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Electromagnetic Wave Propagation in Microstrip Transmission Lines

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Abstract: We present a novel and accurate approach to compute the loss of electromagnetic waves propagating in a Microstrip line. A set of transcendental equation is derived by matching the tangential fields with the surface impedance at the dielectric-air and dielectric-conductor interfaces. The propagation constant is obtained by numerically solving for the root of the equation and substituting the values into the dispersion relation. We found that the loss predicted by our method, though appears to be somewhat higher, is nevertheless still considered to be in agreement with those from the quasi-static method. In our analysis, we also showed that the phase velocity varies with frequencies indicating dispersive effect in the microstrip lines. Since the quasi-static method assumes pure TEM mode of propagation, while our method takes into consideration the coexistence of TE and TM modes, we attribute the higher loss as due to the presence of the longitudinal fields and dispersive effect in a lossy microstrip line.

Key words: Microstrip line, propagation constant, surface impedance, phase velocity, dispersive effect

INTRODUCTION

Microstrip transmission lines have been widely used in Microwave Integrated Circuits (MIC), such as filters (Hsu *et al*., 2005; Ahn *et al*., 2001; Hong and Lancaster, 1997), couplers (Brenner, 1967; Brenner, 1967; Campbell *et al*., 2003) and mixers (Wengler, 1992; Tucker and Feldman, 1985), etc. At low frequencies where the dimensions of the microstrip structure are much smaller than the wavelengths of the signals, the fundamental HE_0 mode resembles closely a TEM wave (Zysman and Varon, 1969). Thus, electrostatic approximations such as the conformal mapping technique (Wheeler, 1964; Wheeler, 1965; Assadourian and Rimai, 1952; Pucel *et al*., 1968; Pucel *et al*., 1968a) have been commonly used to analyze the propagation of waves in the structure. As experimentally verified in (Grunberger *et al*., 1970; Grunberger and Meine, 1971), however, the solutions of these approximation methods deviate from the measurement results at high frequencies. This is because, in reality, the nature of wave propagation is a superposition of both TE and TM modes and the presence of the longitudinal fields cannot be neglected at high frequencies.

Although Mittra and Itoh have considered the co-existence of the hybrid modes using the Spectral Domain Approach (SDA) (Mittra and Itoh, 1971; Itoh and Mittra, 1973, 1974), they have assumed the thickness of the strip to be infinitesimally thin. Hence, their method is only applicable in cases where the thickness of the strip (t_s) is much smaller than the height of the dielectric substrate (s).

In this study, we present a rigorous analysis which incorporates the finite thickness of the strip and groundplane of a microstrip structure. In our method, the superposition of both TE and TM modes are taken into account during formulation. A set of transcendental equation is derived by matching the tangential fields with the surface impedance at the boundaries of the structure. By solving for the root of the equation and substituting the values into the dispersion relation, we are able to compute the attenuation constant of the propagating

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wave. We will demonstrate that our method gives more realistic results as it incorporates the non-TEM characteristics and dispersive nature of the propagating mode.

FORMATION

Fields in the substrate: As shown in Fig. 1, the microstrip structure that we analyze here is assumed to be enclosed by a pair of perfectly conducting walls at both ends of the substrate at x = a/2 and -a/2. The width of the substrate a is taken to approach infinity so that the fields localized at the strip will not be perturbed by the wall and, thus, the strip conductor resembles closely to that of an open microstrip structure.

In a lossless microstrip line, the boundary condition requires that the tangential electric fields E_t and the normal derivative of the tangential magnetic fields $\delta H_t/\delta a_n$ to vanish at the boundary of the conductor. Here, a_n is the normal direction to the conductor wall. Due to the finite conductivities of the strip and groundplane, however, both E_t and $\delta H_t/\delta a_n$ do not decay to zero at the boundary. However, E_t and $\delta H_t/\delta a_n$ at the boundary of a highly conducting strip and groundplane are very small and are only slightly perturbed from the lossless solution. For a microstrip structure having equivalent surface impedance at the boundary of the strip-substrate and groundplane-substrate interfaces, respectively, the skin depth of the fields penetrating into the conductor are the same. Hence, applying the boundary conditions for the fundamental HE_0 mode of the microstrip line at the substrate-conductor interface and solving Helmholtz's homogeneous equation in Cartesian coordinate (Pozar, 2005), the longitudinal fields can be derived as:

$$E_z = E_d \cos\left(\frac{\pi}{a}x\right)\cos(k_y y) \tag{1}$$

$$H_z = H_d \sin\left(\frac{\pi}{a}x\right)\sin(k_y y) \tag{2}$$

where, E_d and H_d are constant coefficients of the fields; while k_y is the transverse wavenumber in the y direction. The usual wave factor in the form of $\exp[j(\omega t - k_z z)]$ is omitted. Here, ω is the angular frequency and k_z is the propagation constant. k_z is a complex variable which comprises a phase constant β_z and an attenuation constant β_z, and can be expressed as:

$$K_Z = \beta_Z - ja_Z \tag{3}$$

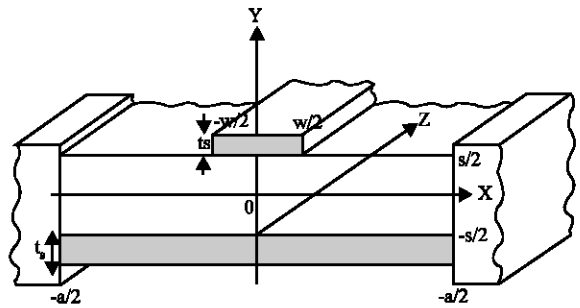

Fig. 1: The cross section of a microstrip structure

The transverse field components E_x and H_x can be derived by substituting the longitudinal fields into Maxwell's source-free curl equations:

$$\nabla \times E - j\omega\mu E \tag{4}$$

$$\nabla \times H = j\omega\varepsilon E \tag{5}$$

where, μ and ε are the permeability and permittivity of the dielectric substrate, respectively. Hence, substituting Eq. 1 and 2 into 4 and 5 and expressing the transverse field components in terms of E_z and H_z, we obtain:

$$H_x = \frac{j(\omega\varepsilon k_y E_d - k_z k_x H_d)}{h^2}\cos\left(\frac{\pi}{a}x\right)\cos(k_y y) \tag{6}$$

$$E_x = \frac{j(k_z k_x E_d + \omega\mu k_y H_d)}{h^2}\sin\left(\frac{\pi}{a}x\right)\sin(k_y y) \tag{7}$$

where, $h^2 = k_x^2 + k_y^2$.

Derivation of the transcendental equation: At the boundary of the conductors, the tangential fields are related through the surface impedance Z_s by (Tham *et al*., 2003; Yeap *et al*., 2009):

$$Z_s = \frac{-E_t}{(a_n \times H_t)} \tag{8}$$

For the strip and groundplane fabricated using the same material and having the same thicknesses, the surface impedance are equivalent. Hence, the surface impedance Z_s can be expressed in terms of the constitutive relations as (Kerr, 1999):

$$Z_{ss} = \frac{jk_s}{\sigma_s}\left[\frac{\exp(jk_s t_s) + \dfrac{\sigma_s Z_\eta - jk_s}{\sigma_s Z_\eta + jk_s}\exp(-jk_s t_s)}{\exp(jk_s t_s) - \dfrac{\sigma_s Z_\eta - jk_s}{\sigma_s Z_\eta + jk_s}\exp(-jk_s t_s)}\right] \tag{9}$$

$$Z_{sg} = \frac{jk_g}{\sigma_g}\left[\frac{\exp(jk_g t_g) + \frac{\sigma_g Z_\eta - jk_g}{\sigma_g Z_\eta + jk_g}\exp(-jk_g t_g)}{\exp(jk_g t_g) - \frac{\sigma_g Z_\eta - jk_g}{\sigma_g Z_\eta + jk_g}\exp(-jk_g t_g)}\right] \qquad (10)$$

where, Z_η is the intrinsic impedance of free space, σ_s and σ_g the conductivities, t_s and t_g the thicknesses, and k_s and k_g the wavenumbers in the strip and groundplane, respectively.

The total surface impedance Z_s of the microstrip structure can be computed by integrating Eq. 8 from x = a/2 to -a/2 at y = s/2 and -s/2, respectively. From Eq. 8, $Z_s = -E_x/H_z = E_z/H_x$ at $y = \pm s/2$. Thus, we have:

$$\int_{-a/2}^{a/2}\frac{-Ex(y=s/2)}{Hz(y=s/2)}dx + \int_{-a/2}^{a/2}\frac{E_{xd}(y=-s/2)}{H_{zd}(y=-s/2)}dx = 2\int_0^{w/2} Z_{ss}(y=s/2)dx + 2\int_{w/2}^{a/2} Z_\eta(y=s/2)dx + 2\int_0^{a/2} Z_{sg}(y=-s/2)dx \qquad (11)$$

$$\int_{-a/2}^{a/2}\frac{H_x(y=s/2)}{E_z(y=s/2)}dx + \int_{-a/2}^{a/2}\frac{-H_x(y=-s/2)}{E_z(y=-s/2)}dx = 2\int_0^{w/2}\frac{1}{Z_{ss}(y=s/2)}dx + 2\int_{w/2}^{a/2}\frac{1}{Z_\eta(y=s/2)}dx + 2\int_0^{a/2}\frac{1}{Z_{sg}(y=-s/2)}dx \qquad (12)$$

Here, we assume that the tangential fields at the air region decay almost instantaneously. Thus, $Z_s = Z_\eta$ in Eq. 8 at the substrate-air boundary.

