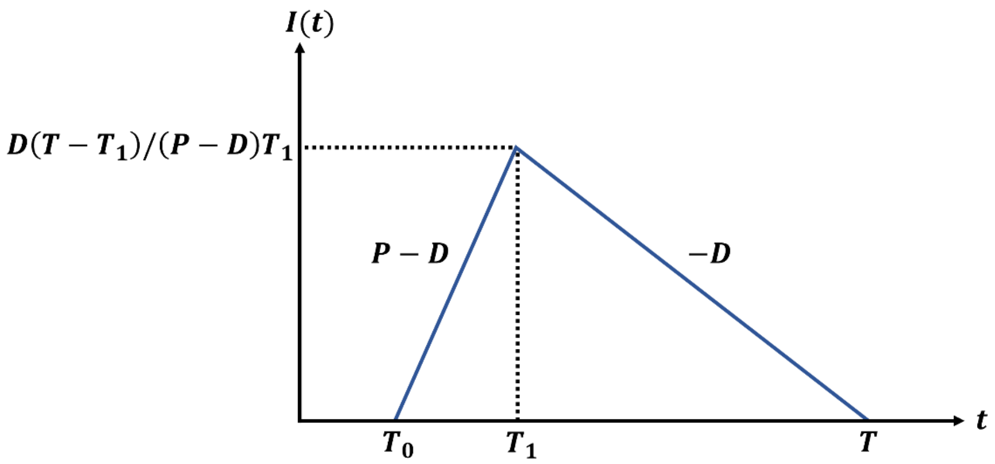


Figure 4.3: Overview of Inventory Status of Basic EPQ Model Over Time.

The total cost per unit time ($TCUT$) is governed by the following equation:

$$TCUT = \frac{1}{T}\left\{K + h\int_{T_0}^{T} I(t)\,dt\right\} \tag{4.9}$$

From the graph above, we notice that the area under graph is an area of triangle. Without loss of generality, set $T_0 = 0$. Then, we obtain:

$$TCUT = \frac{1}{T}\left\{K + h\frac{(P-D)T_1T}{2}\right\} = \frac{K}{T} + \frac{h(P-D)T_1}{2} \tag{4.10}$$

Since the ordered size must be equal to the demand of the cycle. We have:

$$PT_1 = DT \ \rightarrow \ T_1 = \frac{DT}{P} \tag{4.11}$$

$$TCUT = \frac{K}{T} + \frac{h(P-D)DT}{2P} \tag{4.12}$$

Substitute the $T$ obtained from the EOQ model into the equation:

$$TCUT = \frac{KD}{Q} + \frac{h(P-D)Q}{2P} \tag{4.13}$$

Our goal is to determine $Q$ that minimizes $TCUT$. Thus, we differentiate $TCUT$ with respect to $Q$ and let the equation equals to $0$:

$$\frac{d(TCUT)}{dQ} = -\frac{KD}{Q^2} + \frac{h(P-D)}{2P} = 0 \tag{4.14}$$

This gives us the EPQ formula:

$$Q^* = \sqrt{\frac{2KD}{h}\left(\frac{P}{P-D}\right)} = EOQ\sqrt{\frac{P}{P-D}} \tag{4.15}$$

## 4.3 Formulation of EPQ Model with Fully Back-logged Shortages

Assumptions and notations:

1. $I(t)$: Inventory level at time $t$.
2. $D$: Constant demand rate.
3. $P$: Finite replenishment rate, where $P > D$.
4. $K$: Fixed ordering cost per order.
5. $h$: Unit holding cost per unit time.
6. $s$: Unit shortage cost per unit time.
7. Shortages are allowed and are completely backordered.
8. The planning horizon is infinite in length.
9. The inventory policy is cyclic.

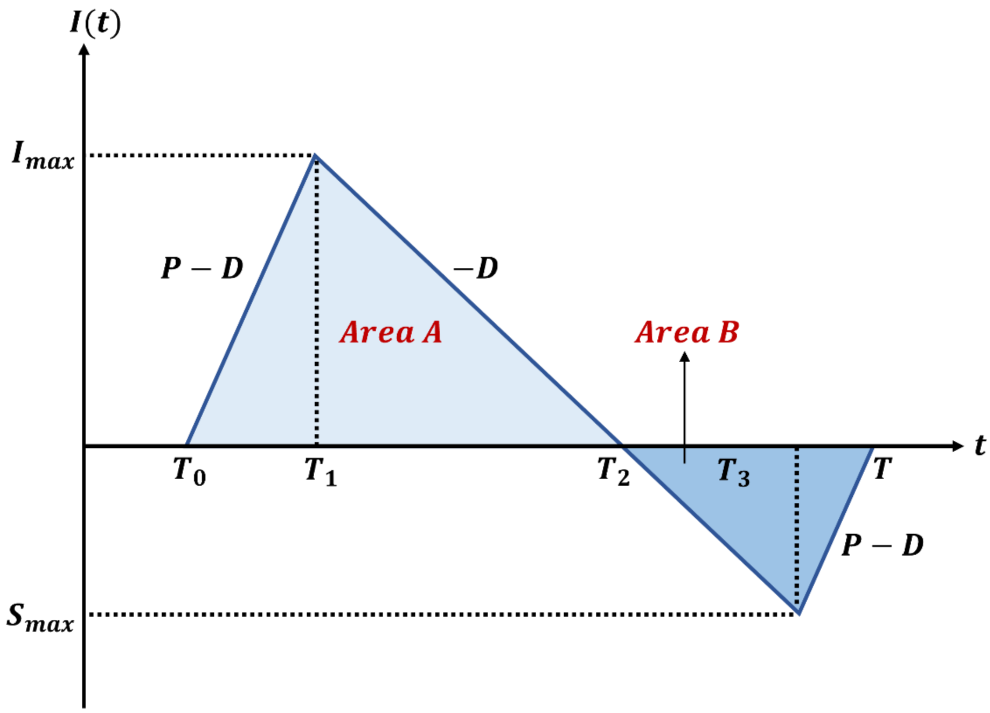


Figure 4.4: Overview of Inventory Status of EPQ Model with Fully Backlogged Shortages Over Time.

The total cost per unit time ($TCUT$) is governed by the following equation:

$$TCUT = \frac{1}{T}\{K + hA + sB\} \tag{4.16}$$

$$TCUT = \frac{1}{T}\left\{K + h\frac{I_{max}(T_2 - T_0)}{2} + s\frac{|S_{max}|(T - T_2)}{2}\right\} \tag{4.17}$$

Note that:

$I_{max} = (P - D)T_1$ and $|S_{max}| = (P - D)(T - T_3)$

Without loss of generality, set $T_0 = 0$. Since the ordered size must be equal to the demand of the cycle. We have:

$$PT_1 = DT_2 \ \rightarrow \ T_1 = \frac{DT_2}{P} \tag{4.18}$$

$$P(T - T_3) = D(T - T_2) \ \rightarrow \ (T - T_3) = \frac{D(T - T_2)}{P} \tag{4.19}$$

Substitute the above equation into $TCUT$ and obtain:

$$TCUT = \frac{K}{T} + \frac{hD(P-D)T_2^2}{2PT} + \frac{sD(P-D)(T-T_2)^2}{2PT} \tag{4.20}$$

To optimize the $TCUT$ function, $T$ and $T_2$ is derived from the following partial differential equations:

$$\frac{\partial(TCUT)}{\partial T} = -\frac{K}{T^2} - \frac{hD(P-D)T_2^2}{2PT^2} + \frac{sD(P-D)(T-T_2)(T+T_2)}{2PT^2} = 0 \tag{4.21}$$

$$\frac{\partial(TCUT)}{\partial T_2} = \frac{hD(P-D)T_2}{PT} - \frac{sD(P-D)(T-T_2)}{PT} = 0 \tag{4.22}$$

By solving equation (4.21), we obtain $T^*$ in terms of $T_2^*$ as

$$T^* = \sqrt{\frac{2PK+D(P-D)(h+s)(T_2^*)^2}{sD(P-D)}} \tag{4.23}$$

By solving equation (4.22), we obtain $T_2^*$ in terms of $T^*$ as

$$T_2^* = \frac{sT^*}{h+s} \tag{4.24}$$

Then, substitute $T_2^*$ into the $T^*$ function, we get:

$$T^* = \sqrt{\frac{2PK(h+s)}{hsD(P-D)}} \tag{4.25}$$

This gives us the EPQ formula:

$$Q^* = DT^* = \sqrt{\frac{2KD}{h}\left(\frac{P}{P-D}\right)\left(\frac{h+s}{s}\right)} \tag{4.26}$$

### 4.3.1 Numerical Example

Let $N = 1$, $D = 100$, $P = 120$, $K = 100$, $h = 10$, and $s = 8$. Then, we conduct a test to verify the optimality of the TCUT function by demonstrating its optimality with respect to the variable Q. Using Python for computation, we plot the TCUT function (4.20) against a range of values of Q from 100 to 300.

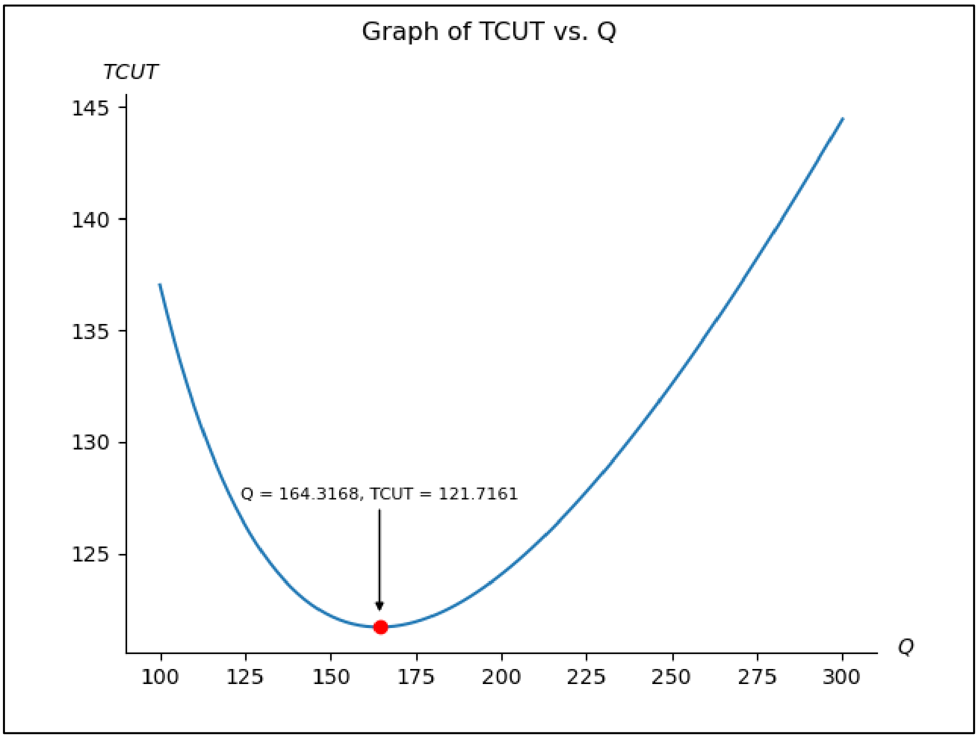


Figure 4.5: Graph of TCUT vs. Q for EPQ Model with Fully Back-logged Shortages.

After computing the TCUT function with Python, we plotted the function against Q ranging from 100 to 300 to demonstrate that it can be optimized by the variable Q. As shown in Figure 4.5, the plot takes the form of a U-shaped curve with a smooth turn at the minimum point. This minimum point indicates that the TCUT function can be optimized by Q. We found that the minimum TCUT of this model is 121.7161, which corresponds to an order size Q of 164.3168.

## 4.4 Formulation of EPQ Model with Constant Deterioration Rate

Assumptions and notations:

1. $I(t)$: Inventory level at time $t$.
2. $D$: Constant demand rate.
3. $\theta$: Constant deterioration rate.
4. $P$: Finite replenishment rate, where $P > D$.
5. $K$: Fixed ordering cost per order.

6. $h$: Unit holding cost per unit time.
7. $d$: Unit deterioration cost per unit time.
8. Shortages are not allowed.
9. The planning horizon is infinite in length.
10. The inventory policy is cyclic.

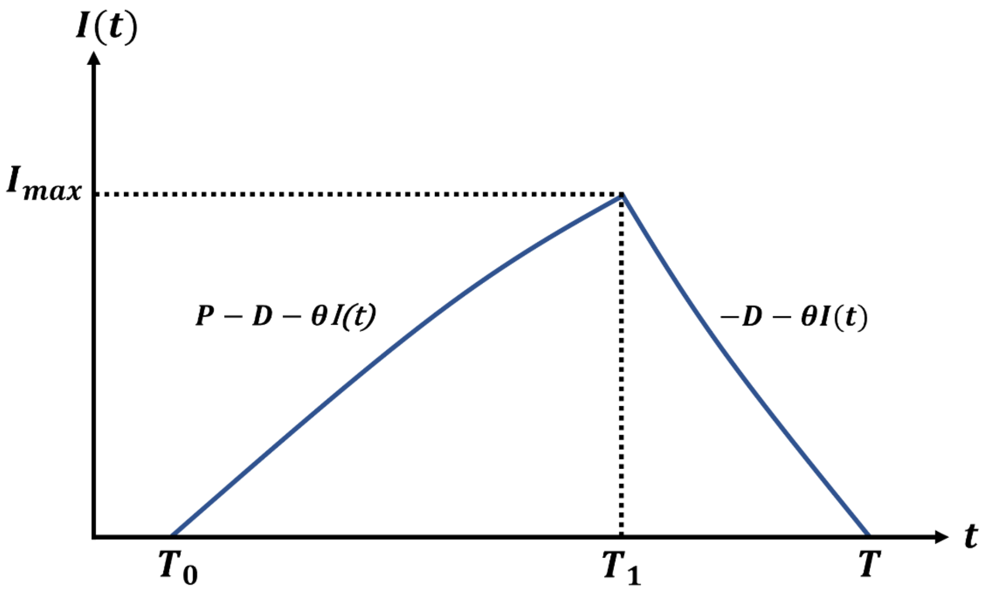


Figure 4.6: Overview of Inventory Status of EPQ Model with Constant Deterioration Rate Over Time.