Substituting the field equations Eq. 1, 2, 6 and 7 into 11 and 12, we obtain:

$$\left[\frac{j2\pi}{h^2}k_z \tan\left(k_y\frac{s}{2}\right)\right]E_d = \left[(w-a)Z_\eta - aZ_{sg} - wZ_{ss} - \frac{j2a}{h^2}\omega\mu k_y \tan\left(k_y\frac{s}{2}\right)\right]H_d \qquad (13)$$

$$\left[\frac{j2\pi}{h^2}k_z \cot\left(k_y\frac{s}{2}\right)\right]H_d = \left[\frac{(w-a)}{Z_\eta} - \frac{a}{Z_{sg}} - \frac{w}{Z_{ss}} + \frac{j2a}{h^2}\omega\epsilon k_y \cot\left(k_y\frac{s}{2}\right)\right]E_d \qquad (14)$$

Equations 13 and 14 admit nontrivial solution only in case where the determinant is zero. Thus, by letting the determinant of the coefficients E_d and H_d vanish, we obtain the following transcendental equation:

$$\left[\frac{(a-w)}{aZ_\eta} + \frac{1}{Z_{sg}} + \frac{w}{aZ_{ss}} - \frac{j2\omega\epsilon k_y}{h^2\tan\left(\frac{k_y s}{2}\right)}\right]\left[\left(1-\frac{w}{a}\right)Z_\eta + Z_{sg} + \frac{wZ_{ss}}{a} + \frac{j2\omega\mu k_y \tan\left(\frac{k_y s}{2}\right)}{h^2}\right] = \left[\frac{j2\pi k_z}{ah^2}\right]^2 \qquad (15)$$

In the transcendental equation, k_y is the unknown to be numerically solved for, since k_z can be expressed in term of k_y using the dispersion relation in Eq. 16 given below:

$$k_z = \sqrt{k_d^2 - (\pi/a)^2 - k_y^2} \qquad (16)$$

where, k_d is the wavenumber in the dielectric substrate.

To compute our results, the Powell Hybrid root-searching algorithm in a NAG routine is used to find the root of k_y. The returned values of k_y depend entirely on the values of the initial guesses given for the search. Since the fundamental mode of the microstrip line is the HE_0 mode, suitable guesses for k_y are clearly values close to zero. It is worthwhile noting that the solution did not always converge to the required mode. It was often necessary to refine initial values slightly in order to force convergence to the correct mode. The attenuation constant α_z can be obtained by substituting k_y into Eq. 16 and extracting the imaginary part of k_z in Eq. 3.

RESULTS AND DISCUSSION

In order to validate our formulation, we have calculated the attenuation constant using the transcendental equation in Eq. 15 based on two sets of microstrip parameters arbitrarily chosen from the results by Pucel *et al.* (1968). Both the strip and groundplane of the microstrip line is made of copper. The attenuation curve as a function of frequency f for rutile substrate with a dielectric constant $\epsilon_r = 105$ is depicted in Fig. 2 and for alumina substrate with $\epsilon_r = 9.35$ in Fig. 3. For the rutile substrate, we have taken w = s = 508.0 μm, and $t_s = t_g$ = 8.382 μm. For the alumina substrate, we have taken w = 3.048 mm, s = 1.27 mm, and $t_s = t_g$ = 990.6 μm. The attenuation constants are compared with those obtained by Pucel *et al.* (1968) (PMH), derived using the quasi-static methods (Wheeler, 1965; Wheeler, 1964; Assadourian and Rimai, 1952) and Wheeler's incremental inductance method (Wheeler, 1942). As illustrated in Fig. 2 and 3, the attenuation curves predicted by our