In Figure 4.6, it is shown that deterioration occurs at a constant rate $\theta$. This indicated that a fraction $\theta$ of the inventory level at any time $t$, $I(t)$, is destroyed by deterioration. The changes in the inventory level can be described by the following differential equations:

$$\frac{d[I(t)]}{dt} = P - D - \theta I(t), \quad T_0 \leq t \leq T_1 \tag{4.27}$$

with the initial condition $I(T_0) = 0$, and

$$\frac{d[I(t)]}{dt} = -D - \theta I(t), \quad T_1 \leq t \leq T \tag{4.28}$$

with the ending condition $I(T) = 0$.

Without loss of generality, set $T_0 = 0$. The solutions of the above differential equations are represented by:

$$I(t) = e^{-\theta t}\int_{T_0}^{T_1}(P-D)e^{\theta u}\,du = \frac{P-D}{\theta}\left(1-e^{-\theta t}\right), \quad 0 \le t \le T_1 \qquad (4.29)$$

$$I(t) = e^{-\theta t}\int_{T_1}^{T} De^{\theta u}\,du = \frac{D}{\theta}\left[e^{\theta(T-t)}-1\right], \quad T_1 \le t \le T \qquad (4.30)$$

respectively, with the integrating factor $\mu(t) = e^{\int \theta\, dt} = e^{\theta t}$.

Hence, the area under the graph is governed by:

$$\int_0^T I(t)dt = \int_0^{T_1}\frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_1}^{T}\frac{D}{\theta}\left[e^{\theta(T-t)}-1\right]dt \qquad (4.31)$$

This leads to

$$\int_0^T I(t)dt = \frac{P-D}{\theta^2}\left(\theta T_1 + e^{-\theta T_1}-1\right) + \frac{D}{\theta^2}\left[\theta(T_1-T) + e^{\theta(T-T_1)}-1\right] \qquad (4.32)$$

Since the ordered size must be equal to the demand of the cycle. We have:

$$(P-D)\left(1-e^{-\theta T_1}\right) = D\left[e^{\theta(T-T_1)}-1\right] \qquad (4.33)$$

$$T_1 = \frac{1}{\theta}ln\left|\frac{De^{\theta T}+P-D}{P}\right| \qquad (4.34)$$

For the moment, the total cost per unit time ($TCUT$) is governed by the following equations:

$$TCUT = \frac{K+h\int_0^T I(t)dt + d\int_0^T \theta I(t)dt}{T} \qquad (4.35)$$

$$TCUT = \frac{K}{T} + \frac{h+\theta d}{T}\int_0^T I(t)dt \qquad (4.36)$$

The $TCUT$ function can be further derived as:

$$TCUT = \frac{K}{T} + \frac{h+\theta d}{\theta^2 T}\{(P-D)\left[\theta T_1 + e^{-\theta T_1}-1\right] +$$

$$D\left[\theta(T_1-T) + e^{\theta(T-T_1)}-1\right]\} \qquad (4.37)$$

### 4.4.1 Numerical Example

Since the TCUT function obtained in (4.37) contains $T_1$, a logarithm function that is in terms of $T$, it is difficult to derive the optimal $T^*$ in a square root formula directly from the function using differentiation. Thus, we test the optimality of the TCUT function through a numerical example.

In the numerical example, let $D = 100$, $P = 120$, $\theta = 0.1$, $K = 100$, $h = 10$, and $d = 15$. Then, we test the optimality of the TCUT function constructed to illustrate that the function can be optimized by the variable $T$. By handling the computation with Python, the TCUT function in (4.37) is plotted against $T$ ranging from 0.5 to 1.5.

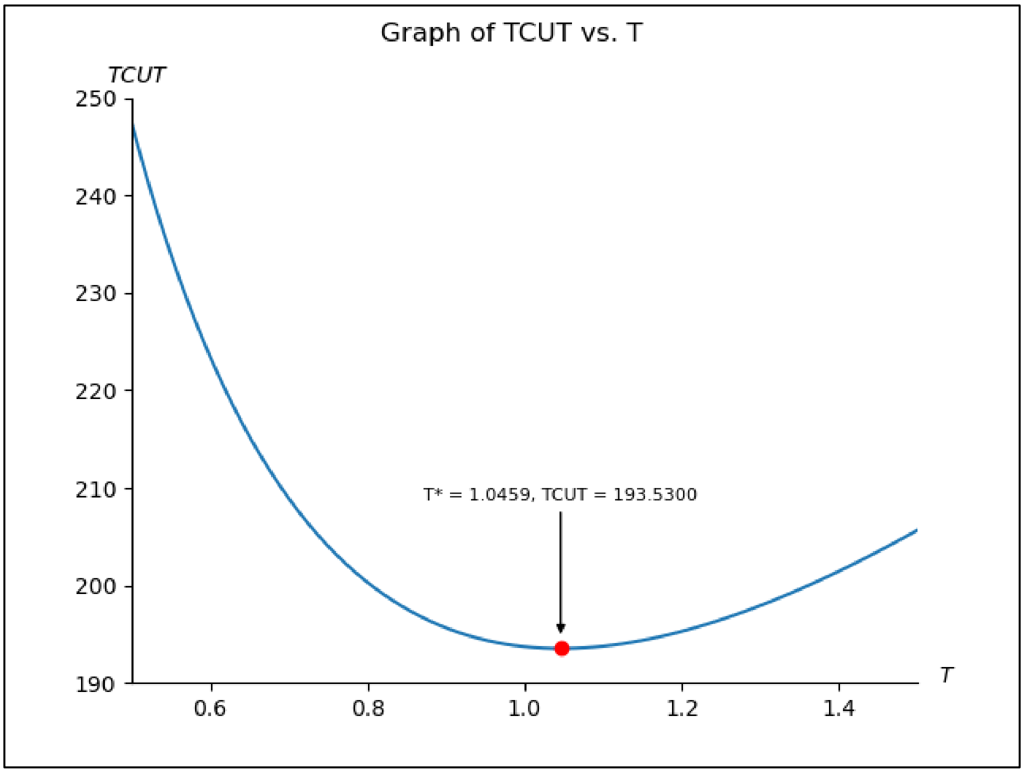


Figure 4.7: Graph of TCUT vs. T for EPQ Model with Constant Deterioration Rate.

The U-shaped plot in Figure 4.7 shows a smooth turn at the minimum point, indicating that the variable T can be optimized in the formulated TCUT function. To determine the minimum value, we used the fmin function from the Scipy library. The result reveals that the minimum TCUT value in this model is 193.5300 at time T of 1.0459.

# CHAPTER 5

# CLASSICAL REVERSE LOGISTICS INVENTORY MODELS

## 5.1 Formulation of (1, 1) Reverse Logistics Inventory Model

Assumptions and notations:

1. Constant demand rate $D$.
2. Constant repairing rate $R$ and production rate $P$, where $R, P > D$.
3. All repaired used items are considered as good as new.
4. Used items are collected at a constant rate $\alpha D$, where $0 < \alpha < 1$.
5. Shortages are not allowed.
6. In each period, there is 1 repairing cycle and 1 production cycle where we call this (1, 1) policy.
7. The planning horizon is infinite in length.
8. The last point of time in the complete cycle is denoted as $T$.
9. The quantity parameters for the service and used items are as follows:

   $Q_p$ = Economic production quantity.

   $Q_r$ = Economic repairing quantity.
10. The cost parameters for the service items are as follows:

    $K_p$ = Production set-up cost.

    $h_p$ = Unit holding cost per unit time.

    $c_p$ = Unit production cost.
11. The cost parameters for the used items are as follows:

    $K_r$ = Repairing set-up cost.

    $h_r$ = Unit holding cost per unit time.

    $c_r$ = Unit repairing cost.

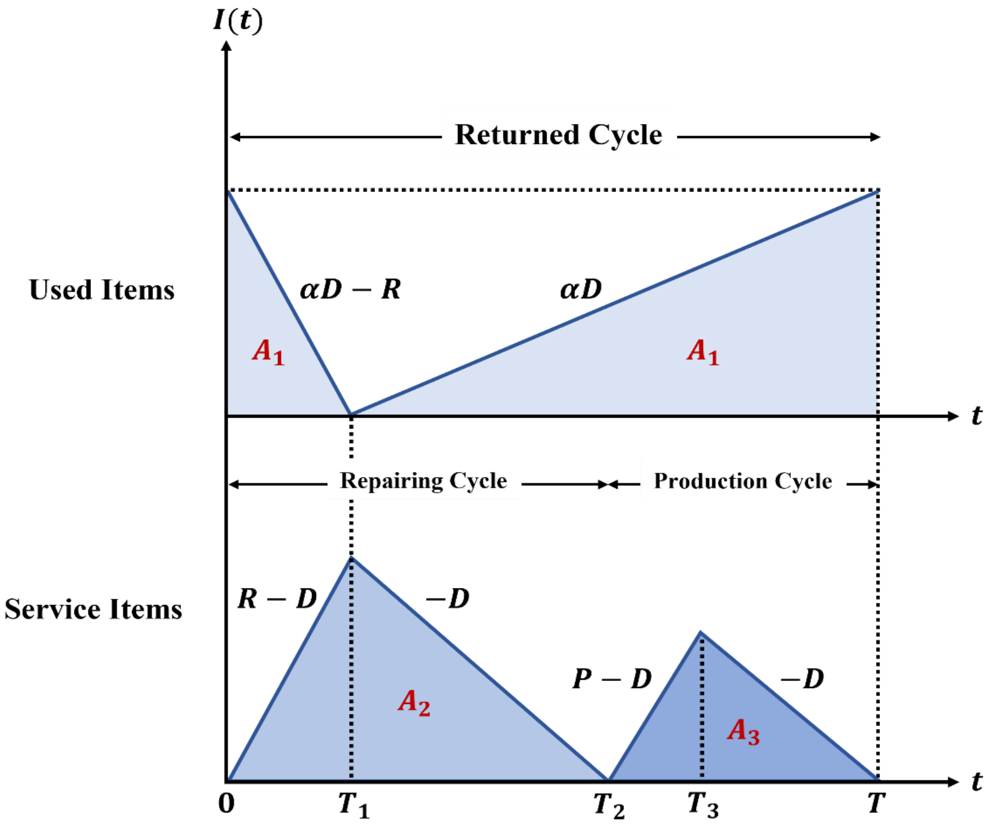


Figure 5.1: Overview of Inventory Status of a Reverse Logistics Inventory Model with (1,1) Policy Over Time.

Since all used items are repaired, we have

$$\alpha DT = RT_1 \quad \rightarrow \quad T_1 = \frac{\alpha DT}{R} \tag{5.1}$$

During the period $[0, T_2]$, the demand is satisfied by repairing. Thus, we obtain

$$DT_2 = RT_1 \quad \rightarrow \quad T_2 = \frac{RT_1}{D} = \alpha T \tag{5.2}$$

During the period $[T_2, T]$, the demand is satisfied by production. Thus, we have

$$D(T - T_2) = P(T_3 - T_2) \tag{5.3}$$

From equation (5.2) and (5.3), we obtain $T_3$ as follow:

$$T_3 = \frac{\alpha P + D(1-\alpha)}{P} T \tag{5.4}$$

The total cost per unit time $(TCUT)$ is governed by the following equation:

$$TCUT = \frac{K_p + K_r + h_p(A_2 + A_3) + h_r A_1 + Q_p c_p + Q_r c_r}{T} \quad (5.5)$$

where

$A_1$, $A_2$ and $A_3$ are the areas of triangles (areas under the graphs of inventory levels – see Figure 5.1).

$Q_r = \alpha DT$

$Q_p = DT - Q_r$

By solving the area of triangles, we obtain the following areas:

$$A_1 = \frac{T(R - \alpha D)T_1}{2} \quad (5.6)$$

$$A_2 = \frac{T_2(R - D)T_1}{2} \quad (5.7)$$

$$A_3 = \frac{(T - T_2)(P - D)(T_3 - T_2)}{2} \quad (5.8)$$

Substituting (5.1) into (5.6) gives

$$A_1 = \frac{(R - \alpha D)\alpha D}{2R} T^2 \quad (5.9)$$

Substituting (5.1) and (5.2) into (5.7) gives

$$A_2 = \frac{(R - D)\alpha^2 D}{2R} T^2 \quad (5.10)$$

Substituting (5.2) and (5.4) into (5.8) gives

$$A_3 = \frac{(P - D)(1 - \alpha)^2 D}{2P} T^2 \quad (5.11)$$

Eventually, we can obtain the $TCUT$ function in the following form:

$$TCUT = \frac{\beta}{T} + \gamma T + (1 - \alpha)Dc_p + \alpha Dc_r \quad (5.12)$$

where

$\beta = K_p + K_r$

$$\gamma = h_p\left[\frac{(R-D)\alpha^2 D}{2R} + \frac{(P-D)(1-\alpha)^2 D}{2P}\right] + h_r\frac{(R-\alpha D)\alpha D}{2R}.$$

Our goal is to determine $T$ that minimizes $TCUT$. Thus, we differentiate $TCUT$ with respect to $T$ and let the equation equals to 0:

$$\frac{d(TCUT)}{dT} = -\frac{\beta}{T^2} + \gamma = 0 \quad (5.13)$$

By rearranging the above equation, the square root formula to find the optimal $T^*$ is easily found as follow:

$$T^* = \sqrt{\frac{\beta}{\gamma}} \quad (5.14)$$

### 5.1.1 Numerical Example

The following parameters are assigned based on the constraints as mentioned in the assumptions:

$D = 10$, $R = 13$, $P = 12$, $\alpha = 0.6$,

$K_p = 160$, $K_r = 150$, $h_p = 5$, $h_r = 0.6 \times h_p$,

$c_p = 50$, $c_r = 0.6 \times c_p$.

In the numerical example, we assume that the repairing rate is higher than the production rate. We also assume that repairing of used items are easier than production. Hence, lower repairing set-up cost are set for used items.

Then, we test the optimality of the TCUT function constructed to illustrate that the function can be optimized by the variable $T$. By handling the computation with Python, the TCUT function in (5.12) is plotted against $T$ ranging from 6.2 to 6.8 as the figure beow.