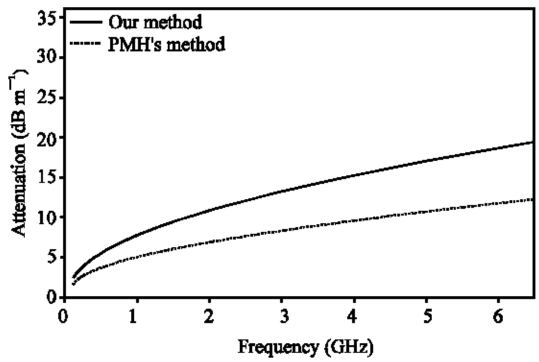

Fig. 2: The loss in a microstrip line with rutile substrate

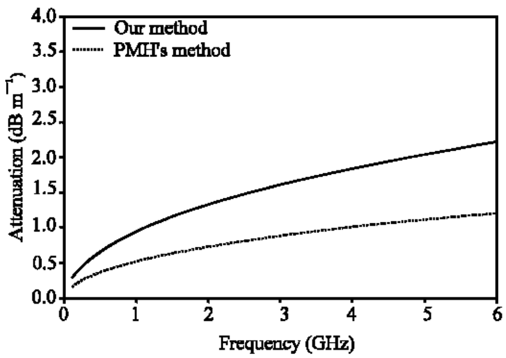

Fig. 3: The loss in a microstrip line with alumina substrate

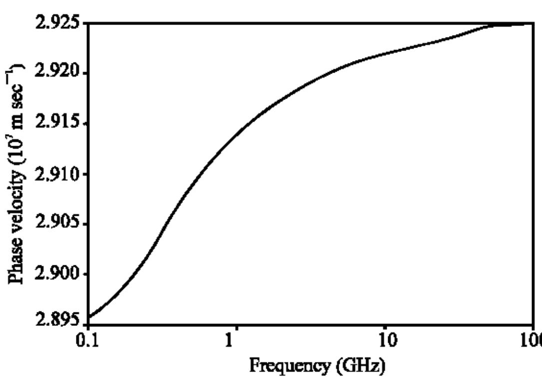

Fig. 4: The phase velocity of waves propagating in a microstrip line with rutile substrate

method are somewhat higher but still considered in agreement with those obtained using PMH's method. Close inspection on theresults shown in (Pucel *et al*., 1968), however, we observe that the measurement results showed higher loss than those predicted by PMH's equation as well. PMH's equation is a quasi-static method which assumes pure TEM mode of propagation; whereas our method is a full-wave analysis which takes into account the coexistence of TE and TM modes. Hence, the

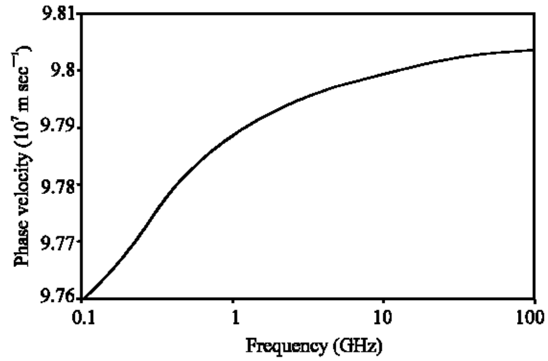

Fig. 5: The phase velocity of waves propagating in a microstrip line with alumina substrate

results suggest strongly that our method gives more accurate prediction of loss.

Next, we have also computed the phase velocity $v_p = \omega/\beta_Z$ for the microstrip lines with rutile and alumina substrates, respectively. As can be clearly seen in Fig. 4 and 5, the phase velocities vary with frequencies, indicating that the lossy microstrip line is dispersive in nature. In the electrostatic solutions, the phase velocity is approximated as (Pozar, 2005):

$$v_p = \frac{c}{\sqrt{\varepsilon_{eff}}} \qquad (17)$$

where, c is the velocity of light in free space and ϵ_{eff} is a constant variable known as the effective dielectric constant. It is apparent that v_p in Eq. 17 is independent of the variation in frequency since both c and ϵ_{eff} are constant variables. Thus, the dispersive effect fails to be accounted for using the quasi-static methods.

CONCLUSION

As a conclusion, we have presented a new and fundamental method to compute the propagation constant of waves in a microstrip transmission line. A set of transcendental equation is derived by integrating the total surface impedance at the boundary of the substrate. The phase and attenuation constants can be calculated by numerically solving for the root of the equation and substituting the value into the dispersion relation.

We have validated our results by comparing with those obtained using quasi-static PMH's equation. Although considered to be in agreement with PMH's results, we observe that the attenuation constants predicted by our method are somewhat higher. Since our method incorporates the superposition of hybrid modes, we attribute such discrepancies to the existence of the longitudinal fields and dispersive effect in our results.

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