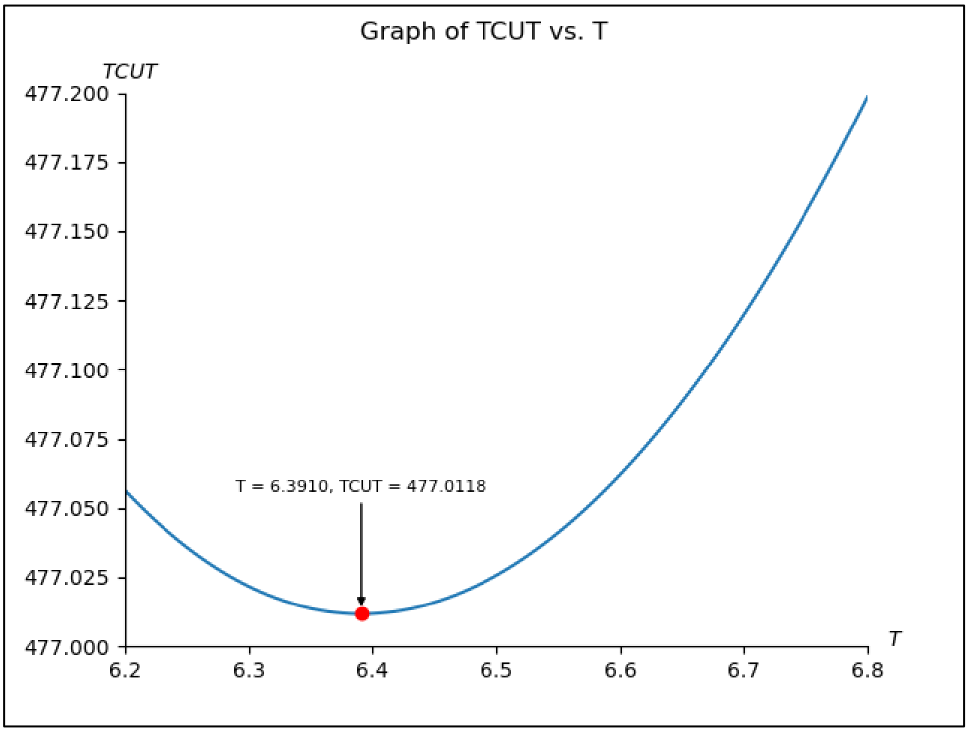


Figure 5.2: Graph of TCUT vs. T for (1,1) Reverse Logistics Inventory Model.

By observing the minimum point in the U-shaped curve plotted in Figure 5.2, it can be inferred that the TCUT function can be optimized by varying T. The existence of a minimum point suggests that the model is capable of finding the optimal value of T. The computed results reveal that the minimum TCUT value of the model is 477.0118, which corresponds to the value of T equal to 6.3910.

## 5.2 Formulation of (1, n) Reverse Logistics Inventory Model

All assumptions and notations are exactly the same as the formulation of (1, 1) reverse logistics inventory model in section 5.1, except that there is only 1 repairing cycle with a number n of production cycle where we call this (1, n) policy. Additionally, the following notations are used for the elapsed time:

1. $T_R$, the elapsed time until the end of the repairing cycle
2. $T_{i,1}$, the elapsed time until the start of the $i^{th}$ production cycle where $i = 1, 2, \dots, n$.
3. $T_{i,2}$, the elapsed time until the end of the $i^{th}$ production cycle where $i = 1, 2, \dots, n$.

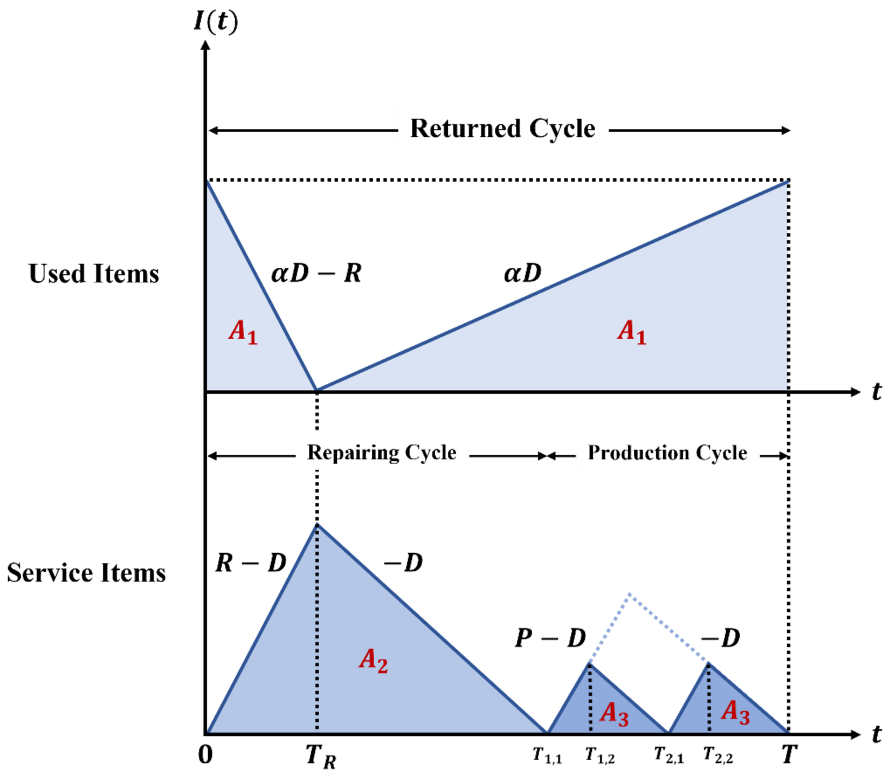


Figure 5.3: Overview of Inventory Status of a Reverse Logistics Inventory Model with (1, n=2) Policy Over Time.

The total cost per unit time ($TCUT$) is governed by the following equation:

$$TCUT = \frac{nK_p + K_r + h_p(A_2 + nA_3) + h_r A_1 + Q_p c_p + Q_r c_r}{T} \tag{5.15}$$

where

$A_1$, $A_2$ and $A_3$ are the areas of triangles (areas under the graphs of inventory levels – see Figure 5.4).

$Q_r = \alpha DT$

$Q_p = DT - Q_r$

Since there is only one repairing cycle per period, the elapsed time $T_R$ and $T_{1,1}$ are equal to the time $T_1$ and $T_2$ in the (1, 1) model. Thus, we have

$$T_R = \frac{\alpha DT}{R} \tag{5.16}$$

$$T_{1,1} = \alpha T \tag{5.17}$$

In Figure 5.3, we assume that each production cycle produces the same batch size of service items. Since the total production quantity, $Q_p = (1-\alpha)DT$, then each production cycle will produce $\frac{(1-\alpha)DT}{n}$ items. Then, we consider the first production cycle that starts at time $T_{1,1}$ and ends at time $T_{1,2}$. We have

$$P(T_{1,2} - T_{1,1}) = \frac{(1-\alpha)DT}{n} \tag{5.18}$$

which gives

$$T_{1,2} = T_{1,1} + \frac{(1-\alpha)DT}{nP} = \alpha T + \frac{(1-\alpha)DT}{nP} \tag{5.19}$$

During the period $[T_{1,1}, T_{2,1}]$, all the demand is satisfied by the first production cycle. Thus, we have

$$D(T_{2,1} - T_{1,1}) = P(T_{1,2} - T_{1,1}) \tag{5.20}$$

which gives

$$T_{2,1} = T_{1,1} + \frac{(1-\alpha)T}{n} = \alpha T + \frac{(1-\alpha)T}{n} \tag{5.21}$$

We know that each subsequent production cycle will have the same production duration and depletion duration as the first production cycle. Thus, we have

$$\text{production duration, } pd = T_{1,2} - T_{1,1} = \frac{(1-\alpha)DT}{nP} \tag{5.22}$$

$$\text{depletion duration, } dd = T_{2,1} - T_{1,2} = \frac{(P-D)(1-\alpha)T}{nP} \tag{5.23}$$

Hence, we have recursive equations to compute $T_{i,1}$ and $T_{i,2}$ for $i = 1, 2, \dots, n$.

$$T_{i,1} = T_{i-1,2} + dd \tag{5.24}$$

$$T_{i,2} = T_{i,1} + pd \tag{5.25}$$

Since the area of $A_1$ and $A_2$ are the same as in (1, 1) model, we can obtain the areas from (5.9) and (5.10)

$$A_1 = \frac{(R-\alpha D)\alpha D}{2R} T^2 \tag{5.26}$$

$$A_2 = \frac{(R-D)\alpha^2 D}{2R} T^2 \tag{5.27}$$

The area of $A_3$ can be solved using geometry and we have

$$A_3 = \frac{(pd+dd)(P-D)pd}{2} \tag{5.28}$$

Substituting (5.22) and (5.23) into (5.28) gives

$$A_3 = \frac{(P-D)D}{2P} \left(\frac{1-\alpha}{n}\right)^2 T^2 \tag{5.29}$$

By substituting the area of $A_1$, $A_2$, and $A_3$ obtained from equation (5.26), (5.27) and (5.29), we can obtain the $TCUT$ function in the following form:

$$TCUT(n, T) = \frac{\beta}{T} + \gamma T + (1-\alpha)Dc_p + \alpha D c_r \tag{5.30}$$

where

$\beta = nK_p + K_r$

$\gamma = h_p \left[\frac{(R-D)\alpha^2 D}{2R} + \frac{(P-D)(1-\alpha)^2 D}{2nP}\right] + h_r \frac{(R-\alpha D)\alpha D}{2R}$

Our goal is to determine $T$ that minimizes $TCUT$. Thus, we differentiate $TCUT$ with respect to $T$ and let the equation equals to 0:

$$\frac{d(TCUT)}{dT} = -\frac{\beta}{T^2} + \gamma = 0 \tag{5.31}$$

By rearranging the above equation, the square root formula to find the optimal $T^*$ is easily found as follow:

$$T^* = \sqrt{\frac{\beta}{\gamma}} \tag{5.32}$$

Futhermore, we concerned to determine the optimal $n^*$ that minimizes $TCUT$ while satisfying the constraints $T^* > LB$ and $n^* > 0$ . $LB$ represents a minimum threshold for the inventory period such that operating below this limit is infeasible. In case we establish a $LB$ for $T^*$, then we must set $T^* = LB$ as $TCUT$ exhibits convexity with respect to $T$ for a particular $n$. Therefore, to obtain the optimal $T^*$ and $n^*$, we suggest the following numerical approach:

1. Initialize $n = 1$.
2. Find $T^*$ for $n = 1$ using (5.32) and let this $T^* = T_a$.
3. If $T_a < LB$, set $T_a = LB$.
4. Find $TCUT(1, T_a)$ using (5.30) and let this $TCUT = TCUT_a$.
5. Increase $n$ by 1.
6. Repeat step 2 for the current $n$ and let this $T^* = T_b$.
7. If $T_b < LB$, set $T_b = LB$.
8. Repeat step 4 to find $TCUT(2, T_b)$ and let this $TCUT = TCUT_b$.
9. If $TCUT_b > TCUT_a$, stop the procedure. Return the current $n - 1$ as $n^*$, $TCUT_a$ as $T^*$, and $TCUT_a$ as the minimum $TCUT$.
10. Otherwise, let $T_b = T_a$ and $TCUT_b = TCUT_a$. Repeat step 5.

### 5.2.1 Numerical Example

The following parameters are assigned based on the constraints as mentioned in the assumptions:

$D = 10$, $R = 13$, $P = 12$, $\alpha = 0.6$,

$K_p = 160,$ $K_r = 150,$ $h_p = 5,$ $h_r = 0.6 \times h_p,$

$c_p = 50,$ $c_r = 0.6 \times c_p.$

All parameter values and the assumptions are exactly the same as in the numerical example in section 5.1.1. The only difference is that we have 2 production cycle run in this numerical example. Note that the lower bound value $LB$ mentioned in the solution procedure is not included in this numerical example. This is because we would like to compare the (1, 1) model and (1, 2) model with the same parameter values.

We test the optimality of the TCUT function constructed to illustrate that the function can be optimized by the variable $T$. By handling the computation with Python, the TCUT function in (5.30) is plotted against $T$ ranging from 7 to 9 as the figure below.

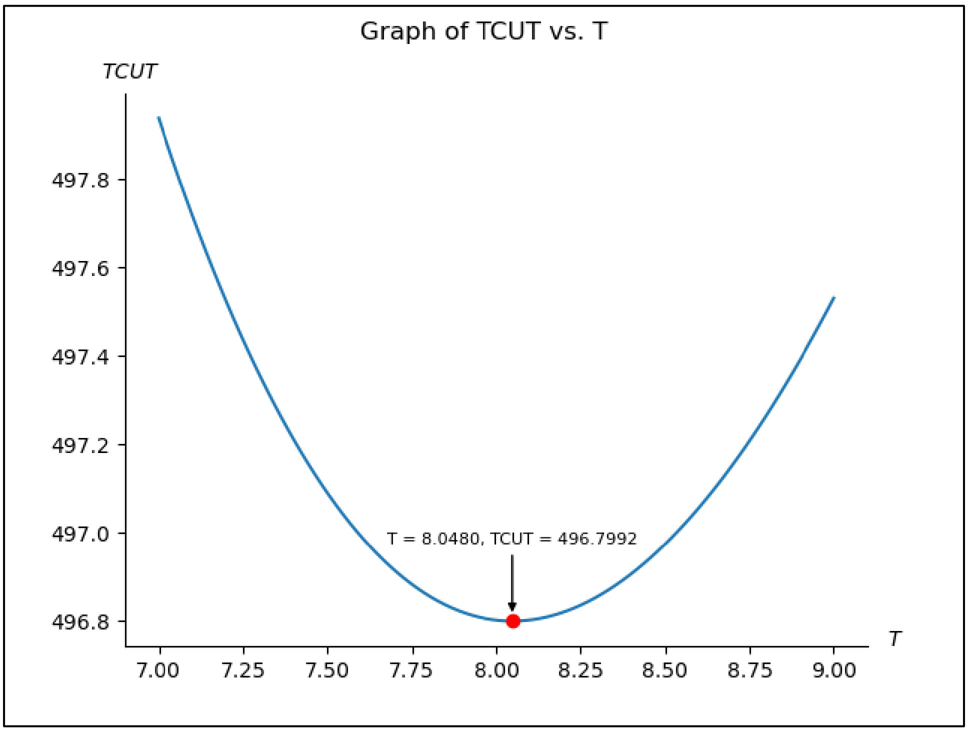


Figure 5.4: Graph of TCUT vs. T for (1,2) Reverse Logistics Inventory Model.

In Figure 5.4, the plot is a U-shapped curve with a smooth turn at the minimum point. By looking at the existance of the minimum point, we conclude that the TCUT function formulated can be optimized by the variable

T. It is shown that the minimum TCUT of this model is 496.7992 corresponding to the time T of 8.0480. From here, we compare the TCUT value and T value of this (1,2) policy to the one of (1,1) policy. It is obvious that the TCUT and time T increases when the number of production cycle increases. In this numerical example, we can emphasise that the TCUT and time T inceases when number of production cycle n increases subject to all other parameter values remain the same. However, this is not necessary true for all (1, n) model because an extra low set-up costs will not impact much on the TCUT even if the n is large.

# CHAPTER 6

# AN INVENTORY MODEL OF CIRCULAR ECONOMY

## 6.1 Formulation of Model

In this chapter, we will extend the (1, 1) reverse logistics inventory model from section 5.1 to incorporate emission costs and a circularity indicator as a decision variable. Let us consider the following assumptions and notations:

1. Constant circularity index of the produxt $\omega$.
2. Deterministic demand rate $\lambda(\omega)$ where $\lambda(\omega) > 0$.
3. Constant repairing rate $R$ and production rate $P$, where $R, P > \lambda(\omega)$.
4. All repaired used items are considered as good as new.
5. Used items are collected at a constant rate $k\omega\lambda(\omega)$, where $0 \leq k\omega \leq 1$ and $k$ is a known constant.
6. Since the demand is deterministic, excess inventory and shortages are not allowed.
7. Carbon emission is the result of warehousing only.
8. There is only one repairing cycle and one production cycle per period.
9. The planning horizon is infinite in length.
10. The last point of time in the complete cycle is denoted as $T$.
11. The quantity parameters for the service and used items are as follows:
    $Q_p$ = Economic production quantity.
    $Q_r$ = Economic repairing quantity.
12. The cost parameters for the service items and used items are as follows:
    $K$ = Total set-up cost.
    $h_p$ = Unit holding cost per unit time for the service items.
    $h_r$ = Unit holding cost per unit time for the used items.
    $w_e$ = Unit carbon emission cost per unit time for the holding items in warehouse.
13. The above costs are positive and are independent of $\omega$.
14. Unit production cost, unit repairing cost, and unit selling price (thus unit gross profits) are functions of $\omega$.
15. The unit gross profit functions are as follows:

$p(\omega)$ = Unit gross profit function for repairing.

$q(\omega)$ = Unit gross profit function for production.

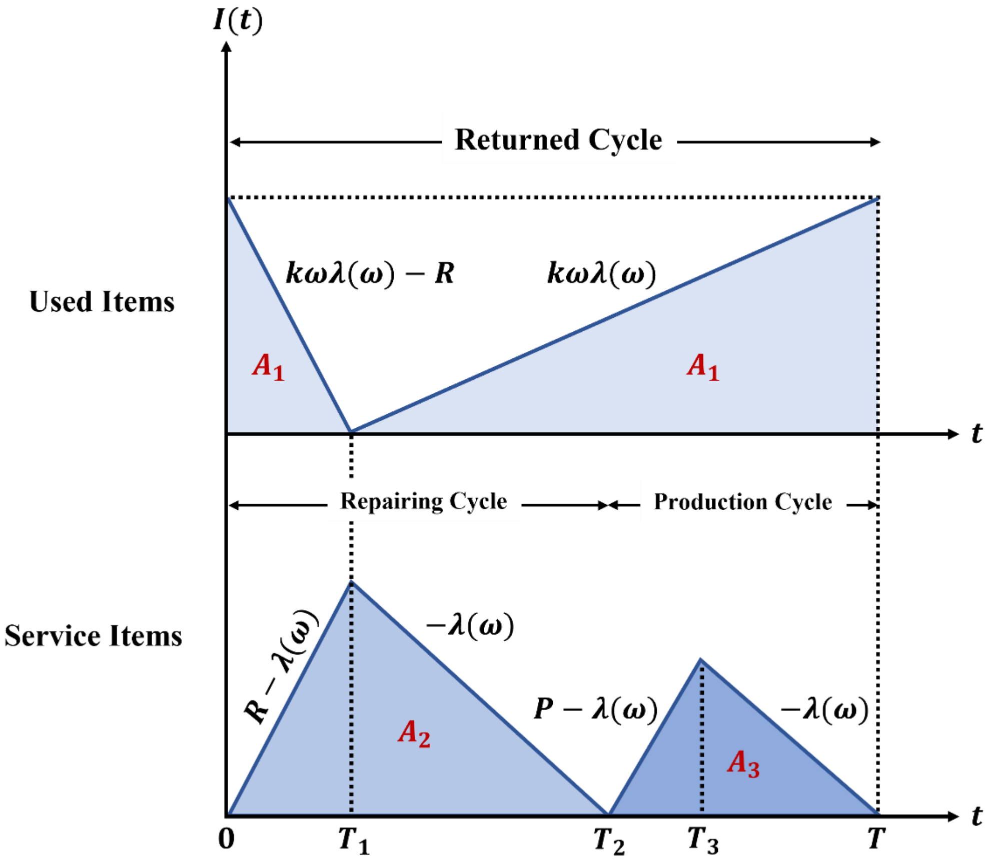


Figure 6.1: Overview of Inventory Status of Inventory Model with Circularity Indicator and Carbon Emission Cost.

The cost of making a circular product, either by repairing or by production, increases with circularity level. Hence, the unit gross profits decrease with circularity level. We propose to express this phenomenon by using exponential functions for the unit gross profits, which we may write as

$$p(\omega) = p_0 - ae^{\alpha(\omega-1)} \tag{6.1}$$

$$q(\omega) = p_0 - be^{\beta(\omega-1)} \tag{6.2}$$

where $p_0$, $a$, $b$, $\alpha$, and $\beta$ are positive parameters.

Similarly, the demand function could be expressed in a nonlinear form. We propose a logarithmic demand function as follows:

$$\lambda(\omega) = \lambda_0 + c\ln(1 + \gamma\omega) \tag{6.3}$$

where $\lambda_0$, $c$, and $\gamma$ are positive parameters.

Since all used items are repaired, we have

$$k\omega\lambda(\omega)T = RT_1 \quad \rightarrow \quad T_1 = \frac{k\omega\lambda(\omega)T}{R} \tag{6.4}$$

During the period $[0, T_2]$, the demand is satisfied by repairing cycle. Thus, we obtain

$$\lambda(\omega)T_2 = RT_1 \quad \rightarrow \quad T_2 = \frac{RT_1}{\lambda(\omega)} = k\omega T \tag{6.5}$$

During the period $[T_2, T]$, the demand is satisfied by production cycle. Thus we have

$$\lambda(\omega)(T - T_2) = P(T_3 - T_2) \tag{6.6}$$

From equation (6.5) and (6.6), we obtain $T_3$ as

$$T_3 = \frac{k\omega P + \lambda(\omega)(1 - k\omega)}{P}T \tag{6.7}$$

The total inventory cost per unit time ($TCUT$) is governed by the unit time sum of the set-up costs, holding costs, and carbon emission cost as the following equation:

$$TCUT = \frac{K + (h_p + w_e)(A_2 + A_3) + (h_r + w_e)A_1}{T} \tag{6.8}$$

where $A_1$, $A_2$, and $A_3$ are area of triangles under the graph of inventory levels (see Figure 6.1).

By implementing the area of triangles obtained from the previous section 5.1 with the current parameters, we have:

$$A_1 = \frac{[R - k\omega\lambda(\omega)]k\omega\lambda(\omega)}{2R}T^2 \tag{6.9}$$

$$A_2 = \frac{[R - \lambda(\omega)](k\omega)^2\lambda(\omega)}{2R}T^2 \tag{6.10}$$

$$A_3 = \frac{[P-\lambda(\omega)](1-k\omega)^2\lambda(\omega)}{2P}T^2 \quad (6.11)$$

Eventually, we can obtain the $TCUT$ function in the following form:

$$TCUT = \frac{K}{T} + \left\{(h_p + w_e)\frac{P[R-\lambda(\omega)](k\omega)^2\lambda(\omega) + R[P-\lambda(\omega)](1-k\omega)^2\lambda(\omega)}{2RP} + (h_r + w_e)\frac{[R-k\omega\lambda(\omega)]k\omega\lambda(\omega)}{2R}\right\}T \quad (6.12)$$

For a particular $\omega$, it can be shown that the optimal $T^*$ is easily found as

$$T^* = \sqrt{\frac{2RPK}{(h_p+w_e)\{P[R-\lambda(\omega)]g(\omega)k(\omega)+R[P-\lambda(\omega)]\lambda(\omega)(1-k\omega)^2\} + (h_r+w_e)g(\omega)P[R-g(\omega)]}} \quad (6.13)$$

where $g(\omega) = k\omega\lambda(\omega)$.

As the $TCUT$ function does not consider the unit production cost and the unit repairing cost, we will focus on profit maximization instead of cost minimization, which is typically done in the classical EOQ model. Therefore, our goal is to identify the ideal period $T^*$in $(0, \infty)$ that maximises the profit at a given circularity level $\omega$ in the range of $[0, 1]$. The function for total profit per unit time (average profit) is expressed as follows:

$$\prod(T,\omega) = \frac{p(\omega)Q_r + q(\omega)Q_p}{T} - TCUT(T,\omega) \quad (6.14)$$

$$\prod(T,\omega) = \lambda(\omega)[p(\omega)k\omega + q(\omega)(1-k\omega)] - TCUT(T,\omega) \quad (6.15)$$

## 6.2 Numerical Example

The following parameters are assigned based on the constraints as mentioned in the assumptions:

| | | | |
|---|---|---|---|
| $p_0 = 100,$ | $\omega = 0.5,$ | $a = 1,$ | $\alpha = 5,$ |
| $b = 4,$ | $\beta = 3,$ | $c = 1,$ | $\gamma = 5,$ |

$k = 1,$ $P = 1500,$ $R = 1200,$ $D_0 = 1000,$

$K = 200,$ $h_p = 5,$ $h_r = 0.6 \times h_p,$ $w_e = 0.8.$

Then, we test the optimality of the Average Profit function constructed to illustrate that the function can be maximized by the variable $T$. By handling the computation with Python, the Average Profit function in (6.15) is plotted against $T$ ranging from 0.3 to 0.7 as the figure below.

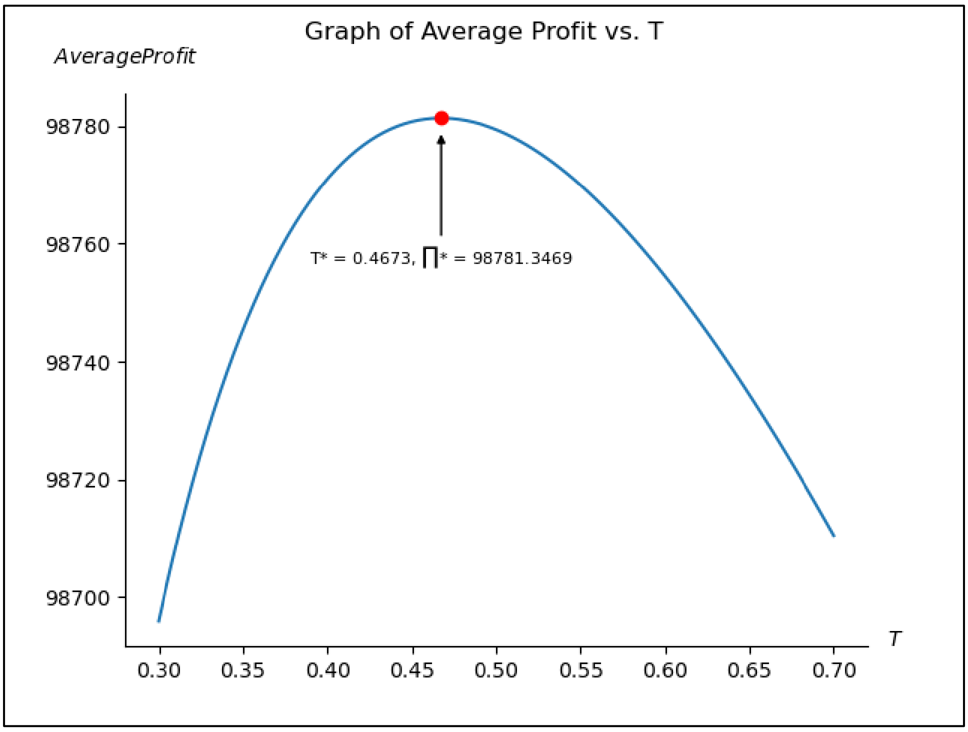


Figure 6.2: Graph of Average Profit vs. T for Inventory Model with Circularity Indicator and Carbon Emission Cost.

In Figure 6.2, the plot is an inverse U-shapped curve with a smooth turn at the maximum point. By looking at the existance of the maximum point, we conclude that the average profit function formulated can be maximized by the variable T. It is shown that the maximum average profit of this model is 98781.3469 corresponding to the time T of 0.4673.

# CHAPTER 7

# CONCLUSIONS AND RECOMMENDATIONS

## 7.1 Conclusion

In chapter 4 of our research, we have delved into the fundamentals of the Economic Order Quantity (EOQ) model, Economic Production Quantity (EPQ) model, and their extensions, which incorporate fully back-logged shortages and constant deterioration. We have formulated the Total Cost per Unit Time (TCUT) function and explained the method to obtain the optimal TCUT. Additionally, we have provided a numerical example to test each model.

Furthermore, we have explored the classical reverse logistics inventory models and their basic extensions to incorporate the reuse of used items that are collected proportionally to the demand rate. The (1, 1) model has been extended to the (1, n) model, where n denotes n production setups per inventory cycle. We have also formulated the TCUT function and provided a numerical example to obtain the optimal TCUT for each model.

Based on our research on the classical inventory models, we have developed a circular economy (1, 1) inventory model, which includes carbon emission costs incurred from holding items in the warehouse and a circularity indicator as the second decision variable. Ultimately, we have built a sustainable inventory management system that aims to maximize profit in a sustainable manner as a rough idea, suitable for real-world business applications in circular economy.

## 7.2 Recommendations of Future Work

As we move towards a more sustainable business landscape, future research may explore the possibilities of extending the circular economy (1, 1) inventory model by integrating carbon emission costs and circularity indicators. Furthermore, researchers can seek to optimize the (1, n) policy by determining the ideal values of $n^*$, $\omega^*$, and $T^*$ to achieve maximum profitability for products in a sustainable manner. To make the model more comprehensive, additional parameters can be incorporated, considering the countless factors that affect inventory management. For instance, the carbon emission cost coverage can be expanded to include all inventory stages, such as transportation and disposal management. Additionally, the model can be extended to account for uncertainties, such as demand fluctuations and supply chain disruptions, by incorporating stochastic modeling techniques.

Overall, the potential for further development and refinement of the circular economy inventory model is vast, offering exciting opportunities for researchers to explore the intersection of sustainability and inventory management.

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